

# Return to-zero feedback insertion in a continuous time Delta-Sigma modulator for excess loop delay compensation

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**Abstract:** Excess loop delay in continuous time (CT) Delta-Sigma modulators (DSM), as a result of the limited DAC response time, leads to SNR degradation so that the optimal tuning of the system will be impossible. In order to solve this problem, this paper proposes a novel return to-zero feedback insertion method. To verify the analytical results extracted in this paper, a second order CT DSM is simulated.

**Keywords:** continuous time, Delta-Sigma, modulator

**Classification:** Integrated circuits

## References

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## 1 Introduction

The continuously decreasing supply voltage of recent CMOS technologies is causing important limitations to the performances of SC circuits. The switch parasitic characteristics such as high switch resistance, settling-time errors, clock feed-through, and charge injection, limit the accuracy and the sampling frequency. In the other hand, continuous-time (CT) circuits do not

suffer from these limitations and are therefore capable of achieving higher performances in recent low-voltage CMOS processes [1].

In switched-capacitor circuits the unity gain frequency of operational amplifiers must be at least three to five times the sampling rate. Therefore a high quiescent current is required to achieve a high bandwidth, leading to higher power consumption in discrete time (DT) modulators. In the other hand, unity gain frequencies of opamp's in CT Delta-Sigma modulators are usually lower than the sampling frequency. As a result, for low-voltage, high frequency, and low-power applications, it can be concluded that a CT Delta-Sigma modulator has better performance than its DT counterpart. However; there are some disadvantages with CT Delta-Sigma modulators such as the excess loop delay, DAC output rise and fall time asymmetry, and the clock jitter [2].

The delay in the feedback signal is mainly due to the comparator response-time. This delay has been found to alter the frequency response and degrade the signal-to-noise ratio (SNR) of CT Delta-Sigma modulators [3]. This delay can also increase the order of the equivalent DT modulators leading to instability of the system. In order to overcome this problem, several methods such as the DAC pulse shape selection, the feedback coefficient tuning, and the additional feedback insertion are proposed in [3, 4].

As a remedy for the damaging effects of the excess loop delay, a novel return to-zero (RZ) feedback insertion method is proposed in this paper, and the equations for the method are obtained precisely. In section II the CT/DT modulator equivalence is described briefly. In section III, the proposed method is described and the analytical results are extracted precisely. Simulation results are illustrated in section IV, and finally the paper is concluded in section V.

## 2 CT/DT modulator equivalence

Knowledge of the equivalence allows us to perform the CT Delta-Sigma loop filter design in the DT domain, exploiting the useful toolboxes that are available for DT modulators.

Once we have chosen,  $H(z)$ , we may find the  $H(s)$  to implement the CT modulator with identical behavior, assuming a certain type of DAC pulse [2, 3]. For simplicity, we assume a perfectly rectangular DAC pulse of magnitude 1 that lasts from  $\alpha$  to  $\beta$  [3]:

$$r_{(\alpha,\beta)}(t) = \begin{cases} 1 & \alpha \leq t \leq \beta, \ 0 \leq \alpha \leq \beta \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

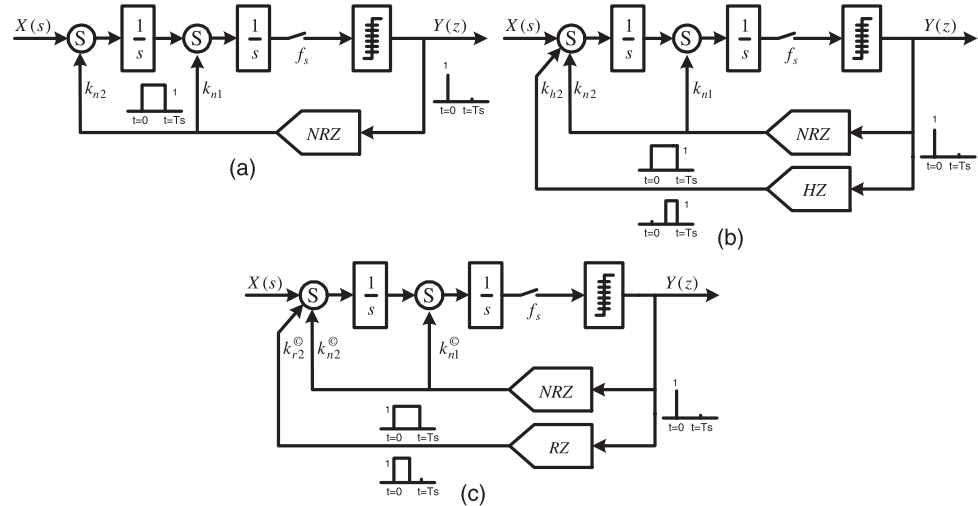
Shoaei, and Cherry have performed a complete analyses, and their results can be concluded as follows:

$$\frac{1}{s} \Rightarrow \frac{y_0}{z-1} \quad y_0 = \beta - \alpha \quad (2)$$

$$\frac{1}{s^2} \Rightarrow \frac{y_1 z + y_0}{(z-1)^2} \quad y_1 = \frac{1}{2}[\beta(2-\beta) - \alpha(2-\alpha)] \quad y_0 = \frac{1}{2}(\beta^2 - \alpha^2) \quad (3)$$

In the cases that  $\beta \geq 1$ , we have [3]:

$$r_{(\alpha,\beta)}(t) = r_{(\alpha,1)}(t) + r_{(0,\beta-1)}(t-1) \quad (4)$$



**Fig. 1.** (a) The structure of a conventional second order CT DSM. (b) The structure of a half-delay return to-zero second order CT DSM. (c) The structure of the proposed second order CT DSM.

### 3 Return to-zero feedback insertion

The block diagram of a second order continuous time Delta-Sigma modulator (CT DSM) is shown in Fig. 1 (a). Utilizing CT-DT equivalence, described in equations (2) and (3), the loop gain of the system can be calculated in CT and DT domain, respectively. Making the z-domain calculated loop-gain equivalent to the optimal loop gain of a second order DT DSM, equation (5), the feedback coefficients of the system, shown in Fig. 1 (a), are determined as follow [3]:

$$H(z) = \frac{-2z + 1}{(z - 1)^2} \quad (5)$$

$$\{k_{n1}, k_{n2}\} = \{-1.5, -1\} \quad (6)$$

CT DSM's are sensitive to the excess loop delay, caused by the DAC in the feedback path, so that it increases the order of the system, leading to instability. The other hazardous influence of the excess loop delay is that the proper coefficients of  $k_{n1}$  and  $k_{n2}$  can not be found to meet the equivalence between the optimal loop-gain, and the affected loop gain. Additional feedback insertion solves this problem [3]. In this paper a novel RZ feedback insertion method is proposed, and the required relationships are extracted. In [3] the author has suggested the half-delay return to-zero (HZ) feedback insertion for compensation the excess loop delay, as shown in Fig. 1 (b), but making a qualitative argument the author has abandoned the RZ feedback insertion for the excess loop delay compensation. In this paper, it is proven

that the RZ feedback can also be employed for this purpose. There are two approaches for analysis of the proposed method. The first approach is to calculate the coefficients of the conventional HZ method, and to extract the RZ coefficients from the HZ coefficients straightforwardly as follows.

In order to calculate the loop gain of the HZ method, shown in Fig. 1 (b), the following transformations, extracted from equations (2), (3), can be used. The  $y_{px}^{ij}$  coefficients are described in Table I.

$$\frac{k_{n1}}{s} \rightarrow \frac{y_{0n}^{10}}{z-1} + z^{-1} \frac{y_{0n}^{11}}{z-1} \quad (7)$$

$$\frac{k_{n2}}{s^2} \rightarrow \frac{y_{1n}^{20}z + y_{0n}^{20}}{(z-1)^2} + z^{-1} \frac{y_{1n}^{21}z + y_{0n}^{21}}{(z-1)^2} \quad (8)$$

$$\frac{k_{h2}}{s^2} \rightarrow \frac{y_{1h}^{20}z + y_{0h}^{20}}{(z-1)^2} + z^{-1} \frac{y_{1h}^{21}z + y_{0h}^{21}}{(z-1)^2} \quad (9)$$

Using the above relations and assuming a given excess loop delay such as  $\tau_d$ , the loop gain of the system shown in Fig. 1 (b) is obtained as the relationship (10). If the equivalence between equations (5) and (10) is met, some linear equations are obtained as described in relationship (11).

