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Optimal multiband spectrum sensing in cognitive radio

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Abstract: In order to maximize the throughput in cognitive radio when primary communication system is an OFDM system, the optimal spectrum sensing is addressed. Firstly the single user sensing is discussed, and it is proved that the optimal sensing time that maximizes the throughput exists. In addition, the cooperative sensing using k-out-of-N fusion rule is considered. By analyzing the mathematical model of the throughput, a joint searching method is proposed to obtain the optimal sensing time and fusion rules which can maximize the throughput. In the simulation results, the correctness of theoretical analysis is verified.

Keywords: cognitive radio, spectrum detection, energy detection, OFDM, throughput
Classification: Science and engineering for electronics

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1 Introduction

Cognitive radio [1] (CR) is an intelligent wireless communication system. In a CR network, the secondary users (SUs) are allowed to utilize the frequency bands of the primary users (PUs) when these bands are not currently being used. Therefore, CR is a potential technique to mitigate the issue of spectrum scarcity, which is caused by traditional fixed spectrum access policy [2].

In order to avoid interfering with PUs, spectrum sensing is a key problem in CR. Since energy detection [3, 4] does not require any a priori knowledge of PUs' signals and has the lowest complexity, it has been widely applied in CR. In order to further improve detection performance, multi-user cooperative spectrum sensing based on energy detection is explored [4, 5].

Besides designing spectrum sensing algorithm, the design of the sensing parameters is also very important. In a CR system, SUs always sense the spectrum periodically. The longer the sensing time, the more opportunities the SUs can obtain. However, due to the fixed frame duration, the less time the SUs could transmit data. Therefore, an optimal parameter setting is expected to maximize the throughput of the CR network. In [6], the authors derive the optimal sensing time for the CR system with single SU sensing in a single frequency band. In this paper, we discuss the case that the primary system is an OFDM system. The contribution is twofold. First of all, we optimize the sensing time to maximize the throughput of the CR system with single SU sensing. Secondly, we consider cooperative spectrum sensing using k-out-of-N fusion rule [5]. We optimize the sensing time and the fusion rules to maximize the throughput, and a joint searching method is proposed to obtain the optimal parameters.

2 Optimal single user sensing

2.1 The system model

Consider the primary communication system is an OFDM system. The number of subcarriers or subchannels is L. A single SU is sensing the OFDM signals. After down conversion, removal of prefix cyclic, serial-to-parallel conversion and FFT, the received signals at the SU are converted to samples in frequency domain. Then energy detector is used at each subchannel. The SU is allowed to access the subchannels at which the PU's absence is detected. We suppose the received noise at the l-th subchannel is complex Gaussian i.i.d. random process with zero mean and variance σ^2. Assume OFDM symbol rate is f_s, and the spectrum sensing time is τ, then the detection performance at the l-th subchannel can be given by

$$P_{f,l}\left(\tau,\lambda_l\right) = Q\left(\frac{\lambda_l - \sigma^2}{\sigma^2}\sqrt{\tau f_s}\right) \tag{1}$$

$$P_{d,l}\left(\tau,\lambda_l\right) = Q\left(\frac{\lambda_l - \sigma^2\left(1+\gamma_l\right)}{\sigma^2\sqrt{1+2\gamma_l}}\sqrt{\tau f_s}\right) \tag{2}$$

where λ_l is the threshold, and $\gamma_l = \sum_{m=1}^{M} \left|h_l S_{l,m}\right|^2 / \left(M\sigma^2\right)$ is the average received signal-to-noise ratio (SNR) at the l-th subchannel.

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2.2 Optimization of the throughput

A frame structure of CR consists of one sensing slot and one data transmission slot. Suppose the sensing duration is τ and the frame duration is T, so the data transmission slot duration is $T-\tau$.

