ELectronics EXpress
IEICE Electronics Express, Vol.2, No.9, 316–320

Blind deconvolution by using phase spectral constraints and natural gradient

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Abstract: A new natural gradient-based algorithm for a blind deconvolution of blurred images is proposed. In the proposed algorithm, phase spectral constraint condition is newly introduced, and a natural gradient descent method on an amplitude spectrum is applied. The effectiveness of the proposed algorithm is confirmed by comparing with the conventional method.
Keywords: blind deconvolution, iterative method, cost function, modulo π phase, image restoration
Classification: Science and engineering for electronics

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1 Introduction

In case where a blurred image is obtained as the convolution of an original image and a point-spread function (PSF), which is a function to blur an image, the objective of a blind deconvolution is to restore the original image

DOI: 10.1587/elex.2.316
Received March 29, 2005
Accepted April 11, 2005
Published May 10, 2005

IEICE Electronics Express, Vol.2, No.9, 316–320

from the blurred image, by using only *a-priori* information such as the constraint conditions of nonnegativity of pixel value and/or a support constraint condition of an image.

The most commonly used method for blind deconvolution is an iterative method called A-D algorithm [1]. The A-D algorithm restores an original image and also a PSF itself by iteratively applying the above mentioned constraint conditions. This algorithm shows a good restoration performance, however, it is not stable and can not always restore the original image with good satisfaction.

To cope with those problems, the gradient-based method and its modified type have been proposed [2], [3]. In the modified gradient-based method, after introducing a cost function defined by the error in a frequency space, and by using the gradient descent method, the blind deconvolution is achieved so that this cost function becomes minimum. This algorithm is stable and shows a better restoration performance than A-D algorithm. However, it tends to be trapped at local minima solutions.

On the other hand, the zero sheet separation algorithm and its modified type have been proposed [4], [5]. These are analytical methods for the blind deconvoulution problem and can obtain good solution without being trapped to a local minimum when an observed image is not corrupted by a noise. However, those algorithms are very complicated and highly mathematical. Further, for an observed image corrupted by a noise, the zero sheet separation algorithm can not be applied easily, and its modified type does not guarantee theoretically the convergence to a global minimum, similar to the references [1], [2] and [3].

In this paper, in order to improve the restoration performance of the modified gradient-based method, we propose a natural gradient-based algorithm for an amplitude spectrum of images by newly employing the phase spectral constraints. It requires no additional observations, and realizes a high blind deconvolution performance.

The effectiveness of the proposed method is verified by applying it to some blind deconvolution problems of images.

2 Proposed blind deconvolution algorithm

Let a blurred image $g(x, y)$ be given by a convolution of an original image $f(x, y)$ and a PSF $h(x, y)$, where (x, y) is a position in an image space. The values of $f(x, y)$ and $h(x, y)$ are assumed to be real and nonnegative. In this case, the Fourier transform $G(u, v)$ of $g(x, v)$ is given as follows:

$$G(u, v) = H(u, v)F(u, v), \tag{1}$$

where $H(u, v)$ and $F(u, v)$ are Fourier transforms of $h(x, y)$ and $f(x, y)$, respectively. If either $h(x, y)$ or $f(x, y)$ is known, the counterpart can be restored by using the inverse filter or by using the Wiener filter. The purpose here is to restore the original image $f(x, y)$ from only the information on the blurred image $g(x, y)$ without knowing PSF $h(x, y)$.

