

# Fast transient analysis method for lossy nonuniform transmission line with nonlinear terminations

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**Abstract:** A fast and effective approach for simulating lossy nonuniform transmission line with the nonlinear terminations is presented. The method based on the improved delay extraction-based macromodeling algorithm (DEPACT) for the lossy uniform transmission line. Meanwhile, by reducing the approximation order, the macromodel is simplified and the simulation time is reduced. The simulation results are compared with the finite difference time domain (FDTD) method and good agreement is obtained.

**Keywords:** fast, transient analysis, lossy nonuniform, transmission line, nonlinear terminations, macromodel

**Classification:** Electromagnetic theory

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## 1 Introduction

The lossy nonuniform transmission lines widely exist in the high speed interconnect systems [1, 2, 3]. The FDTD method is a common way to obtain the transient response of the lossy nonuniform transmission lines [4, 5, 6, 7]. But it becomes complex when the termination is nonlinear load. Combining the spice models of the nonlinear devices, the conventional lumped models can simulate the transient response of the nonlinear terminations. But, as the frequency of operation increases, the conventional lumped models become inadequate. The method will lead to large circuit matrices and the simulation becomes inefficient [8, 9, 10]. Macromodeling of the transmission line is an efficient technique to solve these problems. The method of characteristics (MoC) and the matrix rational approximation (MRA) are two popular techniques for macromodeling. The MoC can get the transient results for long delay lines. Nevertheless, it can not guarantee the passivity of the macromodels. The MRA can provide efficient macromodels for lossy uniform transmission lines. However, it need higher order approximations for the long lossy delay lines [5]. A passive delay extraction-based macromodeling algorithm (DEPACT) has been introduced for simulation of long lossy uniform lines [11, 12]. But the macromodeling of the lossy nonuniform transmission lines have not been reported.

In this paper, we improved the DEPACT for lossy uniform lines. The lossy nonuniform transmission line is divided into a sequence of smaller segments, each of which is approximate uniform. The improved DEPACT macromodel for every segment is built. Then the subcomponent macromodels for every segment are combined to form the macromodel of the lossy nonuniform transmission lines. If the length of the segment is small enough, the approximated order can be 1. The macromodel will be simplified, and the simulation time will be reduced. This method can be used to simulate the transient response of the lossy nonuniform transmission lines with nonlinear terminations fast.

## 2 Proposed macromodeling algorithm for the lossy nonuniform transmission line

Consider the distributed lossy uniform transmission lines represented by the Telegrapher's equation as [13, 14]

$$\begin{aligned}\frac{\partial V(z, t)}{\partial z} &= -RI(z, t) - L \frac{\partial I(z, t)}{\partial t} \\ \frac{\partial I(z, t)}{\partial z} &= -GV(z, t) - C \frac{\partial V(z, t)}{\partial t}\end{aligned}\tag{1}$$

where  $V(z, t)$ ,  $I(z, t)$  are the voltage and current of the transmission lines as a function of position  $z$  and time  $t$ ,  $R$ ,  $L$ ,  $G$ ,  $C$  are the per-unit-length parameter matrices.

The solution of Eq. (1) can be expressed in the matrix-exponential form, in the frequency domain as

$$\begin{bmatrix} V(d, s) \\ I(d, s) \end{bmatrix} = e^{(A+sB)d} \begin{bmatrix} V(0, s) \\ I(0, s) \end{bmatrix} \quad (2)$$

$$A = \begin{bmatrix} 0 & -R \\ -G & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -L \\ -C & 0 \end{bmatrix}$$

where  $d$  is the length of the line.

The exponential term can be approximated using the DEFACT. In the process of building the macromodels of the lossy uniform transmissions, the per-unit-length parameter matrices will be changed with the approximated order in [11]. In this paper, we improved the approximated equation as

$$e^{(A+sB)d} \cong \prod_{k=1}^m (e^{sB \frac{d}{2m}} e^{A \frac{d}{m}} e^{sB \frac{d}{2m}})_k + \varepsilon_m \quad (3)$$

where  $m$  represents the approximation order,  $\varepsilon_m$  is the error. The  $\varepsilon_m$  will be smaller when the  $d/m$  is smaller. In Eq. (3), the per-unit-length parameter matrices are irrelevant with the  $m$ .

The macromodel of the lossy uniform transmission line is considered as a cascade of  $m$  transmission line subnetworks. Each transmission line subnetwork is represented by a cascade of lossy and lossless transmission line equivalent circuit. The length of the lossy section is  $d/m$ , and the equivalent circuit of the lossy section is purely resistive networks as [12]. The length of the lossless section is  $d/2m$ , and the equivalent circuit as [14].

For the lossy nonuniform transmission lines, the per-unit-length parameter matrices vary with the position  $z$ . The lossy nonuniform transmission lines can be divided into a sequence of smaller segments, each of which is approximate uniform. The per-unit-length parameter matrices of every segment are regarded as constant value. Each segment is considered as a cascade of  $m_0$  transmission line subnetworks. as

$$\begin{aligned} e^{(A(z)+sB(z))d} &\approx \underbrace{e^{(A_1+sB_1)\frac{d}{n_0}} e^{(A_2+sB_2)\frac{d}{n_0}} \dots e^{(A_{n_0}+sB_{n_0})\frac{d}{n_0}}}_{n_0} \\ &\approx \underbrace{\prod_{k=1}^{m_0} (e^{sB_1 \frac{d}{2m_0 n_0}} e^{A_1 \frac{d}{m_0 n_0}} e^{sB_1 \frac{d}{2m_0 n_0}})_k \prod_{k=1}^{m_0} (e^{sB_2 \frac{d}{2m_0 n_0}} e^{A_2 \frac{d}{m_0 n_0}} e^{sB_2 \frac{d}{2m_0 n_0}})_k \dots \prod_{k=1}^{m_0} (e^{sB_{n_0} \frac{d}{2m_0 n_0}} e^{A_{n_0} \frac{d}{m_0 n_0}} e^{sB_{n_0} \frac{d}{2m_0 n_0}})_k}_{n_0} \end{aligned} \quad (4)$$

