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Transient and DC approximate expressions for diode circuits

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Abstract: In general terms, it is not possible to establish symbolic explicit analytic expressions of the operating point and transient analysis for circuits containing diodes modelled using an exponential function. Therefore, this work propose replacing the diode for an equivalent circuit obtained by using a power series and a Taylor series consecutively. Finally, we present a symbolic solution for some circuits that include diodes; resulting for the best case: for DC analysis a relative error of 1E-11 and for transient analysis a relative error $\leq$ 5E-4.

Keywords: circuit analysis, nonlinear circuits
Classification: Electron devices, circuits, and systems

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1 Introduction

Circuit designers do not posses analytic explicit expressions to perform analysis of diode circuits; that is the reason they analyse circuits, first, by using crude approximations and, second, by an iterative process of numerical simulations allowing them to reach the required specifications of the design.

In [1], an explicit analytic expression for the current of a basic circuit containing a voltage source, a resistor, and a diode was proposed. Nevertheless, such approximation cannot be generalized to circuits with several meshes containing diodes. Therefore, due to the exponential characteristic of the diode model, in general, it is not possible to establish symbolic explicit analytic expressions of the operating point and transient analysis of circuits containing diodes. In [2] an analytic approximate solution of the AC behaviour for a rectifier circuit based on a perturbation method is exposed; nevertheless, the provided expression is not handy and not suitable to be applied on large circuits. In consequence, this work propose a procedure to generate approximate analytic expressions for transient and DC analyses for circuits including diodes; which is based on power series and can be extended to VLSI circuits.

This paper is organized as follows. In Section 2, we will perform the DC and transient analyses of a basic circuit with one diode. In Section 3, the analyses will be done for a circuit with two diodes. In Section 4, we present some numerical simulations. In Section 5, we summarize our findings and suggest possible directions for future investigations. Finally, a brief conclusion is given in Section 6.

2 Circuit analysis of the basic nonlinear circuit

Fig. 1 (a) shows a circuit containing a voltage source (V), a resistor (R), an inductor (L), and a diode (D). The voltage drop at the diode is

$$V_D = V_T \ln(i(t)/I_s + 1), \tag{1}$$

where I_s is the saturation current of the diode and V_T is the thermal voltage.

Now, we establish the nonlinear differential equation that describes the transient behaviour for the circuit

$$Ri(t) + L\tfrac{di(t)}{dt} + V_T \ln(i(t)/I_s + 1) - V = 0, \qquad i(0) = A. \tag{2}$$

Equation (2) does not have analytic solution due to the natural logarithm term from the diode model. Nevertheless, it is possible to establish a reasonable approximation expanding (1) to a power series with respect to I_s, giving

