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A special complex-valued simplicial canonical piecewise linear function for amplifier and predistorter nonlinearity representation

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Abstract: In this paper, a special complex-valued simplicial canonical piecewise linear (CSCPWL) function is presented for power amplifier and digital predistorter nonlinearity representation. The proposed function is derived from the simplicial canonical piecewise linear (SCPWL) function approximates to polynomial basis and can be easily applied for baseband modeling with complex-valued signal. Through the experiment simulation, the modeling capacity, algorithm complexity and numerical stability of the proposed function are discussed. Finally, the conclusion that the special CSCPWL function is a compromise between the system performance and hardware cost is given.

Keywords: simplicial canonical piecewise linear function (SCPWL), polynomial function, nonlinear modeling, normalized mean square error (NMSE).
Classification: Microwave and millimeter wave devices, circuits, and systems

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1 Introduction

Piecewise linear function is an approximate representation of a nonlinear function. It is an attractive tool for nonlinear modeling. It replaces the global nonlinear function by a series of linear subfunctions that are defined in properly partitioned subregions of the original definition region of the nonlinear function. Recently, a piecewise linear function known as the simplicial canonical piecewise linear (SCPWL) function has been proposed for modeling power amplifier and digital predistorter characteristics [1, 2, 3].

The conventional SCPWL function in [4] only fits for R^1 domain and allows the representation of any arbitrary continuous nonlinearity between two real-valued variables. Thus, the polar structure becomes the inevitable choice for baseband complex-valued signal modeling, especially in the digital predistortion system. In the polar structure, the coordinate conversion and independent representation of amplitude and phase will increase hardware cost and implementation difficulty. For reducing the cost, complex-valued SCPWL (CSCPWL) function, which is a high level canonical piecewise linear function, is introduced in [5] and [6]. The CSCPWL function can be directly utilized for baseband data processing without coordinate conversion and can represent the whole nonlinearity of amplitude and phase. However, compare with SCPWL, CSCPWL has more parameters number and higher algorithm complexity. If we assume the subregions (sectors) number is P, the polar structure of SCPWL shall have $2(1 + P)$ parameters, but the parameters number of CSCPWL function shall be $1 + 2P + P^2$. All in all, both functions suffer from different drawbacks. So the goal of the work is to find a special CSCPWL function that realizes the trade-off between hardware cost and algorithm complexity.

2 Special CSCPWL function description

Recently, complex-valued polynomial model is one of the most common functions for amplifier and predistorter modeling which is widely discussed and studied in much paper. It can be expressed as

$$y(n) = \sum_{k=1}^{K} a_k x(n)|x(n)|^{k-1} \tag{1}$$

where $x(n)$ and $y(n)$ are the input and output complex-valued signal, a_k $(k = 1, 2, \ldots, K)$ are the complex-valued coefficients and K is the polynomial order. The $|x(n)|$ represents the absolute value calculation of $x(n)$. From the Eq. (1), assuming a_k priori known, we can attain the transfer function of

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amplifier or predistorter.

$$\frac{y(n)}{x(n)} = \sum_{k=1}^{K} a_k |x(n)|^{k-1} = a_1 + a_2|x(n)| + \cdots + a_K |x(n)|^{K-1} \tag{2}$$

The transfer function can be described as the linear combination of a series of basis, $|x(n)|^k$. According to SCPWL characters, we can adopt SCPWL function to represent the basis as follows.

$$|x(n)|^k \approx b_{k0} + \sum_{i=1}^{P} b_{ki}\lambda_i(|x(n)|) = \mathbf{b_k^T}\boldsymbol{\lambda}(|\mathbf{x(n)}|) \tag{3}$$

where $|x(n)|$ and $|x(n)|^k$ are the input and output of SCPWL function, P is the sectors number, $\mathbf{b_k} = [b_{k0}, b_{k1}, \ldots, b_{kP}]^T$ is the real-valued weight vector, $\boldsymbol{\lambda}(|\mathbf{x(n)}|) = [1, \lambda_1(|x(n)|), \ldots, \lambda_P(|x(n)|)]^T$ is the SCPWL basis vector. The SCPWL basis is described as

$$\lambda_i(|x(n)|) = \begin{cases} (|x(n)| - \beta_i + ||x(n)| - \beta_i|)/2, & |x(n)| \le \beta_{P+1} \\ (\beta_{P+1} - \beta_i + |\beta_{P+1} - \beta_i|)/2, & |x(n)| > \beta_{P+1} \end{cases} \tag{4}$$

Because the sectors number is P, the amplitude dynamic range of input signal has P + 1 breakpoints. Then the breakpoints $\boldsymbol{\beta} = [\beta_1, \beta_2, \ldots, \beta_{P+1}]^T$ can be predefined and partitioned by equal interval.

Hence, substitute Eq. (3) into Eq. (2), the Eq. (2) can be rewritten as

$$\begin{aligned} \frac{y(n)}{x(n)} &= a_1 + a_2|x(n)| \cdots + a_K|x(n)|^{K-1} \\ &= a_1 + a_2\left(b_{10} + \sum_{i=1}^{P} b_{1i}\lambda_i(|x(n)|)\right) \cdots \\ &\quad + a_K\left(b_{(K-1)0} + \sum_{i=1}^{P} b_{(K-1)i}\lambda_i(|x(n)|)\right) \\ &= (a_1 + a_2 b_{10} \cdots + a_K b_{(K-1)0}) + \sum_{i=1}^{P}\left(\sum_{k=2}^{K} a_k b_{(k-1)i}\right)\lambda_i(|x(n)|) \\ &= c_0 + \sum_{i=1}^{P} c_i \lambda_i(|x(n)|) \end{aligned} \tag{5}$$

From Eq. (5), we get the transfer function that is the linear combination of the SCPWL basis and the complex-valued weights $c_i (i = 0, 1, \ldots, P)$. So the special CSCPWL function is

$$y(n) = c_0 x(n) + \sum_{i=1}^{P} c_i \lambda_i(|x(n)|) x(n) = \mathbf{c^H g(n)} \tag{6}$$

where $\mathbf{g(n)} = [x(n), \lambda_1(|x(n)|)\ x(n), \ldots, \lambda_P(|x(n)|)\ x(n)]^T$ is the special CSCPWL basis vector. The number of parameters is 1 + P, which less than SCPWL and CSPWL.

As mentioned above, the special CSCPWL function, whose implementation structure is shown as Fig. 1, is derived from the SCPWL function approximates to polynomial basis. The proposed function can be described

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as the linear representation of the special CSCPWL basis. The basis is calculated by the input signal and breakpoints $\boldsymbol{\beta}$. The weight vector can be extracted by the linear filter theories, such as LS, RLS, LMS, etc. The special CSCPWL function is based on memoryless polynomial model, so we can easily extend the memory special CSCPWL function, as Eq. (7), according to memory polynomial model.

$$y(n) = \sum_{q=0}^{Q}\sum_{k=1}^{K} a_{kq} x(n-q)|x(n-q)|^{k-1} = \sum_{q=0}^{Q} y_q(n-q) = \sum_{q=0}^{Q} \mathbf{c_q^H g(n-q)} \quad (7)$$

where Q is memory depth, $\mathbf{g(n-q)} = [\mathrm{x(n-q)}, \lambda_1(|\mathrm{x(n-q)}|)\ \mathrm{x(n-q)}, \ldots, \lambda_P(|\mathrm{x(n-q)}|)\ \mathrm{x(n-q)}]^T$ is the special CSCPWL basis vector of q-th memory branch, and $\mathbf{c_q}$ is corresponding weight vector. So the special CSCPWL function can also model the amplifier and predistorter nonlinearity with memory.

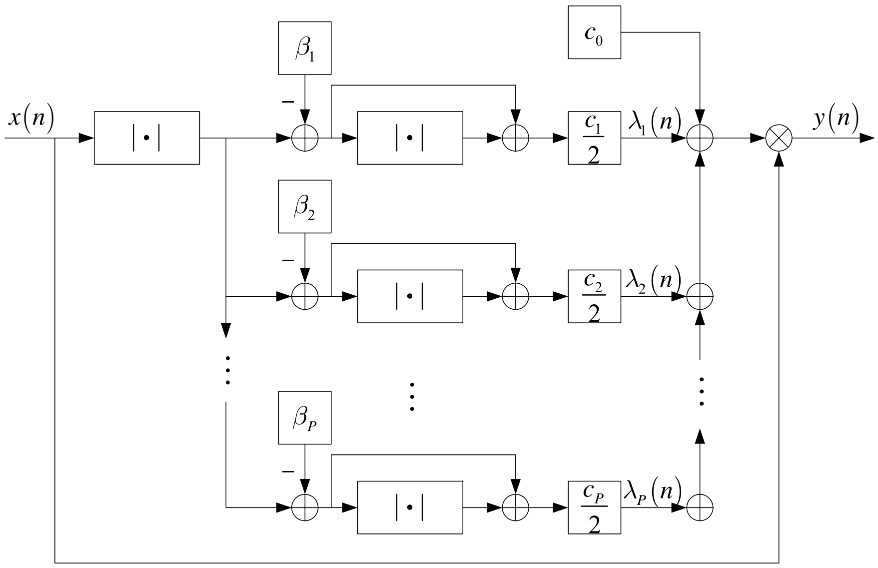

Fig. 1. The block diagram of special CSCPWL structure.

3 Simulation results and discussion

The foundation of special CSCPWL function is SCPWL function approximates to polynomial basis, as (3). Fig. 2 (a) shows the different polynomial basis, such as $|x(n)|^4$, $|x(n)|^8$ and $|x(n)|^{12}$, approximated by the SCPWL function, and consequently verifies the validity of approximation. And Fig. 2 (b) displays the modeling capability of different functions, including SCPWL, CSCPWL, special CSCPWL, polynomial function only with odd orders (Polynomial_1) and polynomial with odd and even orders (Polynomial_2), for Saleh nonlinearity behavioral model that adapts NMSE as figure of merit. The graph shows the Polynomial_2 has the best capability when the order is 10, while special CSCPWL takes the second place.

For the same input signal and same sectors or orders, the condition number of class-correlation matrix is Polynomial_2 > Polynomial_1 > CSCPWL >

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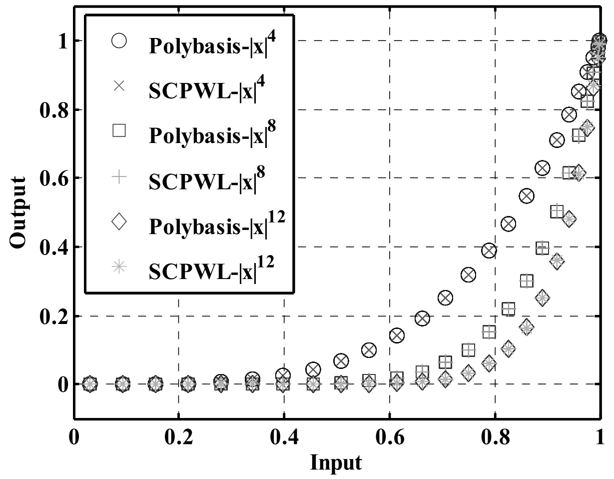
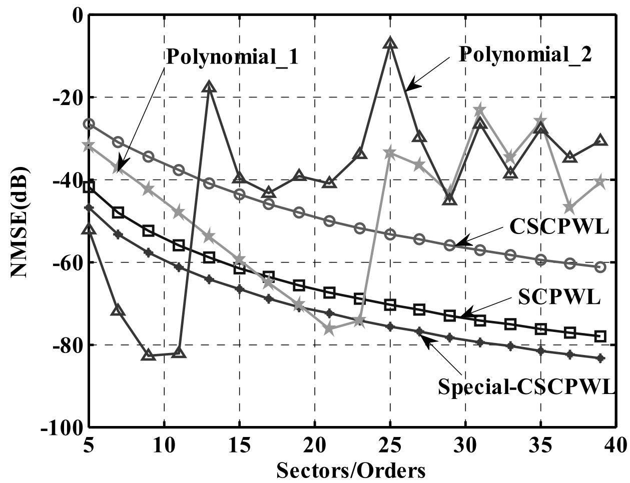


