IEICE Electronics Express, Vol.8, No.4, 252–258

On the performance of partial relay selection in dual-hop relaying with beamforming

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Abstract: A scheme combining partial relay selection and beamforming in the dual-hop relaying system is proposed. We present the performance analysis of the scheme in Nakagami-m fading by providing closed-form expressions for the outage probability and average bit error probability as well as simple approximations for the two metrics to quantify the performance in high signal-to-noise ratio regime. In the numerical results, the correctness of the theoretical results is validated and the superior performance of the proposed scheme is also shown.
Keywords: partial relay selection, beamforming, outage probability, average bit error probability
Classification: Science and engineering for electronics

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DOI: 10.1587/elex.8.252
Received January 22, 2011
Accepted February 01, 2011
Published February 25, 2011

IEICE Electronics Express, Vol.8, No.4, 252–258

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1 Introduction

Selecting the appropriate relay(s) according to some network parameters to cooperate can bring in remarkable performance improvements for cooperative communications, which has attracted considerable interest. Among diversiform relay selection schemes, the single relay selection scheme known as partial relay selection (PRS) that only requires the single hop channel state information (CSI) is a promising solution to significantly minimize the implementation cost and required complexity, especially in practical ad-hoc and sensor cooperative networks [1]. In [2], the asymptotic performance analysis of amplify-and-forward (AF) with PRS in Rician fading is presented. And in [3], the outage probability (OP) and average bit error probability (BEP) of the PRS with feedback delay are analyzed. Moreover, the inferior performance of PRS is shown in [4], by the performance comparison with opportunistic relaying. And then, a scheme combining PRS and antenna diversity in the dual-hop relaying system is proposed to obtain performance improvement [5]. On the other hand, using beamforming at the multi-antenna sink in the dual-hop relaying system can yield a good performance [6].

This letter proposes a new scheme in the dual-hop AF relaying system. In the scheme, PRS is used in the first hop to select a relay to cooperate based on the instantaneous partial channel knowledge, and beamforming is employed for the multi-antenna destination collecting signals in the second hop. The performance of the scheme is analyzed in Nakagami-m fading, which often gives the best fit to land-mobile and indoor-mobile multi-path propagation, as well as scintillating ionospheric radio links and covers a broad variety of fading scenarios, including the Rayleigh fading (the fading severity parameter $m = 1$), the close approximation for the Rician fading ($m = \frac{(1+K)^2}{1+2K}$ with the Rician fading parameter $K \geq 0$), the nonfading (in the limit as $m \to +\infty$), etc. [7]. Closed-form expressions for the OP and average BEP as well as simple approximations for the two metrics to quantify the performance in the high signal-to-noise ratio (SNR) regime are derived, of which the correctness is verified by the comparison with Monte Carlo simulations. The numerical results indicate the superior performance of the proposed scheme.

Mathematical notations and functions: Vectors are shown with bold letters. $[\cdot]^T$ is the transpose operator, $\|\cdot\|_F$ denotes the Frobenius norm, $F_X(\cdot)$ and $f_X(\cdot)$ represent the cumulative distribution function (CDF) and probability density function (PDF) of the random variable X, respectively. $\Gamma(\cdot)$ and $\gamma(\cdot,\cdot)$ indicate the gamma function [8, Eq. 8.310.1] and incomplete gamma function [8, Eq. 8.350.1], respectively. $K_\nu(\cdot)$ stands for the νth order modified Bessel function of the second kind [8, Eq. 8.446] and $F(\cdot,\cdot;\cdot;\cdot)$ denotes the Gauss hypergeometric function [8, Eq. 9.100].

