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ON THE STRENGTH OF FIBRE-REINFORCED SOILS

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ABSTRACT

Fibre reinforced soils have been investigated for several decades and different models have been suggested to estimate their improved shear strength. The shear strength of such composite materials is affected by the micro and macro mechanical characteristics of both the fibres and the soils (e.g., relative sizes of fibres and soil grains, fibres aspect ratio, stress state, mechanical properties of the fibres), yet no model is available to explicitly take all of them into account. The aim of this work is to establish a new expression for the shear strength of the reinforced material, able to consider the main characteristics of the soil and the fibres as well as the effect of fibre to grains relative dimensions. Data from triaxial tests carried out on fibre reinforced soils with distinct grain size distributions (from clayey sands to sandy gravels) and from previous experimental works were considered and have been analysed successfully within the proposed framework.

Key words: failure envelope, fibres, micromechanical properties, reinforced soil, triaxial testing (**IGC**: D6/D10/M4)

INTRODUCTION

Experimental research conducted on fibre-reinforced materials (e.g., Michalowski and Cermák, 2003; Consoli et al., 2007a; Ahmad et al., 2010; Diambra et al., 2010) has demonstrated that the addition of discrete fibres improves the mechanical behaviour of granular soils, increasing their strength and ductility and reducing the post peak-strength loss. The macroscopic effect of reinforcement is governed by the content in fibres, their orientation, their geometrical (length L_f and diameter d_f) and mechanical (tensile strength and stiffness) characteristics, as well as the intrinsic (grading, mineralogy, grain shape) and state (density and applied stresses) properties of the soil (e.g., Gray and Al-Refeai, 1986; Consoli et al., 2009). Furthermore, experimental observations (e.g., Michalowski and Cermák, 2003) have highlighted that reinforcement is more effective when (for a given value of the aspect ratio $\rho = L_f/d_f$) the fibre length is large compared to the size of the grains. As the length of fibres reduces, their beneficial effect reduces as well, eventually fading away when it approaches the size of grains.

One of the main problems in the mechanical characterization of fibre reinforced materials is related with the difficulty of quantifying and taking into account the effect of fibre orientation. Diambra et al. (2007) have shown that the fibres in compacted specimens are preferentially oriented horizontally, and therefore the composite material is strongly anisotropic. Michalowski (2008) has proposed an analytical approach to take fibre orientation into account in the definition of an anisotropic yield surface, clearly showing its relevance in the solution of boundary value problems.

There is significant experimental evidence that the failure envelope of a fibre-reinforced soil is non linear (Consoli et al., 2007a; Santos et al., 2010). In the case of a bilinear schematisation, which is often adopted to represent this non linearity, the normal stress at which the slope of the envelope changes is defined as a 'critical normal stress' ($\sigma_{n,crit}$) and grossly represents a change in the fibre to grain interaction mechanism. For stress levels below such a normal stress, the failure mechanism of the composite material mainly implies slippage at the soil-fibre interface. Most authors also assume that, for stresses larger than $\sigma_{n,crit}$, the shear stresses at the soil-fibre interface mobilise the tensile strength within the fibre, involving extensive fibre breakage, even though alternative explanations of the change in slope of the failure envelope have been reported (Michalowski, 2008).

In the discrete framework proposed by Zornberg (2002), the critical normal stress $\sigma_{n,crit}$ is a function of the tensile strength of fibres ($\sigma_{f,y}$), the fibre aspect ratio ρ, the soil shear strength and two soil-fibre interface shear strength coefficients ($c_{i,c}$ and $c_{i,\varphi}$). By adopting such an approach, for $\sigma < \sigma_{n,crit}$ the equivalent friction angle of the reinforced soil (φ_r) is estimated as a function of the geometrical properties of the fibres (aspect ratio ρ and volumetric fibre content χ), of the shear strength of the soil (φ) and of the soil-fibre interface shear strength coefficient:

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$$\tan(\varphi_r) = (1 + \alpha \cdot \rho \cdot \chi \cdot c_{i,\varphi}) \cdot \tan\varphi \quad (1)$$

where α is an empirical coefficient that accounts for the orientation of the fibres and the efficiency with which they were mixed ($0 < \alpha < 1$). Michalowski and Cermák (2003) proposed another expression of φ_r for the first part of the bilinear scheme ($\sigma < \sigma_{n,crit}$) that has proved to be satisfactory for relatively short fibres (Sadek et al., 2010) but cannot be used for large or very large aspect ratios or fibre contents, since the values of $\sigma_{n,crit}$ proposed by Michalowski and Cermák (2003) may become negative in these cases (and therefore have no physical sense). An alternative approach would be to consider a single non linear failure envelope (Gray and Maher, 1989). As noted by Sadek et al. (2010), however, the available expressions of shear strength do not take into account the relevant role played by fibre length (in addition to fibre aspect ratio) or soil grain size, and this is certainly a major drawback.

Since all the experimental evidence has highlighted the relevance of the micromechanical interaction among soil grains and fibres (the fibre-grain 'scale effect') on the shear strength of the composite material and, as previously said, no expression is available to explicitly take it into account, an experimental program was planned to better understand the role of micromechanical mechanisms, eventually considering them into a new expression of the failure envelope of fibre reinforced soils. To this aim, a range of soil gradings (from clayey sand to sandy gravel) and fibre dimensions were used in the experimental activity. The proposed approach does not take into account the effect of anisotropy in fibre orientation, as all the results refer to an increase of the deviatoric stress normal to the preferential (horizontal) orientation of fibres determined by the compaction of the specimens.

