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An analytical model of code-tracking performance for next-generation GNSS modulations

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Abstract: Precorrelation bandlimiting, sampling, and quantization (BSQ) are three fundamental functions of receivers for global navigation satellite system (GNSS) signal processing. Analytical models have been developed to evaluate the implementation losses due to different combinations of these parameters for legacy modulations such as BPSK. However, the performance metric was losses of effective carrier-to-noise ratio in dB, instead of performance degradation of a delay-locked loop (DLL) used for code-tracking. This paper proposes an analytical model to predict the DLL performance with the loss of RMS tracking error in meters as the performance metrics. The signal model & assumptions and canonical form of DLL are first introduced. Then, analytical model is derived for DLL tracking accuracy and the performance metrics, tracking loss is defined. Finally, simulation results of two next-generation modulations for GPS and Galileo, namely CBOC and TMBOC, are presented.

Keywords: BSQ, bandlimiting, quantization, sampling, DLL

Classification: Electronic instrumentation and control

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1 Introduction

There has long been interest in studying the joint effect of precorrelation filtering, sampling, and quantization in GNSS receivers. Theoretical, numerical and experimental results have been developed for decades, with initial emphasis on 1-bit quantization [1] and infinite precorrelation bandwidth through recent investigations on the combined effects of bandlimiting, sampling and quantization (BSQ) [2]. However, the results in [1, 2] are for receivers using sampling frequencies that are commensurate with the desired signal chipping rate, i.e. with an integer number of samples per chip. This is not the case since contemporary receivers use incommensurate sampling frequency values. Besides, in [2] sampling rates are constrained to Nyquist and twice Nyquist rate. In [3, 4] an analytical model for evaluating receiver implementation losses is proposed in which incommensurate sampling and arbitrary number of output levels of quantizer are taken into consideration. Literature closely related to our work includes [5], in which the joint effects of front-end filtering and quantization are deeply studied but the effect of sampling is neglected.

This paper proposes a generic analytical model to predict DLL tracking accuracy when the combined effects of BSQ are studied with incommensurate sampling and arbitrary number of output levels of quantizer.

The paper is structured as follows. The second section defines the problem being analyzed, introduces the notation and assumptions that are employed, and introduces the canonical form of a DLL. The signal is assumed to be known (to the receiver) and is buried in additive Gaussian white noise (AWGN) and non-white interference with relatively flat spectra within the precorrelation bandwidth. Dot Product (DP) is chosen as the discriminator for its popularity in GNSS receiver. The third section presents the analytical expression of the DLL tracking accuracy without and with BSQ. The DLL tracking loss is defined as the difference of these two parameters by using SNR losses model proposed in [3]. The fourth section describes the computer simulation approach to validating the aforementioned analytical model. The SNR losses computed using method mentioned in [3] very closely matched the Monte Carlo simulation results with an average error below 0.01 dB. This level of accuracy is sufficient for the code-tracking precision degradation eval-

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uation.

It is proved that wider precorrelation bandwidth tends to lower the loss and decrease the DLL tracking error while fewer bits of quantization tend to increase the loss and the code-tracking error. During these simulations, different sampling frequencies are assumed and it is shown that within a proper frequency range, the higher the sampling frequency, the higher the code-tracking accuracy. The comparison between legacy and next-generation modulations shows that the latter outperforms the former and therefore the new modulations are extremely suitable for high-accuracy and safety-of-life applications.

2 Problem formulation

2.1 Signal model

Complex baseband representations are used for the received data being analyzed. It is assumed that the received data is composed of GNSS signal $s(t)$, noise $n(t)$ and interference $l(t)$:

$$x(t) = e^{i\theta} s(t - t_0) + n(t) + l(t). \tag{1}$$

where t_0 is the time of arrival and θ the carrier phase. The signal is unaffected by other channel effects such as multipath that introduce distortion correlated to the signal. The noise and interference are zero-mean, uncorrelated from the signal, wide-sense stationary and circularly symmetric Gaussian random processes. The noise is white with power spectral density (PSD) N_0.