There are two scenarios for which SU can access the l-th subchannel. Scenario 1: when the l-th subchannel is not occupied and no false alarm is generated by the SU. The achievable throughput of the CR system is $\frac{T-\tau}{T}C_{0,l}$, here $C_{0,l}$ (bits/s/Hz) is the capacity per unit bandwidth of the CR network in this scenario. The probability of this scenario is $P\left(H_{0,l}\right)\left(1-P_{f,l}\right)$, here $P\left(H_{0,l}\right)$ is the probability for which the l-th subchannel is not occupied. Scenario 2: when the l-th subchannel is occupied but miss detection is generated by the SU. The achievable throughput of the CR system is $\frac{T-\tau}{T}C_{1,l}$, here $C_{1,l}$ (bits/s/Hz) is the capacity per unit bandwidth of the CR network in this scenario. The probability of this scenario is $P\left(H_{1,l}\right)\left(1-P_{d,l}\right)$, here $P\left(H_{1,l}\right)$ is the probability for which the l-th subchannel is occupied.

We define
$$R_0\left(\tau,\boldsymbol{\lambda}\right)=\frac{T-\tau}{T}\sum\nolimits_{l=1}^{L}C_{0,l}P\left(H_{0,l}\right)\left[1-P_{f,l}\left(\tau,\lambda_l\right)\right] \tag{3}$$
$$R_1\left(\tau,\boldsymbol{\lambda}\right)=\frac{T-\tau}{T}\sum\nolimits_{l=1}^{L}C_{1,l}P\left(H_{1,l}\right)\left[1-P_{d,l}\left(\tau,\lambda_l\right)\right] \tag{4}$$

where $\boldsymbol{\lambda}=[\lambda_1,\lambda_2,\cdots,\lambda_L]$, then the average throughput of the CR system can be given by

$$R\left(\tau,\boldsymbol{\lambda}\right)=R_0\left(\tau,\boldsymbol{\lambda}\right)+R_1\left(\tau,\boldsymbol{\lambda}\right) \tag{5}$$

where $\boldsymbol{\lambda}=[\lambda_1,\lambda_2,\cdots,\lambda_L]$. Under the constraint that the PU is sufficiently protected, we establish the optimization problem as

$$\begin{aligned}\max_{\tau,\boldsymbol{\lambda}}\quad & R\left(\tau,\boldsymbol{\lambda}\right)=R_0\left(\tau,\boldsymbol{\lambda}\right)+R_1\left(\tau,\boldsymbol{\lambda}\right)\\ s.t.\quad & P_{d,l}\geq P_{d,l}^{0}\end{aligned} \tag{6}$$

where $P_{d,l}^{0}$ is the lower limit of the detection probability at the l-th subchannel. It can be proved that the optimal solution to (6) is achieved when $P_{d,l}=P_{d,l}^{0}$, and the throughout in scenario 1 dominates the achievable throughput. Therefore, using (1) and (3), (6) can be approximated by

$$\max_{\tau}\quad R_0\left(\tau\right)=\frac{T-\tau}{T}\sum\nolimits_{l=1}^{L}C_{0,l}P\left(H_{0,l}\right)\left[1-Q\left(\alpha_l+\beta_l\sqrt{\tau_s}\right)\right] \tag{7}$$

where $\alpha_l=\sqrt{1+2\gamma_l}Q^{-1}\left(P_{d,l}^{0}\right)$, $\beta_l=\gamma_l\sqrt{f_s}$.

Proposition 1 In (7), there exists a unique optimal sensing time $\tau^*\in(0,T)$ to maximize the throughput $R_0\left(\tau\right)$.

Proof. According to [7], Proposition 1 can be proved by the following conditions:

1) When τ is very small, $R_0\left(\tau\right)$ increases with τ.
   The first derivative of $R_0\left(\tau\right)$ with respect to τ is

$$\begin{aligned}R_0'\left(\tau\right)=-\frac{L}{T}+\sum_{l=1}^{L}\Bigg\{&\frac{1}{T}Q\left(\alpha_l+\beta_l\sqrt{\tau}\right)\\&+\frac{\beta_l}{2\sqrt{2\pi}}\left(\frac{1}{\sqrt{\tau}}-\frac{\sqrt{\tau}}{T}\right)\exp\left[-\left(\alpha_l+\beta_l\sqrt{\tau}\right)^2/2\right]\Bigg\}\end{aligned} \tag{8}$$

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Obviously we have $\lim_{\tau\to 0} R_0'(\tau) \to +\infty > 0$.