DOI: 10.1587/elex.8.252
Received January 22, 2011
Accepted February 01, 2011
Published February 25, 2011

IEICE Electronics Express, Vol.8, No.4, 252–258

2 System and channel model

We consider a dual-hop AF relaying system, where a source $\mathcal{S}$ communicates with a destination $\mathcal{D}$ through the help of L relays $\mathcal{R}_l$ $(l = 1, 2, \ldots, L)$. The source and relays are single-antenna devices, and the destination is equipped with N receive antennas. There is no direct link between $\mathcal{S}$ and $\mathcal{D}$ due to the unsatisfactory channel quality. The PRS [1] is used in the system and the relay $\mathcal{R}_M$ with the highest $\mathcal{S} \to \mathcal{R}_l$ instantaneous SNR is selected to cooperate. All nodes in the system are assumed to operate in a half-duplex mode. Thus, each transmission between $\mathcal{S}$ and $\mathcal{D}$ contains two phases. In the first phase, $\mathcal{S}$ transmits signals to the relay $\mathcal{R}_M$. In the second phase, $\mathcal{D}$ knows the perfect CSI and employs beamforming [6] to collect signals relayed by $\mathcal{R}_M$ using the AF protocol.

Let us define $\{h_l\}_{l=1}^{L}$ and $\{g_l^n\}_{n=1}^{N}$ as the channel coefficients of the channels from $\mathcal{S}$ to $\mathcal{R}_l$ and from $\mathcal{R}_l$ to the nth antenna of $\mathcal{D}$, respectively. Then, the $N \times 1$ channel vector from $\mathcal{R}_l$ to $\mathcal{D}$ is $\mathbf{g}_l = [g_l^1, g_l^2, \ldots, g_l^N]^T$. We assume that $\{h_l\}_{l=1}^{L}$ are subject to independent and identically distributed (i.i.d.) Nakagami-m fading with the fading severity parameter m_1 and average fading power Ω_1. Similarly, $\{g_l^n\}_{n=1}^{N}$ $(l = 1, 2, \ldots, L)$ are assumed to be i.i.d. Nakagami-m fading random variables with the fading severity parameter m_2 and average fading power Ω_2. We also assume the average SNRs of the first and second hops are ρ_1 and ρ_2, respectively. Then, the instantaneous end-to-end (e2e) SNR can be given by

$$\gamma_{\text{end}} = \frac{\gamma_{1M}\gamma_{2M}}{\gamma_{1M} + \gamma_{2M} + 1} \tag{1}$$

where $M = \arg \max_{l=1,2,\ldots,L} \rho_1 |h_l|^2$, $\gamma_{1M} = \rho_1 |h_M|^2$ and $\gamma_{2M} = \rho_2 \|\mathbf{g}_M\|_F^2$.

3 Performance analysis

Assuming m_1 to be an integer and with the help of [8, Eq. 3.351.1], [8, Eq. 1.111] and [8, Eq. 0.334], the CDF of γ_{1M} can be written as

$$\begin{aligned} F_{\gamma_{1M}}(\gamma_1) &= \prod_{l=1}^{L} \Pr\left(\rho_1 |h_l|^2 < \gamma_1\right) = \left[\frac{\gamma\left(m_1, m_1'\gamma_1\right)}{\Gamma(m_1)}\right]^L \\ &= 1 + \sum_{k=1}^{L} \binom{L}{k} (-1)^k \exp(-m_1'\gamma_1) \sum_{i=0}^{k(m_1-1)} a_i^k (m_1'\gamma_1)^i \end{aligned} \tag{2}$$

where $m_1' = \frac{m_1}{\rho_1\Omega_1}$, $a_0^k = 1$, $a_{k(m_1-1)}^k = \left[\frac{1}{\Gamma(m_1)}\right]^k$ and $a_i^k = \sum_{p=1}^{q} \frac{pk+p-i}{ip!} a_{i-p}^k$ with $q = \min(i, m_1 - 1)$ and $1 \le i \le k(m_1 - 1) - 1$. And the PDF of γ_{2M} is given in [6] as

$$f_{\gamma_{2M}}(\gamma_2) = \frac{m_2'^{Nm_2} \gamma_2^{Nm_2-1}}{\Gamma(Nm_2)} \exp(-m_2'\gamma_2) \tag{3}$$

where $m_2' = \frac{m_2}{N\rho_2\Omega_2}$. Then, the CDF of instantaneous e2e SNR can be expressed as

$$F_{\gamma_{\text{end}}}(\gamma) = \Pr\left(\frac{\gamma_{1M}\gamma_{2M}}{\gamma_{1M} + \gamma_{2M} + 1} < \gamma\right)$$

