

# Design and Analysis of dual-band matching networks using composite right/left handedness for arbitrary impedances

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**Abstract:** In this paper, a mathematical design for dual-band matching of arbitrary impedances is presented based on Composite Right/Left Handedness (CRLH) and conventional transmission line theories. This design process of transforming arbitrary impedances to 50 ohms at two frequencies suggests an analytic approach to dual-band matching for multi-band devices. As well, bandwidth analysis of the proposed method is presented for wideband applications. For the verification, CRLH transmission lines were built successfully to match the output impedances of a commercial transistor at 824 MHz and 2.5 GHz. This work is expected to be useful for the extraction of general solutions for the dual-band matching network of arbitrary impedances.

**Keywords:** Composite Right/Left Handedness, dual-band, matching, transmission line

**Classification:** Microwave and millimeter wave devices, circuits, and systems

## References

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## 1 Introduction

Today's trend of supporting global roaming in mobile terminals necessitates multi-band capabilities in the radio frequency paths. However, this multi-band support is done mostly by switching of separate signal paths for different frequencies, which requires a redundancy of components and a large amount of layout space. In most cases, dual-band circuits have quite separate impedances at two frequencies due to the frequency dependency of the parasitic components in devices. This makes it difficult to find general solutions for a perfect matching. So far, though a few analytic approaches for dual-band matching have been done, they are mostly case-by-case approaches or for real-valued impedances [1, 2, 3]. Even Composite Right/Left Handedness (CRLH)-based matching has been mainly focused on dual-band phase shift [4]. Therefore, in this paper, a general design method to match analytically arbitrary impedances at two frequencies is given, which is based on the CRLH and conventional transmission-line theories. The proposed idea was then verified with a commercial transistor using both simulation and measurements.

## 2 Step 1: Matching with conventional transmission line

At each frequency, the conventional stub-tuning technique is applied to match arbitrary load impedances to the 50 ohm source termination at each frequencies so that the required dual-band property can be achieved by merging these results into the CRLH transmission line theory.

Assuming that the load impedance at the first frequency is defined as  $Z_{L1} = R_1 + jX_1 = 1/(G_L + jB_L)$  at  $f_1$ , the initial step is to transform  $Z_{L1}$  into  $R_o + jX$  with the feeding transmission line of  $Z_o$  and the phase shift,  $\Phi_{feed}$  as shown in Fig. 1[5]. The impedance  $Z_{feed}(= R + jX)$  at the input of the feeding line can be calculated as

$$R = \frac{G_L(1 + \tan^2(\Phi_{feed}))}{G_L^2 + (B_L + \tan(\Phi_{feed})/Z_o)^2} \quad (1a)$$

$$X = \frac{G_L^2 \tan(\Phi_{feed}) - (Y_o - B_L \tan(\Phi_{feed}))(tY_o + B_L)}{Y_o [G_L^2 + (B_L + \tan(\Phi_{feed})Y_o)^2]}, \quad (1b)$$

where  $Y_o = 1/Z_o$ .

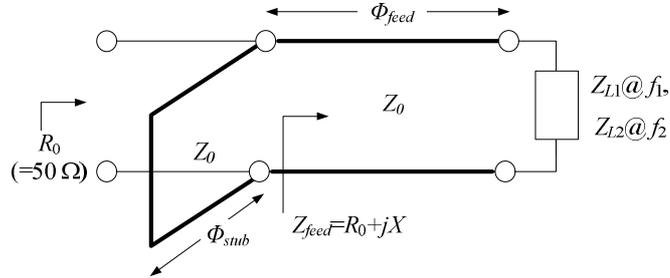
The transmission line length is chosen to meet  $R = R_o$ , and thereby the phase shift for the quadratic equation of Eq. (1a) is calculated as

$$\Phi_{feed} = \tan^{-1} \left( \frac{B_L \pm \sqrt{G_L [(Y_o - G_L)^2 + B_L^2] / Y_o}}{G_L - Y_o} \right), \text{ for } G_L \neq Y_o. \quad (2)$$