2) When τ approaches T, $R_0(\tau)$ decreases with τ.

$$\lim_{\tau\to T} R_0'(\tau) = -\frac{L}{T} + \sum_{l=1}^{L} \frac{1}{T} Q\left(\alpha_l + \beta_l\sqrt{\tau}\right) < 0 \tag{9}$$

3) $R_0(\tau)$ is a concave function for $\tau^* \in (0,T)$.

In (8), $Q\left(\alpha_l + \beta_l\sqrt{\tau}\right)$, $\dfrac{1}{\sqrt{\tau}} - \dfrac{\sqrt{\tau}}{T}$ and $\exp\left[-\left(\alpha_l + \beta_l\sqrt{\tau}\right)^2/2\right]$ are all decreasing functions of τ, so $R_0'(\tau)$ is a decreasing function of τ. Therefore $R_0(\tau)$ is a concave function for $\tau^* \in (0,T)$ [7]. ■

According to proposition 1, we can obtain the optimal τ^* by one-dimensional search method [7], such as bisection method, the golden mean method, and so on.

3 Optimal multi-user cooperative sensing

3.1 The system model

Consider a CR network with N SUs. Each SU detects the PU's OFDM signals by the energy detectors as in 2.1. Each SU makes decision (1-bit decision, "0" denotes the subchannel is not occupied and "1" denotes the subchannel is occupied) at each subchannel, and sends its decision results at the L subchannels to a fusion center (FC) over a control channel. At the FC, the decisions from N SUs is fused to make a final decision at each subchannel using certain hard decision rule, which is assumed to be k_l-out-of-N rule at the l-th subchannel. If the FC decides "0" at certain subchannels, the FC can allocate these subchannels to certain SUs for data transmission. We suppose that the size of CR network is much smaller than the distance between the PU and the CR network, and the average received SNRs of N SUs are all γ_l at the l-th subchannel. Without loss of generality, we assume the received noise of each SU at each subchannel is complex Gaussian i.i.d. random process with zero mean and variance σ^2.

Let the thresholds of the L SUs at the l-th subchannel be η_l, we can get each SU's detection performance at each subchannel as

$$P_{f,l}(\tau,\eta_l) = Q\left(\frac{\eta_l - \sigma^2}{\sigma^2}\sqrt{\tau f_s}\right), \quad P_{d,k}(\tau,\eta_l) = Q\left(\frac{\eta_l - \sigma^2(1+\gamma_l)}{\sigma^2\sqrt{1+2\gamma_l}}\sqrt{\tau f_s}\right) \tag{10}$$

Since k_l-out-of-N rule is used by the FC at the l-th subchannel, the final detection pefprmance can be calculated as [5]

$$P_{F,l}(\tau,\eta_l,k_l) = \sum\nolimits_{i=k_l}^{N} C_N^{k_l}\left[P_{f,l}(\tau,\eta_l)\right]^{k_l}\left[1 - P_{f,l}(\tau,\eta_l)\right]^{N-k_l} \tag{11}$$

$$P_{D,l}(\tau,\eta_l,k_l) = \sum\nolimits_{i=k_l}^{N} C_N^{k_l}\left[P_{d,l}(\tau,\eta_l)\right]^{k_l}\left[1 - P_{d,l}(\tau,\eta_l)\right]^{N-k_l} \tag{12}$$

where C_m^n denotes combination formula and $C_m^n = m!/\left(n!\,(m-n)!\right)$.