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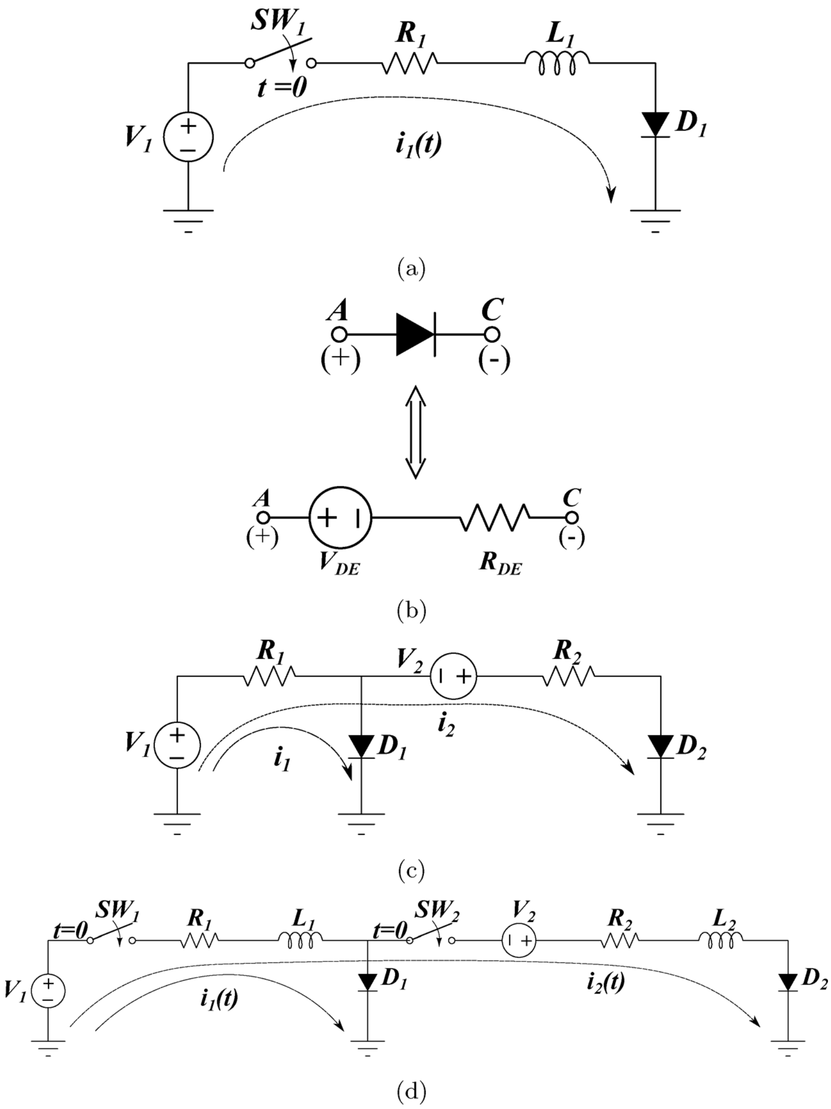

Fig. 1. a) Basic cell, b) Proposed equivalent diode model, c) DC scheme of two mesh circuit, and d) Transient scheme of two mesh circuit.

as result

$$V_T \ln\left(\tfrac{i(t)}{Is}\right) + \tfrac{V_T}{i(t)} Is - \frac{1}{2}\frac{V_T}{(i(t))^2} Is^2 + \frac{1}{3}\frac{V_T}{(i(t))^3} Is^3 + O\left(Is^4\right). \tag{3}$$

Given that I_s has values in the order of 1E-12 (for silicon diodes), it is possible to discard all the terms but first in (3)

$$V_D = V_T \ln\left(\tfrac{i(t)}{Is}\right). \tag{4}$$

Expanding (4) the result is

$$\begin{aligned} V_D = V_T \ln\left(\frac{B}{Is}\right) + \frac{V_T}{B}(i(t) - B) - \frac{1}{2}\left(\frac{V_T}{B^2}\right)(i(t) - B)^2 \\ + \frac{1}{3}\left(\frac{V_T}{B^3}\right)(i(t) - B)^3 + O\left((i(t) - B)^4\right) \end{aligned} \tag{5}$$

where B is the current at the expansion point. If $i(t)$ is close to B, then it is possible to use the first two terms of the series and discard the rest of the

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terms.

$$V_D = V_T \ln\left(\tfrac{B}{Is}\right) + \tfrac{V_T}{B}\left(i(t) - B\right), \quad B > 0 \tag{6}$$

Therefore, using (6), we can rewrite equation (2)

$$Ri(t) + L\tfrac{di(t)}{dt} + V_T \ln(B/I_s) + (V_T/B)(i(t) - B) - V = 0,\ i(0) = A. \tag{7}$$

The solution for (7) is

$$i(t) = i_{dc} + (A - i_{dc}) \exp\left(-\frac{(RB + V_T)t}{BL}\right), \tag{8}$$

where i_{dc} is

$$i_{dc} = \frac{B(V - V_T \ln(B/I_s) + V_T)}{RB + V_T}, \quad V \geq V_T \ln(B/I_s) - V_T. \tag{9}$$

Equation (8) represents the transient for the nonlinear circuit in Fig. 1 (a) and (9) represents the steady state of the transient, that is, the DC current value of the circuit. By inspecting (5) or (6) we can conclude that the best approximation for (9) is when we select B right at the operating point, thus, equalling $i_{dc} = B$ in (9) and solving for B, we obtain

$$i_{dc} = B = I_s \exp\left(V/V_T - W((RI_s/V_T)\exp(V/V_T))\right). \tag{10}$$

where W represents the Lambert W function.

