Propagation-constant matching based broadband permittivity extraction from S-parameter

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Abstract: This paper presents a coplanar waveguide (CPW) method to extract the permittivity of dielectric materials. The extraction is implemented by specifying an estimated permittivity and then calibrating it repeatedly. The calibrating process lies on matching the simulated and measured propagation-constants, i.e., narrowing their difference until it is less than 1%. A single variable strategy is also proposed to accelerate the calibration. Compared with the transmission-line method, our method shows good agreement over a broad frequency range for silicon substrate, while it is easier to be implemented.

Keywords: permittivity, scattering parameter, propagation-constant, coplanar waveguide

Classification: Electronic materials, semiconductor materials

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1 Introduction

In the design and fabrication of the microwave monolithic circuits and devices, the permittivity of dielectric materials is an important character. Its real indicates dielectric reflection characteristics, while its imaginary indicates dielectric transmission properties. Several techniques have been proposed for extracting the permittivity, such as resonator method [1, 2, 3], on-wafer coplanar-waveguide (CPW) method [4, 5], interdigital capacity (IDC) method [6], and free-space method [7]. CPW and IDC methods are convenient for permittivity extraction. However, IDC method is limited in frequency range, while CPW method is suitable for broadband permittivity extraction. Therefore, we mainly focused on CPW methods. The quasi-TEM analysis [4] is an analytical CPW method extracting permittivity through a series of complex equations. The ELIF [5] CPW method is based on 2D (2-Dimensional) finite element simulation. Both of these two methods assume that there is only one mode of propagation, which will limit the dimensions design of the CPWs. 3D (3-Dimensional) simulation would help to overcome this limitation. However, it will result in significant time and memory consumption.

In order to obtain the advantages of 3D simulation, a propagation-constant matching based 3D simulation CPW method is proposed. Furthermore, for reducing the consumption, a single variable fast-calibration strategy is proposed to accelerate the matching process. The method starts with initializing an estimated permittivity ϵ_{est}, and then calibrates ϵ_{est} repeatedly according to the differences between the simulated propagation-constant γ_{sim} and the measured one γ_{meas}. γ_{meas} is calculated from the scatter parameter (S-parameter) of the CPWs fabricated on the dielectric substrate, while γ_{sim} is predicted by the 3D model with ϵ_{est} as input. The fast-calibration strategy is used for ϵ_{est} calibration to accelerate the matching. In the strategy, the imaginary and real of ϵ_{est} are calibrated with different formulas, respectively.

The proposed method is applied to extract the permittivity of silicon(100) substrate at the frequency range from 1 GHz to 110 GHz. The Au/Cr electrodes of CPWs with the same cross-section and different lengths are fabricated onto the substrate. The S-parameter of CPWs is measured by the Anritsu ME7838A Vector Network Analyzer (VNA). γ_{meas} is calculated from the S-parameter with the Multiline Thru-Reflect-Line (Multi-TRL) approach [8]. The building and simulating model are realized by the Matrix Laboratory (MatLab) coupling with the High Frequency Structure Simulator (HFSS). Experimental results show that the difference between γ_{sim} and γ_{meas} is less than 1%, and it is less than 0.6% in most cases. Compared with the analytic CPW method [4], the proposal shows good agreement over a broad frequency range.

2 Propagation-constant matching based extraction method

The proposed method extracts the permittivity by matching γ_{sim} and γ_{meas} of the CPWs fabricated on the measured material, where the matching means narrowing their difference $\Delta\gamma$. The extraction process can be divided into four steps: (a) Calculate γ_{meas} from S-parameter. (b) Predict γ_{sim} through HFSS model with an initial ϵ_{est}. (c) Calibrate the ϵ_{est} based on $\Delta\gamma$. (d) Repeat the predicting and calibrating processes until $\Delta\gamma$ is small enough.

In order to calculate γ_{meas}, CPWs with the same cross-section and different lengths are needed. The S-parameter of each CPW is measured by the VNA. γ_{meas} can be calculated from the S-parameters with Multi-TRL approach [8].