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3.2 Optimization of the throughput

We define
$$R_0(\tau, \boldsymbol{\eta}, \boldsymbol{k}) = \frac{T-\tau}{T}\sum\nolimits_{l=1}^{L} C_{0,l} P(H_{0,l})\left[1 - P_{F,l}(\tau, \eta_l, k_l)\right] \quad (13)$$
$$R_1(\tau, \boldsymbol{\eta}, \boldsymbol{k}) = \frac{T-\tau}{T}\sum\nolimits_{l=1}^{L} C_{1,l} P(H_{1,l})\left[1 - P_{D,l}(\tau, \eta_l, k_l)\right] \quad (14)$$

where $\boldsymbol{\eta} = [\eta_1, \eta_2, \cdots, \eta_L]$, $\boldsymbol{k} = [k_1, k_2, \cdots, k_L]$. Similar to section 2.2, the optimization problem can be established as

$$\begin{aligned} \max_{\tau, \boldsymbol{\eta}, \boldsymbol{k}} \quad & R(\tau, \boldsymbol{\eta}, \boldsymbol{k}) = R_0(\tau, \boldsymbol{\eta}, \boldsymbol{k}) + R_1(\tau, \boldsymbol{\eta}, \boldsymbol{k}) \\ s.t. \quad & P_{D,l} \geq P_{D,l}^{0} \end{aligned} \quad (15)$$

we can derive that the optimal throughput is achieved when $P_{D,l} = P_{D,l}^{0}$. Similar to section 2.2, the optimization problem can be written as

$$\max_{\tau, \boldsymbol{k}} \quad R_0(\tau, \boldsymbol{k}) = \frac{T-\tau}{T}\sum\nolimits_{l=1}^{L} C_{0,l} P(H_{0,l})\left[1 - P_{F,l}(\tau, k_l)\right] \quad (16)$$

Define $R_{0,l}(\tau, k_l) = \frac{T-\tau}{T} C_{0,l} P(H_{0,l})\left[1 - P_{F,l}(\tau, k_l)\right]$, two propositions are given as below.

Proposition 2 For a fixed $\boldsymbol{k}$, $R_0(\tau, \boldsymbol{k})$ is a concave function of τ, and it exists a unique $\tau' \in (0, T)$ to maximize $R_0(\tau, \boldsymbol{k})$.

Proposition 3 For a fixed τ, the vector $\boldsymbol{k}'$, consisting of $k_l'(l = 1, 2, \cdots, L)$ that maximize $R_{0,l}(\tau, k_l)\,(l = 1, 2, \cdots, L)$, optimizes $R_0(\tau, \boldsymbol{k})$.

The proofs are omitted here. Based on proposition 2, we can obtain the optimal τ' by one-dimensional search method to maximize $R_0(\tau, \boldsymbol{k})$ for a fixed $\boldsymbol{k}$. $\boldsymbol{k}$ is a vector composed with integers. For a fixed τ, a search over all possible $\boldsymbol{k}$ is required to obtain the optimal $\boldsymbol{k}'$, and the search times are N^L. However, according to proposition 3, we can reduce the search times to $N \times L$.

From the conclusions above, we propose the following joint search method in figure 1.

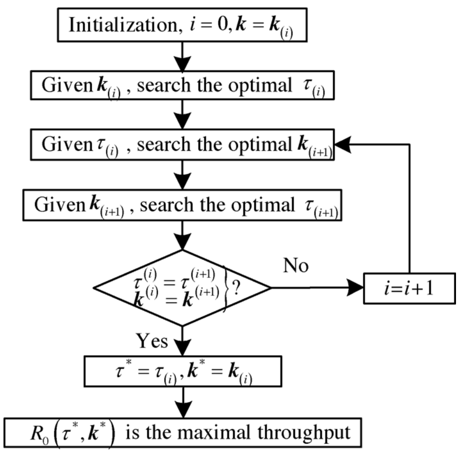

Fig. 1. the joint search method

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4 Simulation results

In this section, the simulation results are presented. We list the parameters as following.

