

# An adaptive predistortion for power amplifier nonlinearity in the presence of measurement noise

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**Abstract:** In this paper, we propose a robust adaptive predistortion (PD) scheme based on a hybrid direct learning (HDL) structure for mitigating the effects of measurement noise derived from the feedback path of power amplifier (PA). In particular, by means of an additional gradient adaptive term, a modified normalized least mean square (MNLMS) algorithm was developed to identify the coefficients of PA forward model with relative long-term memory in the noisy feedback environments. The performance of this PD scheme is validated with numerical simulations.

**Keywords:** predistortion (PD), power amplifier (PA), hybrid direct learning (HDL), modified normalized least mean square (MNLMS), measurement noise

**Classification:** Microwave and millimeter wave devices, circuits, and systems

## References

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## 1 Introduction

Among all linearization techniques, adaptive digital predistortion (PD) is a



highly cost-effective approach to compensate for the nonlinear distortions of power amplifiers (PAs) existing in wireless communication systems. As the signal bandwidth increases, such as OFDM (orthogonal frequency division multiplexing) system, PA memory effects can no longer be ignored [1]. The PD implementations in current literatures mostly focus on the compensation of distortions mutually caused by the PA nonlinearity and memory effects [2]. Considering the computational burden and memory requirements, most of available PD methods are based on the indirect learning structure, which can effectively avoid the need for PA model assumption and corresponding coefficients estimation. However, a major drawback of indirect learning is that it's particularly susceptible to the measurement noise derived from actual PA output owing to analog devices, e.g. down-conversion mixer and A/D converter. The noisy signal fed back to the coefficients identifying unit may make the adaptive filtering converges to biased values [3, 4], which deteriorates PD performance, leading to higher spectral regrowth. Besides, it was proved that the effects of measurement noise is increasingly magnified as the order of PA nonlinearity increases in modern wideband applications.

In order to resolve this fundamental disadvantage, Morgan proposed an improved an indirect learning technique in [4], which demonstrates adequate performance in noisy feedback conditions. Nevertheless, it is complicated in configuration and computation, and could not remedy the issue of cascade order caused by post-inverse filter. As an alternative approach, several PD schemes based on direct learning structure including the adjoint nonlinear least-mean-square algorithm (NALMS) and nonlinear filtered-x recursive least square (NFXRLS) [5], were developed to overcome such limitations. By using the concept of virtual filter, their structures can be simplified or even unified to a linear form, while achieving better performance than the PD based on indirect learning. Unfortunately, such schemes are only suitable for the PAs with weak nonlinearity and memory effects.

In this correspondence, we propose a robust adaptive digital PD scheme based on the hybrid direct learning (HDL) structure to mitigate the effects introduced by measurement noise. Specifically, a modified normalized least mean square (MNLMS) algorithm is developed to identify the forward model of PA in the noisy feedback cases. Moreover, to fully exploit the potential of the PD scheme for wideband signals, deep PA memory effects was also taken into account. Simulations confirm its effectiveness and robustness.

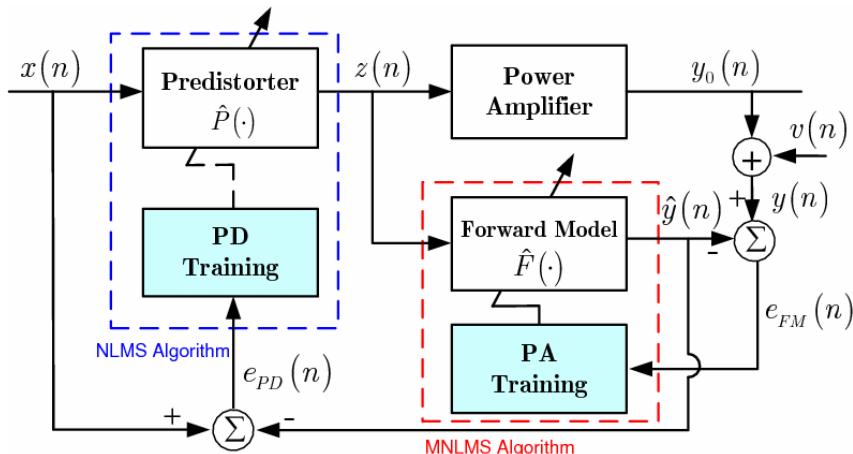
The organization of the paper is as follows. The proposed PD scheme and MNLMS training algorithm developed for updating the coefficients of PA forward model are presented in Section 2. Section 3 shows simulation results, and Section 4 concludes the paper.