where  $n_0$  is the number of the segments. And each segment is closer approximate uniform when the  $d/n_0$  is smaller. Consequently, the macromodel of the lossy nonuniform transmission is considered as a cascade of  $n_0$  lossy uniform transmission line segment macromodel. The length of the lossy section is  $d/m_0 n_0$ , and the lossless section is  $d/2m_0 n_0$ . If the  $m_0$  is 1 and the  $n_0$  is large enough. the macromodel will be simplified as

$$e^{(A(z)+sB(z))d} \approx \underbrace{(e^{sB_{1\frac{d}{2N}}A_{1\frac{d}{N}}e^{sB_{1\frac{d}{2N}}}})(e^{sB_{2\frac{d}{2N}}A_{2\frac{d}{N}}e^{sB_{2\frac{d}{2N}}}}) \dots (e^{sB_{N\frac{d}{2N}}A_{N\frac{d}{N}}e^{sB_{N\frac{d}{2N}}}})}_N \quad (5)$$

The precision can be guaranteed when the  $N \geq n_0 m_0$ .

### 3 Method validation

In order to prove the accuracy and efficiency of the proposed method, a lossy nonuniform transmission network is considered as Fig. 1. The ‘Vs’ is the active voltage source, which is a trapezoidal pulse with rise/fall time of 0.5 ns, pulse width of 30 ns, and amplitude of 200 V. The ‘D’ represents the transient voltage suppressor, and the type is 1.5KE39CA. The values of resistance and capacitance are shown in the figure. The parameters of T1 and T2 are the same as [6]

$$\mathbf{R} = \begin{bmatrix} r(z) & 0 \\ 0 & r(z) \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} l(z) & l_m(z) \\ l_m(z) & l(z) \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} c(z) & c_m(z) \\ c_m(z) & c(z) \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} g(z) & 0 \\ 0 & g(z) \end{bmatrix}.$$

where

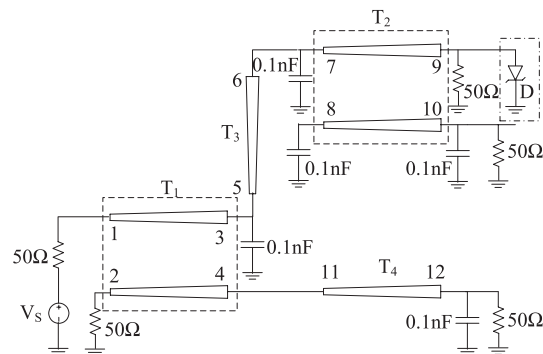
$$r(z) = \frac{30}{[1 + k(z)]} \Omega/\text{m}, \quad l(z) = \frac{387}{[1 + k(z)]} \text{ nH/m},$$

$$c(z) = \frac{104.3}{[1 - k(z)]} \text{ pF/m}, \quad g(z) = \frac{0.001}{[1 - k(z)]} \text{ S/m},$$

$$l_m(z) = k(z)l(z) \text{ nH/m}, \quad c_m(z) = -k(z)c(z) \text{ pF/m},$$

$$k(z) = 0.25[1 + \sin(6.25\pi z + 0.25\pi)].$$

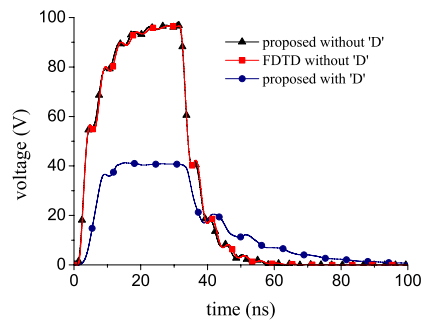
The parameters of T3 and T4 are  $r(z)$ ,  $l(z)$ ,  $c(z)$ ,  $g(z)$ . The length of T1, T2, T3, and T4 is 0.04 m. We divided each line of the network into ten segments. In other words, the  $N$  is 10.



**Fig. 1.** Lossy nonuniform transmission network

Fig. 2 shows the transient response at the end port ‘9’. As seen from the figure, the responses for the proposed method and the FDTD method are highly agreed when the ‘D’ does not exist. The computational time of the FDTD method is 20 s when the spatial step is 0.004 m and the time step is 0.01 ns. Moreover, the computational time of the proposed method is 8 s, which is less than the half of

the FDTD method when they are operated in the same condition. In addition, the amplitude of the response when the ‘D’ exists is lower than the result when the ‘D’ does not exist. It is below the maximum clamping voltage of the transient voltage suppressor, and which agree well with the theoretical analysis results. Therefore, from the above comparison, we can conclude that our proposed method is effective and fast to simulate the lossy nonuniform transmission with nonlinear terminations easily.



**Fig. 2.** Transient response at the end port ‘9’.

#### 4 Conclusion

Based on the improved DEFACT macromodels for the lossy uniform transmission lines, a new and fast method for simulating the lossy nonuniform transmission lines has been presented. For a lossy nonuniform transmission line network, the simulation results are good agreement with the FDTD method and the simulation time is much shorter. Consequently, using this approach, the transient response for the lossy nonuniform transmission with nonlinear terminations could be obtained fast and accurately.

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