The value obtained for B by evaluating (10) can be used as expansion point for the approximate current formula (9).

In [1], an analytic expression for the current in the circuit shown in Fig. 1 (a) is formulated

$$i_{dc} = I_s + (V_T/R)W((I_sR/V_T)\exp\left(\frac{(V + I_sR)}{V_T}\right). \tag{11}$$

In fact, using a numerical example we will show in Section 4 that the error between the exact current (11) and the approximate (10) is quite low.

By using (6) is possible to build an approximate circuital model for the diode (as it can be seen in Fig. 1 (b))

$$\left.\begin{aligned} V_{DE} &= V_T \ln\left(\frac{B}{I_s}\right) - V_T, \\ R_{DE} &= \frac{V_T}{B}, \end{aligned}\right\} V_{th} \geq V_{DE} \tag{12}$$

where V_{DE} is an independent voltage source, R_{DE} an equivalent linear resistor and V_{th} results from the Thevenin equivalent seen between the diode terminals. Such model may be employed to perform approximate analysis on DC and transient by means of mesh circuit analysis techniques, MNA [3], among others.

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3 DC and transient analysis for two mesh circuit with diodes

Fig. 1 (c) shows a circuit composed by two identical diodes (D_1 and D_2), two resistors (R_1 and R_2), and two voltage sources (V_1 and V_2). Applying loop analysis and approximate circuital model (see Fig. 1 (b)), we obtain the following solution

$$\begin{aligned} i_1 = (1/D)(&-R_1B_1B_2(V_1+V_2-V_T\ln(B_2/I_s)+V_T) \\ &-B_1((R_1+R_2)B_2+V_T)(V_1-V_T\ln(B_1/I_s)+V_T)), \end{aligned} \tag{13}$$

and

$$\begin{aligned} i_2 = (1/D)(&B_2(R_1B_1+V_T)(V_1+V_2-V_T\ln(B_2/I_s)+V_T) \\ &-B_1B_2R_1(V_1-V_T\ln(B_1/I_s)+V_T)), \end{aligned} \tag{14}$$

where

$$D = (R_1B_1+V_T)((R_1+R_2)B_2+V_T) - B_1B_2R_1^2, \tag{15}$$

where B_1 and B_2 are Taylor series expansion points for currents i_1 and i_2, respectively.

The exact equations for the nonlinear transient of the circuit shown in Fig. 1 (c) are established as

$$R_1(i_1(t)+i_2(t)) + L_1(\tfrac{di_1(t)}{dt}+\tfrac{di_2(t)}{dt}) + V_T\ln(i_1(t)/I_s+1) - V_1 = 0, \tag{16}$$

and

$$\begin{aligned} &R_1(i_1(t)+i_2(t)) + L_1(\tfrac{di_1(t)}{dt}+\tfrac{di_2(t)}{dt}) + \\ &R_2i_2(t) + L_2\tfrac{di_2(t)}{dt} + V_T\ln(i_2(t)/I_s+1) - V_1 - V_2 = 0. \end{aligned} \tag{17}$$

where $i_1(0) = A_1$ and $i_2(0) = A_2$.