**Notation:** the superscripts  $(\cdot)^T$ ,  $(\cdot)^H$  and  $(\cdot)^*$  denote the transpose, Hermitian transpose, and a complex conjugate, respectively.

## 2 Proposed PD scheme

The baseband equivalent model of proposed adaptive PD scheme based on HDL structure is illustrated in Fig. 1. As shown in this figure,  $x(n)$  denotes





**Fig. 1.** Baseband equivalent model of the proposed PD structure.

the baseband input to the predistorter, and  $z(n)$  is its corresponding output, where  $n$  is the time index.  $y(n)$  is PA output signal for the feedback path, and it has been added measurement noise  $v(n)$ , which is independent of the PA output  $y_0(n)$ . In the HDL structure, the forward model of PA  $\hat{P}(\cdot)$  is firstly identified to eliminate the effects of measurement noise. Then, a noiseless inverse model  $\hat{P}(\cdot)$  is estimated as the predistorter coefficients. The two-step processor was derived in following subsections.

## 2.1 PA identification

In the first step, we identify the forward model of PA, which is unbiased because its input  $z(n)$  is noiseless. Considering the memory polynomial model of Kim and Konstantinou [6], the forward model  $\hat{F}(\cdot)$  can be modeled as

$$\hat{y}(n) = \sum_{k=1}^K \sum_{q=0}^Q \omega_{kq} z(n-q) |z(n-q)|^{k-1}, \quad (1)$$

where  $\omega_{kq}$  are the coefficients specifying this model,  $K$  is the highest order of nonlinearity and  $Q$  is memory depth. The difference between  $\hat{y}(n)$  and  $y(n)$  is used to adapt its coefficients with the instantaneous error  $e_{FM}(n) = y(n) - \hat{y}(n)$ . The cost function performed on a sample-by-sample basis is expressed as

$$J(\mathbf{w}) = |e_{FM}(n)|^2 = |y(n) - \hat{y}(n)|^2 = |y(n) - \mathbf{w}(n)\mathbf{S}(n)|^2, \quad (2)$$

where  $\mathbf{w}(n) = [w_{10}, w_{20}, \dots, w_{K0}, \dots, w_{1Q}, w_{2Q}, \dots, w_{KQ}]^T$  is the coefficient vector of  $\hat{F}(\cdot)$ , and  $\mathbf{S}(n) = [z(n), z(n)|z(n)|, \dots, z(n)|z(n)|^{K-1}, \dots, z(n-Q)|z(n-Q)|^{K-1}]^T$ . It is obvious that minimizing the derivative of  $J(\mathbf{w})$  with respect to  $\mathbf{w}(n)$  will achieve the optimum coefficients, i.e.

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{1}{2} \mu_1 \frac{\partial J(n)}{\partial \mathbf{w}} = \mathbf{w}(n) + \mu_1 \mathbf{S}(n) e_{FM}^*(n) \quad (3)$$

where  $\mu_1$  is the step size (learning rate). The classic steepest descent methods such as NLMS algorithm [7] is described as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu_1}{\|\mathbf{S}(n)\|_2^2 + \varepsilon} \mathbf{S}(n) e_{FM}(n), \quad (4)$$

where  $\|\cdot\|_2$  denotes the Euclidean norm, and  $\varepsilon$  is a small positive constant to preserve the stability for close-to-zero input vectors. Compared with LMS algorithm, NLMS can change its learning rate adaptively according to the dynamics of input vectors in the non-stationary signal environments, so as to achieve faster convergence and smaller steady-state mis-adjustment. In the proposed PD scheme, considering the effects of noise, the MNLMS algorithm for the unbiased PA forward model identification is derived as