The γ_{sim} predicting is performed by HFSS coupled with Matlab. The structure of the CPW model with ϵ_{est} as input is defined in the Matlab, then an HFSS script is created. γ_{sim} is predicted by running the script. The propagation-constant is determine by the cross-section of the CPWs, and has nothing to do with the length and impedance match. Therefore, only one model is needed, which has the same cross-section as the fabricated CPWs. And the length of the model is defined as short as possible to reduce the simulating overhead.

The ϵ_{est} calibrating process lies on the difference between γ_{meas} and γ_{sim}. A single variable strategy is proposed to accelerate the calibration, which will be introduced in detail in the next section.

We define two variables, $\Delta\gamma'$ and $\Delta\gamma''$, to describe the relative differences between γ_{meas} and γ_{sim}.

$$\Delta\gamma' = \frac{|\gamma'_{sim} - \gamma'_{meas}|}{\gamma'_{meas}} \tag{1}$$

$$\Delta\gamma'' = \frac{|\gamma''_{sim} - \gamma''_{meas}|}{\gamma''_{meas}} \tag{2}$$

where, γ'_{sim} and γ''_{sim} are the real and imaginary of the γ_{sim} respectively, while γ'_{meas} and γ''_{meas} are that of γ_{meas}. A threshold of the relative differences, $\Delta\gamma_{thr}$, is also specified. The predicting and calibrating processes above will be repeated until $\Delta\gamma'$ and $\Delta\gamma''$ are both smaller than $\Delta\gamma_{thr}$.

3 Single variable calibrating strategy

The complex permittivity ϵ ($\epsilon = \epsilon' - j\epsilon''$) can be described in two variables, which are dielectric constant ϵ_r ($\epsilon_r = \epsilon'$) and loss tangent $\tan\delta$ ($\tan\delta = \epsilon''/\epsilon'$). In the proposed strategy, ϵ_r and $\tan\delta$ are calibrated independently. ϵ_r is calibrated according to γ'' matching, while $\tan\delta$ according to γ'. γ' and γ'' denote the real and imaginary of the propagation-constant.

γ'' equals to the phase shift, $\Delta\phi$, of the signal along a transmission line within unit length.

$$\gamma'' = \frac{\Delta\phi}{L} = -\frac{\omega}{c} \cdot \sqrt{\epsilon_{eff}} \tag{3}$$

where, ϵ_{eff} is the effective dielectric constant of the CPWs, and L is the length of transmission line. γ'' is primarily determined by ϵ_r, since ϵ_r of the substrate are mainly determined by ϵ_{eff} when the section parameters of the CPWs are fixed. Therefore, it is effective to simplify the strategy by calibrating ϵ_r and $\tan\delta$ independently.

For reducing the time overhead, we also develop several formulas to improve the calibrating accuracy.

3.1 ϵ_r calibration formula

According to reference [4], the relationship between ϵ_r and cross-section parameters of CPWs can be derived as follow:

$$\epsilon_r = \frac{1}{2\omega\epsilon_0 F_{low}} \cdot imag\left(\frac{\gamma^2}{R + j\omega L}\right) - \frac{F_{up}}{F_{low}} \tag{4}$$

where, ϵ_0 is the permittivity of vacuum, ω is the angular frequency, γ is the propagation-constant, $imag$ means the imaginary part, and R, L, F_{up}, F_{low} are the functions of the section parameters of the CPWs which are defined in the references [4, 5, 6]. By expanding Eq. (4), we can get a more detail expression about ϵ_r as shown in Eq. (5).

$$\epsilon_r = \frac{2R\gamma'\gamma'' - \omega L\gamma'^2 + \omega L\gamma''^2}{2\omega\epsilon_0 F_{low}(R^2 + \omega^2 L^2)} - \frac{F_{up}}{F_{low}} \tag{5}$$

When the frequency is greater than 1 GHz, $\omega > 10^9$, the term $2R\gamma'\gamma''$ can be negligible in Eq. (5). For low loss substrates, γ' is much less than γ'', $\omega L\gamma'^2$ is far less than $\omega L\gamma''^2$. Therefore, as shown in Eq. (6), ϵ_r is approximately proportional to γ''^2.