Likewise, by using (12) the approximate equations for the nonlinear transient for the circuit in Fig. 1 (c) are

$$\begin{aligned} &R_1(i_1(t)+i_2(t)) + L_1(\tfrac{di_1(t)}{dt}+\tfrac{di_2(t)}{dt}) \\ &+V_T\ln(B_1/I_s) + V_T/B_1)(i_1(t)-B_1) - V_1 = 0, \end{aligned} \tag{18}$$

and

$$\begin{aligned} &R_1(i_1(t)+i_2(t)) + L_1(\tfrac{di_1(t)}{dt}+\tfrac{di_2(t)}{dt}) + R_2i_2(t) + \\ &L_2\tfrac{di_2(t)}{dt} + V_T\ln(B_2/I_s) + (V_T/B_2)(i_2(t)-B_2) - V_1 - V_2 = 0. \end{aligned} \tag{19}$$

where $i_1(0) = A_1$, $i_2(0) = A_2$, B_1, and B_2 are Taylor series expansion points for currents i_1 and i_2.

Solving the equations system (18) and (19), and separating the steady state term for i_1 and i_2, we find exactly the same equations (13) and (14), respectively.

4 Numerical simulations

For all the above cases we considered the diodes to have identical thermal voltage $V_T = 0.02585$V and saturation current I_s=1E-12A. By choosing an arbitrary expansion point $B = 1$ in (8), transient analysis was performed for

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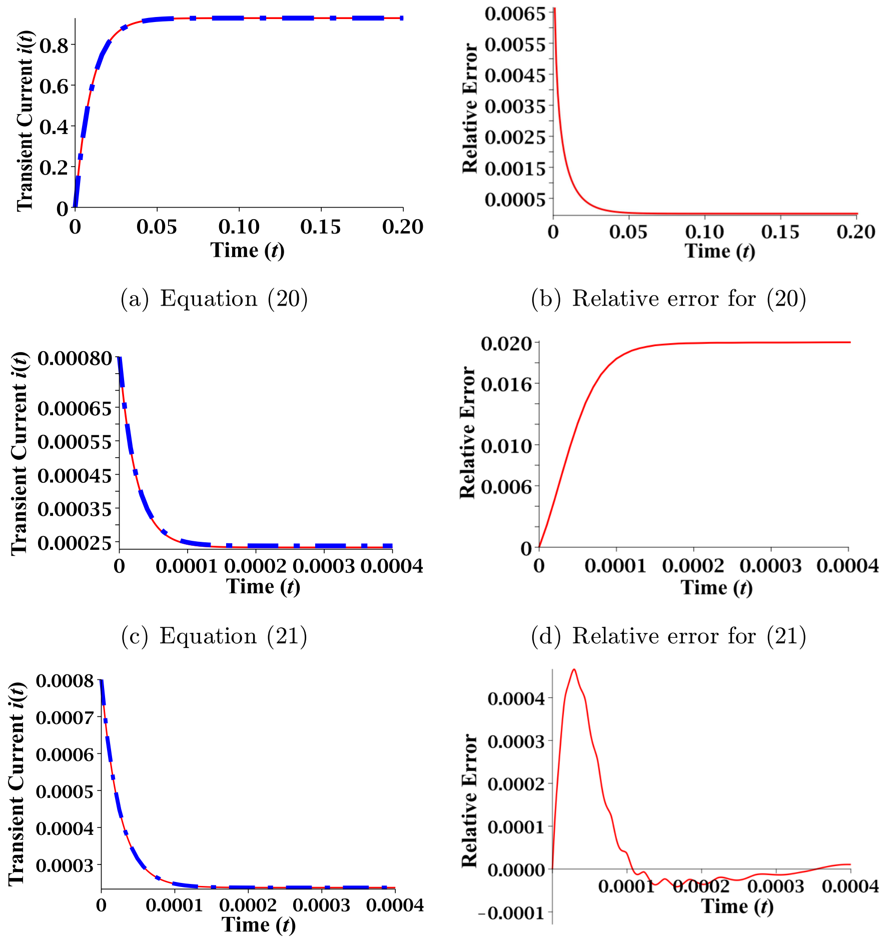

Fig. 2. Numerical solution for (2) (dash-dot) against approximate solution (continuous line) (20), (21) and (22). Time is in seconds and current in Amperes.

the circuit in Fig. 1 (a) for two different cases (see Fig. 2 (a) and Fig. 2 (c)). The results are the following approximations

$$i(t) = 0.9287579710 - 0.9287579710 \exp(-100.2585t), \tag{20}$$

considering $V_1 = 10\text{V}$, $L_1 = 0.1\text{H}$, $R_1 = 104\,\Omega$ and $A = 0\text{A}$; and

$$i(t) = 0.0002327895521 + 0.0005672104479 \exp(-40000.02585t). \tag{21}$$

considering $V_1 = 10\text{V}$, $L_1 = 1\text{H}$, $R = 40\,\text{k}\Omega$ and $A = 0.8\text{E-3A}$.

