

A MODIFIED PROCEDURE TO EVALUATE EARTHQUAKE-INDUCED DISPLACEMENT OF SLOPES CONTAINING A WEAK LAYER

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ABSTRACT

A procedure based on the Janbu method for slope stability analysis and Newmark's sliding block method for displacement calculation is proposed to evaluate rationally the earthquake-induced displacement of slopes containing a weak layer. In the procedure, the effects of the irregular geometry of the sliding plane and generation of excess pore water pressure in the weak layer which forms partly the sliding plane are considered. In the computation following the proposed procedure, a negative yield seismic coefficient to induce a safety factor of unity can be obtained because of the effect of excess pore water pressure, while the residual displacement can become a finite value because the sliding plane becomes flatter in general after the slope undergoes a large displacement.

Key words: earthquake, excess pore water pressure, Janbu method, Newmark's sliding block method, slope stability, weak layer (IGC: E8/E12/E13)

INTRODUCTION

Since its proposal in 1965, Newmark's sliding block method (Newmark, 1965) and its modified versions have been used extensively to calculate the earthquake-induced sliding displacements in practice (e.g., Richards and Elms, 1979; Whitman, 1990; Wartman et al., 2003; Wartman et al., 2005; RTRI, 2007).

The feature of Newmark's sliding block method is that, when the inertia force induced by an earthquake load is sufficiently large, the safety factor of the slope will become less than unity, causing the slope to start to slide as a rigid block and undergo a permanent displacement.

The procedure of Newmark's sliding block method in its original version is schematically described in Fig. 1. When the ground acceleration exceeds $K_y g$, the product of the yield seismic coefficient K_y to induce a safety factor of unity and the acceleration of gravity g , a sliding event will be triggered. The sliding continues until the sliding block has the same velocity as that of the foundation (i.e., the relative velocity of the sliding block becomes zero). Such a sliding event may occur several times during one earthquake. The relative sliding displacement can be calculated by integrating the relative velocity between the sliding block and its foundation.

In the original Newmark method, two assumptions are employed: the assumptions of response as a rigid block and a constant value of K_y during sliding. Some modifica-

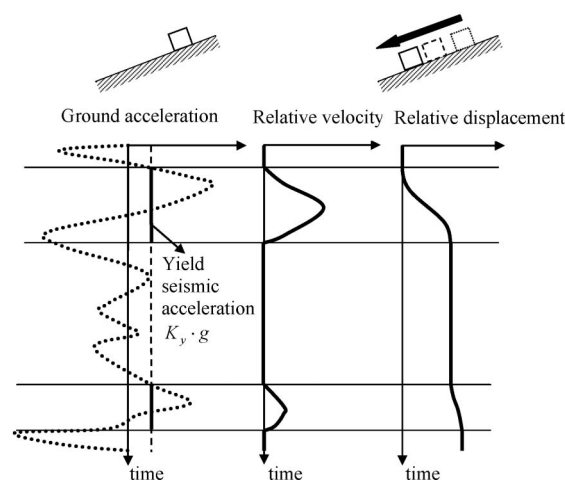


Fig. 1. Newmark's sliding block method

tions were made on the assumption of the rigid block (e.g., Makdisi and Seed, 1978; Rathje and Bray, 2000). Other modifications were also made on the yield seismic coefficient K_y . For example, in order to consider the post-peak strain-softening properties of dense granular soils, Okuyama et al. (2003) and Nakajima (2008) calculated the displacements of earthfill dam and retaining wall structures, respectively, using K_y values which decrease with the amount of sliding displacement.

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However, in evaluating the value of K_y to be used in the displacement computation of slopes containing a weak layer (Deng, 2008), two additional issues are worth noting. One is the effect of the irregular geometry of the sliding plane; the other is the effect of excess pore water pressure generation within a saturated weak layer during the earthquake. The sliding plane typically consists of more gentle or flatter conditions on the down slope side, so the shape of the sliding plane to be used in the computation would change with the amount of the sliding displacement. The excess pore water pressure would not dissipate significantly if the sliding movement takes place during the limited duration of the earthquake event. These two issues affect the value of the yield seismic coefficient K_y .

In view of the above, this study aims to propose a modified procedure to evaluate more rationally the earthquake-induced displacement of slopes containing a weak layer. First, the original Newmark method (denoted as original NM method herein) is extended by considering the effect of the irregular geometry of the sliding plane in a simplified manner and that of excess pore water pressure generation within the weak layer on the yield seismic coefficient K_y (denoted as extended NM method). Second, in order to consider the effect of the irregular geometry of the sliding plane in a generalized manner, the extended method is further modified by combining it with the widely employed Janbu method (Janbu, 1957), used for slope stability analysis (denoted as extended NJ method).