MICROMECHANICAL CONSIDERATIONS

If fibres having length and diameter L_f and d_f are inserted into a granular material, they will have an effect during a deformation process of the composite material if there is a fibres to grains interaction, with normal and shear stresses exerted by the grains on the fibres surface, and a subsequent tensile stress induced into them. Michalowski and Cermák (2003), for instance, have nicely described the possible situations (no fibre slippage, slippage or fibre yielding) which rule the interaction at different confining stresses, stating that the fibre to grain interaction is effective if L_f is one order of magnitude larger than the size of the grains. This is certainly reasonable, because a sufficiently large number of fibre-to-grain contacts are needed to allow the interaction. Micromechanical analyses of slender flexible inclusions within granular materials (De Gennes, 1979) have shown that the effective length L^* (intended as the length truly interested by the mechanical interaction with the surrounding soil) of the inclusions is smaller than the nominal one (L_f). The physical reason is that the fibres usually assume an irregular, bended position, and are consequently stressed from the surrounding soil only for a limited part of their length. De Gennes (1979) indicates that, considering $L_f > 1$, the effective length may be taken as the square root of the nominal one. To be dimensionally consistent, such evidence can be formally expressed as: $L^* = [(L_{ref}/L_f)^{0.5} \cdot L_f]$, where $(L_{ref}/L_f) < 1$ (for instance, $L_{ref} = 1$ mm, with L_f expressed in millimetres).

For the sake of simplicity, let's then assume that a single diameter d^* represents soil grading. The true fibre to grain interaction mechanism is very complex and difficult to idealize, but it is necessary to recall that not all grains are equally stressed, and stress chains within the granular material carry most of the load. Such chains are intrinsically highly unstable, and continuously rearrange during a loading process. By uniformly inserting fibres within the soil mass, the rearrangement of stress chains is modified as long as the fibres are long enough to intersect more than one of these chains, thus likely reducing their instability, and, consequently, modifying the macroscopic mechanical behaviour of the reinforced material. Then, the mechanical effect of fibres is not confined to the fibre interface, and therefore not only to the grains directly in contact with them. A much larger number of grains is involved, and this can be realistically conceptualized as extending to the volume of soil surrounding the effective length L^* of the single fibre. If the number of grains directly in contact with the fibres is proportional to the ratio L^*/d^*, it is then reasonable to assume that the (larger) number of grains whose mechanical behaviour is influenced by the fibre is proportional to the cubic power of such a ratio, $(L^*/d^*)^3$ (Nicodemi, 2010, personal communication).

Keeping this simple micromechanical consideration in mind, it is interesting to make some considerations about the parameter $w_f \cdot \rho$ (where w_f is the fibre content by weight, and ρ is the fibre aspect ratio) often adopted in literature (e.g., Michalowski and Cermák, 2003) to take the effect of fibres on the equivalent friction angle of the reinforced soil at the macro scale into account. In particular, such a parameter can be formally expressed in terms of the geometrical properties of both the soil (considered as an equivalent monogranular material having diameter d^*) and the fibres; it is simple to demonstrate (*see* APPENDIX) that:

$$w_f \cdot \rho = \left(\frac{3}{2} \cdot \frac{\gamma_f}{\gamma_s}\right) \cdot \frac{n_f}{n_g} \cdot \frac{d_f}{d^*} \cdot \left(\frac{L_f}{d^*}\right)^2 \quad (2)$$

where n_f and n_g are respectively the number of fibres and the number of grains within the specimen.

As previously mentioned, the fibre to grain interaction mechanism should be ruled by $(L^*/d^*)^3$, and therefore it seems attractive to propose a first improvement for the macroscopic parameter $w_f \cdot \rho$, multiplying it for the ratio L_f/d^* and getting:

$$w_f \cdot \rho \cdot \frac{L_f}{d^*} = \left(\frac{3}{2} \cdot \frac{\gamma_f}{\gamma_s}\right) \cdot \frac{n_f}{n_g} \cdot \frac{d_f}{d^*} \cdot \left(\frac{L_f}{d^*}\right)^3 \quad (3)$$

In which the expected dependency on the cubic power of the ratio between fibre length and grains dimension is formally introduced. However, the effective length L^* should be used instead of L_f. Recalling that L^* is proportional to the square root of L_f ($L^* = [(L_{ref}/L_f)^{0.5} \cdot L_f] = (L_{ref})^{0.5} \cdot (L_f)^{0.5}$), the simplest possible modification of Eq. (3) to consider the really relevant length of the fibres is:

$$\sqrt{w_f \cdot \rho \cdot \frac{L_f}{d^*}} = \sqrt{\left(\frac{3}{2} \cdot \frac{\gamma_f}{\gamma_s}\right) \cdot \frac{n_f}{n_g} \cdot \frac{d_f}{d^*} \cdot \left(\frac{L_f}{d^*}\right)^3} \tag{4}$$

in which the term $(L_{ref})^{0.5}$, previously introduced just for making dimensionally correct the relationship between L^* and L_f, has been dropped.

The goal of the qualitative micromechanical considerations reported above is not to give a complete description of the exact fibre to grains interaction mechanism. They have been advocated simply to provide a hint for a rational way to get a relevant comprehensive parameter (Eq. (4)) for the description of the mechanical behaviour of the reinforced soil. This 'conceptual' parameter takes into account the effect of fibre reinforcement from a geometrical point of view. As previously mentioned, however, often the shear strength envelope of reinforced soils is non linear even at small confining stress well below the critical stress value $\sigma_{n,\,crit}$. In such a case, the effect of stress state must be explicitly taken into account in expressing the shear strength of the composite material, along with the geometrical parameter given by Eq. (4).