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\mu_1}{\|\mathbf{S}(n)\|_2^2 + \eta(n)} \mathbf{S}(n) e_{FM}^*(n), \quad (5)$$

The compensation term  $\eta(n)$  in the denominator of learning rate is given by

$$\eta(n) = \frac{\rho(\|\mathbf{S}(n)\|_2^2 + \|e_{FM}(n)\|_2^2)}{\|\mathbf{S}(n)\|_2^2}, \quad (6)$$

where  $\rho$  is a special adaptation factor to control the variation rate of  $\eta(n)$ , which can effectively remedy the effects of corrupting measurement noise. As observed, the algorithm is converging along with the variation of additive noise; the additional gradient adaptive term  $\eta(n)$  can decrease monotonically with the error performance surface decided by  $e_{FM}(n)$ . When the noise  $v(n)$  is strong, namely, a large  $e_{FM}(n)$ , a large step-size is preferred to provide faster convergence. On the contrary,  $\eta(n)$  gradually decreases along with  $e_{FM}(n)$  decreasing for a low steady-state error. Thus, by means of  $\eta(n)$ , an additional stabilization and faster convergence can be introduced.

## 2.2 Predistorter identification

In the sequel, using the above forward model  $\hat{F}(\cdot)$ , a noiseless inverse model  $\hat{P}(\cdot)$  is then identified as the predistorter coefficients, which are defined as

$$z(n) = \sum_{l=1}^L \sum_{m=0}^M a_{lm} |x(n-m)|^{l-1} x(n-m) \quad (7)$$

where  $L$  is the highest nonlinearity order, and  $M$  is memory depth. Here, the NLMS algorithm is employed to update the predistorter coefficients.

$$\mathbf{a}(n+1) = \mathbf{a}(n) + \frac{\mu_2}{\|\mathbf{P}(n)\|_2^2 + \varepsilon} \mathbf{P}(n) e_{PD}^*(n) \quad (8)$$

where  $e_{PD}(n) = x(n) - \hat{y}(n)$ ,  $\mu_2$  is the step size to control the learning rate, and  $\mathbf{a}(n) = [\alpha_{10}, \alpha_{20}, \dots, \alpha_{L0}, \dots, \alpha_{1M}, \alpha_{2M}, \dots, \alpha_{LM}]^T$  is the coefficient vector of model  $\hat{P}(\cdot)$ , and  $\mathbf{P}(n) = [x(n), x(n)|x(n)|, \dots, x(n)|x(n)|^{L-1}, \dots, x(n-M)|x(n-M)|^{L-1}]^T$ .

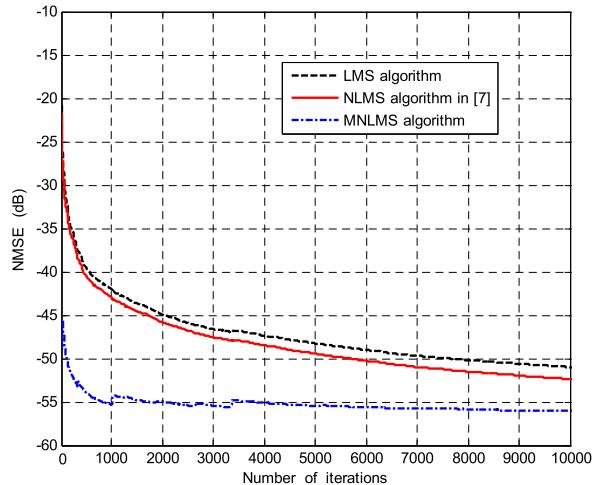
## 3 Simulation results

In this section, the numerical simulations were carried out to evaluate the effectiveness of this measurement noise remediation scheme proposed