DOI: 10.1587/elex.8.252
Received January 22, 2011
Accepted February 01, 2011
Published February 25, 2011

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$$= F_{\gamma_{2M}}(\gamma) + \underbrace{\int_{\gamma}^{\infty} F_{\gamma_{1M}}\left(\frac{\gamma(x+1)}{x-\gamma}\right) f_{\gamma_{2M}}(x)dx}_{I_1} \quad (4)$$

Substituting Eq. (2) and Eq. (3) into Eq. (4), we have

$$\begin{aligned} I_1 &= 1 - F_{\gamma_{2M}}(\gamma) + \sum_{k=1}^{L} \sum_{i=0}^{k(m_1-1)} \frac{(-1)^k a_i^k \gamma^i m_1'^i m_2'^{Nm_2}}{\Gamma(Nm_2)\exp(km_1'\gamma + m_2'\gamma)} \\ &\times \underbrace{\int_0^{\infty} \exp\left(-\frac{km_1'\gamma(\gamma+1)}{t} - m_2' t\right) \frac{(t+\gamma+1)^i (t+\gamma)^{Nm_2-1}}{t^i} dt}_{I_2} \end{aligned} \quad (5)$$

Assuming an integer Nm_2 and using [8, Eq. 1.111] and [8, Eq. 3.471.9], after some algebraic manipulations, we can obtain that

$$\begin{aligned} I_2 &= 2\sum_{j=0}^{i} \binom{i}{j} \sum_{\mu=0}^{Nm_2+j-1} \binom{Nm_2+j-1}{\mu} \left(\frac{km_1'\gamma(\gamma+1)}{m_2'}\right)^{\frac{\mu-i+1}{2}} \\ &\times \gamma^{Nm_2+j-\mu+1} K_{\mu-i+1}\left(2\sqrt{km_1'm_2'\gamma(\gamma+1)}\right) \end{aligned} \quad (6)$$

Substituting Eq. (5) and Eq. (6) into Eq. (4), the CDF of γ_{end} can be determined as

$$\begin{aligned} F_{\gamma_{\text{end}}}(\gamma) &= 1 + \sum_{k=1}^{L} \sum_{i=0}^{k(m_1-1)} \sum_{j=0}^{i} \sum_{\mu=0}^{Nm_2+j-1} \binom{L}{k}\binom{i}{j}\binom{Nm_2+j-1}{\mu} \\ &\times \left(\frac{km_1'}{m_2'}\right)^{\frac{\mu-i+1}{2}} \frac{2m_2'^{Nm_2}(-1)^k a_i^k m_1'^i \gamma^{Nm_2+i+j-\mu-1}}{\Gamma(Nm_2)\exp(km_1'\gamma + m_2'\gamma)} \\ &\times [\gamma(\gamma+1)]^{\frac{\mu-i+1}{2}} K_{\mu-i+1}\left(2\sqrt{km_1'm_2'\gamma(\gamma+1)}\right) \end{aligned} \quad (7)$$

3.1 Outage Probability

OP is defined as the probability that the instantaneous e2e SNR falls below the given threshold γ_{th}. Therefore OP of the system can be calculated as $P_{\text{OP}} = F_{\gamma_{\text{end}}}(\gamma_{\text{th}})$.

3.2 OP in the high SNR regime

In the high SNR regime ($\rho_1^{-1}, \rho_2^{-1} \to 0$), OP can be approximated as

$$\begin{aligned} P_{\text{OP}} &\approx \Pr(\min(\gamma_{1M}, \gamma_{2M}) < \gamma_{\text{th}}) \\ &= F_{\gamma_{1M}}(\gamma_{\text{th}}) + F_{\gamma_{2M}}(\gamma_{\text{th}}) - F_{\gamma_{1M}}(\gamma_{\text{th}})F_{\gamma_{2M}}(\gamma_{\text{th}}) \end{aligned} \quad (8)$$

Since $\gamma(\alpha, u)$ with $u \to 0$ can be approximated by using [8, Eq. 8.354.1] as $\gamma(\alpha, u) \approx \alpha^{-1}u^{\alpha}$, in the high SNR regime, we have

$$F_{\gamma_{1M}}(\gamma_{\text{th}}) = \left[\frac{\gamma(m_1, m_1'\gamma_{\text{th}})}{\Gamma(m_1)}\right]^L \approx \left[\frac{m_1^{m_1}}{\Gamma(m_1+1)}\left(\frac{\gamma_{\text{th}}}{\Omega_1}\right)^{m_1}\right]^L \rho_1^{-Lm_1} \quad (9)$$

and

$$F_{\gamma_{2M}}(\gamma_{\text{th}}) = \frac{\gamma(Nm_2, m_2'\gamma_{\text{th}})}{\Gamma(Nm_2)} \approx \left(\frac{m_2\gamma_{\text{th}}}{N\Omega_2}\right)^{Nm_2} \frac{\rho_2^{-Nm_2}}{\Gamma(Nm_2+1)} \quad (10)$$

DOI: 10.1587/elex.8.252
Received January 22, 2011
Accepted February 01, 2011
Published February 25, 2011

Assuming $\rho_2 = \xi\rho_1$ and using Eq. (8), Eq. (9) and Eq. (10), the OP in the high SNR regime can be expressed as

$$P_{\rm OP}^{\infty} = \beta\rho_1^{-\min(Lm_1,Nm_2)}\gamma_{\rm th}^{\min(Lm_1,Nm_2)} \tag{11}$$

where

$$\beta = \begin{cases} \left[\frac{m_1^{m_1}}{\Gamma(m_1+1)\Omega_1^{m_1}}\right]^L, & Lm_1 < Nm_2 \\ \left[\frac{m_1^{m_1}}{\Gamma(m_1+1)\Omega_1^{m_1}}\right]^L + \frac{m_2^{Nm_2}}{\Gamma(Nm_2+1)(N\xi\Omega_2)^{Nm_2}}, & Lm_1 = Nm_2 \\ \frac{m_2^{Nm_2}}{\Gamma(Nm_2+1)(N\xi\Omega_2)^{Nm_2}}, & Lm_1 > Nm_2 \end{cases} \tag{12}$$

From Eq. (11), we can see that the diversity order of the system is $d = -\lim_{\rho_1\to\infty}\frac{\log P_{\rm OP}^{\infty}}{\log\rho_1} = \min(Lm_1, Nm_2)$.

3.3 Average BEP

The average BEP can be evaluated as the expectation of the conditional BEP, which is obtained as

$$P_b = \frac{a}{2\sqrt{2\pi}}\int_0^{\infty}\exp\left(-\frac{t}{2}\right)F_{\gamma_{\rm end}}\left(\frac{t}{c}\right)t^{-\frac{1}{2}}dt \tag{13}$$

where a and c are constants determined by the modulation format as in [3] (eg., for the quadrature phase shift keying (QPSK) modulation, $a = 1$ and $c = 1$). Though Eq. (7) is complicate, we use the tractable form of it in the high SNR regime ($\gamma \approx \gamma+1$) instead. Substituting Eq. (7) into Eq. (13) with $\gamma \approx \gamma+1$ and using [8, Eq. 6.621.3], a closed-form approximate expression for the average BEP can be derived as follow:

$$\begin{aligned} P_b \quad = \quad & \frac{a}{2} + \frac{a m_2'^{Nm_2}\sqrt{8c}}{\Gamma(Nm_2)}\sum_{k=1}^{L}\sum_{i=0}^{k(m_1-1)}\sum_{j=0}^{i}\sum_{\mu=0}^{Nm_2+j-1}\binom{L}{k}\binom{i}{j}\binom{Nm_2+j-1}{\mu} \\ & \times\frac{\Gamma\left(Nm_2-i+j+\mu+\frac{3}{2}\right)\Gamma\left(Nm_2+i+j-\mu-\frac{1}{2}\right)(-1)^k a_i^k m_1'^{\mu+1}}{\Gamma(Nm_2+j+1)4^{i-\mu}k^{i-\mu-1}\phi^{Nm_2-i+j+\mu-\frac{3}{2}}} \\ & \times F\left(Nm_2-i+j+\mu+\frac{3}{2}, \mu-i+\frac{3}{2}; Nm_2+j+1; \frac{\varphi}{\phi}\right) \end{aligned} \tag{14}$$

where $\varphi = km_1' + m_2' + \frac{c}{2} - 2\sqrt{km_1'm_2'}$ and $\phi = \varphi + 4\sqrt{km_1'm_2'}$.

3.4 Average BEP in the high SNR regime

By using Eq. (11) and Eq. (13), the average BEP in the high SNR regime $(\rho_1^{-1}, \rho_2^{-1} \to 0)$ can be evaluated as

$$P_b^{\infty} = \frac{a\beta}{2\sqrt{2\pi}(c\rho_1)^{\min(Lm_1,Nm_2)}}\int_0^{\infty}\exp\left(-\frac{t}{2}\right)t^{\min(Lm_1,Nm_2)-\frac{1}{2}}dt \tag{15}$$

With the help of [8, Eq. 3.326.2], we can express the average BEP in the high SNR regime as

$$P_b^{\infty} = \frac{a\beta 2^{\min(Lm_1,Nm_2)-1}\Gamma\left(\min(Lm_1,Nm_2)+\frac{1}{2}\right)}{\sqrt{\pi}(c\rho_1)^{\min(Lm_1,Nm_2)}} \tag{16}$$

DOI: 10.1587/elex.8.252
Received January 22, 2011
Accepted February 01, 2011
Published February 25, 2011

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4 Numerical results

In this section, Monte Carlo simulations are performed to validate our theoretical results. Without the loss of generality, the situation of $\Omega_1 = 0.5$, $\Omega_2 = 0.2$ and $\rho_1 = \rho_2$ is considered here. The theoretical and simulated curves of the OP and average BEP are plotted in the following figures for several values of the fading severity parameters m_1 and m_2, the number of relays L, and the number of destination antennas N.

Fig. 1 shows OP versus the average first hop SNR ρ_1 for several values of m_1, m_2, L and N with $\gamma_{\mathrm{th}} = 5\,\mathrm{dB}$. The exact expression and high SNR

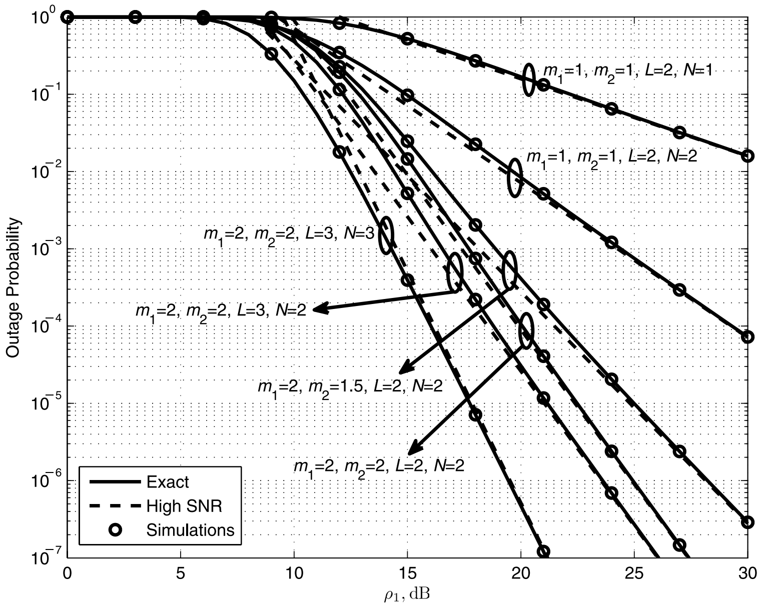

Fig. 1. OP versus ρ_1 for several values of m_1, m_2, L and N with $\gamma_{\mathrm{th}} = 5\,\mathrm{dB}$.