The receiver's front-end is approximated by a precorrelation ideal low-pass filter with two-sided bandwidth B_r. The signal's PSD is denoted by:

$$C_s G_s(f) = |S_{kT}(f)|^2 / T. \tag{2}$$

where T is the integration period, $G_s(f)$ is the normalized PSD and C_s is the signal carrier power, $C_s/N_0 = 40\,\text{dB-Hz}$:

$$\int_{-\infty}^{+\infty} G_s(f) df = 1. \tag{3}$$

The interference has power spectral density $C_l G_l(f)$ where $G_l(f)$ is the normalized PSD and C_l is the interference power:

$$\int_{-\infty}^{+\infty} G_l(f) df = 1. \tag{4}$$

It is convenient to work with the sum of noise and interference, denoted $w(t) = n(t) + l(t)$, whose PSD is:

$$G_w(f) = N_0 + C_l G_l(f). \tag{5}$$

2.2 Canonical form of DLL with a Dot Product discriminator

This paper evaluates precision of DLL that complies with the canonical form of DLL shown in Fig. 1 (a) [6, 7]. The received signal plus noise plus interference enter a time-of-arrival (TOA) estimator. The TOA estimator uses

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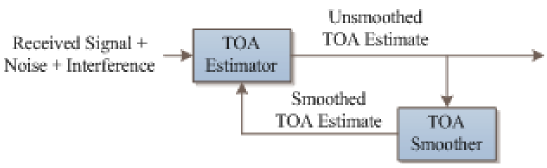
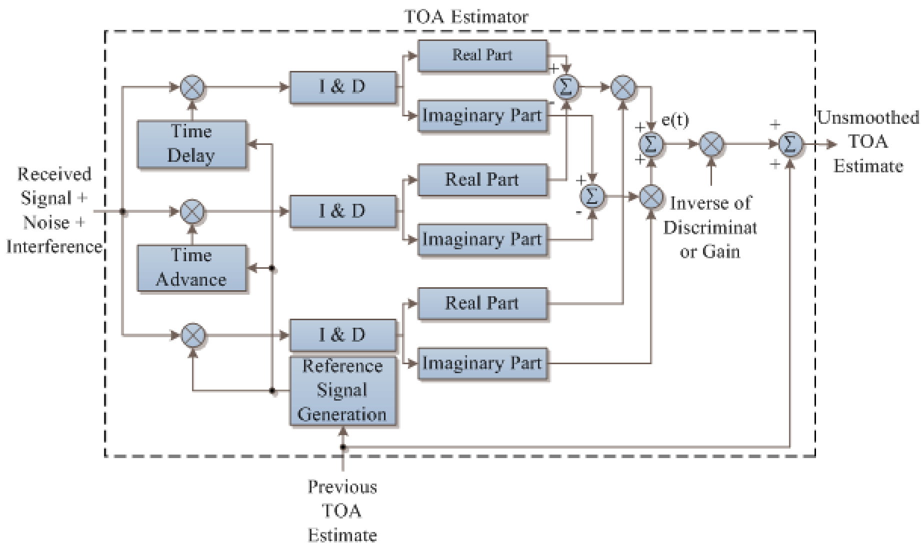
T seconds to produce an unsmoothed TOA estimate τ_k^u and this estimate is passed through a smoothing filter to form the smoothed TOA estimate τ_k^s, which is fed into the TOA estimator to update the error signal to generate the next unsmoothed TOA estimate. The TOA estimator in Fig. 1 (a) can be described in more details in Fig. 1 (b). The 'I & D' in Fig. 1 (b) means 'Integrate and Dump' operation in correlation process.



(a) Canonical form of DLL



(b) TOA estimator

Fig. 1. Canonical form of DLL.