$$\epsilon_r + \frac{F_{up}}{F_{low}} \propto \gamma''^2 \tag{6}$$

As mentioned above, the calibrated ϵ_r, denoted as ϵ_{r_cal}, can be calculated by the formula as follow.

$$\epsilon_{r\_cal} = \left(\epsilon_{r\_est} + \frac{F_{up}}{F_{low}}\right) \cdot \left(\frac{\gamma''_{meas}}{\gamma''_{sim}}\right)^2 - \frac{F_{up}}{F_{low}} \tag{7}$$

3.2 tan δ extraction formula

According to reference [4], $\tan\delta$ is proportional to G, which is the conductance of CPWs.

$$\tan\delta = \frac{G}{2\omega\epsilon_0\epsilon_r F_{low}} = \frac{real\left(\frac{\gamma^2(R - j\omega L)}{R^2 + \omega^2 L^2}\right)}{2\omega\epsilon_0\epsilon_r F_{low}} \tag{8}$$

By expanding Eq. (8), we can obtain a detail expression about $\tan\delta$.

$$\tan\delta = \frac{\gamma'^2 R - \gamma''^2 R + 2\omega L\gamma'\gamma''}{2\omega\epsilon_0\epsilon_r F_{low}} \tag{9}$$

When the frequency is greater than 1 GHz, $\omega > 10^9$, there are $2\omega L\gamma'\gamma'' \gg \gamma'^2 R$ and $2\omega L\gamma'\gamma'' \gg \gamma''^2 R$. Furthermore, when ϵ_r of substrate is given, γ'' is approximately a constant. Therefore, $\tan\delta$ is approximatively proportional to γ', and the calibrated $\tan\delta$ can be calculated as follow.

$$\tan\delta_{cal} = \tan\delta \cdot \frac{\gamma'_{meas}}{\gamma'_{sim}} \tag{10}$$

4 Experimental results

4.1 CPWs fabrication and propagation-constant calculation

The CPWs with the same cross-section and different lengths are fabricated onto the silicon(100) substrate. The CPWs are made of Au/Cr electrodes, which are patterned by conventional thermal-evaporated, optical ultraviolet lithography, and

lift-off processes. The cross-section parameters and the SEM views of the CPWs are shown in Fig. 1.

The thickness of electrodes and substrate, defined as t and $h1$, are 140 nm and 0.5 mm respectively. The central conductor and the planar ground planes have widths of 28 um and 190 um. The width of slot between them is 3.4 um. The lengths of CPWs are 3 mm, 1.6 mm, 0.8 mm and 0.4 mm respectively.
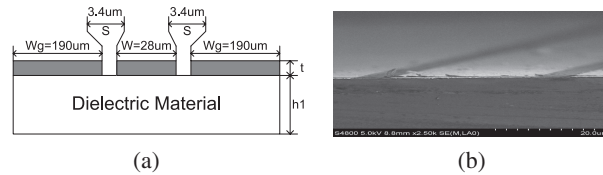

Fig. 1. Cross-section of the CPWs. (a) Cross-section parameters. (b) SEM view of the cross-section.

The S-parameters measurements are performed by the Anritsu ME7838A Vector Network Analyzer (VNA) with the Cascade Micro-tech Summit 12 K probe station. The measurement frequencies are swept from 1 GHz to 110 GHz, with a step of 0.1 GHz.

In order to eliminate the probe pads effect, the Multi-TRL calibration [8] is applied to calculate γ_{meas} from the measured S-parameter. Multi-TRL calibration is a de-embedding technique to remove the effect of unknown factors and the impedance mismatching. With the designs of *thru*, *reflect*, and *line*, the pure response of CPWs can be de-embedded, as illustrated in Fig. 2. The influence of the reference planes from the probe pads to the dashed lines is removed, and therefore only the segment between the reference planes is considered.
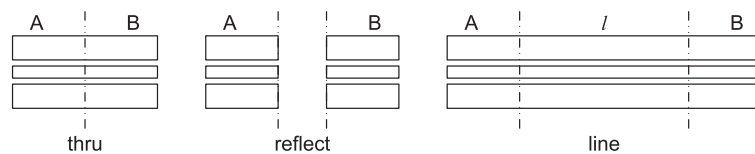

Fig. 2. The *thru*, *reflect* and *line* CPW transmission-lines for Multi-TRL calibration.

