LETTER

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Noncoherency correction algorithm for removing spectral leakage in ADC spectral test

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Abstract: A noncoherency correction algorithm is proposed to remove the spectral leakage caused by noncoherent sampling in ADC spectral test. The coherent data is reconstructed from the original data by an additional FFT and only a few simple time domain operations. Then accurate spectral testing results can be obtained by performing normal FFT on the reconstructed data. Compared with windowing techniques, the proposed method can acquire better spectral testing accuracy without any prior knowledge. Theoretical analysis, simulation and experimental results demonstrate that the developed method can achieve the estimation accuracy comparable to that of coherent sampling method but without requiring coherent sampling.

Keywords: analog-to-digital converter, noncoherent sampling, spectral analysis, spectral leakage

Classification: Integrated circuits

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1 Introduction

Analog-to-Digital converters (ADCs) are among the world's largest volume mixed-signal integrated circuit (IC) products [1]. Spectral test of high performance ADCs is a well-known important and challenging problem facing the semiconductor industry [2, 3]. Discrete Fourier transform (DFT) or fast Fourier transform (FFT) is the most prevalent approach for ADC spectral test. However, there is an implicit assumption of periodicity when using FFT. If the periodicity of the data doesn't match the periodicity of the FFT, the results will be corrupted by the spectral leakage [4, 5].

To suppress spectral leakage, both IEEE standard and best practice in industry are to require coherent sampling whenever possible [2, 6, 7]. Coherent sampling provides accurate and repeatable test result while avoiding the more complex signal processing stages. However, coherent sampling requires expensive instruments, such as high performance signal generators and frequency synthesizers, which will increase the test cost significantly. Furthermore, for state-of-the-art ADCs, coherent sampling is challenging to achieve since an extremely stringent synchronization between excitation signal sources and ADC sampling clock is required, to ensure that an integer number of cycles exist in the predefined sample window.

In the past decades, many attempts have been made to suppress spectral leakage and reduce test cost, such as sinewave fitting technique [8, 9], singular value decomposition [10], 2-D FFT [11], filter banks [12], etc. These methods are accurate, but they are computationally inefficient. Another two methods that are widely used are windowing techniques [13, 14, 15] and the windowed interpolated DFT techniques [16, 17]. However, the selection of window functions involves the trade-off of several factors, such as main lobe width, maximum side lobe level, side lobe roll-off rate, and equivalent noise bandwidth, which requires prior

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knowledge, and the amount of leakage suppression is limited [13]. In particular, the two methods fail to obtain correct results in some applications. For example, when the spectral component is lower than side lobes or masked by main lobe, neither of the two methods can detect it.

In this letter, a noncoherency correction algorithm is proposed for removing the spectral leakage during ADC spectral test. In the proposed method, the noncoherent fundamental component in the original data is first identified and replaced by a coherent sinusoidal component in order to reconstruct a coherent data. This procedure is called noncoherency correction. Then the normal FFT is performed on the reconstructed data. As a result, the spectral leakage is removed and accurate spectral results can be obtained only at the cost of an additional FFT with only a few simple time domain operations. No assumed periodicity over the observation time is needed, hence the stringent requirement of coherent sampling is eliminated. In comparison with windowing techniques, the presented method can achieve better spectral testing accuracy without any prior knowledge. Therefore, the developed method offers a low-cost approach for high resolution ADC spectral test.

2 Mechanism of spectral leakage in ADC test

In this section, the mechanism of spectral leakage in ADC test is discussed in detail, which provides a theoretical foundation for the proposed non-coherency correction algorithm.

In mixed-signal integrated circuits test, ADCs under test are generally weakly nonlinear systems. Therefore, when applying a pure sinewave to the ADC under test, the ADC output will be a low-distortion sinusoidal signal as in (1).