EXTENSION OF NEWMARK METHOD

Outline of Extended NM Method

The sliding of a single rigid element is studied in the extended NM method by considering the effect of excess pore water pressure and also the effect of the irregular geometry of the sliding plane on yield seismic coefficient K_y and residual displacement. The effect of excess pore water pressure on residual displacement has been considered in recent research (e.g., Sassa et al., 1996; Deng et al., 2007).

In the extended NM method, when the excess pore water pressure exceeds a threshold value, the value of K_y can decrease to be negative. Refer to APPENDIX 1 for the formulation of the K_y value. As will be demonstrated in Case 1 later, such negative K_y value in theory yields an infinite value of the residual displacement. However, by considering the decrease in the slope angle with the accumulation of the sliding displacement, the K_y value can be recovered to be positive, inducing a finite value of residual displacement.

Calculation Example

Two example cases (Fig. 2) were considered to demonstrate the calculation process in the extended NM method.

The slope in Case 1 is a regular infinite slope, on which a sliding block of a finite width is placed, while in Case 2, the slope is irregular with a flat horizontal plane on its toe located at a horizontal distance of 50 m from the block

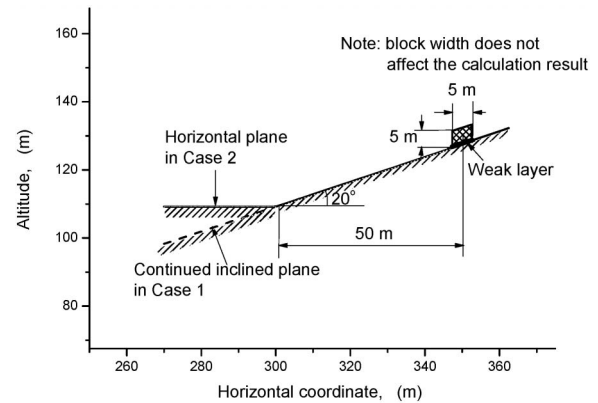


Fig. 2. A block on a slope in Cases 1 and 2

(Fig. 2). Therefore, the effects of the irregular geometry of the sliding plane on the yield seismic coefficient K_y as formulated in Eq. (A11) and residual displacement can be discussed in Case 2.

Both in Cases 1 and 2, a saturated weak layer that partly forms the sliding plane is assumed to exist at the bottom of the sliding block, and the ground water level within the block is set to be 2 m above the sliding plane. For simplicity, the excess pore water pressure ratio R_u of the weak layer is assumed to change from zero to a specified value immediately after a certain moment ($t = 17.42$ s) when the ground acceleration exceeds 0.25 g and to be kept constant until the end of sliding. Here, $R_u = \Delta u / \sigma'_0$ is the ratio of excess pore water pressure Δu to the initial effective stress σ'_0 .

The value of R_u to be used for the practical application of the proposed procedure would be evaluated based on relevant laboratory cyclic loading tests on the samples retrieved from the concerned slope (e.g., Deng et al., 2007). Otherwise, empirical methods using the relationship between the R_u value and the liquefaction resistance factor (e.g., JGS 1998) could be also employed, while the effect of initial shear stress mobilized within the weak layer on its properties of excess pore water pressure generation needs further studies.

In the calculation, the block is regarded as a single rigid element of density $\rho = 1.73$ g/cm³. The density of pore water ρ_{water} is set to be 1.0 g/cm³. The assumed values for the other parameters are listed in Table 1.

As the input ground acceleration, a strong motion record at Ojiya Station (EW component) in the 2004 Niigata-ken Chuetsu Earthquake in Japan (NIED, 2004) is employed, as shown in Fig. 3. The time interval of ground acceleration history is 0.01 second.

A comparison of the residual displacement calculation results in Case 1 and Case 2 is made in Fig. 4.

Figure 4 shows that the effect of excess pore water pressure on residual displacement is significant in both Case 1 and Case 2. In Case 2, for $R_u = 0$ where the excess pore water pressure is not considered based on the original NM method, the horizontal residual displacement is 1.39 m, while for $R_u = 0.6$, the horizontal residual displacement is 70.5 m.

Table 1. Parameters and calculation results

Case	Strength parameters			Slope conditions		Horizontal residual displacement (m)	
	c'	ϕ'	$R_u^{*)}$	Ground water level ^{**) (m)}	Block height (H , m)		Slope geometry
Case 1	0	39°	0~0.6	2	5	One inclined plane	1.39~∞
Case 2				2	5	One inclined plane and one horizontal plane	1.39~70.5
Case 3				0	0~10.0	One arc plane and one horizontal plane	5.3~75.9

^{*}) This value is set to be zero before ground motion exceeds 0.25 g.