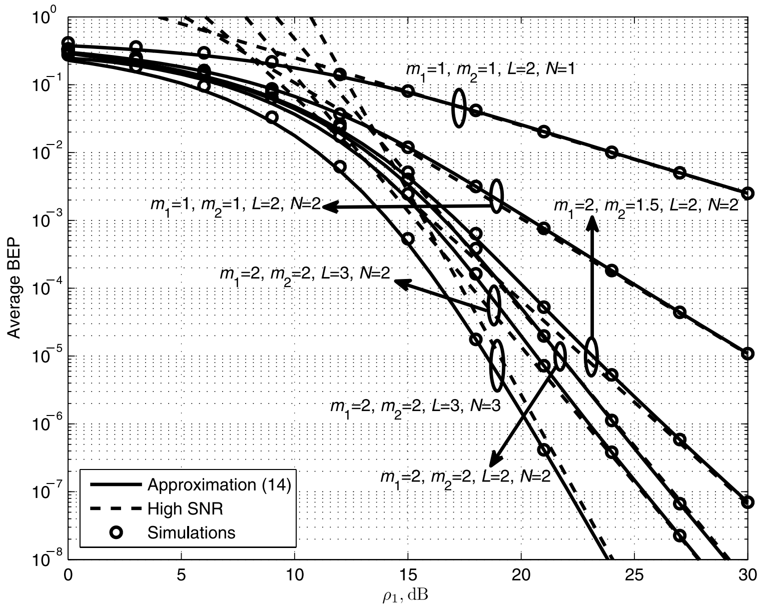

Fig. 2. QPSK modulated average BEP versus ρ_1 for several values of m_1, m_2, L and N.

DOI: 10.1587/elex.8.252
Received January 22, 2011
Accepted February 01, 2011
Published February 25, 2011

IEICE Electronics Express, Vol.8, No.4, 252–258

approximation for OP are calculated by using Eq. (7) and Eq. (11), respectively. In Fig. 2, the QPSK modulated average BEP versus ρ_1 for several values of m_1, m_2, L and N is depicted. The high SNR approximation for the average BEP is calculated by using Eq. (16). It can be seen that the theoretical results match well with the simulation results. As observed from the figures, the diversity order of the system is $\min(Lm_1, Nm_2)$ (eg., $d = 1$ for the case of $m_1 = m_2 = 1$, $L = 2$ and $N = 1$; $d = 4$ for the case of $m_1 = m_2 = 2$ and $L = N = 2$), and larger $\min(Lm_1, Nm_2)$ results in better performance. Specifically, in Rayleigh fading ($m_1 = m_2 = 1$), our proposed scheme in the case of $L = 2$ and $N = 2$ can achieve more than 10 dB SNR gain over the conventional two relays dual-hop relaying with PRS ($L = 2$ and $N = 1$) while guaranteeing $P_{\mathrm{OP}} = 10^{-2}$ or $P_b = 10^{-3}$.

5 Conclusions

In this letter, a scheme combining PRS and beamforming in the dual-hop relaying system is proposed. The performance analysis of the scheme is presented in Nakagami-m fading. Closed-form expressions and high SNR approximations for the OP and average BEP are derived. Our theoretical results are verified by Monte Carlo simulations. And, withal, the numerical results demonstrate that the proposed scheme can achieve better performance than the conventional dual-hop relaying with PRS.

Acknowledgments

This work was supported by the National Natural Science Foundations of China under Grant No. 61001095 and No. 61072103.

DOI: 10.1587/elex.8.252
Received January 22, 2011
Accepted February 01, 2011
Published February 25, 2011