$$x(t) = A\cos(2\pi f_{in}t + \varphi) + \sum_{h=2}^{H} A_h \cos(2\pi h f_{in}t + \varphi_h) + w(t) \tag{1}$$

where A, f_{in} and φ are the amplitude, frequency and initial phase of the fundamental component respectively, and $A \approx 1$, $\varphi \in [0, 2\pi)$. H is the number of harmonics. A_h and φ_h are the amplitude and initial phase of the h-th harmonic respectively, and $\varphi_h \in [0, 2\pi)$ for all $2 \leq \mathrm{h} \leq H$. The $w(t)$ is the white Gaussian noise with zero mean and variance σ_w^2. And these parameters satisfy

$$\sum_{h=2}^{H} A_h^2 \ll A^2 \tag{2}$$

$$\left| \sum_{h=2}^{H} A_h \cos(2\pi h f_{in}t + \varphi_h) + w(t) \right| \ll A \tag{3}$$

$$\sigma_w^2 \ll A^2 \tag{4}$$

Notice that in (1), the ADC output is intentionally expressed as continuous-time signal, but it will be converted into discrete-time signal in the next paragraph. Furthermore, the gain error and offset of ADC are assumed to have been calibrated.

To convert the continuous-time signal $x(t)$ into discrete-time signal, we suppose that f_s is the sampling clock frequency, M is the data record length, J is the number of periods of the fundamental component in the whole data record. Then the four parameters f_{in}, f_s, J and M are related by

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$$\frac{f_{in}}{f_s} = \frac{J}{M} = \frac{J_{\text{int}} + \Delta}{M} \tag{5}$$

where J_{int} and Δ are the integer part and fractional part of J respectively, and $\Delta \in (-0.5, 0.5]$. When Δ equals to zero, the whole data is coherent, otherwise it is noncoherent. Then the corresponding discrete time signal of $x(t)$, i.e. the ADC output sequence $x[k]$, can be given by

$$\begin{aligned} x[k] &= A\cos\left(2\pi\frac{J_{\text{int}} + \Delta}{M}k + \varphi\right) + \sum_{h=2}^{H} A_h \cos\left(\frac{2\pi hJ}{M}k + \varphi_h\right) + w(k) \\ &= x_1[k] + \sum_{h=2}^{H} x_h[k] + x_w[k], k = 0, 1, \ldots, M-1 \end{aligned} \tag{6}$$

where $x_1[k]$, $x_h[k]$ and $x_w[k]$ are the fundamental, the *h-th* harmonic and noise, respectively. The task of spectral test is to estimate all frequency components from the spectrum, such as harmonic tones, spurious tones, total distortion level and the noise level. For simplicity, we only discuss the computation of the spectral parameters such as SFDR (spurious-free dynamic range), THD (total harmonic distortion), SNR (signal-to-noise ratio), SINAD (signal-to-noise-and-distortion ratio) and ENOB (effective number of bits), etc [2].

Performing DFT on $x[k]$, we have

$$\begin{aligned} X[n] &= DFT(x[k]) = DFT\left(x_1[k] + \sum_{h=2}^{H} x_h[k] + x_w[k]\right) \\ &= DFT(x_1[k]) + \sum_{h=2}^{H} DFT(x_h[k]) + DFT(x_w[k]) \\ &= X_1[n] + \sum_{h=2}^{H} X_h[n] + X_w[n], \end{aligned} \tag{7}$$

where $X_1(n)$, $X_h(n)$ and $X_w(n)$ denote the DFT of $x_1[k]$, $x_h[k]$ and $x_w[k]$, respectively. Using DFT formula

$$X[n] = \frac{1}{M}\sum_{k=0}^{M-1} x[k]e^{-j\frac{2\pi}{M}nk}, \quad n = 0, 1, \ldots, M-1, \tag{8}$$

then the DFT of the fundamental, $X_1[n]$ can be derived as follows

$$\begin{aligned} X_1[n] &= \frac{A}{2}[e^{j\varphi}\delta(n - J_{\text{int}}) + e^{-j\varphi}\delta(n - M + J_{\text{int}})] \\ &\quad + \frac{A}{2M}\left[e^{j\varphi}\sum_{k=0}^{M-1}(e^{j\frac{2\pi}{M}\Delta k} - 1)e^{j\frac{2\pi}{M}(J_{\text{int}}k - nk)}\right. \\ &\quad \left. + e^{-j\varphi}\sum_{k=0}^{M-1}(e^{-j\frac{2\pi}{M}\Delta k} - 1)e^{-j\frac{2\pi}{M}(J_{\text{int}}k + nk)}\right] \\ &= ideal\_term(1) + skirt(1). \end{aligned} \tag{9}$$