The total bandwidth: 2.5 MHz;

The number of subchannels: 8;

The symbol period: $4\,\mu s$;

$P\left(H_{0,l}\right) = 0.8, \quad l = 1, 2, \cdots, L$

Received SNRs at the 8 subchannels (dB): -10, -15, -18, -13, -12, -10, -20, -15;

$C_{0,l}, \quad l = 1, 2, \cdots, L$ (bits/s/Hz): 3, 6, 6, 8, 2, 3, 3, 4;

$C_{1,l} = 0.8 \times C_{0,l}, \quad l = 1, 2, \cdots, L;$

CR frame duration: 20 ms.

Firstly, we consider the single SU sensing scenario. In figure 2, $P_{d,l}^0$, $l = 1, 2, \cdots, L$ for different subchannels are same, which is written in the figure. We can see from figure 2 that the curves of theoretical results and the Mont Carlo simulation results are almost coincident, and the throughput is a concave function with respect to τ. When $P_{d,l}^0 = 0.95$, 0.99, 0.995, $l = 1, 2, \cdots, L$, the maximal throughputs are respectively 9.2097, 6.0061 and 5.1082, and the corresponding optimal sensing durations are 6 ms, 7.5 and 8 ms.

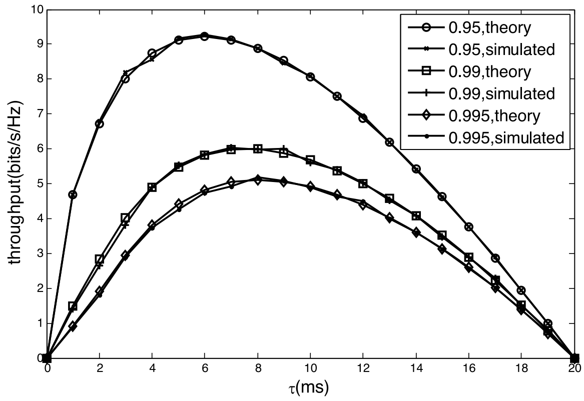

Fig. 2. Single SU sensing

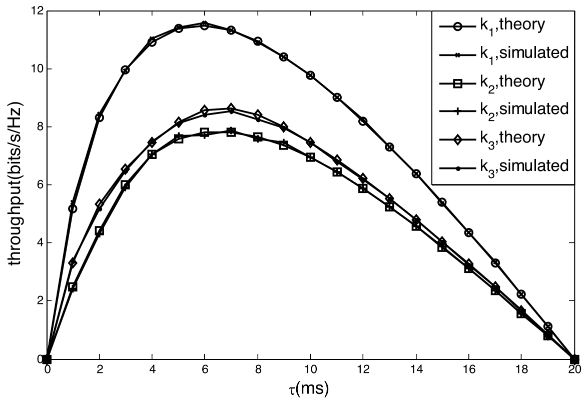

Fig. 3. Multi-user cooperative sensing

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Next we consider multi-user cooperative spectrum sensing. Assume the number of cooperative SUs is 5. Figure 3 shows the throughput versus τ, and $P_{d,l}^0 = 0.99$, $l = 1, 2, \cdots, L$, $\boldsymbol{k}_1 = [3\ 2\ 2\ 3\ 3\ 3\ 2\ 2]$, $\boldsymbol{k}_2 = [4\ 1\ 1\ 5\ 3\ 1\ 2\ 1]$, $\boldsymbol{k}_3 = [2\ 3\ 5\ 1\ 3\ 2\ 1\ 1]$. It is easy to see from the figure that the throughput is a concave function of τ. When $\boldsymbol{k} = \boldsymbol{k}_1$ and $\tau = 5.8$ ms, the optimal throughput 11.49 is achieved.

5 Conclusion

In this paper we discuss the optimal spectrum sensing in CR when the primary system is an OFDM system. We firstly study the single SU sensing, and prove that there exits an optimal sensing time to maximize the throughput and give search methods. Then we study the multi-user cooperative sensing for OFDM signals, and the FC adopts k-out-of-N rule to fuse the decisions from SUs. We proposed a joint search method to obtain the optimal sensing time and fusion rules to maximize the throughput. The simulation results are consistent with the theoretical analysis.

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