LABORATORY TESTING PROGRAM

An experimental laboratory program consisting of triaxial tests was carried out on three granular materials: a uniformly graded sand (Osorio Sand, OS), a clayey sand (Botucatu Residual Soil, BRS) and a sandy gravel (SG). The grain size distributions of the tested materials are presented in Fig. 1. In all cases, the fibre-reinforced specimens were prepared by hand mixing dry soil, water and fibres. The fibres were added progressively to ensure a uniform distribution throughout the soil mass. The fibre content by weight (w_f) is defined as:

$$w_f = 100\,\frac{W_f}{W_s} \tag{5}$$

being W_f and W_s respectively the weight of fibres and of dry soil. Circular cross section polypropylene fibres with five different geometries (L_f or d_f) were used in the tests (Table 1). Figure 2 presents pictures of a typical monofilament polypropylene fibre used in present research and an exhumed fibre-reinforced BRS specimen. OS and BRS were tested using 100 × 200 mm specimens; the coarser material (SG) was tested using 200 × 400 mm specimens. In all cases the specimens were saturated in the triaxial cell and then tested along drained monotonic stress paths with constant confining pressure. The tests were carried out at constant strain rates, chosen for each tested material to guarantee fully drained conditions. Axial strain was always measured by means of external LVDTs, and volumetric strain by means of volume gauges connected to the specimen. In all cases, the reported results in terms of shear strength pertain to the end of the tests.

Osorio Sand (OS)

This soil is a non-plastic uniform fine sand (European Standard, EN ISO 14688-2, 2004) having $G_s = 2.62$, $e_{max} = 0.90$, $e_{min} = 0.60$. Table 2 summarises the test procedure. Three monotonic drained triaxial tests were carried out on unreinforced specimens (OS in Table 2) and five drained triaxial tests were conducted on reinforced specimens (ROS in Table 2), the latter using a single kind of polypropylene fibre (type 1, Table 1). The unreinforced and fibre-reinforced Osorio Sand specimens were statically compacted in three layers into a split mould 100 mm in

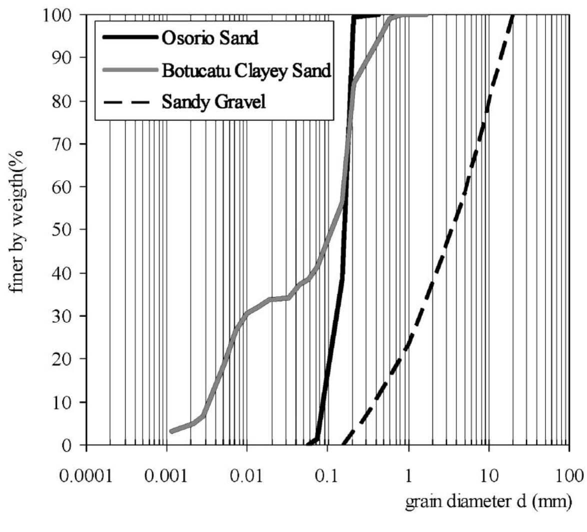

Fig. 1. Grading of the tested soils

Table 1. Properties of the tested monofilament polypropylene fibres

Fibre type	Soil	Specific gravity, G_f	Fibre length, L_f (mm)	Diameter of fibre, d_f (mm)	Aspect ratio, $\rho = L_f/d_f$	Stiffness of fibre, $E_{s(average)}$ (GPa)	Tensile strength of fibre, $\sigma_{y,\,f(average)}$ (MPa)
1	SG, OS	0.91	50	0.1	500	10	100
2	SG	0.91	50	0.076	658	10	100
3		0.91	75	0.076	987	10	100
4		0.91	100	0.076	1316	10	100
5	BRS	0.91	24	0.023	1043	3	120

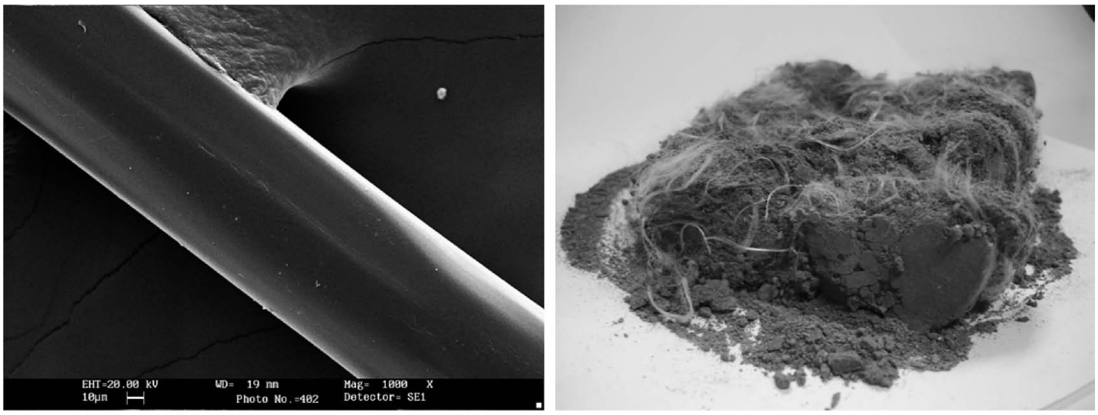

Fig. 2. Pictures of (a) typical monofilament polypropylene fibre used in present research and (b) exhumed fibre-reinforced BRS specimen