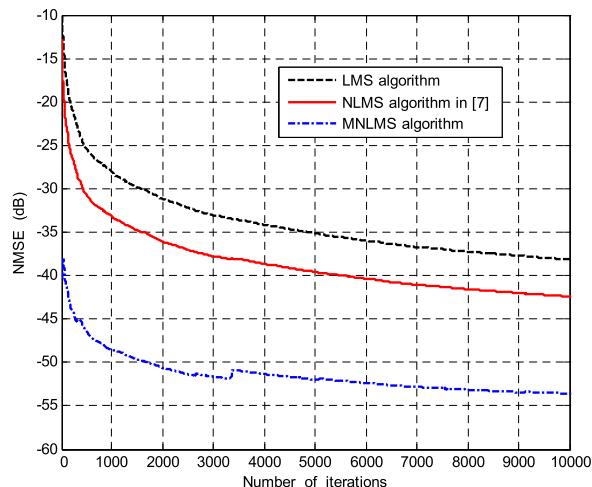
above. Comparisons were also made with the traditional LMS, NLMS algorithms, indirect learning method [1] and Morgan's technique [4]. The baseband input is a long-term evolution (LTE) OFDM signal of total bandwidth 20 MHz, with 2048 subcarriers, 64-quadrature amplitude modulation (64-QAM), and 8-times up-sampled. We will show that the proposed HDL structure can be utilized to linearize different PA behavioral models with deep memory effects in the presence of measurement noise, hence, demonstrating its robustness. The one PA obeys an odd-order-only memory polynomial model

$$y_0(n) = \sum_{k=1}^K \sum_{q=0}^Q c_{kq} z(n-q) |z(n-q)|^{k-1}, \quad (9)$$

whose coefficients are defined as [8]



(a)

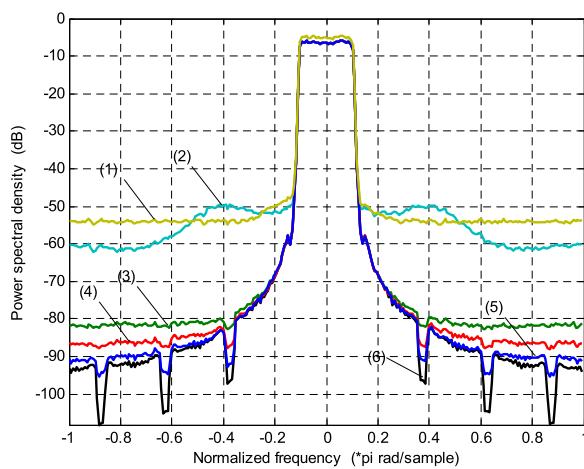


(b)

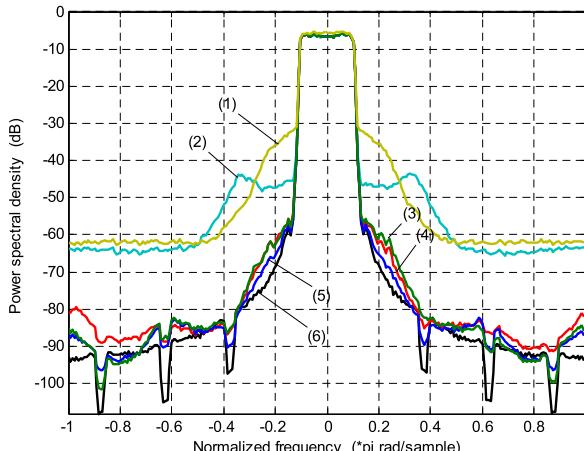
**Fig. 2.** Learning trajectories consisting of the NMSEs between PA output  $y_0(n)$  and its forward model output  $\hat{y}(n)$  using the HDL structure when measurement noise exists. (a) Memory polynomial PA model. (b) Wiener PA model.