Then, the phase shift  $\Phi_{sub1}$  of the shunt stub to cancel the reactance in Eq. (1b) is calculated at  $f_1$  as

$$\Phi_{sub1} = -\tan^{-1} \left( \frac{X}{Z_o} \right). \quad (3)$$

Similarly, for the second impedance at  $f_2$  of  $Z_{L2} = R_2 + jX_2 = 1/(G_{L2} + jB_{L2})$ , two more phase shifts ( $\Phi_{feed2}$  and  $\Phi_{stub2}$ ) are calculated to match to 50 ohm. As a result, two phase-shifts at each frequency are calculated to match arbitrary impedances using conventional transmission lines.



**Fig. 1.** Matching with the series-shunt stub tuning method.

### 3 Step 2: Matching with CRLH transmission lines

A CRLH transmission line in a balanced condition can be designed precisely to have roughly independent phase shifts at two frequencies based on the purely left-handed (PLH) and the purely right-handed (PRH) transmission line properties. Hence, from the relations of resonating components of the unit cell of Fig. 2 (a) to the propagation constant, characteristic impedance, and the balance condition, three equations can be set as below [4].

$$\beta^{\text{CRLH}} = \omega \sqrt{L_R C_R} - \frac{1}{\omega \sqrt{L_L C_L}} \quad (4)$$

$$Z_t = \sqrt{\frac{L_R}{C_R}} \quad (5)$$

$$Z_t = \sqrt{\frac{L_L}{C_L}} \quad (6)$$

where  $L_R$ ,  $C_R$ ,  $L_L$ , and  $C_L$  are resonating components for the unit cell, and  $Z_t$  is the characteristic impedance of the CRLH transmission line.

Additionally, the second propagation constant at the second frequency sets the fourth equations for the unknowns to solve for the  $L_S$  and  $C_S$  for the CRLH cell. Thus, when the dual-band CRLH transmission lines are built with the  $N$  repetition of the unit cell, the final components can be found as below [4].

$$L_R = \frac{Z_t [\Phi_1(\omega_1/\omega_2) - \Phi_2]}{N\omega_2 [1 - (\omega_1/\omega_2)^2]} \quad (7a)$$

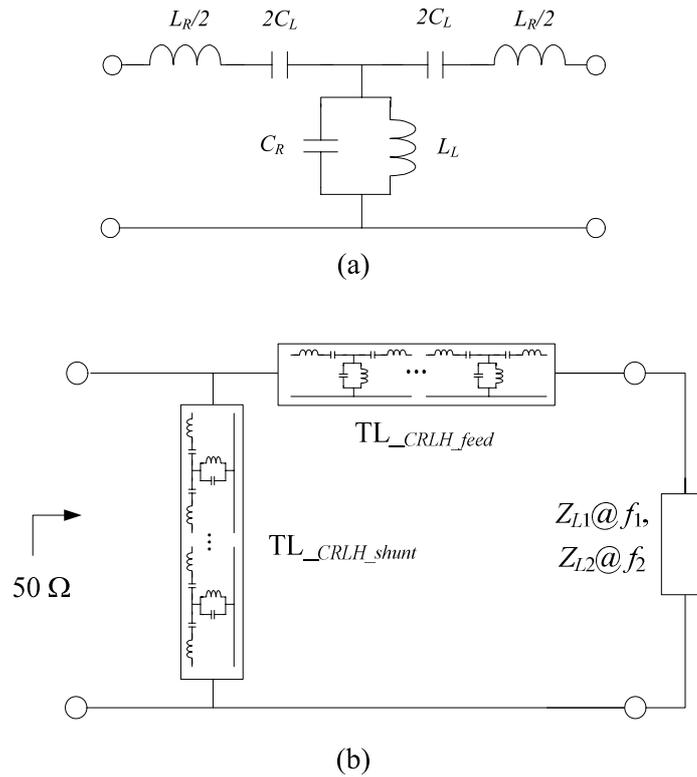
$$C_R = \frac{[\Phi_1(\omega_1/\omega_2) - \Phi_2]}{N\omega_2 Z_t [1 - (\omega_1/\omega_2)^2]} \quad (7b)$$