Table 2. Triaxial tests on Osorio Sand (OS) and Reinforced Osorio Sand (ROS)

Soil	Test name	Initial void ratio, e_0	Fibre type	Fibre length/mean grain size, L_f/d_{50}	Fibre content by weight, w_f (%)	Effective confining pressure, σ'_c (kPa)*
	OS1					50
Osorio Sand (OS)	OS2	0.75	—	—	—	100
	OS3					200
	ROS1					20
	ROS2					100
Reinforced Osorio Sand (ROS)	ROS3	0.75	1	250	0.5	200
	ROS4					400
	ROS5					550

* All tests were carried out with a constant confining stress (σ'_c).

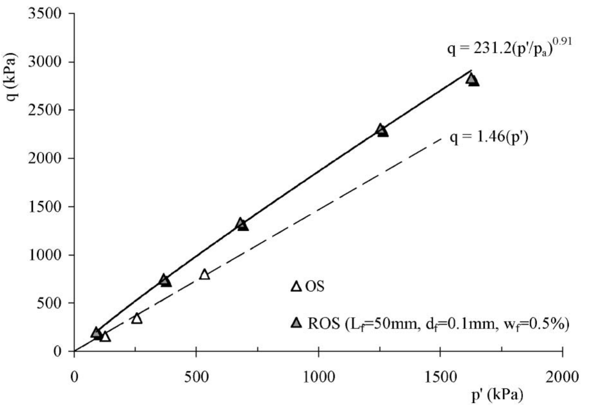

Fig. 3. Shear strength envelopes for unreinforced (OS) and reinforced Osorio Sand (ROS)

diameter and 200 mm high, to a moisture content of 10.0% and relative density $D_{r0} = 50\%$ (equivalent to a void ratio (e_0) 0.75). Each sample was compacted in a mould on the triaxial pedestal by applying a static load via the loading platen. The final height of the sample was controlled to ensure a relative density of 50%. Consistent with other experimental works on such a soil (Consoli et al., 2007b, 2009), the addition of fibres was shown to increase significantly the strength of the soil. In Fig. 3 the shear failure envelopes of OS and ROS are plotted in the deviatoric plane ($q = \sigma_1 - \sigma_3$, $p' = (\sigma_1 + 2\sigma_3)/3$). The results on the reinforced material have been fitted with the power expression reported in the figure, while the results on non reinforced specimens have been linearly fitted ($q/p' = 1.46$, $\varphi' = 36°$).

Botucatu Residual Soil (BRS)

The soil is classified as a low plasticity clayey sand (European Standard, EN ISO 14688-2, 2004) having $G_s = 2.64$ and a plasticity index $P_I = 10$. Table 3 summarises the test procedure. Four drained triaxial tests were carried out on unreinforced specimens (BRS) and four drained triaxial tests were conducted on fibre-reinforced BRS specimens (RBRS), the latter using a single kind of polypropylene fibre (type 5, Table 1). The unreinforced and fibre-reinforced BRS specimens were statically compacted in three layers into a split mould 100 mm in diameter and 200 mm high, to an optimum moisture content of 16.0% and maximum dry unit weight of 17.4 kN/m^3 ($e_0 = 0.55$). These values were obtained from standard Proctor compaction tests carried out on both unre-

Table 3. Triaxial tests on Botucatu Residual Soil (BRS) and on Reinforced Botucatu Residual Soil (RBRS)

Soil	Test name	Initial void index, e_0	Fibre type	Fibre length/mean grain size, L_f/d_{50}	Fibre content by weight, w_f (%)	Effective confining pressure, σ'_c (kPa)*
Botucatu Residual Soil (BRS)	BRS1	0.55	—	—	—	20
	BRS2					60
	BRS3					100
	BRS4					200
Reinforced Botucatu Residual Soil (RBRS)	RBRS1	0.55	5	220	0.5	20
	RBRS2					60
	RBRS3					100
	RBRS4					200

* All tests were carried out with a constant confining stress (σ'_c).

Table 4. Triaxial tests on Sandy Gravel (SG) and Reinforced Sandy Gravel (RSG)

Soil	Test name	Initial void index, e_0	Fibre type	Fibre length/mean grain size, L_f/d_{50}	Fibre content by weight, w_f (%)	Effective confining pressure, σ'_c (kPa)*
Sandy Gravel (SG)	SG-A	0.56	—	—	—	50
	SG-B	0.48				
	SG-C	0.41				
	SG-D	0.30				
Reinforced Sandy Gravel (RSG)	RSG-1	0.71	1	16.67	0.2	50
	RSG-2	0.79		16.67	0.2	
	RSG-3	0.68		16.67	0.2	400
	RSG-4	0.392	2	16.67	0.1	50
	RSG-5	0.409		16.67	0.2	
	RSG-6	0.396	3	25.00	0.1	
	RSG-7	0.384		25.00	0.2	
	RSG-8	0.356	4	33.33	0.1	
	RSG-9	0.368		33.33	0.2	

* All tests were carried out at a constant confining stress (σ'_c).