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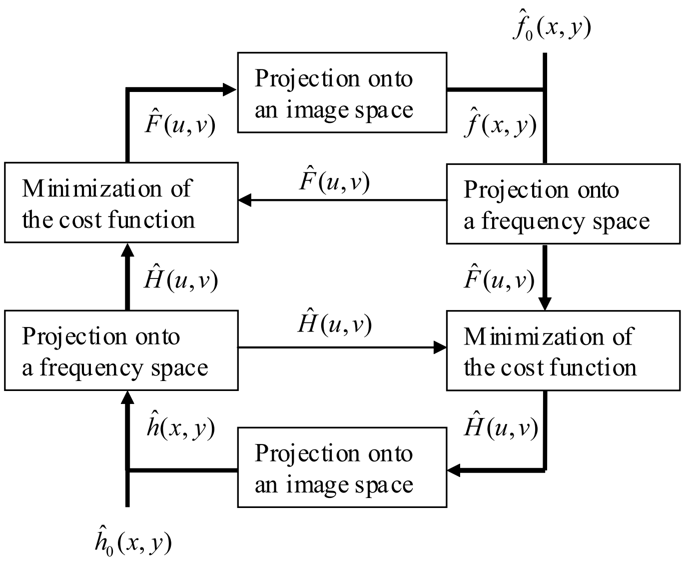

Fig. 1. Block diagram of the proposed algorithm

The proposed algorithm consists of the following three procedures, which are 1) a projection onto a frequency space imposing the constraint condition of phase spectrum, 2) minimization of a cost function, and 3) a projection onto an image space imposing the nonnegative and support constraint conditions. Fig. 1 shows a block diagram of the above proposed algorithm

In this algorithm, both of the initial estimates $\hat{h}_0(x,y)$ and $\hat{f}_0(x,y)$ are first set by random numbers. Then, the procedures 1), 2) and 3) are iterated in due order until both the estimates $\hat{h}(x,y)$ and $\hat{f}(x,y)$ converge. Owing to a page limitation, the procedures only for updating $\hat{h}(x,y)$ are described in the following section, though $\hat{f}(x,y)$ is also updated in the similar manner.

2.1 Projection onto a frequency space

In the procedure 1), $\hat{F}(u,v)$ is updated by using the proposed new constraint, i.e., a phase spectrum of the original image. This phase spectrum is obtained by the error reduction algorithm [6] for the amplitude spectrum of the blurred image, $|G(u,v)|$. However, its phase spectrum $\phi(u,v)$ is modulo π. Therefore, it must be determined which to employ $\phi(u,v)$ or $\phi(u,v)+\pi$. In the proposed algorithm, the closer spectrum to the phase spectrum of $\hat{F}(u,v)$ is chosen to update $\hat{F}(u,v)$. The update of $\hat{F}(u,v)$ is achieved by replacing the phase spectrum as:

$$\theta_{\hat{F}}^{new}(u,v) = \phi(u,v), \text{ or } \phi(u,v)+\pi, \tag{2}$$

where $\theta_{\hat{F}}^{new}$ is a phase spectrum for an estimated $\hat{F}(u,v)$.

2.2 Minimization of a cost function

In the procedure 2), $\hat{H}(u,v)$ is adjusted in the same manner as in [2], [3] so that the following cost function becomes minimum:

$$E = \sum_{u,v} |G(u,v) - \hat{H}(u,v)\hat{F}(u,v)|^2. \tag{3}$$

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Concretely, a natural gradient descent method [7] with respect to amplitude spectrum is employed here in order to obtain a fast convergence. Keeping the phase spectrum updated in section 2.1, $\hat{H}(u,v)$ is in here updated by:

$$\hat{H}^{new}(u,v) = \hat{H}^{old}(u,v) - \alpha_{\hat{H}} D_{\hat{H}}(u,v) \tag{4}$$

with

$$D_{\hat{H}}(u,v) = \frac{\partial E}{\partial |\hat{H}^{old}(u,v)|} |\hat{H}^{old}(u,v)|^T |\hat{H}^{old}(u,v)|, \tag{5}$$

where $\alpha_{\hat{H}}$ is a descent parameter, and T is a transpose operator. The descent parameter $\alpha_{\hat{H}}$ is determined to minimize the cost function.