Note that in (8), $1/M$ is a normalization factor. In (9), $\delta(n)$ is the unit impulse function which is equal to unit at $n = 0$ and is zero elsewhere. The first term, ideal_term(1), corresponds to two separate spectral lines located at the $(J_{\text{int}} + 1)$-*th* and $(M - J_{\text{int}} + 1)$-*th* frequency bins in the spectrum. The second term $skirt(1)$ is

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used to denote the spectral leakage due to the non-coherency of the fundamental. The word "skirt" is chosen since the shape of the spectral leakage visually looks like a skirt. The *skirt*(1) will be nonzero when Δ is nonzero, i.e., the data is noncoherent, which is the reason why the second term leads to spectral leakage. Another important observation is that (9) indicates that the spectral leakage term *skirt*(1) is proportional to the amplitude of fundamental A.

Similarly, the DFT of the h-order harmonic, $X_h[n]$, can be derived to be

$$\begin{aligned} X_h[n] &= \frac{A_h}{2}[e^{j\varphi_h}\delta(n - J_{\text{int}\_h}) + e^{-j\varphi_h}\delta(n - M + J_{\text{int}\_h})] \\ &+ \frac{A_h}{2M}\left[e^{j\varphi_h}\sum_{k=0}^{M-1}(e^{j\frac{2\pi}{M}k\Delta_h} - 1)e^{j\frac{2\pi}{M}(J_{\text{int}\_h}k - nk)}\right. \\ &\left. + e^{-j\varphi_h}\sum_{k=0}^{M-1}(e^{-j\frac{2\pi}{M}k\Delta_h} - 1)e^{-j\frac{2\pi}{M}(J_{\text{int}\_h}k + nk)}\right] \\ &= ideal\_term(h) + skirt(h) \end{aligned} \tag{10}$$

where J_{int_h} and Δ_h equal to $\text{mod}[round(h \cdot J), M/2]$ and $h \cdot J - round(h \cdot J)$, respectively. Here $\text{mod}(x_1, x_2)$ returns the remainder of x_1 divided by x_2, $\text{round}(x)$ rounds the element x to the nearest integer. Note that for those harmonics beyond Nyquist interval, their corresponding aliased frequencies in Nyquist interval are considered. Similarly, in (10), the first term ideal_term(h) corresponds to two separate spectral lines located at the $(J_{\text{int}\_h} + 1)$-*th* and $(M - J_{\text{int}} + 1)$-*th* frequency bins in the spectrum, respectively. The second term *skirt*(h) is used to denote the spectral leakage due to the non-coherency of the *h-th* harmonic. The *skirt*(h) will be nonzero when Δ_h is nonzero. Similarly, the spectral leakage term *skirt*(h) is proportional to the amplitude of the h-th harmonic A_h. In (7), substituting $X_1[n]$ and $X_h(n)$ with (9) and (10) gives

$$\begin{aligned} X[n] &= ideal\_term(1) + \sum_{h=2}^{H} ideal\_term(h) \\ &+ skirt(1) + \sum_{h=2}^{H} skirt(h) + X_w[n]. \end{aligned} \tag{11}$$

In realistic ADC spectral testing, noncoherent sampling is frequently encountered and usually results in erroneous spectrum estimation. The goal of this paper is trying to achieve accurate spectral testing results from noncoherent data. In the case of noncoherent sampling, Δ is nonzero, so Δ_h is possibly nonzero as well. Consequently, *skirt*(1) is nonzero, and *skirt*(h) is possibly nonzero as well.

As mentioned above, *skirt*(1) is proportional to the amplitude of fundamental A, similarly *skirt*(h) is proportional to the amplitude of the h-th harmonic A_h $(h = 2, 3, \ldots, H)$ as well. Based on this observation, using (2) and (3), it can be concluded that

$$\sum_{h=2}^{H} skirt(h) \ll skirt(1) \tag{12}$$

This means the noncoherency in the fundamental component is the major contributor to the spectral leakage, and the contribution of the noncoherency in the

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2^{nd} *and higher order harmonics are negligible.* Therefore, $X[n]$ can be approximated as

$$X[n] \approx ideal\_term(1) + \sum_{h=2}^{H} ideal\_term(h) + skirt(1) + X_w[n] \tag{13}$$

The $skirt(1)$ could be so large that all harmonic distortion components are masked in the spectrum, and hence the true spectrum of the signal can not be estimated correctly. *To remove the spectral leakage, we only need to remove skirt(1).* Notice that equations (12) and (13) are the theoretical foundation of the noncoherency correction method which will be introduced in next section. Furthermore, it should be mentioned that, equation (9) also shows that the $skirt(1)$ is a linear combination of M different frequency terms and therefore has M independent basis functions. As a result, the $skirt(1)$ can exhibit different shapes. This fact indicates that any attempts to identify the $skirt(1)$ in frequency domain with a reduced number of basis functions will be ineffectual.