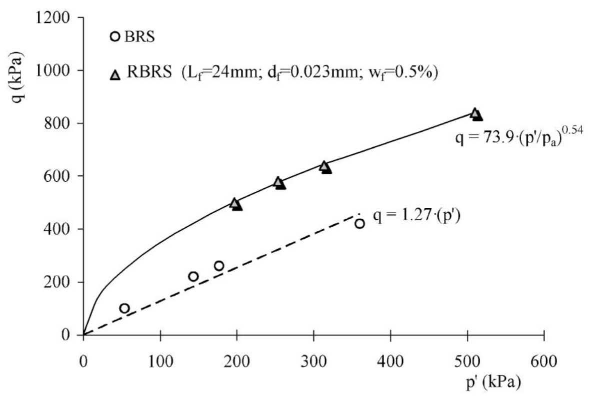

Fig. 4. Shear strength envelopes for unreinforced (BRS) and reinforced Botucatu Residual Soil (RBRS)

inforced BRS and BRS–fibre mixtures.

Also for this material the addition of fibres has a clear beneficial effect on shear strength, with a non linear failure envelope (Fig. 4) for the reinforced soil, and a friction angle $\varphi = 31.4°$ for the unreinforced soil.

Sandy Gravel (SG)

The soil is classified as a sandy gravel (European Standard, EN ISO 14688-2, 2004) having $G_s = 2.72$, $e_{max} = 0.60$, $e_{min} = 0.19$. Table 4 summarises the test procedure. Four monotonic triaxial tests were carried out on unreinforced specimens (SG) prepared by wet tamping (at a water content $w = 10\%$) at different initial void ratios. The mass of the adopted tamper is 10.6 kg, the diameter of the hitting end is 20 cm and the height of drop is about 40 cm. By compacting the gravel in layers with a thickness of 6 cm, different values of the specific energy were

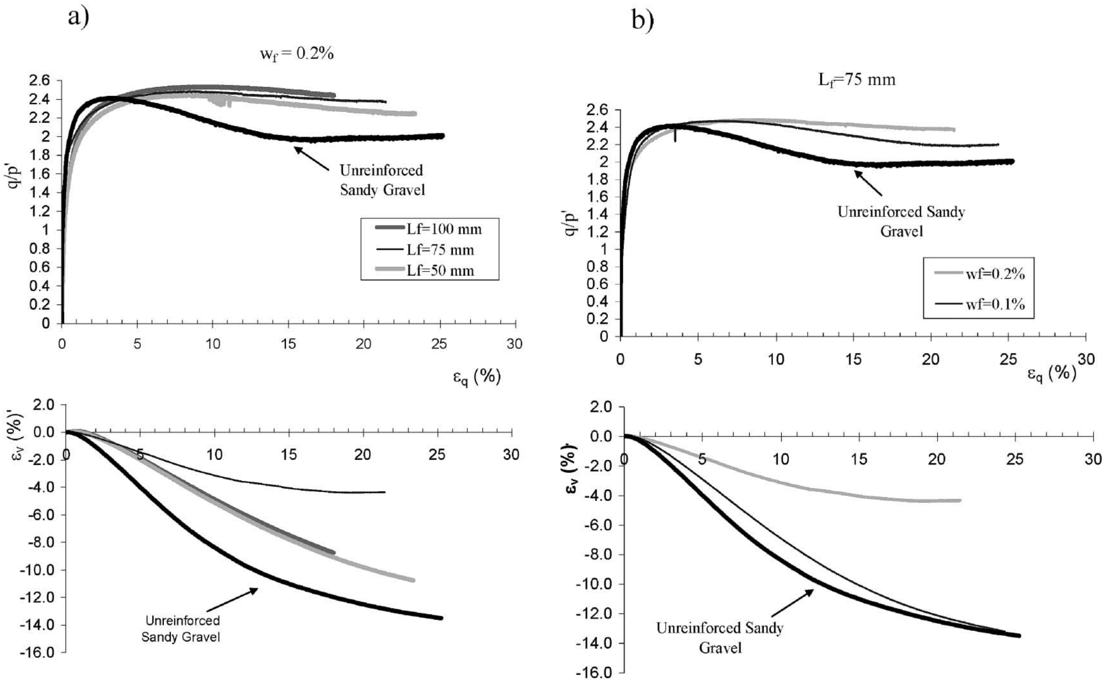

Fig. 5. Results of triaxial tests on unreinforced and reinforced Sandy Gravel (SG): a) effect of fibre length ($w_f = 0.2\%$); b) effect of fibre content ($L_f = 75$ mm)

applied to the soil, depending on the number of blows per stratum (up to $E_{max} = 113$ kJ/m^3 for the densest configuration, obtained with 30 blows per stratum; for comparison, the Standard Proctor Energy for 25 blows is 605 kJ/m^3). Nine drained triaxial tests were conducted on reinforced specimens (RSG) with non-uniform polypropylene fibres (Table 1); in particular, fibres with two different diameters ($d_f = 0.076$–0.1 mm) and three different lengths ($L_f = 50$, 75, 100 mm) were used. In this case, the adopted fibre contents ($w_f = 0.1$–0.2%) are lower than usual, depending on the coarseness of the soils ($w_f = 0.2\%$ was the largest one to avoid extensive tangling). The results are the first to be published on the fibre reinforcement of such a coarse material: the stress-strain behaviour of SG and RSG specimens is shown in Fig. 5. The failure envelope is not reported, since all but one of the tests were carried out at the same confining pressure (*see* Table 4). The behaviour of the SG specimens was typical of coarse grained soils, with high shear strength (represented by the stress obliquity ratio $\eta = q/p'$) and a state dependent peak reached at medium strain level, followed by a subsequent reduction in shear strength. The specimens always showed a dilative behaviour. Even though the peak strength of the RSG did not increase very much, the increase in the shear strength of the reinforced soil at large strains was very clear. As expected, the fibre-reinforced specimens showed a more ductile behaviour with reduced dilatancy. Our findings were consistent with the findings reported in the literature (e.g., Consoli et al., 2009) and also with the simple micromechanical considerations mentioned earlier: the longer the fibres, the greater the effect of the fibres (Fig. 5(a)). Furthermore, for a given fibre length, reinforcement is more effective when a larger amount of fibres is used (Fig. 5(b)).