3 Analytical results

3.1 DLL tracking error without BSQ losses

DLL tracking error variance without BSQ losses can be derived as:

$$\sigma_s^2 = \frac{B_L T(1-0.5B_L T)\int_{-B_r/2}^{B_r/2} G_s(f)G_w(f)\sin^2(\pi f\Delta)df}{(2\pi)^2 C_s\left(\int_{-B_r/2}^{B_r/2} fG_s(f)\sin(\pi f\Delta)df\right)^2} \times\left[1+\frac{1}{T\frac{C_s}{N_0}\int_{-B_r/2}^{B_r/2}G_s(f)df}+\frac{\int_{-B_r/2}^{B_r/2}G_s(f)G_l(f)df}{T\frac{C_s}{C_l}\left(\int_{-B_r/2}^{B_r/2}G_s(f)df\right)^2}\right]. \quad (6)$$

where $B_L = 1\,\text{Hz}$ is the one-sided equivalent bandwidth of the DLL and Δ is the early-late spacing.

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3.2 DLL tracking error with BSQ losses

DLL tracking error variance with BSQ losses is proposed as:

$$\sigma^2_{s,BSQ} = \frac{B_L T(1-0.5B_L T)\int_{-B_r/2}^{B_r/2} G_s(f)G_w(f)\sin^2(\pi f\Delta)df}{(2\pi)^2 C_s\left(\int_{-B_r/2}^{B_r/2} fG_s(f)\sin(\pi f\Delta)df\right)^2} \times\left[1+\left(\frac{1}{T\frac{C_s}{N_0}\int_{-B_r/2}^{B_r/2}G_s(f)df}+\frac{\int_{-B_r/2}^{B_r/2}G_s(f)G_l(f)df}{T\frac{C_s}{C_l}\left(\int_{-B_r/2}^{B_r/2}G_s(f)df\right)^2}\right)L_{SNR}\right]. \tag{7}$$

where L_{SNR} is loss of effective carrier-to-noise ratio [3]:

$$L_{SNR} = \frac{\int_{-B_T/2}^{B_T/2} G_s(f)df \Big/ \left(\bar{b}_1\left(\int_{-B_r/2}^{B_r/2}G_s(f)df\right)^2\right)}{N_0 \Big/ \left(T_s \sum_{n=-(M-1)}^{M-1} R_{\tilde{r}^q}[n]R_s[n]\right)}. \tag{8}$$

where $B_T = 30.69\,\text{MHz}$ is the transmit bandwidth of the GNSS signal, T_s is the sampling interval, M is the number of samples in each integration interval. We use 1 PRN code period $R_s[n]$ and $R_{\tilde{r}^q}[n]$ are autocorrelation function of locally generated signal and quantized received signal, respectively. $\bar{b}_1$ is a parameter selected to minimize signal power losses for a uniform quantizer with N output levels [3].

3.3 DLL tracking loss

The code-tracking loss is defined as:

$$L_{BSQ} = c\cdot(\sigma_{s,BSQ} - \sigma_s). \tag{9}$$

where c is the speed of light in vacuum.

4 Simulation results

In this section, simulation results are presented for legacy civil signal, BPSK(1) and next-generation civil signal, CBOC(6, 1, 1/11) and TMBOC(6, 1, 4/33). The precorrelation bandwidth is 16 MHz, number of output levels of the uniform quantizer is assumed as 2, 3, 4, 8, and 16. L_{BSQ} is computed using Eq. (6-9) and plotted versus different sampling frequencies in Fig. 2 and listed in Tab. I. Note that the bottom right subplot of Fig. 2 assumes a sampling frequency of twice the Nyquist rate, i.e. 32 MHz.