After the update of $\hat{H}(u,v)$, the cost function becomes

$$E = \sum_{u,v} \left| G(u,v) - \hat{F}(u,v)\hat{H}^{new}(u,v) \right|^2 . \tag{6}$$

E is a quadratic convex function with respect to $\alpha_{\hat{H}}$. For the minimization of the above cost function, $\alpha_{\hat{H}}$ should satisfy:

$$\frac{\partial E}{\partial \alpha_{\hat{H}}} = 0, \tag{7}$$

and then, $\alpha_{\hat{H}}$ is obtained as:

$$\alpha_{\hat{H}} = -\frac{\sum_{u,v} \Re\left[D_{\hat{H}}(u,v)\hat{F}(u,v)\left(G(u,v) - \hat{F}(u,v)\hat{H}^{old}(u,v) \right)^* \right]}{\sum_{u,v} \left| D_{\hat{H}}(u,v) \right|^2 \left| \hat{F}(u,v) \right|^2}, \tag{8}$$

where $\Re$ is an operator to extract the real part of the complex number, and $*$ means a complex conjugate.

2.3 Projection onto an image space

In the procedure 3), $\hat{h}(x,y)$ is obtained by an inverse Fourier transform of $\hat{H}(u,v)$. $\hat{h}(x,y)$ is then updated by imposing the nonnegative and support constraints as follows:

$$\hat{h}^{new}(x,y) = \begin{cases} \hat{h}^{old}(x,y), & (x,y) \notin \gamma_{\hat{h}} \\ 0, & (x,y) \in \gamma_{\hat{h}}, \end{cases} \tag{9}$$

where $\gamma_{\hat{h}}$ is a set of coordinates with negative pixel values and outside of the support of $h(x,y)$.

3 Results

The standard images often used for a bench mark test of an algorithm "Girl," "Lena," and "Barbara" are employed as the original images. Each image is constituted with 64×64 pixels, and its resolution is 8 bit/pixel gray-level. As a PSF, an image shown in Fig. 2 (a) is employed. This PSF is constituted with 7×5 pixels, and its resolution is the same as the original image. Fig. 2 (b) shows a blurred image "Barbara" produced by a convolution of the original image and the PSF shown in Fig. 2 (a). Fig. 2 (c) shows the restored image

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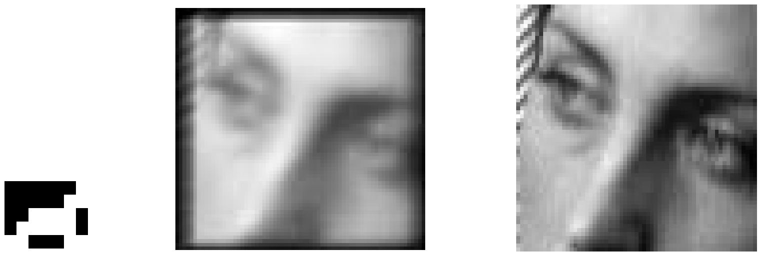

(a) (b) (c)

Fig. 2. Experimental results. (a) A point-spread function. (b) A blurred image "Barbara." (c) A restored image "Barbara."

by the proposed algorithm. From Fig. 2 (c), it is seen that the deconvolution is achieved with good precision.

Table I shows the average root mean square error (RMSE) in an image space for 30 trials achieved by changing the initial estimates of algorithm. For a comparison, the conventional modified gradient-based method [3] was also applied. The stability of convergence was better for the proposed algorithm, and also as for the restoration performance the proposed method was much superior to that of the conventional one.

Table I. The average RMSE obtained for 30 trials achieved by changing the initial estimates of algorithm.

Test images	Average RMSE	
	Modified gradient-based method	Proposed method
Girl	9.7×10^{-2}	8.3×10^{-4}
Lena	1.3×10^{-1}	3.5×10^{-3}
Barbara	1.3×10^{-1}	1.4×10^{-3}

4 Conclusions

In this paper, a new blind deconvolution method based on a natural gradient descent method has been proposed, which has the support and nonnegativity constraints and the additional constraints concerning the phase spectra of images.

Through the experiments, it has been confirmed that the proposed algorithm achieves the blind deconvolution far better than the conventional one. It can be said that the newly introduced constraint, i.e., phase spectra of images, helps converge to an optimal solution, not being trapped to a local minimum.

Future works are, in order for increasing the practicality of the proposed method, to remove the constraints, e.g., support of images, and to extend the algorithm to be applicable to the image corrupted by a noise.

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