A POSSIBLE INTERPRETATION OF THE EFFECT OF FIBRE REINFORCEMENT

The micromechanical observations previously reported highlight the role of the different geometrical parameters of the fibres and of the soil on the fibre to the grain interaction mechanism and hence on the macroscopic shear strength. As a first step to verify if further improvements of the available approaches are needed, Eq. (1) has been first used to interpret the test results. Since we analysed samples with a rather large range of grain sizes and fibres of varying characteristics, it is of some interest to check if such an equation fits reasonably well all the experimental data. To increase the experimental data set, results were also retrieved from the literature (*see* Table 5) and processed along with the results obtained in this work (Sivakumar et al., 2007; Michalowski and Cermák, 2003; Diambra et al., 2010; Sadek et al., 2010). The equivalent friction angles of the reinforced specimens (φ_r) were calculated by means of Eq. (1), and are plotted against the measured values in Fig. 6. Since the value of the parameter α is not known and is difficult to calibrate, two extreme values ($\alpha = 0.1$ and $\alpha = 1$) were adopted for it. A very large scatter can be observed for most of the data in both cases.

In order to use the approach suggested by the micromechanical considerations previously shown, there is the need to define d^*, for which there are a number of possible choices. A simple and reasonable assumption is $d^* = d_{50}$, but, in principle, other choices are possible. Then, the macro variable can be expressed following Eq.

(4) as:

$$\beta = \sqrt{w_f \cdot \rho \cdot \frac{L_f}{d_{50}}} \tag{6}$$

In discussing some of the experimental results, it was shown that the shear strength envelope of the reinforced soils may be non linear even before the critical stress $\sigma_{n,\,crit}$ (Figs. 3 and 4). This is also a rather typical feature of peak strength in non reinforced granular soils, which is dependent on state variables, and there are different elegant ways in literature to express such a dipendency (e.g., Bolton, 1986; Gajo and Muir Wood, 1999). To emphasize the influence of the stress state, the shear strength of the reinforced soils (ROS an RBRS) shown in Figs. 3 and 4 was expressed as:

$$q = a\left(\frac{p'}{p_a}\right)^b \tag{7}$$

where p_a is the atmospheric pressure (introduced for dimensional consistency), a is a parameter which has the dimensions of a stress and the exponent b is non dimensional. A similar non linear expression of the shear strength envelope of the reinforced soils could be also written in terms of the stress ratio $\eta = q/p'$. For natural soil with no fibres, as previously shown (in Figs. 3 and 4), the failure envelope can be considered linear (and therefore, φ = constant, and η = constant).

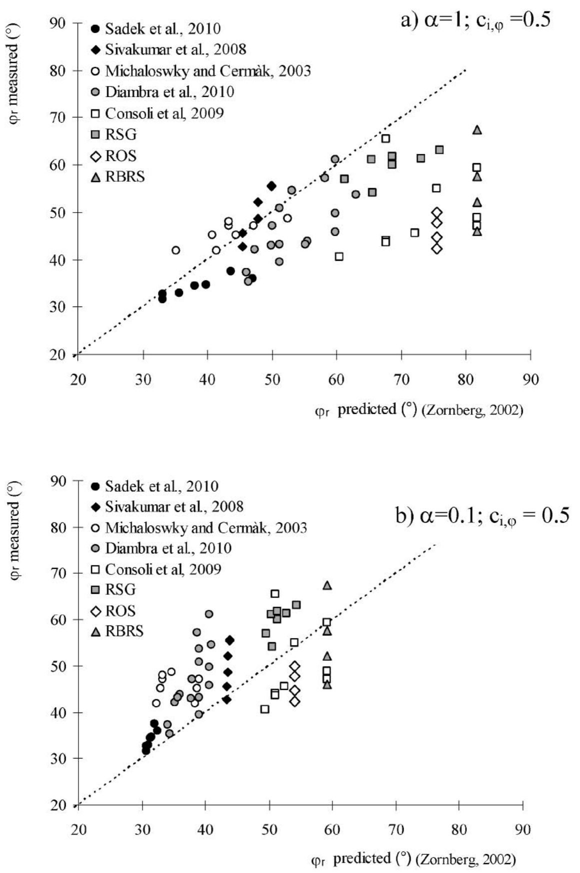

Fig. 6. Measured versus predicted (Eq. (1)) friction angles of fibre-reinforced specimens (φ_r)

It is possible to express the non linear failure envelope of the reinforced soil by explicitly taking the mechanical effect of fibres and the stress state into account and considering it as the sum of two terms, as has been done for instance by other authors (e.g., Zornberg, 2002): one linear term, relative to the natural soil, and a non linear one, related to the effect of fibres and stress state. As previously discussed, the latter must depend on a geometrical parameter (β), on the stress state (for instance, via the confining pressure σ'_c) and on the mechanical properties of fibres. In principle, the fibre to grains interface friction angle, the tensile strength ($\sigma_{y,\,f}$) and the stiffness of the fibres should all play a role. From an engineering point of view, however, considering that most times the only easily known mechanical property of the fibres $\sigma_{y,\,f}$, it is tempting to use it to represent the overall properties of the.