$$L_L = \frac{N Z_t [1 - (\omega_1/\omega_2)^2]}{\omega_1 [\Phi_1 - \Phi_2(\omega_1/\omega_2)]} \quad (7c)$$

$$C_L = \frac{N [1 - (\omega_1/\omega_2)^2]}{\omega_1 Z_t [\Phi_1 - \Phi_2(\omega_1/\omega_2)]} \quad (7d)$$

where  $\phi_k = -\beta_k^{CRLH}$  ( $k = 1, 2$ ).

Finally, the stub-tuning phase shifts from (2) and (3) are merged into (7a)~(7d) to design a dual-band impedance matching network, which is based on the dual-band phase shifts of the CRLH. The final matching network is structured as shown in Fig. 2 (b).



**Fig. 2.** (a) The symmetric unit cell of the CRLH transmission line. (b) Simplified structure of CRLH dual-band matching network.

#### 4 Matching sensitivity over frequency

In previous sections, it was shown that the exact matching conditions can be found by stub tuning at each frequency, and the CRLH theory can make dual-band solution using the dual-band phase shift properties. Subsequently, to check out the frequency dependency of the circuit, the performance of dual-band matching networks with CRLH transmission lines is analyzed. For simplicity, the analysis on the matching sensitivity over frequency was performed on the CRLH feeding transmission line, on which the reflection coefficient can be estimated based on transmission line theory [4]:

$$\begin{aligned} \Gamma_{in} &\cong \Gamma_1 + \Gamma_L \exp(-j2\Phi_{feed}) \\ &= \frac{Z_t - Z_o}{Z_t + Z_o} + \frac{Z_L - Z_t}{Z_L + Z_t} \cdot \exp \left\{ -j2 \left( \omega Z_t C_R - \frac{C_L}{Z_t \omega} \right) \right\}, \end{aligned} \quad (8)$$

where  $Z_o$  is the characteristic impedance of the transmission line.

Assuming that the  $Z_o$  and  $Z_t$  are identical, the matching sensitivity of the structure can be calculated as

$$\text{Matching Sensitivity} = \left| \frac{\partial \Gamma_{in}}{\partial \omega} \right| \cong \left| \frac{Z_L - Z_t}{Z_L + Z_t} \right|^2 \left( Z_t C_R + \frac{C_L}{Z_t \omega^2} \right). \quad (9)$$

Referring to (9), it is interesting to note that the matching gets less sensitive as the frequency is increased with the given phase shift, from which it can be estimated that the matching at the higher frequency would be more tolerant than that at the lower frequency when all the other frequency-related conditions are assumed to be constant. Also, it can be noted that the influence on the sensitivity of the left-handed transmission, which is represented by  $C_L$ , diminishes as the frequency increases. As a result, the bandwidth of the matching network asymptotically approaches toward that of the conventional transmission line as the frequency gets higher.

## 5 Simulation and Measurement

For dual-band matching at  $f_1 = 824$  MHz and  $f_2 = 2.5$  GHz using a commercial transistor (ATF-53189) with input impedances of  $Z_{L1} = 19.76 - j4.48$  at  $f_1$  and  $Z_{L2} = 22 + j8.27$  at  $f_2$ , the required phase shifts for the shunt and series transmission lines are calculated with (2) and (3), resulting in  $\Phi_{feed1} = -38.084^\circ$ ,  $\Phi_{feed2} = -201.618^\circ$ ,  $\Phi_{stub1} = 45.797^\circ$ , and  $\Phi_{stub2} = -131.357^\circ$ .