(a)



(b)

Fig. 2. Modeling capacity comparison: (a) SCPWL function approximates to polynomial basis $|x(n)|^4$, $|x(n)|^8$ and $|x(n)|^{12}$; (b) The modeling capability of different function.

special CSCPWL > SCPWL. So the numerical stability of polynomial function is poor, which can also be proved from the figure that the NMSE curves of polynomial dramatic changes as order increasing. Through the simulation results and discussion, the special CSCPWL function is not optimal for any aspects among NMSE, numerical stability and parameters number, but the proposed function is the compromise of them.

The Fig. 3 shows the linearization performance of memory special CSCPWL predistorter which express as Eq. (7). The predistorter not only compensates the nonlinearity of amplifier, but also corrects the memory distortion. The figure further proves the efficiency of the special CSCPWL function which can flexibly represents kinds of amplifier and predistorter.

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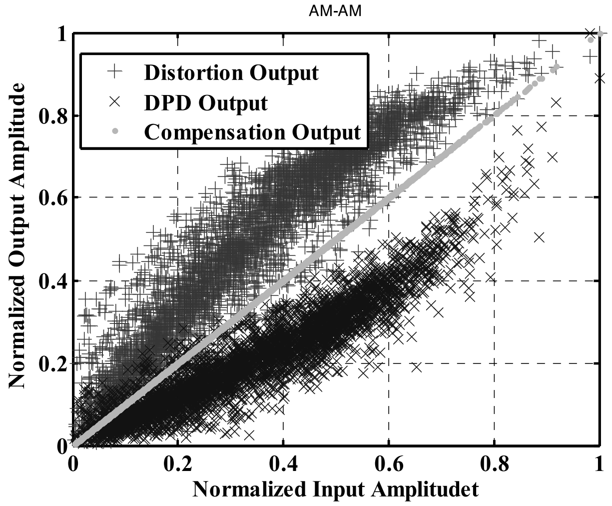
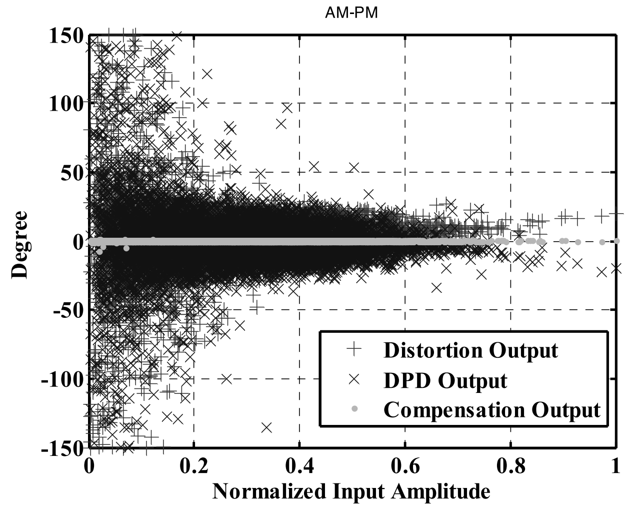


(a)



(b)

Fig. 3. The memory special CSCPWL predistorter compensates for the nonlinearity and memory distortion of power amplifier.

4 Conclusions

In the letter, we present a special CSCPWL function which has less model parameters, higher modeling capability, and better numerical stability. The function is a compromise between the system performance and hardware cost. And the model can be easily applied to more complex structure, such as parallel memory structure, Hammerstein and Wiener structure, for amplifier model and predistortion.

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