From Fig. 2 (a) and Fig. 2 (c) can be seen that the exact solution (2) and approximations ((20) and (21)) are very similar and exhibit a typical asymptotic behaviour for this type of circuit. Besides, in general terms, the present solution encompass good accuracy in the range of amperes down to micro-amperes with an acceptable relative error margin (see relative error from Fig. 2 (b) and Fig. 2 (d)). The worst case for the relative error is found in Fig. 2 (d), which has a maximum value of 0.02. Therefore, in case that a

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better precision is required, (10) should be evaluated, giving as result $B = 0.0002375365310$ (see Fig. 2 (e)); thus, the value for the current is

$$i(t) = 0.000237536531 + 0.000562463468 \exp(-40108.8253662t). \tag{22}$$

Now, Fig. 2 (f) shows that the maximum relative error of (22) has been reduced to 0.00045 against the relative error 0.02 obtained by (21). The DC relative error value represented by a steady state term in (20) and (21) is 7E-6 and 2E-2, respectively.

Considering that the expansion point for the current was $B = 1$ (see Fig. 2 (a) and Fig. 2 (c)), becomes clear that as i_{dc} moves away from that value, the relative error increases. Nevertheless, for this case, the range of practical values around the expansion point is quite wide because having Taylor expansion at $B = 1$, an acceptable relative error was reached.

Also, for (22) the steady state absolute relative error is 1E-11, meaning that using (10) to calculate expansion point B helps to increase accuracy, where the exact DC value was calculated using (11).

Solving (18) and (19) using these parameter values $V_1 = 15$V,$V_2 = 7$V, $L_1 = 0.1$H, $L_2 = 0.01$H, $R_1 = 30\,\Omega$, $R_2 = 60\,\Omega$, $A_1 = 0.2$A and $A_2 = 0$ A, we obtain the following transients

$$\begin{aligned} i_1(t) = {} & 0.1166513312 \exp(-6005.170115t) \\ & -0.2766226475 \exp(-300.2583830t) + 0.3599713163, \end{aligned} \tag{23}$$

and

$$\begin{aligned} i_2(t) = {} & -0.1166460458 \exp(-6005.170115t) \\ & -0.0001253995687 \exp(-300.2583830t) + 0.1167714453. \end{aligned} \tag{24}$$

At steady state for (23) and (24), the relative error is 1.4E-4 and 3.2E-3, respectively. Besides, the relative error of the transient for currents $i_1(t)$ and $i_2(t)$ shown in Fig. 3 (b) and Fig. 3 (d), is quite low.

In Fig. 3 (a) and Fig. 3 (c) are shown currents $i_1(t)$ and $i_2(t)$ for the exact transient (16) and (17)), and the approximate transient (23) and (24), respectively. The expansion point for both cases is $B_1 = 1$ y $B_2 = 1$, providing a low relative error (see Fig. 3 (b) and Fig. 3 (d)); even though the currents for the steady state are $i_1 = 0.3$A and $i_2 = 0.12$A.