^{**}) This value is defined as the elevation of the ground water level above the surface of the slope.

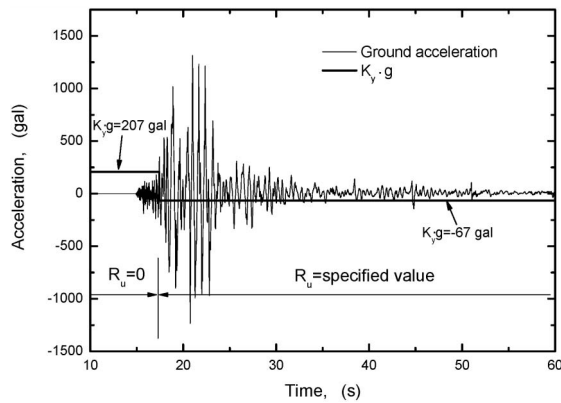
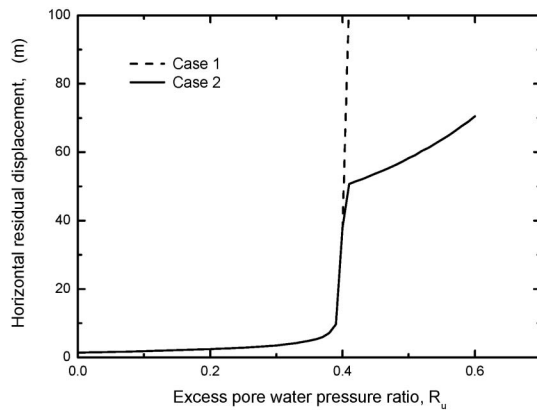
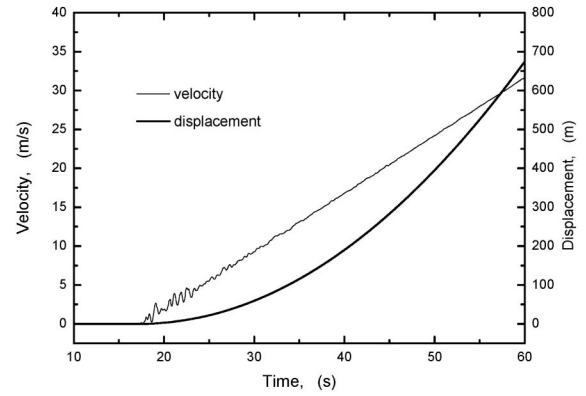
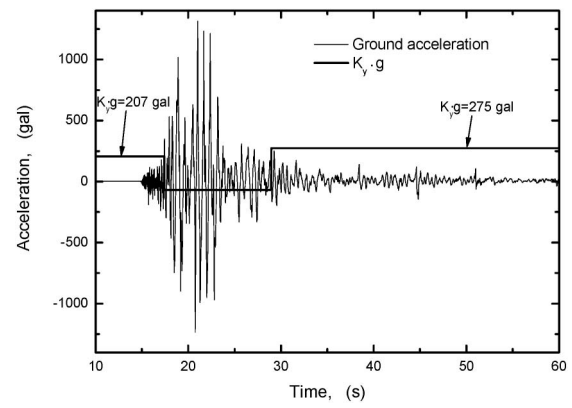
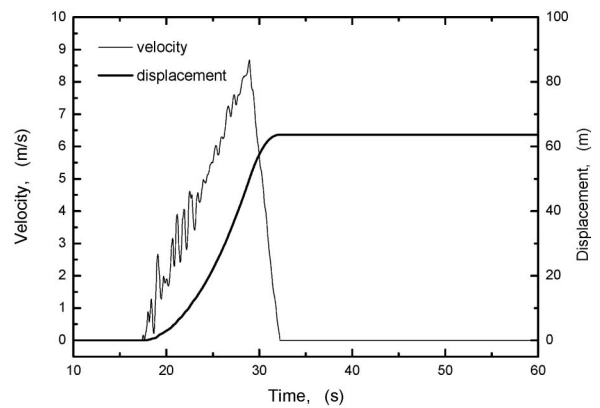
**Fig. 3. Time histories of ground acceleration and yield condition in Case 1 (for $R_u = 0.55$)****Fig. 4. Relation between R_u and horizontal residual displacement in Cases 1 and 2**

Figure 4 also shows that the effect of irregular geometry should be taken into consideration in evaluating a large sliding. For Case 1, the horizontal residual displacement is infinite when R_u exceeds 0.415. This threshold value corresponds to a condition which yields the value of K_y to be zero (Eq. (A11)). Typical time histories of K_y and