4.2 Permittivity extraction

The permittivity is extracted with initial $\epsilon_{est} = 10$ and $\Delta\gamma_{thr} = 0.01$. The extracted ϵ_r is about 11.1–12 at the frequency range from 1 GHz to 110 GHz, close to the manufacturer spec 11.9 at 1 MHz. It is nearly constant when above 5 GHz. The $\tan\delta$ gradually rises up to 0.014 as the frequency increases. The proposed method is compared against the U.Arz's method [4], which is based on the same CPWs transmission line. As shown in Fig. 3, the blue dashed line is ϵ extracted by U.Arz's method, while the red solid line is by our method. These two results show good agreement.
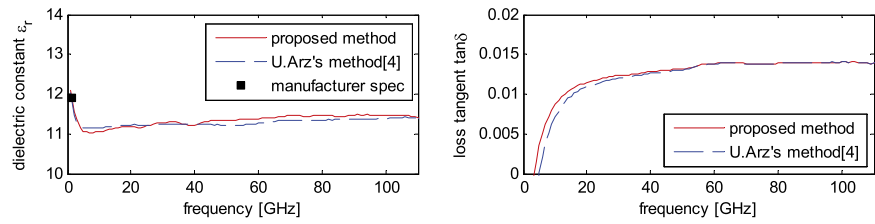
Fig. 3. The permittivity extracted in the proposed method.

As shown in Fig. 4, the $\Delta\gamma'$ and $\Delta\gamma''$ are below 1%, which are less than 0.5% at most frequency point. The error of the γ''_{sim} in our CPWs model is about 1 rad/m, and the imaginary is about 90 rad/m at 1 GHz while up to 6000 rad/m at 110 GHz. Therefore, $\Delta\gamma''$ is less than 1% from 1 GHz to 3 GHz.
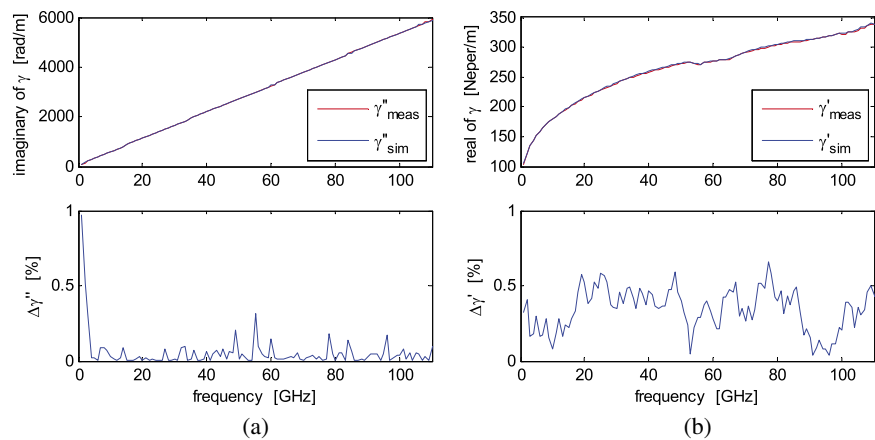

(a)   (b)

Fig. 4. The relative difference between the γ calculated from S-Parameter and simulated from HFSS. (a) Imaginary of the γ. (b) Real of the γ.

5   Conclusion

This paper proposes a propagation-constant matching based method to extracting the permittivity. The method is applied to the silicon(100) substrate, and $\Delta\gamma'$ and $\Delta\gamma''$ are both less than 1%. Compared with the analytical transmission-line method, it shows good agreement over a broad frequency range. The proposed method not only suitable for dielectric substrate, but also for thin film materials deposited on the substrate. In further work, we will applied it to extract the thin film permittivity by modifying the calibration formulas.