Based on these values and by using (7a)~(7d), CRLH transmission lines were implemented with the lumped  $L$  and  $C$  components for the left-handed section whereas conventional transmission lines were used for the right-handed section. Since the circuits with more than two cells for both the feeding and the stub transmission lines all showed successful results in simulation,  $N = 2$  was chosen for the CRLH transmission lines and the resulting  $L$  and  $C$  values are as following:

For TL-CRLH-feed,

$L_R = 5.8919$  (nH),  $C_R = 2.357$  (pF),  $L_L = 34.772$  (nH), and  $C_L = 13.909$  (pF).

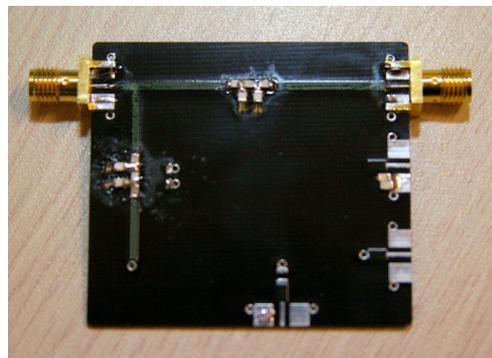
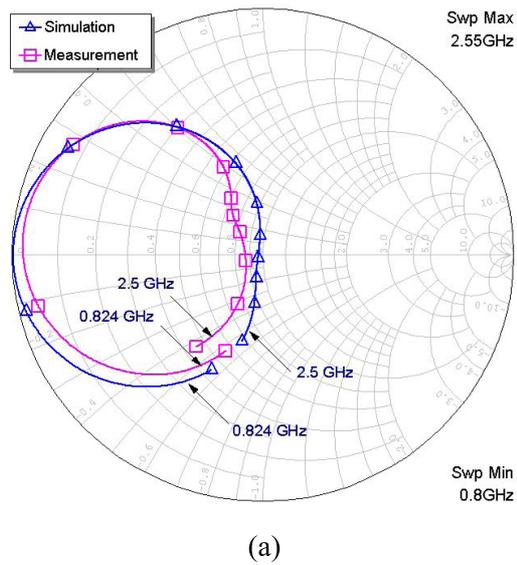
For TL-CRLH-shunt,

$L_R = 4.564$  (nH),  $C_R = 1.826$  (pF),  $L_L = 11.072$  (nH), and  $C_L = 4.428$  (pF).

Fig. 3(a) shows the simulated and measured reflection coefficients of the dual-band matching network, and it is evident that arbitrary-impedance matching at dual-frequencies is successful with the CRLH transmission line of  $N = 2$ . Fig. 3(b) shows the implemented matching network at 824 MHz and 2.5 GHz for the transistor.

## 6 Conclusion

In order to design dual-band matching networks with arbitrary load impedances, an analytic design method combined with the CRLH theory and conventional transmission line theory is presented for the first time. Initially, with the given frequencies and load impedances, the analytic single-frequency matching solutions are extracted by the series and shunt transmission lines



**Fig. 3.** (a) Measured results of dual-band network to match ATF-53189 transistor at 824MHz and 2.5GHz. (b) Manufactured CRLH dual-band network to match a transistor at 824MHz and 2.5 GHz.

at each frequency, and these solutions are merged by the CRLH transmission lines by using their dual-frequency phase shifting property. Then, the matching sensitivity over frequency is analyzed to estimate the bandwidth of the designed circuit. In order to verify this concept, a commercial transistor was matched at 824Hz and 2.5 GHz using the suggested design method. As a result, the designed circuit successfully matched the complex impedances to 50 ohms at the desired frequencies with reasonable bandwidths. In an actual circuit design, a relatively large number of resonating elements can be alleviated if distributed components were chosen for reactive elements. This concept provides accurate solutions to dual-band matching with arbitrary impedances at two frequencies.

## **Acknowledgments**

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