**Fig. 5. Time histories of relative velocity and displacement in Case 1 (for $R_u = 0.55$)****Fig. 6. Time histories of ground acceleration and yield condition in Case 2 (for $R_u = 0.55$)****Fig. 7. Time histories of relative velocity and displacement in Case 2**

the corresponding horizontal residual displacement (Figs. 3 and 5, for the case of $R_u = 0.55$) confirm that the negative and constant K_y value could induce a significantly large displacement, which would accumulate into an infinite value of the residual displacement. On the other hand, as the irregular geometry of the sliding plane was considered in the extended NM method (Case 2), the recovery of the K_y value to a positive one could induce a finite value of the horizontal residual displacement as

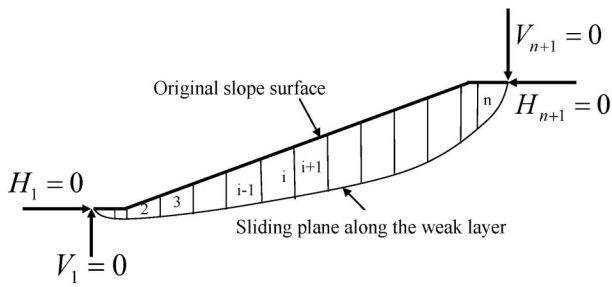


Fig. 8. A slope with vertical slices

typically shown in Figs. 6 and 7 for $R_u = 0.55$. When the horizontal displacement exceeds 50 m, the sliding block arrives at the horizontal plane (Fig. 2), yielding the recovery of the K_y value. Thus, the sliding block could finally come to a stop after an additional decelerated moving.

COMBINATION WITH THE JANBU METHOD

Outline of Extended NJ Method

In the extended NM method, the sliding mass is simplified as a single rigid block. However, in order to consider a progressive change in the angle of the sliding plane that would take place at the bottom of a relatively long sliding plane, the sliding mass needs to be divided into multiple blocks. In the present study, it was made by employing several vertical slices, as typically shown in Fig. 8, while assuming the following simplified conditions; 1) all the slices share a common horizontal velocity; 2) the slope of each slice base changes with the slope displacement to match the current sliding plane (i.e., the base part of the slice is not rigid); and 3) the relative vertical displacement between the adjacent slices is allowable. Thus, a more general and realistic slope could be considered.

First, the value of the yield seismic coefficient K_y is calculated (refer to APPENDIX 2 for the details). Fundamentally, the procedure to calculate K_y is based on the original Janbu method, while a modification is made by considering the effects of the earthquake inertia force and excess pore water pressure. Second, after the K_y value has been obtained, horizontal displacement is obtained by following the procedure of the Newmark method (Fig. 1). Therefore, the above procedure is an extended version of the combination of the Newmark and the Janbu methods (named the extended NJ method).

It should be noted that the angle of sliding plane for each slice may change with the slope displacement. Generally, it induces an increase in the values of the yield seismic coefficient K_y because the sliding plane becomes flatter after undergoing a certain displacement.

Calculation Example

Calculations of displacement of a finite slope with a horizontal plane at the toe (Case 3) were performed in this section to demonstrate the importance of considering the effects of the irregular geometry of the sliding plane and excess pore water pressure generation within the weak layer on the residual displacement.

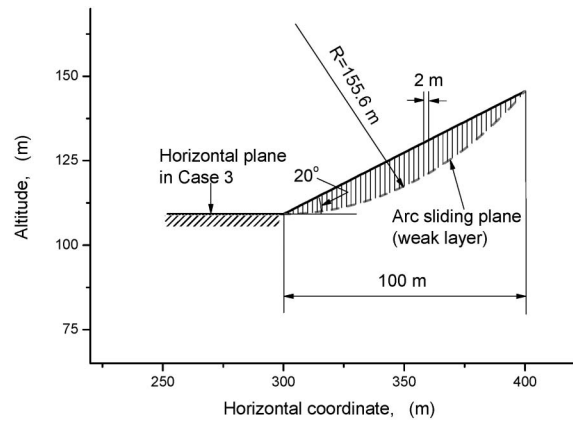


Fig. 9. A slope with existing weak layer in Case 3

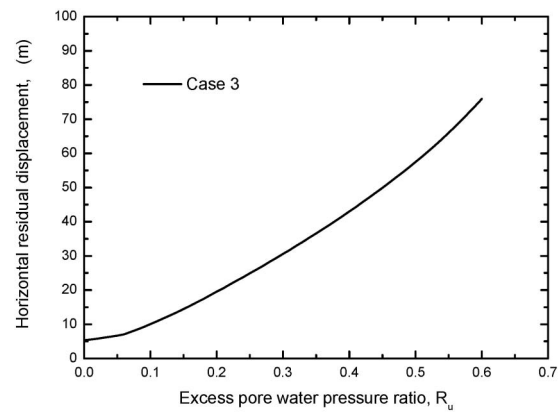


Fig. 10. Relation between R_u and horizontal residual displacement

A slope containing a weak layer in an arc shape is shown in Fig. 9. The sliding is assumed to occur along this existing weak layer.