$$H(z, \tau_d) = \frac{(y_{0n}^{10} + y_{1n}^{20} + y_{1h}^{20})z^2 + (-y_{0n}^{10} + y_{0n}^{11} + y_{0n}^{20} + y_{0h}^{20} + y_{1n}^{21} + y_{1h}^{21})z + (-y_{0n}^{11} + y_{0n}^{21} + y_{0h}^{21})}{z(z-1)^2} \quad (10)$$

$$\begin{bmatrix} (1 - \tau_d) & \frac{1}{2}(1 - 2\tau_d + \tau_d^2) & \frac{1}{2}(\frac{1}{4} - \tau_d + \tau_d^2) \\ (-1 + 2\tau_d) & \frac{1}{2}(1 + 2\tau_d - 2\tau_d^2) & \frac{1}{2}(\frac{3}{4} + \tau_d - 2\tau_d^2) \\ -\tau_d & \frac{1}{2}\tau_d^2 & \frac{1}{2}\tau_d^2 \end{bmatrix} \times \begin{bmatrix} k_{n1} \\ k_{n2} \\ k_{h2} \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \quad (11)$$

For an ideal system,  $\tau_d = 0$ , the matrix of coefficients is singular and any answer satisfying the first, and second equations of the above matrix is

**Table I.** The coefficients of relationships (7), (8), and (9)

Coefficients of 1 <sup>st</sup> order NRZ loop-gain terms	Coefficients of 2 <sup>nd</sup> order NRZ loop-gain terms	Coefficients of 2 <sup>nd</sup> order HZ, and RZ loop-gain terms
$y_{0n}^{10} = (1 - \tau_d)k_{n1}$	$y_{1n}^{20} = \frac{1}{2}(1 - 2\tau_d + \tau_d^2)k_{n2}$ $y_{0n}^{20} = \frac{1}{2}(1 - \tau_d^2)k_{n2}$	$y_{1h}^{20} = \frac{1}{2}(0.25 - \tau_d + \tau_d^2)k_{h2}$ $y_{1r}^{20} = \frac{1}{2}(0.75 - \tau_d)k'_{r2}$ $y_{0h}^{20} = \frac{1}{2}(0.75 - \tau_d - \tau_d^2)k_{h2}$ $y_{0r}^{20} = \frac{1}{2}(0.25 - \tau_d)k'_{r2}$
$y_{0n}^{11} = \tau_d k_{n1}$	$y_{1n}^{21} = \frac{1}{2}(2\tau_d - \tau_d^2)k_{n2}$ $y_{0n}^{21} = \frac{1}{2}\tau_d^2 k_{n2}$	$y_{1h}^{21} = \frac{1}{2}(2\tau_d - \tau_d^2)k_{h2}, y_{1r}^{21} = 0$ $y_{0h}^{21} = \frac{1}{2}\tau_d^2 k_{h2}, y_{0r}^{21} = 0$

valid. For a non-zero excess loop delay,  $\tau_d \neq 0$ , a unique answer for feedback coefficients is obtained as bellow:

$$k_{n1} = \tau_d \frac{1.25 + \tau_d}{0.5 - \tau_d} \quad (12)$$

$$k_{n2} = \frac{-3.5}{0.5 - \tau_d} \quad (13)$$

$$k_{h2} = \frac{6 + 2\tau_d}{0.5 - \tau_d} \quad (14)$$

Now, it is straightforward to find the feedback coefficients of the proposed system shown in Fig. 1 (c). It is readily known that there is a linear relationship between return to-zero (RZ), half-delay return to-zero (HZ), and non-return to-zero (NRZ) pulse shapes as follows:

$$HZ = NRZ - RZ \quad (15)$$

Therefore, the feedback coefficients of Fig. 1 (c) are obtained as follows:

$$k'_{n1} = k_{n1} = \tau_d \frac{1.25 + \tau_d}{0.5 - \tau_d} \quad (16)$$

$$k'_{n2} = k_{n2} + k_{h2} = \frac{2.5 + 2\tau_d}{0.5 - \tau_d} \quad (17)$$

$$k'_{r2} = -k_{h2} = -\frac{6 + 2\tau_d}{0.5 - \tau_d} \quad (18)$$

The other new idea of this paper is to present the relations (17) and (18) between the HZ-NRZ and RZ-NRZ feedback coefficients so that one can replace an HZ-NRZ feedback with a RZ-NRZ feedback. Furthermore utilizing relation (15), an HZ feedback can be replaced with a RZ-NRZ feedback, and vice versa. Making use of the above analysis, it can be argued that: since in [3] the coefficients of an HZ-NRZ feedback are calculated for the excess loop delay compensation, the corresponding RZ-NRZ feedback coefficients can also be derived for the excess loop delay compensation from relations (16)–(18) in this paper. Although this argument is consistent, however; to remove any ambiguity about it, as the second approach, the coefficients of the system with RZ-NRZ feedback, shown in Fig. 1 (c), can be calculated directly. To do so, relations (7) and (8) are utilized as before but instead of (9) the relation (19), in bellow, is used. The loop gain of the proposed system is described in (20). Making use the equivalence between relations (20) and (5), equation (21) is obtained. After solving equation (21), the coefficients of the proposed method are derived to be exactly as (16), (17), and (18).