$$\begin{aligned} c_{10} &= 1.0108 + 0.0858j, c_{30} = 0.0879 - 0.1583j, \quad c_{50} = -1.0992 - 0.8891j, \\ c_{11} &= 0.1179 + 0.0004j, c_{31} = -0.1818 + 0.0391j, c_{51} = 0.1684 + 0.0034j, \quad (10) \\ c_{12} &= 0.0473 - 0.0058j, c_{32} = 0.0395 + 0.0283j, \quad c_{52} = -0.1015 - 0.0196j. \end{aligned}$$

Another PA is Wiener model whose  $H(z) = 1 + 0.35z^{-1} + 0.11z^{-2} + 0.06z^{-3} + 0.01z^{-4}$ , and FIR filtering parameters are  $b_1 = 1$ ,  $b_3 = 0.0731 - 0.1628j$ ,  $b_5 = -1.1538 - 0.7817j$ .  $v(n)$  is Gaussian white noise with an SNR of 35-dB. For the proposed PD scheme, the adaptation factor  $\rho$  was set to 4.0, step size  $\mu_1$  (for LMS, NLMS, and MNLMS in PA training) and  $\mu_2$  were set to 0.50 and 0.25, respectively. Both initial weight vectors  $\mathbf{w}(0)$  and  $\mathbf{a}(0)$  were given by  $[1, 0, \dots, 0]^T$ .



(a)



(b)

**Fig. 3.** Simulated PSDs of the PA output  $y_0(n)$  using different PD methods when measurement noise exists. (a) Memory polynomial PA model. (b) Wiener PA model. (1) Noisy output without PD. (2) Classic indirect learning method using LMS [1]. (3) Morgan's indirect technique in [4]. (4) HDL using NLMS identifying  $\hat{F}(\cdot)$ . (5) HDL using MNLMS identifying  $\hat{F}(\cdot)$ . (6) Original OFDM signal.

**Simulation 1:** Learning trajectories showing the normalized mean square errors (NMSEs) between the PA output  $y_0(n)$  and PA forward model output  $\hat{y}(n)$  using the proposed HDL structure are depicted in Fig. 2, where (a) is for the memory polynomial PA model, and (b) is for Wiener PA model. For comparison, three different algorithms for identifying  $\hat{F}(\cdot)$  were considered, including LMS, NLMS and MNLMS presented in (5). All NMSE curves were obtained through 1000 independent trials. As observed, MNLMS algorithm exhibits faster convergence speed and smaller steady-state performance in the noisy feedback environment for both PAs, and significantly outperforms the other two training algorithms.

**Simulation 2:** The power spectral densities (PSDs) of the PA output of the proposed HDL structure using NLMS (curve 4) and MNLMS (curve 5) for identifying  $\hat{F}(\cdot)$  are depicted in Fig. 3, where (a) and (b) are for the memory polynomial and Wiener models, respectively. For comparison, PSDs of the noisy PA output without PD (curve 1), classic indirect learning method using LMS [1] (curve 2), and Morgan's indirect technique [4] (curve 3), and original OFDM input (curve 6) were also plotted.

As shown, the out-of-band PSDs of the proposed scheme are considerably lower than those of the other methods for both PA model cases. For example, compared with Morgan's indirect learning technique [4], HDL brings down the level of spectral regrowth more than 5-dB in nearby adjacent channels. In addition, HDL structure using MNLMS for identifying PA forward model provides the best PSD performance.

As revealed by simulations above, an excellent convergence behavior and a low spectral regrowth can be offered by using HDL structure to remedy the effects of corrupting measurement noise in the feedback path.

#### 4 Conclusion

In the paper, an adaptive PD scheme based on HDL structure for mitigating the effects of measurement noise was proposed. By introducing an additional gradient adaptive term for the learning rate, the MNLMS algorithm was developed in the PA training unit. Computer simulations for an LTE OFDM signal with 20 MHz bandwidth were presented. It is shown that the proposed PD is robust to linearize the PAs with relative long-term nonlinearity and memory in the presence of measurement noise.

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