## DISCUSSIONS

# EVALUATING SUPERPOSITION ERRORS IN BEARING CAPACITY FACTORS FROM SOKOLOVSKI'S METHOD OF CHARACTERISTICS[i]

Discussion by NIELS MORTENSEN[ii]

## INTRODUCTION

The bearing capacity problem has been addressed by Yoshimichi Tsukamoto in "Soil and Foundation", Volume 45, No. 3, 2005, pp. 161–165. The comments given below have been limited to cover the unit weight and surcharge contribution as formulated by Terzaghi (1943) (neglecting the cohesive contribution):

$$b=0.5\gamma' B N_\gamma + q' N_q \quad (7)$$

or by Lundgren and Mortensen (1953):

$$b=(0.5\gamma B+q')N_{\gamma q} \quad (8)$$

In Eqs. (7) and (8), $\gamma'$ represents the effective unit weight, $B$ represents the foundation width, $q'$ represents the vertical surcharge acting at foundation level, $N_\gamma$ and $N_q$ are the bearing capacity factors for the unit weight contribution and the surcharge contribution, respectively, whereas $N_{\gamma q}$ is the combined bearing capacity factor when unit weight and surcharge are included at the same time when defining the bearing capacity factor.

Only plane strain ideal plasticity has been considered for a horizontal soil surface and a homogeneous isotropic soil.

## DISCUSSION OF BEARING CAPACITY FACTORS $N_q$ AND $N_\gamma$

The value of $N_q$ was defined by Prandtl (1920) as a closed form expression obeying the upper and lower bound theorem, valid for associated flow only. The $N_q$-formulas from Tsukamoto (2005), Eqs. (5) and (6), are identical and lead to the known Prandtl formula for $N_q$. The value of $N_q$ depends only on the value of $\varphi'$ and not on the value of the interface strength between the foundation base and the soil and thus, $\varphi'=30°$ implies $N_q=18.40$ for $0°\leq\delta\leq\varphi'$.

The value of $N_\gamma$ has been subjected to discussion throughout the years. A covering description of the subject shall not be given here, but it shall be mentioned that:

a) For a rough foundation base Lundgren and Mortensen (1953) established a mesh of stress characteristics as shown on Fig. 10, left, representing slip lines along which force equilibrium was found in accordance with Kötter (1903). A more detailed description of the Lundgren and Mortensen method may be found in Hansen (1998).
b) Bønding (1977) showed that the Lundgren and Mortensen solution would be kinematically admissible for a wide range of realistic combinations of $\varphi'$ and the dilatancy angle, $\psi$ and thus, the Lundgren and Mortensen procedure for evaluating $N_\gamma$ represents either an exact solution or a solution on the unsafe side (computed too high).
c) The rupture figure obtained by a Plaxis analysis applying the Mohr-Coulomb model with $\varphi'=\delta=\psi=30°$ is shown on Fig. 10, right. The analysis lead to $N_\gamma=$ 15.3 or slightly higher than the value from Lundgren and Mortensen with $N_\gamma=14.75$.
d) The Lundgren and Mortensen rupture figure considered for $\delta=0°$ resembles the rupture figure from Fig. 11 in Tsukamoto (2005).

The issues a) through c) mentioned above all indicate that a bearing capacity factor, $N_\gamma$, of approximately 15 shall be found for $\varphi'=\delta=30°$. A closed form expression for $N_\gamma$ with $\varphi'=\delta$ from Lundgren and Mortensen (1953) has, however, never been established although a very accurate approximation has been given in DS 415 (1998):

$$N_\gamma=[(N_q-1)\cos\varphi]^{3/2}/4 \quad (9)$$

The results for $N_\gamma$ in Tsukamoto (2005) are surprising in the sense that the bearing capacity factor, $N_\gamma$ is found in the range of 15 to 17 for a soil with $\varphi'=30°$ and $\delta=0°$.

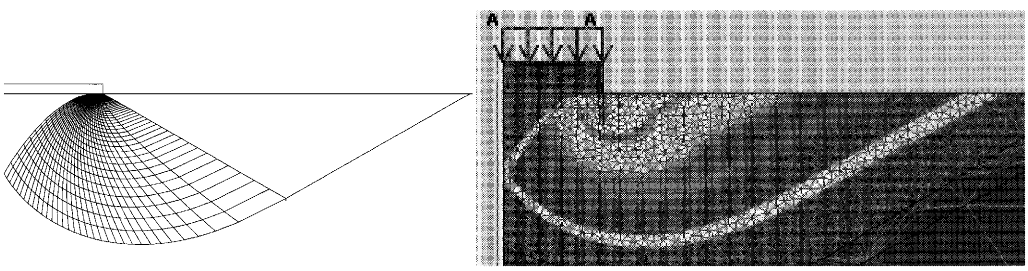


**Fig. 10. The Lundgren-Mortensen rupture figure (left, $N_\gamma=$ 14.75) and shear strain contour plot from Plaxis (right, $N_\gamma=15.3$): Figures computed for $\varphi'=\delta=30°$**

i) By Yoshimichi Tsukamoto, Vol. 45, No. 3, June 2005, pp. 161–165.
ii) Norwegian Geotechnical Institute, Box 3930 Ullevål Stadion, 0806 Oslo, Norway.