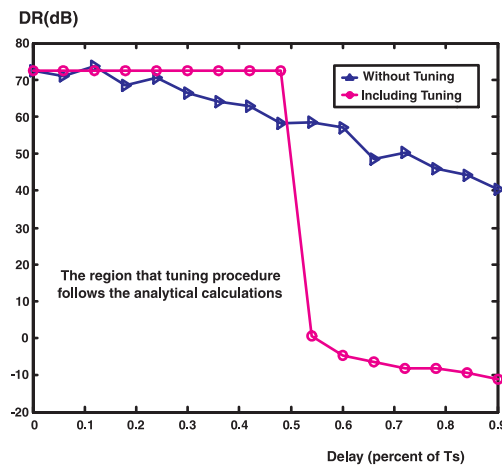
$$\frac{k'_{r2}}{s^2} \rightarrow \frac{y_{1r}^{20}z + y_{0r}^{20}}{(z - 1)^2} \quad (19)$$

$$H'(z, \tau_d) = \frac{(y_{0n}^{10} + y_{1n}^{20} + y_{1r}^{20})z^2 + (-y_{0n}^{10} + y_{0n}^{11} + y_{0n}^{20} + y_{0r}^{20} + y_{1n}^{21})z + (-y_{0n}^{11} + y_{0n}^{21})}{z(z - 1)^2} \quad (20)$$

$$\begin{bmatrix} (1 - \tau_d) & \frac{1}{2}(1 - 2\tau_d + \tau_d^2) & \frac{1}{2}(\frac{3}{4} - \tau_d) \\ (-1 + 2\tau_d) & \frac{1}{2}(1 + 2\tau_d - 2\tau_d^2) & \frac{1}{2}(\frac{1}{4} + \tau_d) \\ -\tau_d & \frac{1}{2}\tau_d^2 & 0 \end{bmatrix} \times \begin{bmatrix} k'_{n1} \\ k'_{n2} \\ k'_{r2} \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \quad (21)$$

#### 4 Simulation results

In this section, in order to verify the analytical results of the proposed method, shown in Fig. 1 (c), with respect to the traditional method, shown in Fig. 1 (a), some MATLAB simulations are performed. For systems shown in Fig. 1 (a) and Fig. 1 (c) the dynamic range (DR) of the ADC versus the excess loop delay is simulated, and the simulation results are shown in Fig. 2. In these simulations, the over-sampling ratio (OSR) is assumed 128, and an 8192-points FFT is used. Considering the results shown in Fig. 2, it is summarized that like the additional half delay return to-zero feedback insertion method, the additional return to-zero feedback insertion method can also be employed for the excess loop delay compensation. However; one drawback of the systems shown in Fig. 1 (b), and Fig. 1 (c) is: for large values of delay in the system loop gain, the over-load factor parameter of the ADC is affected. The reason is that for delay values beyond  $0.5T_s$ , the feedback coefficients increase too much, enhancing the output voltage of the integrators, and reducing the over-load factor parameter.



**Fig. 2.** A comparison between the dynamic range of the un-tuned (Fig. 1 (a)) and tuned (Fig. 1 (c)) CT DSM.

#### 5 Conclusion

Excess loop delay in continuous time (CT) Delta-Sigma modulators (DSM), as a result of the limited DAC response time, leads to SNR degradation so that the optimal tuning of the system will be impossible. In order to solve this problem, this paper proposes a novel RZ feedback insertion method. The

coefficients of the proposed system are calculated analytically. To verify the analytical results, extracted in this paper, a second order CT DSM is simulated. The main advantage of the approach introduced in this paper is that there is no systematic difference between RZ-NRZ and HZ-NRZ feedbacks so that the coefficients of one can be used to extract those of the other. This paper proves that like the HZ feedback insertion method, the RZ feedback insertion method can be employed for the excess loop delay compensation.