In Case 3, conditions of the material densities, the input ground acceleration and the assumed values of R_u are set to be the same as those in Case 1 and Case 2. The ground water level is set at the surface of the slope. The width of the slice is 2 m. The other parameter values are listed in Table 1.

The horizontal residual displacements in Case 3 are shown in Fig. 10. The effect of R_u on the residual displacement is very significant. For the case of $R_u = 0$, where the excess pore water pressure is not generated, the horizontal displacement is 5.3 m; while for the case of $R_u = 0.6$, the horizontal residual displacement is as large as 75.9 m.

It should be noted that, by considering the effect of the irregular geometry of the sliding plane, finite values of the residual displacements could be also obtained in Case 3.

For example, for the case of $R_u = 0.55$, even though the yield seismic coefficient K_y becomes a negative value temporarily, it will increase gradually to be positive (Fig. 11), because the sliding plane has an irregular geometry and the horizontal plane at the slope toe can serve as a brake. Therefore, the residual displacement is finite, as shown in Fig. 12.

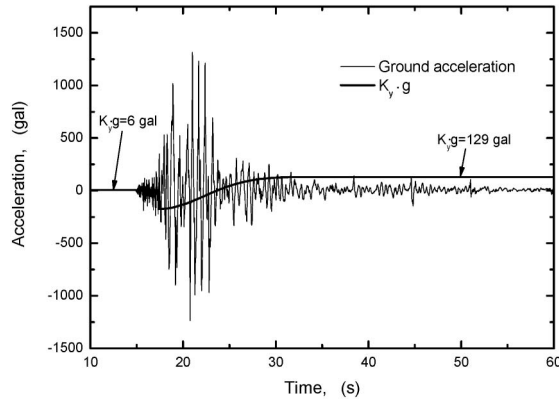


Fig. 11. Time histories of ground acceleration and yield condition in Case 3 (for $R_u = 0.55$)

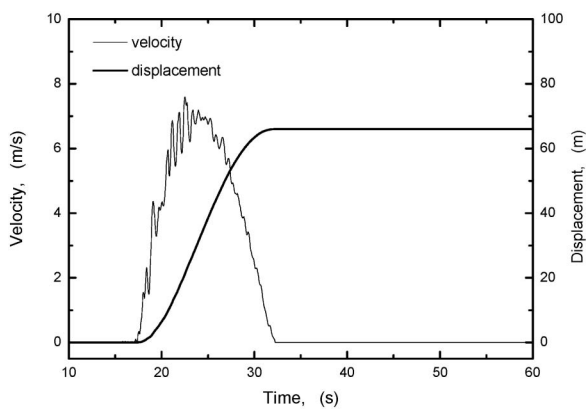


Fig. 12. Time histories of relative velocity and displacement in Case 3 (for $R_u = 0.55$)

CONCLUSIONS

In order to evaluate the earthquake-induced residual displacement of slopes containing a weak layer as a sliding plane, a modified procedure was proposed by combining the Janbu method for slope stability analysis and Newmark's sliding block method for displacement computation. In the proposed method, the sliding mass is divided into several vertical slices, and all the slices are assumed to share a common horizontal velocity. It has the following advantages over the original Newmark's sliding block method.

- 1) The effects of excess pore water pressure generation along the sliding plane can be considered. Thus, the proposed method can simulate possible reduction in the yield seismic coefficient before and/or during the sliding process.
- 2) The effect of irregular geometry of the sliding plane, which typically consists of more gentle or flat condition on the downslope side, can be considered. Thus, the proposed method can yield a finite amount of residual displacement even when the yield seismic coefficient is reduced temporarily to negative values before and/or during the sliding process.

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NOTATION

- α : slope of the base
- α_i : average slope of the base of slice i
- b_i : width of slice i
- Δh_i : difference between h_i and h_{i+1} of slice i
- ΔH_i : difference between two horizontal interslice forces on slice i
- ΔU : resultant force due to excess pore water acting on the block base
- ΔU_i : resultant force due to excess pore water acting on the base of slice i
- Δu : excess pore water pressure on the block base
- Δu_i : excess pore water pressure on the base of slice i
- h_i, h_{i+1} : parameter for location of H_i, H_{i+1} respectively
- H_i, H_{i+1} : horizontal interslice forces on two sides of the slice i
- H_w : distance between ground water level and sliding plane
- H_{wi} : distance between ground water level and sliding plane in slice i
- K_y : yield seismic coefficient
- l : length of block on its base
- l_i : length of slice i on its base
- l_{Ni} : parameter for location of N_i
- N : resultant force due to initial effective normal stress acting on the block base
- N_i : resultant force due to initial effective normal stress acting on the base of the slice i
- ρ : density of soil
- ρ_{water} : density of water
- R_u : excess pore water pressure ratio
- S : shear force on the block base
- S_i : shear force on the base of the slice i
- σ'_0 : initial effective stress on the block base
- σ'_{i0} : initial effective stress on the base of slice i
- u : pore water pressure on the block base
- u_i : pore water pressure on the base of slice i
- U : resultant force due to static pore water pressure acting on the block base
- U_i : resultant force due to static pore water pressure acting on the base of slice i
- W : weight of block
- W_i : weight of soil slice i