Considering these assumptions, the following expression of the failure stress ratio of the reinforced soil is proposed:

$$\eta_r = \eta\left(1 + \lambda \cdot \beta \cdot \left(\frac{\sigma_{y,\,f}}{\sigma'_c}\right)^{\delta}\right) \tag{8}$$

where λ and δ are two parameters. In Fig. 7, all the experimental results obtained in this work are plotted along with Eq. (8), drawn with the best fitting parameters $\lambda = 0.004$ and $\delta = 0.2$. Despite the large differences in the dimensions of the fibres and in the grading of the soils for the results shown in the figure, Eq. (8) nicely fits them, indicating that it is capable of taking all the main relevant factors into account. Then, the same equation was used to interpret the available data retrieved from literature. By carrying out the best fitting procedure for each single dataset, assuming a unique value $\delta = 0.2$ in all cases, it was found that the coefficient λ is related to $\sigma_{y,\,f}$ (Fig. 8), increasing as it increases (Table 5). A power equation was therefore adopted to link the parameter λ to $\sigma_{y,\,f}$, obtained by fitting the data reported in Table 5:

$$\lambda = 0.00004\left(\frac{\sigma_{y,\,f}}{p_a}\right)^{0.65} \tag{9}$$

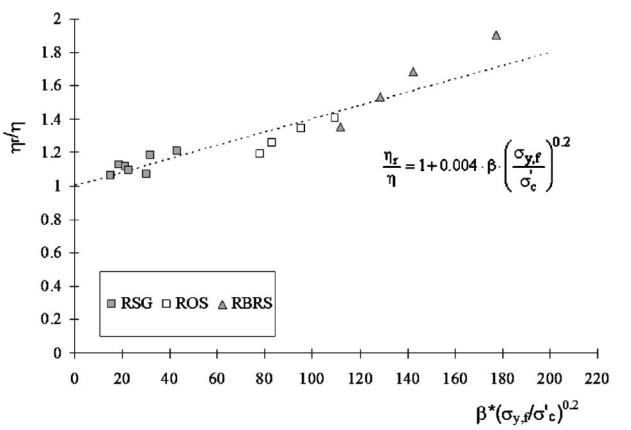

Fig. 7. Calibration of parameters λ and δ (Eq. (10)) based on the experimental results

Introducing Eq. (9) into Eq. (8) and assuming $\delta = 0.2$, a more general form of the expression of the failure envelope of the reinforced soil is obtained:

$$\eta_r = \eta\left(1 + 0.00004\cdot\beta\cdot\frac{\sigma_{y,f}^{0.85}}{p_a^{0.65}\cdot\sigma_c'^{0.2}}\right) \quad (10)$$

Equation (10) can be also written in terms of friction angle of the reinforced soil φ_r as:

$$\varphi_r = \sin^{-1}\left(\frac{3\cdot\sin(\varphi)\cdot\left(1 + 0.00004\cdot\beta\cdot\frac{\sigma_{y,f}^{0.85}}{p_a^{0.65}\cdot\sigma_c'^{0.2}}\right)}{3 + \sin(\varphi)\left(1 + 0.00004\cdot\beta\cdot\frac{\sigma_{y,f}^{0.85}}{p_a^{0.65}\cdot\sigma_c'^{0.2}}\right)}\right) \quad (11)$$

Figure 9 reports the predicted (Eq. (11)) and measured values of φ_r for all the tests considered (either produced in this work or retrieved from the literature). The overall agreement is certainly satisfactory, since the results obtained on different soils, different fibres and under different stress levels are reasonably well predicted by Eq. (11). It must be recalled that, due to compaction, the measured friction angles in each of the tests pertain to a condition (triaxial compression) in which the preferential bedding plane of the fibres is normal to the maximum principal stress. As a consequence, the results cannot give any information regarding the effect of the orientation of the fibres (which is always the same) on shear strength. It is only possible to say that the measured values of friction angles are likely to be the largest possible ones since the fibres are oriented in the direction of the minimum principal stress, even though—because of anisotropy—this direction does not necessarily coincide with the direction of the minimum principal strain. Any other bedding orientation of fibres within the specimens would result in smaller values of the friction angle.

CONCLUSIONS

A number of results of triaxial tests on fibre reinforced soils has been presented, either directly obtained in this work or retrieved from the literature. The results pertain to soils with very different gradings (from clayey sand to sandy gravel) reinforced with polypropylene fibres with a wide range of mechanical and geometrical properties. With reference to shear strength, the results provided a clear picture of the behaviour of the soil-fibre composite material in relation to that of the host soil. The strength of the reinforced material is larger than that of the natural soil, even for very small fibre contents and for coarse gradings, and an expression (Eqs. (10) and (11)) has been proposed to calculate it as a function of some relevant parameters: the fibre content w_f and aspect ratio ρ, the fibre tensile strength $\sigma_{y,f}$, the effective confining stress σ_c' and the fibre to grain relative dimension ratio L_f/d_{50}. The