5 Discussion

Approximate solutions for DC and transient analysis for circuits in Fig. 1 (a), Fig. 1 (c), and Fig. 1 (d) showed low relative errors, even for regions away from the Taylor series expansion point. Also, it was shown how to reduce, significantly, the relative error by expanding Taylor series for the current to a value closer (10) to the exact (11) (see Fig. 2 (e) and Fig. 2 (f)). Besides, the approximate diode model (12) (see Fig. 1 (b)) has the advantage that can be used to explicitly formulate the currents for circuits containing several meshes that includes diodes; while (11) does not allow performing general analyses to larger circuits with diodes. In a future research, we will extend

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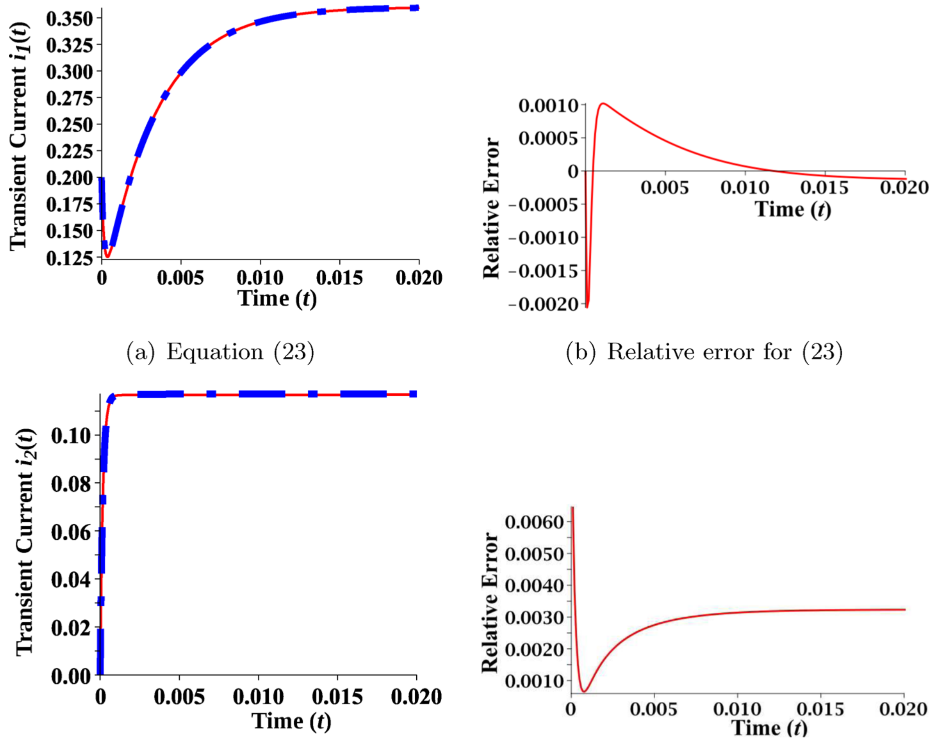

(a) Equation (23) (b) Relative error for (23)

(c) Equation (24) (d) Relative error for (24)

Fig. 3. Numerical solution of i_1 (16) and i_2 (17) (dash-dot) against approximate solutions (continuous line) (23) and (24), respectively. Time is in seconds and current in Amperes.

the present work to analyse circuits with more devices like capacitors, current sources, among others.

For a given diode circuit, if choosing arbitrarily expansion points and evaluating numerically the expressions for the currents, we obtain non-satisfactory results (in terms of accuracy); it is possible to use the results as the new expansion points. This iterative procedure may produce a lower relative error.

Model (12) should be improved to overcome the restriction $V_{th} \geq V_{DE}$, in order to perform symbolic analysis for VLSI circuits composed by bipolar transistors using the Ebers-Moll model.

Finally, analytic proposed approximations for DC and transient may be used to perform more complex analysis like power consumption at the transient, operating point sensibility, temperature effects analysis, symbolic small signal analysis [4], among others.

6 Conclusions

This work showed that by using power series is possible to establish an approximate circuital equivalent for the diode. Therefore, explicit expressions can be obtained for transient and DC regimen, for electrical variables emanating from circuits containing: diodes, resistors, inductors, and voltage sources. The main advantages for the obtained results are the low relative error and the analytic expressions accurately reproduce the behaviour of the

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diodes in a circuit; thus, designers are capable to analyse, quantitatively, the performance of the circuits under study.

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