V_i, V_{i+1} : tangential interslice forces on two sides of the slice i
 z : height of block
 z_i : height of slice i

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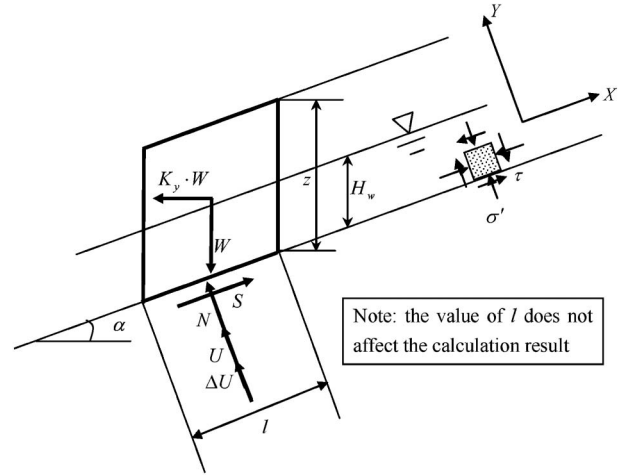


Fig. A1. A single block on a slope

APPENDIX 1: CALCULATION OF K_y IN EXTENDED NM METHOD

A single block on a slope as shown in Fig. A1 is considered. Definitions of notations in Fig. A1 are summarized in the list of notations. The equilibrium equations can be expressed as Eqs. (A1) and (A2):

Y direction:

$$-W \cdot \cos \alpha + K_y \cdot W \cdot \sin \alpha + N + U + \Delta U = 0 \quad (\text{A1})$$

X direction:

$$-W \cdot \sin \alpha - K_y \cdot W \cdot \cos \alpha + S = 0 \quad (\text{A2})$$

The safety factor F_s is defined as

$$F_s = \frac{c \cdot l + \sigma' \cdot \tan \phi' \cdot l}{S}$$

The critical condition to induce the sliding can be expressed by

$$F_s = \frac{c \cdot l + \sigma' \cdot \tan \phi' \cdot l}{S} = 1$$

or

$$S = c \cdot l + \sigma' \cdot \tan \phi' \cdot l \quad (\text{A3})$$

In Eq. (A3)

$$\sigma' = \sigma - u - \Delta u \quad (\text{A4})$$

where σ is total normal stress; u is static water pressure; Δu is excess pore water pressure generated during earthquake.

Referring to Eq. (A1), the total stress is evaluated as

$$\sigma = (N + U + \Delta U) / l = W \cdot (\cos \alpha - K_y \cdot \sin \alpha) / l \quad (\text{A5})$$

where the total weight W of the block is given as

$$W = \rho \cdot g \cdot z \cdot l \cdot \cos \alpha \quad (\text{A6})$$

In a similar manner, by considering the effect of horizontal inertia of pore water, the static water pressure is evaluated as

$$u = U/l = \rho_{\text{water}} \cdot g \cdot H_w \cdot (\cos \alpha - K_y \cdot \sin \alpha) \cdot \cos \alpha \quad (\text{A7})$$

The excess pore water pressure is evaluated by introducing the excess pore water pressure ratio R_u defined as

$$R_u = \Delta u / \sigma'_0$$

or

$$\Delta u = R_u \cdot \sigma'_0 \quad (\text{A8})$$

where σ'_0 is the value of effective stress σ' at a normal state without earthquake.