3 Noncoherency correction algorithm

Instead of identifying and removing the spectral leakage in frequency domain, we try to solve this challenging problem indirectly in time domain. In the proposed method, the fundamental component is first identified from the original data $x[k]$. Then a new coherent data $\hat{x}[k]$ is reconstructed by replacing the identified noncoherent fundamental component with a new coherent sinusoidal component. This new component has the same amplitude and phase as the identified fundamental but its frequency is modified slightly to make it coherent with the sampling clock. In mathematics, $\hat{x}[k]$ is obtained through subtracting the original data $x[k]$ by the identified sinusoidal component and adding a new sinusoidal component with the same A, J_{int}, and φ, but Δ is set to be zero, as shown in (14),

$$\hat{x}[k] = x[k] - A\cos\left(2\pi\frac{J_{\text{int}} + \Delta}{M}k + \varphi\right) + A\cos\left(2\pi\frac{J_{\text{int}}}{M}k + \varphi\right) \tag{14}$$

The above procedure is called noncoherency correction. It should be mentioned again, the idea of noncoherency correction is proposed based on equations (12) and (13). After noncoherency correction, the usual FFT spectral analysis is performed on $\hat{x}[k]$. As a result, the spectral leakage caused by noncoherency of the fundamental component will be removed, and the true spectrum of the signal will be recovered.

Note that, in the process of data reconstruction, the 2^{nd} and higher order harmonics remain unchanged. Therefore, the leakage effects of these harmonics are not removed. However, they will not affect the spectral estimation because, as we mentioned in (12), the leakage effects from harmonics are too small compared with the leakage caused by the fundamental noncoherency.

With noncoherency correction, the problem for preventing spectral leakage is simplified to identify the fundamental component from the original data. In other words, the following task is to estimate the three parameters f_{in}, A and φ. Since $J = Mf_{in}/f_s = J_{\text{int}} + \Delta$, to identify f_{in} is equivalent to estimate J_{int} and Δ. Because the DFT of real-valued signals has the property of Hermitian symmetry, J_{int} can be

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estimated by searching the peak in the half DFT magnitude spectrum, excluding the DC component $X(0)$, as shown in (15).

$$\hat{J}_{\text{int}} = \arg\max\{|X[n]|\}, \ \ 1 \leq \text{n} \leq M/2 \tag{15}$$

where argmax$\{f(x)\}$ denotes the value of the argument x for which $f(x)$ attains the maximum value. Next, we'll discuss how to estimate Δ. According to (6), we have

$$\begin{aligned} x[k] &= A\cos\left(2\pi\frac{J_{\text{int}}+\Delta}{M}k+\varphi\right)+\sum_{h=2}^{H}x_h[k]+x_w[k] \\ &= \frac{A}{2}e^{j(2\pi\frac{J_{\text{int}}+\Delta}{M}k+\varphi)}+\frac{A}{2}e^{-j(2\pi\frac{J_{\text{int}}+\Delta}{M}k+\varphi)}+\sum_{h=2}^{H}x_h[k]+x_w[k]. \end{aligned} \tag{16}$$

To derive the estimator for Δ, let's consider the three DFT samples $X[J_{\text{int}}-1]$, $X[J_{\text{int}}]$, $X[J_{\text{int}}+1]$ around the fundamental bin. It can be shown that for low-distortion sinusoidal signal, the contributions of the last two terms in (16) in these three DFT samples can be ignored. That is, in (7), $X[n] \approx X_1[n]$ for $n = J_{\text{int}}-1, J_{\text{int}}$ and $J_{\text{int}}+1$. Then, the following equation can be derived to estimate Δ [18].

$$\hat{\Delta} = \frac{\tan(\pi/M)}{\pi/M}\text{Re}\left\{\frac{X[J_{\text{int}}-1]-X[J_{\text{int}}+1]}{2X[J_{\text{int}}]-X[J_{\text{int}}-1]-X[J_{\text{int}}+1]}\right\} \tag{17}$$

where $Re\{x\}$ represents the real part of x.

After J_{int} and Δ have been identified, the least square method (LSM) is then used to estimate A and φ. The procedure of the proposed method can be illustrated by the flowchart shown in Fig. 1.

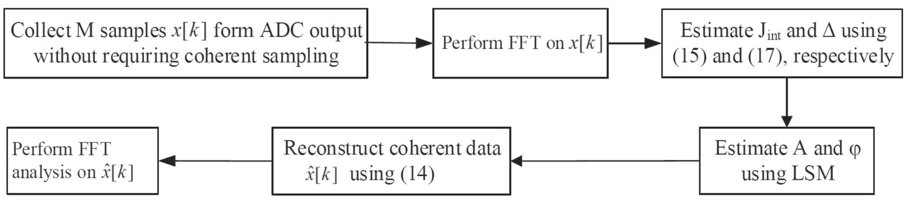

Fig. 1. Flowchart of the proposed method.

The algorithm presented above is computationally efficient. The dominant part of the computation is for performing the two FFTs in steps 2 and 6. As a great advantage, the choice of data length M in the proposed method is very flexible, as long as the FFT processing gain is large enough [7]. It's well-known that FFT is computationally efficient when the data length M is a power of two. Therefore the computational complexity can be easily bounded to O($M\log 2M$), which is the same as that of coherent sampling method [2]. Note that, in step 1, i.e. data acquisition stage, coherent sampling is not required, hence the test can be implemented with inexpensive instruments. Consequently, the proposed method is cost-effective.

4 Simulation and experimental results

In this section, a simulation example is first presented to validate the proposed algorithm. Then the proposed method is compared with Blackman-Harris (B-H)

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window [13]. The reliability of the proposed method is also investigated. Finally experimental data from a commercial 16-bit ADC are used to verify the effectiveness of the algorithm.

Since ADC spectral test is essentially to analyze the spectrum of a low-distortion sinusoidal signal. Therefore, in MATLAB simulation, ADC output is modeled as a low-distortion sinusoidal signal quantized by an ideal N-bit digitizer. For the tested signal, the amplitude of the fundamental is set to be slightly smaller than half of the digitizer input range in order to avoid clipping, even in the presence of noise; the amplitudes of the 2^{nd} and higher order harmonics are on the order of one LSB (least significant bit) in order to characterize the nonlinearity of ADC; and the standard deviation of the white noise is within half LSB. From these setup, the true spectral parameters of the tested signal can be calculated.

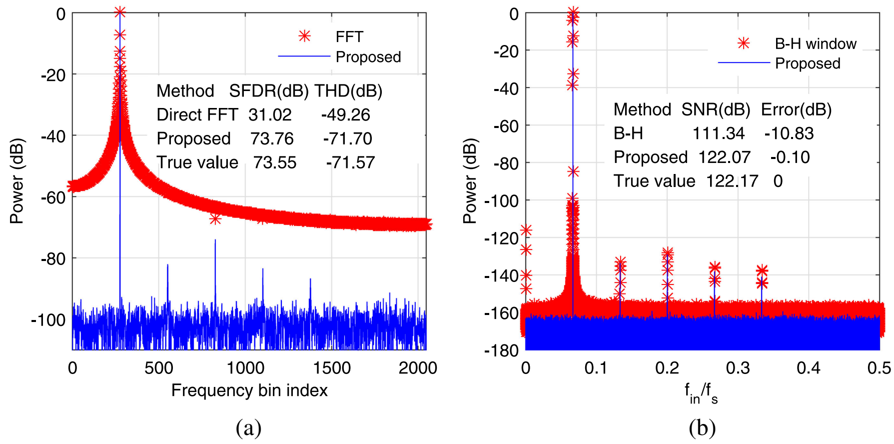

Fig. 2. (a) Spectra obtained by direct FFT and the proposed method (digitizer resolution: 12, RMS noise: 0.5 LSB) (b) spectra obtained by the proposed method and B-H window (digitizer resolution: 21, RMS noise: 0.5 LSB)

Fig. 2(a) illustrates the estimated spectra obtained by performing FFT directly and the proposed method on a noncoherent data where $J = 275.3$, $M = 4096$. When FFT is employed directly, all harmonics are masked since there is a large spectral leakage around the fundamental bin. However, after the data is processed by the proposed method, the spectral leakage is eliminated and all harmonics are clearly visible. The estimated SFDR error and THD error are 0.21 dB and −0.13 dB respectively, which are very small. This shows that the proposed method can estimate the spectrum accurately.

The proposed method is also compared with windowing techniques. As discussed in Section I, the selection of window functions involves the compromise of several factors, including main lobe width, maximum side lobe level, side lobe roll-off rate, and equivalent noise bandwidth, which requires prior knowledge, and the amount of leakage suppression is limited. To evaluate ADC spectral performance accurately, a rule of thumb is to select a window such that the power of secondary lobes of the selected window is less than ADC noise floor by some margin. Otherwise, erroneous results will be obtained. Nevertheless, in the proposed method, there is no need to consider these issues. The following example

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will illustrate this problem. The minimum 4-term B-H window is well-known to have a very low side lobe level (−92 dB) in the window family and able to analyze medium accuracy signal, e.g. 13-bit [13]. However, this window is not adequate to resolve the spectrum for those signal/ADC whose accuracy is above 16-bit. To clarify this issue, both the proposed method and 4-term B-H window are applied on a high purity signal with 21-bit accuracy. As illustrated in Fig. 2(b), the proposed method achieves a lower FFT noise floor. Specifically, the SNR in the proposed method is 122.07 dB, which is close to the true value 122.17 dB, while in B-H window it is 111.34 dB, deviating from the true value by about 11 dB. This shows the proposed method has the ability to analyze high purity signal, but the B-H window does not.

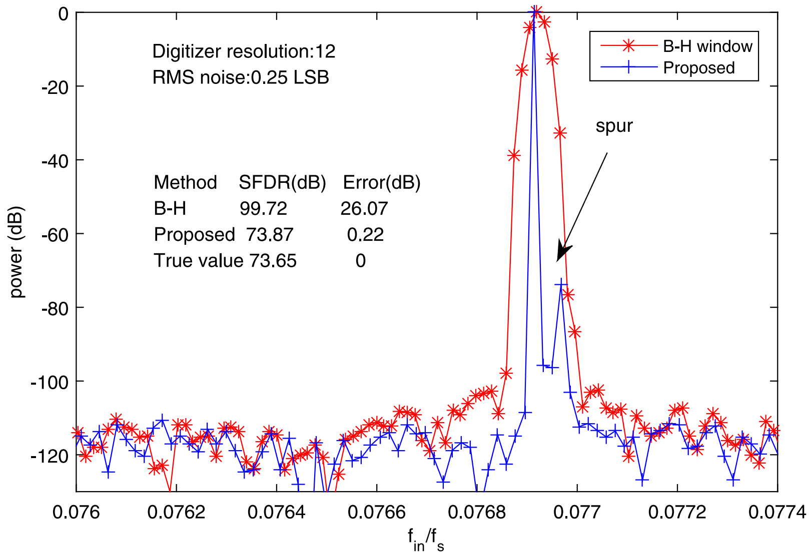

Fig. 3. Spectra around the fundamental bin

Another disadvantage of windowing techniques is that they will fail to obtain correct results when the spectral component is lower than side lobes or masked by main lobe, no matter whether the power of secondary lobes of the selected window is less than ADC noise floor or not. Fig. 3 gives an example where a spur occurs near the fundamental bin. It can be seen that the minimum 4-term B-H window can not detect the spur although the accuracy of the tested signal is only 12 bit, whereas the proposed method has the capability to detect it since the leakage is completely removed by the noncoherency correction. This example shows that the proposed method can achieve better spectral resolution than B-H window.

To investigate the reliability of the proposed method with respect to signal frequency, 1,000 random runs were conducted. In the simulation, M = 4096, the resolution of the ideal digitizer is 12. And f_{in} is uniformly distributed in $[0.01 f_s, 0.49 f_s]$ where the extremely low frequencies in the Nyquist interval are excluded because the parameter identification accuracy is difficult to maintain in these frequencies [19].

Simulation results from 1,000 runs show that there are similar reliability patterns in SFDR, SNR, THD, SINAD, ENOB etc. Due to space limitation, here

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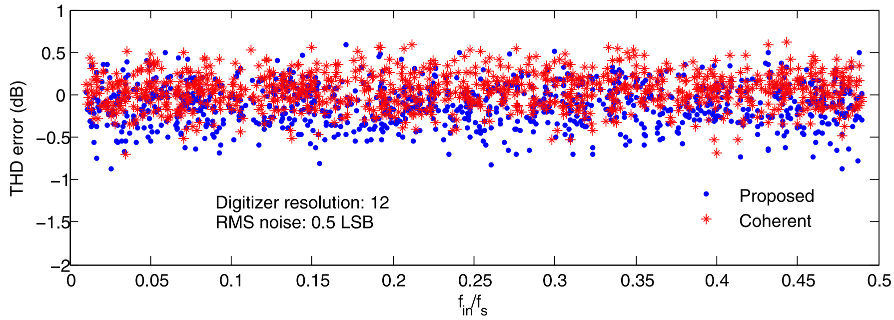

Fig. 4. THD errors in 1,000 runs

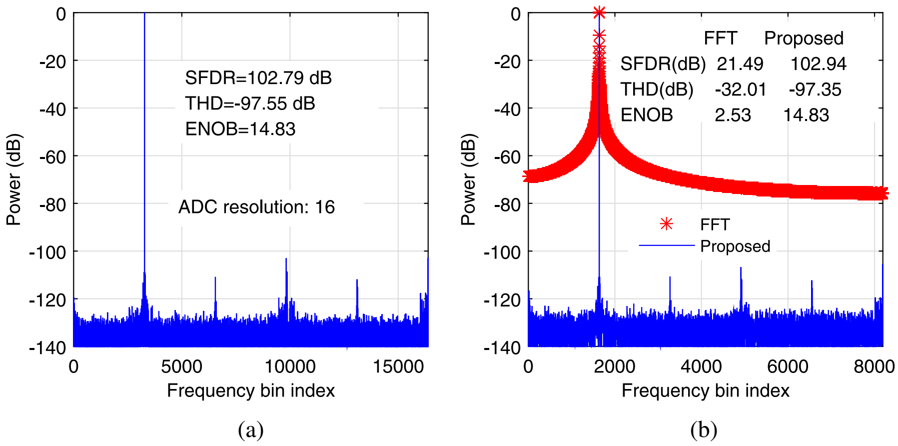

Fig. 5. (a) Spectrum obtained from 32768 coherent samples (b) spectra obtained by direct FFT and the proposed method from 16384 noncoherent samples

we only use THD to present the robustness patterns of the proposed method. Fig. 4 illustrates that the THD errors in the proposed method are bounded within ±1 dB in the 1,000 runs, which is the same as coherent sampling method. This indicates that the proposed method is reliable which can achieve the estimation accuracy comparable to that of the coherent sampling method for any signal frequency.

Next, the experimental data are adopted, which come from a commercial 16-bit ADC output, to evaluate the effectiveness of the proposed method. The collected raw data consist of 32768 coherent samples. And the corresponding spectrum can be obtained by performing FFT directly on the whole raw data, as shown in Fig. 5(a). In order to verify the capability of the proposed method in analyzing the noncoherent data, we will use a truncated subset from the raw data for the spectral analysis. Fig. 5(b) shows the spectra obtained by direct FFT and the proposed method when only 16384 consecutive samples of the raw data are applied.

For the direct FFT, the spectral leakage completely masks all harmonic components. However, the spectrum obtained by the proposed method still exhibits all the spectral contents. The experimental results show that the proposed method can achieve the estimation accuracy comparable to that of the coherent sampling method. Similar process is also applied on a truncated data containing 8192 consecutive samples, and the proposed method works great as well. In fact,

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accurate spectrum estimation can still be obtained when continuing reducing the truncated data length (M) as long as it has enough FFT processing gain [7].

5 Conclusion

A noncoherency correction method is proposed for removing the spectral leakage caused by noncoherent sampling during ADC spectral test. The proposed method is computationally efficient since only an additional FFT and a few simple time domain operations are involved. Simulation and experimental results show the presented method is well suited for high precision spectral analysis and can achieve better spectral testing accuracy than windowing techniques. In addition, it can obtain the estimation accuracy comparable to that of coherent sampling method. Since the stringent requirement of coherent sampling is eliminated, the developed method offers an alternative approach for testing precision ADC with inexpensive instruments, hence the test cost can be reduced greatly.

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