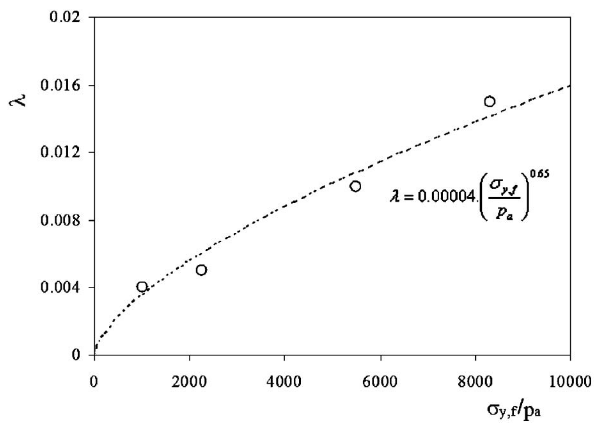

Fig. 8. Relationship between λ and $\sigma_{y,f}$ (Table 5, Eq. (11))

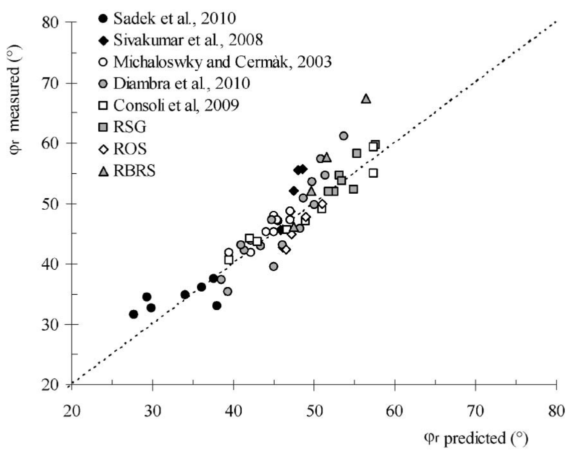

Fig. 9. Measured and predicted (Eq. (11)) friction angles of fibre-reinforced specimens (φ_r)

Table 5. Values of parameter λ (Eq. (11)) for some experimental data derived from literature

Data from literature	Soils	Fibres			Fibre length/mean grain size, L_f/d_{50}	λ
	Mean grain size, d_{50} (mm)	Fibre length, L_f (mm)	Aspect ratio, $\rho = L_f/d_f$	Tensile strength of fibre, $\sigma_{y,f}$ (MPa)		
Sivakumar et al., 2008	0.1	15	60	100	150	0.004
Consoli et al., 2009	0.16	12, 24, 36	120–1043	120	72.7–218.2	0.004
Diambra et al., 2010	0.32	35	350	225	109.4	0.005
Michaloswki and Cermák, 2003	0.2, 0.9	25.4	84.7	550	28.2, 127	0.01
Sadek et al., 2010	0.39	7, 27	40–150	830	17.9, 69.2	0.015

proposed relation stems from some basic micromechanical considerations, and is the first to express the shear strength of fibre reinforced soils explicitly by taking the grain to fibre relative dimensions and the mechanical properties of the fibres ($\sigma_{y,f}$) into account. Even though Eq. (10) or (11) has been written based on an oversimplified interpretation of the true micromechanical interaction mechanism between the grains and fibres, it showed to be capable of predicting the shear strength of a large variety of fibre reinforced soils. It also has the advantage of being a rather general expression that can be used even if only some simple and basic information on the host soil and the fibres are known.

The experimental results did not provide any new insight into the effect of the orientation of the fibres on shear strength, since the preferential orientation of fibres in all the tests was normal to the maximum principal stress, and therefore anisotropy could not be considered in the proposed expression. Further laboratory tests are needed to experimentally investigate the effect of anisotropy. Research is underway to meet this aim.

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APPENDIX

The fibres content by weight (w_f) is defined (Eq. (5)) as the ratio between the weight of fibres W_f and the dry weight of soil. W_f can be written as:

$$W_f = \gamma_f \cdot V_{f,i} \cdot n_f = \gamma_f \cdot \pi \frac{d_f^2}{4} L_f \cdot n_f \tag{1a}$$

where γ_f is the specific weight of fibre, $V_{f,i}$ the volume of fibre (d_f and L_f are respectively the diameter and length of fibre), and n_f the number of fibres within the specimen.

In the same way, by assuming an equivalent monogranular material having spherical particles of diameter d^*, the soil dry weight (W_s) can be written as:

$$W_s = \gamma_s \cdot V_{g,i} \cdot n_g = \gamma_s \cdot 4\pi \frac{(d^*/2)^3}{3} \cdot n_g \tag{2a}$$

where γ_s is the specific weight of the soil, $V_{g,i}$ the volume of the (representative) soil grain and n_g the number of grains within the specimen. Therefore, the fibres content by weight can be written as:

$$w_f = \frac{\gamma_f \cdot \pi \dfrac{d_f^2}{4} L_f \cdot n_f}{\gamma_s \cdot 4\pi \dfrac{(d^*/2)^3}{3} \cdot n_g} = \frac{\gamma_f}{\gamma_g} \cdot \frac{3}{2} \cdot \frac{n_f}{n_g} \cdot \frac{d_f^2 \cdot L_f}{(d^*)^3} \tag{3a}$$

and, as a consequence, the parameter $w_f \cdot \rho$ can be written as:

$$w_f \cdot \rho = \frac{\gamma_f}{\gamma_s} \cdot \frac{3}{2} \cdot \frac{n_f}{n_g} \cdot \frac{d_f}{d^*} \cdot \left(\frac{L_f}{d^*}\right)^2 \tag{4a}$$