According to Eq. (A1) while setting $K_y = 0$ for the normal state, σ'_0 is evaluated as

$$\sigma'_0 = N_0/l = (W \cdot \cos \alpha - U_0)/l \quad (\text{A9})$$

where N_0 and U_0 denote for the reaction forces mobilized by the effective stress and pore water pressure, respectively, at the normal state. The value of U_0 is given as

$$U_0 = \rho_{\text{water}} \cdot g \cdot H_w \cdot \cos^2 \alpha \cdot l \quad (\text{A10})$$

Thus, by combining Eqs. (A2) through (A10), the yield seismic coefficient can be obtained as

$$K_y = \frac{c + \rho \cdot g \cdot z \cdot \cos \alpha \cdot ((1 - R_u) \cdot \cos \alpha \cdot \tan \phi' - \sin \alpha) - \rho_{\text{water}} \cdot g \cdot H_w \cdot (1 - R_u) \cdot \cos^2 \alpha \cdot \tan \phi'}{\rho \cdot g \cdot z \cdot \cos \alpha \cdot (\cos \alpha + \sin \alpha \cdot \tan \phi') - \rho_{\text{water}} \cdot g \cdot H_w \cdot \sin \alpha \cdot \cos \alpha \cdot \tan \phi'} \quad (\text{A11})$$

APPENDIX 2: CALCULATION OF K_y IN EXTENDED NJ METHOD

It is assumed that the sliding plane is along an existing weak layer (Fig. 8). Thus the procedure to determine the location and the shape of the sliding plane is skipped herein.

Equilibrium Condition

A typical vertical slice is shown in Fig. A2 with a definition of X and Y directions. It is assumed in this study that each slice starts to slide at the same value of K_y , so all the slices share the same horizontal velocity and displacement. The definitions of notations employed in Fig. A2 are summarized in the list of notations.

The equilibrium equations of the slice with respect to Y and X directions can be expressed as Eqs. (A12) and (A13), respectively.

$$\begin{aligned} -W_i \cdot \cos \alpha + K_y \cdot W_i \cdot \sin \alpha_i + N_i + U_i + \Delta U_i \\ + \Delta H_i \cdot \sin \alpha_i - \Delta V_i \cdot \cos \alpha_i = 0 \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} -W_i \cdot \sin \alpha - K_y \cdot W_i \cdot \cos \alpha_i + S_i - \Delta H_i \cdot \cos \alpha_i \\ - \Delta V_i \cdot \sin \alpha_i = 0 \end{aligned} \quad (\text{A13})$$

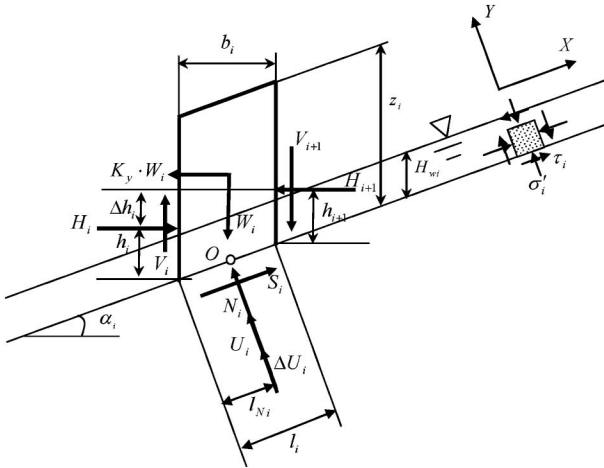


Fig. A2. Forces acting on a vertical slice

where,

$$W_i = \rho \cdot g \cdot z_i \cdot l_i \cdot \cos \alpha_i;$$

$$U_i = \rho_{\text{water}} \cdot g \cdot H_{wi} \cdot (\cos \alpha_i - K_y \cdot \sin \alpha_i) \cdot \cos \alpha_i \cdot l_i$$

Accumulation of all the horizontal internal forces shall be zero:

$$\sum \Delta H_i = 0 \quad (\text{A14})$$

Moment equilibrium of the slice i can be described as:

$$\begin{aligned} V_i \cdot \frac{b_i}{2} + (V_i + \Delta V_i) \cdot \frac{b_i}{2} + H_i \cdot \left(h_i - \frac{b_i}{2} \cdot \tan \alpha_i \right) \\ - (H_i + \Delta H_i) \cdot \left(h_i + \Delta h_i - \frac{b_i}{2} \cdot \tan \alpha_i \right) \sin \alpha_i - K_y \cdot W_i \cdot \frac{z_i}{2} = 0 \end{aligned} \quad (\text{A15})$$

By simplifying the preceding equation for the vertical force V_i , Eq. (A16) can be obtained.

$$V_i = \left(H_i \cdot \Delta h_i + \Delta H_i \cdot h_i + K_y \cdot W_i \cdot \frac{z_i}{2} \right) / b_i \quad (\text{A16})$$

In deriving Eq. (A16), it is assumed that h_i is equal to one third of the height of slice i .

As $V_i = 0$ is a known condition, Eq. (A16) will not be used for the first slice with $i = 1$.