Simulation shows that in all cases, the code-tracking performance of TMBOC and CBOC signal surpasses that of BPSK, the legacy signal. This proves that with more power in high frequency component, TMBOC and CBOC *do* outperform BPSK, and this is one of the major reasons at their design phase. However, this advantage gradually diminishes when each of the three parameters, i.e. number of output levels of quantizer, sampling

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frequency, and precorrelation bandwidth, increases. This is because of two reasons. Firstly, more signal power is included in processing when precorrelation bandwidth increases. Secondly, the effective noise power spectral density decreases due to less aliasing between signal and noise occurs when higher sampling frequency or larger number of output levels of quantizers are used. In Tab. I, the loss value for TMBOC and CBOC slightly increases when sampling frequency changes from 28 to 32 MHz, this is because at some sampling frequencies, such modulations with non-negligible power on high-frequency sidelobes will have aliasing on the noise, which results in increased effective noise power spectral density.

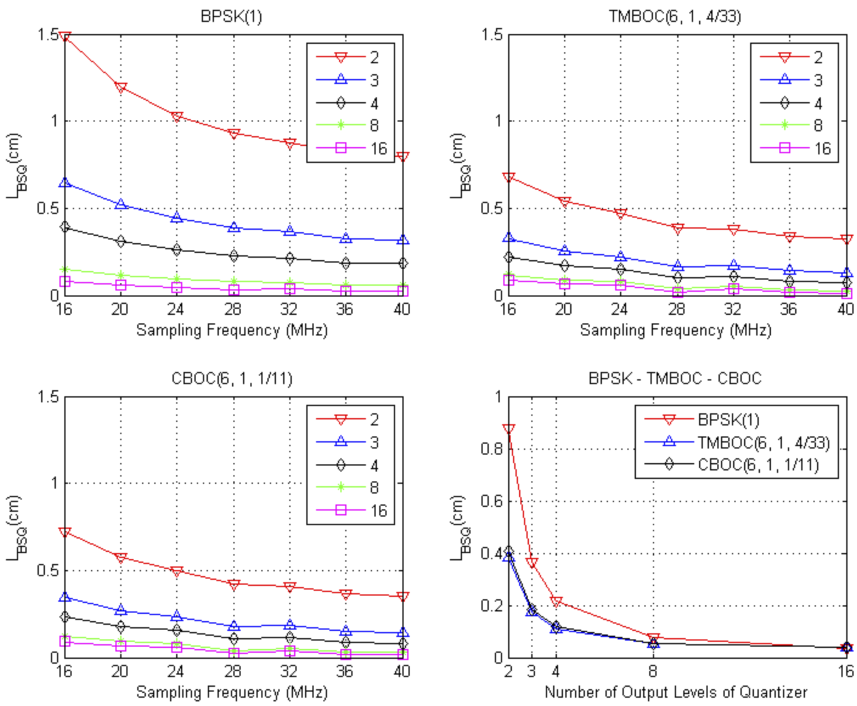

Fig. 2. DLL tracking degradation in cm due to BSQ effects for different modulations.

Table I. DLL tracking degradation in cm due to BSQ effects for different modulations ($N = 4$).

Modulation	Sampling Frequency, MHz						
	16	20	24	28	32	36	40
BPSK(1)	0.391	0.312	0.263	0.229	0.215	0.188	0.183
TMBOC (6, 1, 4/33)	0.222	0.171	0.149	0.100	0.112	0.083	0.076
CBOC (6, 1, 1/11)	0.232	0.179	0.154	0.108	0.117	0.088	0.083

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5 Conclusion

This paper proposes tracking loss for evaluating the BSQ effects on DLL tracking accuracy of a GNSS receiver. This is more direct than the traditional method using loss of effective carrier-to-noise ratio as performance metrics. The analytical model can accommodate arbitrary types of (legacy and next-generation) GNSS signal, noise and interference, arbitrary sampling frequency and arbitrary number of output levels of quantization. The simulation results show that CBOC and TMBOC modulations offer great advantages over BPSK when receiver DLL performance is a matter of significance. This makes these two signals extremely suitable for high-accuracy and safety-of-life applications.

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