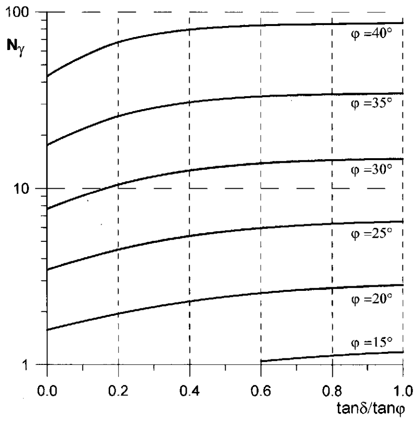


**Fig. 11. The bearing capacity factor, $N_\gamma$, computed in accordance with Lundgren and Mortensen (1953) versus the relative roughness along the foundation base**

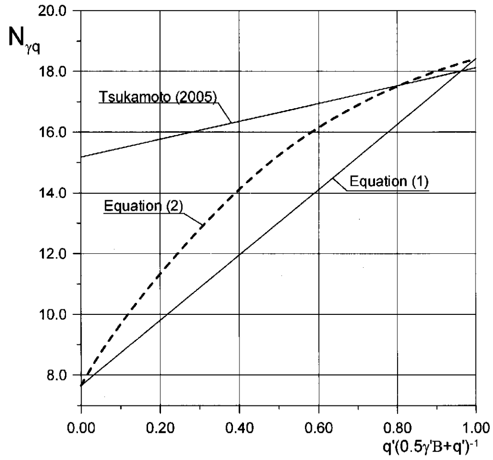


**Fig. 12. The combined bearing capacity factor $N_{\gamma q}$ versus the effective surcharge normalised with respect to $[0.5 \cdot \gamma \cdot B + q']$: All curves are shown corresponding to $\varphi' = 30°$ and $\delta = 0°$**

In addition, reference is made to Brinch Hansen (1970) in order to verify the computed values. The following comments may be given:

- Brinch Hansen (1970) has not considered footings with $\delta = 0°$. Equation (6) in Tsukamoto (2005), adopted from Brinch Hansen (1970), is an approximation to $N_\gamma$ for $\varphi' = \delta$ and where $N_\gamma$ is computed in accordance with Lundgren and Mortensen (1953). The Brinch Hansen results can therefore not be used as a sufficient reference.
- Repeating the analyses from Lundgren and Mortensen (1953) implies $N_\gamma = 7.65$ for $\varphi' = 30°$ and $\delta = 0°$ whereas $\varphi' = \delta = 30°$ implies $N_\gamma = 14.75$. There is a significant difference between the Tsukamoto results and the Lundgren and Mortensen results for $\varphi' = 30°$ and $\delta = 0°$.
- The Lundgren and Mortensen value of $N_\gamma$ has been computed for various values of the friction angle, $\varphi'$, and the interface strengths, $\delta$, along the foundation base. The results are depicted on Fig. 11.

Figure 11. indicates that $N_\gamma$ is more or less constant for $\tan\delta/\tan\varphi' > 0.5$.

## COMBINED UNIT WEIGHT AND SURCHARGE

If the analyses from Tsukamoto (2005) is performed applying the principles from Lundgren and Mortensen (1953), the result for $\varphi = 30°$ and $\delta = 0°$ have been depicted in Fig. 12. The x-axis in Fig. 12 is based on the normalised factor $k = q'/[0.5 \cdot \gamma'(\mathrm{B} + \mathrm{q}']$ and thus, the pure $\gamma$-case is found for $k = 0.0$ and the pure $q'$-case is found for $k = 1.0$.

The straight line in Fig. 12 representing Eq. (7) is the result of linear superposition as applied by Terzaghi (1943) using the bearing capacity factors $N_\gamma = 7.65$ and $N_q = 18.40$. The curve representing Eq. (8) is the result from the Lundgren and Mortensen analyses, whereas the straight line representing Tsukamoto (2005) is the authors interpretation of the Tsukamoto results applying $N_\gamma = 15.18$ and $N_q = 18.11$.

The reason for the difference comparing the Tsukamoto-results with the other curves in Fig. 12 has not been found. It has, however, been assumed that the Lundgren and Mortensen results are not only more accurate but also in line with finite element analyses.

Tsukamoto (2005) shortly discusses the amount of nodes applied in the analyses and indicates that 11 nodes are used to define the line OA on Fig. 10 in Tsukamoto (2005). The results presented in Fig. 12 above have been based on 154 nodes along line OA and a total amount of nodes of 57399 in the entire rupture figure.

*References*


11) Bønding, N. (1977): Kinematically admissibility of the pure $N_\gamma$ rupture figure, *Proc. 9th ICSMFE*, Tokyo 1977, **1**, 415–418.
12) DS 415 (1998): *Danish Code of Practise for Foundation Engineering*, Dansk Standard, 4th ed.
13) Hansen, B. (1998): *Advanced Theoretical Soil Mechanics*, the Technical University of Denmark, Lecture Notes, Later issued through www.danskgeotekniskforening.dk as dgf-Bulletin number 20.
14) Kötter, F. (1903): *Die Bestimmung des Drucks an gekrümmten Gleitflächen, eine Aufgabe aus der Lehre vom Erddruck*, Sitzungsber. Kgl. Preuss, Akad. Der Wiss., Berlin 1903.
15) Lundgren, H. and Mortensen, K. (1953): Determination by the theory of plasticity of the bearing capacity of continuous footings on sand, *Proc. 3rd ICSMFE*, Zürich 1953, **1**, 409–412.
16) Prandtl, L. (1920): *Über die Härte plastischer Körper*, Nachr. D. Ges. D. Wiss., math.-phys. Kl., Göttingen 1920.
17) Terzaghi, K. (1943): *Theoretical Soil Mechanics*, Wiley, New York.
18) Tsukamoto, Y. (2005): *Soils and Foundations*, **45** (3), 161–165.