Critical Condition to Induce Sliding

Assuming the soil property along the slice base to be perfectly plastic described by the Coulomb failure criterion, the local safety factor F_{si} , which is equal to unity under the critical condition, is defined as

$$F_{si} = \frac{c_i \cdot l_i + \sigma'_i \cdot \tan \phi'_i \cdot l_i}{S_i} = 1 \quad (\text{A17})$$

Normal stress σ_i , pore water pressure u_i and effective stress σ'_i can be expressed by Eqs. (A18), (A19) and (A20) respectively.

$$\sigma_i = (N_i + U_i + \Delta U_i) / l_i \quad (\text{A18})$$

$$u_i = U_i / l_i = \rho_{\text{water}} \cdot g \cdot H_{wi} \cdot (\cos \alpha_i - K_y \cdot \sin \alpha_i) \cdot \cos \alpha_i \\ = A_{1i} - A_{2i} \cdot K_y \quad (\text{A19})$$

where,

$$A_{1i} = \rho_{\text{water}} \cdot g \cdot H_{wi} \cdot \cos^2 \alpha_i, \\ A_{2i} = \rho_{\text{water}} \cdot g \cdot H_{wi} \cdot \cos \alpha_i \cdot \sin \alpha_i, \\ \sigma'_i = \sigma_i - u_i - \Delta u_i = N_i / l_i \quad (\text{A20})$$

The definition of excess pore water pressure ratio follows the one as has been made by Eq. (A8) for extended NM method. For extended NJ method, it is further assumed that all slices have the same R_u value. So the expression of R_u is:

$$R_u = \Delta u_i / \sigma'_{i0}$$

or

$$S_i = \frac{1}{m_\alpha} \cdot [(c_i - R_u \cdot \sigma'_{i0} \cdot \tan \phi'_i) \cdot b_i + (W_i + \Delta V_i - (A_{1i} - A_{2i} \cdot K_y) \cdot b_i) \tan \phi'_i] \quad (\text{A24})$$

where, $m_\alpha = \cos \alpha_i + \sin \alpha_i \cdot \tan \phi'_i$.

By substituting Eq. (A24) into Eq. (A13), expression of ΔH_i can be deduced:

$$\Delta H_i = \frac{1}{m_\alpha \cdot \cos \alpha_i} \cdot [(c_i - R_u \cdot \sigma'_{i0} \cdot \tan \phi'_i) \cdot b_i + (W_i + \Delta V_i - (A_{1i} - A_{2i} \cdot K_y) \cdot b_i) \tan \phi'_i] - (W_i + \Delta V_i) \tan \alpha_i - K_y \cdot W_i \quad (\text{A25})$$

By substituting Eq. (A25) into Eq. (A14), expression of K_y can be obtained:

$$K_y = \frac{\sum_{i=1}^n \left(\frac{1}{m_\alpha \cdot \cos \alpha_i} \cdot [(c_i \cdot b_i + (W_i + \Delta V_i - (A_{1i} + R_u \cdot \sigma'_{i0}) \cdot b_i) \tan \phi'_i] - (W_i + \Delta V_i) \tan \alpha_i \right)}{\sum_{i=1}^n \left(W_i - \frac{A_{2i} \cdot \tan \phi'_i \cdot b_i}{m_\alpha \cdot \cos \alpha_i} \right)} \quad (\text{A26})$$

Iteration Procedure to Evaluate the K_y Value

It needs a process of trial-and-error iteration to find K_y in Eq. (A26). The process which is similar to that used in JGS (2006), involves Eqs. (A25), (A26) and (A16):

1) Assuming $\Delta V_i = 0$ in Eq. (A26), substitute other parameters into this equation to find $(K_y)_0$.

2) Substitute $\Delta V_i = 0$ and $(K_y)_0$ into Eq. (A25) to find ΔH_i , and calculate horizontal forces H_i on each slice by $H_i = \sum \Delta H_{i-1}$;

$$\Delta u_i = \Delta U_i / l_i = R_u \cdot \sigma'_{i0} \quad (\text{A21})$$

where σ'_{i0} is effective stress σ'_i at normal state without earthquake, which is obtained by the original Janbu method.

Deduction of Yield Seismic Coefficient

Expression of S_i (Eq. (A22)) can be deduced from Eqs. (A17) and (A20).

$$S_i = c_i \cdot l_i + \sigma'_i \cdot \tan \phi'_i \cdot l_i = c_i \cdot l_i + (N_i / l_i) \cdot \tan \phi'_i \cdot l_i \quad (\text{A22})$$

Equation (A23) can be deduced from Eqs. (A12) and (A13):

$$S_i \cdot \sin \alpha_i = W_i - (N_i + U_i + \Delta U_i) \cdot \cos \alpha_i + \Delta V_i \quad (\text{A23})$$

After eliminating N_i in Eqs. (A22) and (A23), and substituting the expressions of U_i (Eq. (A19)) and ΔU_i (Eq. (A21)), expression of S_i can be obtained: