

Enhanced Lattice-Reduction aided detection for MIMO systems with QRD-M detector

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Abstract: In this letter, an efficient MIMO detection scheme combined with a QRD-M and a complex LR-aided detection scheme is proposed. For the first T stages, the QRD-M detection is executed, and then the complex LR-aided detection is executed for the last $N_t - T$ stages. Simulation results show that the proposed scheme provides the comparable detection performance to the QRD-M. Also, the proposed scheme can significantly reduce the computational complexity compared with the QRD-M because the computations for the QRD-M is limited by the parameter T . The value of T is determined by the required system performance.

Keywords: QRD-M, LR-aided detector, CLLL algorithm

Classification: Science and engineering for electronics

References

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1 Introduction

Recently, the vertical Bell Laboratories layered space time (V-BLAST) scheme has been widely used in rich scattering wireless environment for its

high spectral efficiency [1]. Since the optimum detection scheme, maximum likelihood (ML) detection has very high complexity [2], a lot of multiple-input multiple-output (MIMO) detection schemes have been investigated by many researchers because it is important to apply a proper MIMO detection scheme with low complexity and high performance.

A linear detection is a popular way to detect the transmitted signal with low complexity but it has marginal error performance [1]. Also, a lattice reduction (LR)-aided detection scheme significantly improves the performance of the linear detection but it still not approach the optimal performance [3, 4]. For the high detection performance with the practical implementation, a QRD-M detection has been proposed [5]. However, its detection complexity is still highly increased by the number of transmit/receive antennas, constellation level, and the number of survivor symbol candidates.

In this letter, to achieve a good trade-off between performance and complexity, we propose an efficient MIMO detection scheme combined with a QRD-M and a complex LR-aided detection scheme.

2 System model

We consider a V-BLAST system with N_t transmit and N_r receive antennas. The $N_r \times 1$ receive signal vector, $\mathbf{y} = [y_{N_r} \ y_{N_r-1} \ \cdots \ y_1]^T$, is given by

$$\mathbf{y} = \sum_{i=1}^{N_t} \mathbf{h}_i \cdot x_i + \mathbf{w} = \mathbf{H} \cdot \mathbf{x} + \mathbf{w}, \quad (1)$$

where \mathbf{x} means $N_t \times 1$ vector of transmit symbol, $[x_{N_t} \ x_{N_t-1} \ \cdots \ x_1]^T$, \mathbf{w} is zero-mean Gaussian noise, and $\mathbf{H} = [\mathbf{h}_{N_t} \ \mathbf{h}_{N_t-1} \ \cdots \ \mathbf{h}_1]$ is an i.i.d random complex matrix of multipath channel with $\mathbf{h}_i = [h_{N_r,i} \ h_{N_r-1,i} \ \cdots \ h_{1,i}]^T$.

3 Conventional detection scheme

3.1 QRD-M detection scheme

The QRD-M detection is based on QR decomposition, $\mathbf{H} = \mathbf{QR}$ [5]. By multiplying \mathbf{y} with \mathbf{Q}^H , the $N_t \times 1$ output vector can be expressed as

$$\mathbf{z} = \mathbf{Q}^H \mathbf{y} = \mathbf{Q}^H \mathbf{H} \mathbf{x} + \mathbf{Q}^H \mathbf{w} = \mathbf{R} \mathbf{x} + \bar{\mathbf{w}}$$

$$\begin{bmatrix} z_{N_t} \\ z_{N_t-1} \\ \vdots \\ z_1 \end{bmatrix} = \begin{bmatrix} r_{N_t,N_t} & r_{N_t,N_t-1} & \cdots & r_{N_t,1} \\ 0 & r_{N_t-1,N_t} & \cdots & r_{N_t-1,1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_{1,1} \end{bmatrix} \begin{bmatrix} x_{N_t} \\ x_{N_t-1} \\ \vdots \\ x_1 \end{bmatrix} + \begin{bmatrix} \bar{w}_{N_t} \\ \bar{w}_{N_t-1} \\ \vdots \\ \bar{w}_1 \end{bmatrix}. \quad (2)$$

At first, the all L symbols, $\mathbf{c} = [c^{(1)} \ c^{(2)} \ \cdots \ c^{(L)}]$, of L -QAM are considered for the symbol candidates. The metric based on the squared Euclidian distance between z_1 and $c^{(l)}$ is calculated as $e_1^{(l)} = ||z_1 - r_{1,1} \cdot c^{(l)}||^2$. The whole metric can be written as $\mathbf{e}_1 = [e_1^{(1)}, \dots, e_1^{(L)}]$. In this stage, M ($M \leq L$) symbols are selected among L symbol candidates from the one with the smallest branch metric. The surviving symbol candidates, $\hat{\mathbf{x}}_1 = [\hat{x}_1^{(1)} \ \hat{x}_1^{(2)} \ \cdots \ \hat{x}_1^{(M)}]$, are transferred to the next stage. In the n -th stage, the accumulated branch

metrics (path metrics) are updated for all $M \cdot L$ combinations of M surviving symbol candidates. The path metric for m -th surviving symbol candidates $[\hat{x}_1^{(m)}, \hat{x}_2^{(m)}, \dots, \hat{x}_{n-1}^{(m)}]$ and l -th symbol candidate is calculated as

$$e_n^{(m,l)} = \left\| z_n - \left[r_{n,n} \cdot c^{(l)} + \left(\sum_{i=1}^{n-1} r_{n,i} \cdot \hat{x}_i^{(m)} \right) \right] \right\|^2 + E_{n-1}^{(m)} \quad (3)$$

where $E_{n-1}^{(m)}$ is the path metric of m -th surviving symbol candidates of $(n-1)$ -th stage. Similar to the first stage, M ($M \leq M \cdot L$) candidates are selected from the one with the smallest path metric as the surviving candidates, along with their accumulated branch metrics as $\mathbf{E}_n = [E_n^{(1)} \dots E_n^{(M)}]$. The surviving candidates of n -th stage can be written as

$$\hat{\mathbf{x}}_n = [\{\hat{x}_1^{(1)} \dots \hat{x}_n^{(1)}\}, \{\hat{x}_1^{(2)} \dots \hat{x}_n^{(2)}\}, \dots, \{\hat{x}_1^{(M)} \dots \hat{x}_n^{(M)}\}]^T. \quad (4)$$

The process is repeated up to the last stage. Finally, the surviving candidate with the smallest path metric is selected among M surviving candidates.

3.2 LR-aided detection scheme

The one of the LR-aided detection schemes, the complex Lenstra-Lenstra-Lovasz (CLLL) algorithm transforms a given basis \mathbf{H} into a new basis $\tilde{\mathbf{H}}$ with vectors of shortest length or into a basis consisting of roughly orthogonal basis vectors, equivalently. The CLLL algorithm requires lower computational complexity and the whole algorithm is presented in [3, 4]. A new channel $\tilde{\mathbf{H}}$ is obtained as $\tilde{\mathbf{H}} = \tilde{\mathbf{Q}}\tilde{\mathbf{R}} = \mathbf{H}\mathbf{U}$. Using this new channel, Eq. (1) is rewritten as follows.

$$\mathbf{y} = \mathbf{H}\mathbf{U}\mathbf{U}^{-1}\mathbf{x} + \mathbf{w} = \tilde{\mathbf{H}}\mathbf{U}^{-1}\mathbf{x} + \mathbf{w} = \tilde{\mathbf{H}}\mathbf{s} + \mathbf{w}, \quad (5)$$

where $\mathbf{s} = \mathbf{U}^{-1}\mathbf{x}$.

If the LR-aided ZF equalizer is applied to Eq. (5), the output signal can be expressed as follows.

$$\tilde{\mathbf{s}} = \tilde{\mathbf{G}}\mathbf{y} = \mathbf{U}^{-1}\mathbf{x} + \tilde{\mathbf{G}}\mathbf{w} = \mathbf{s} + \tilde{\mathbf{w}}, \quad (6)$$

where $\tilde{\mathbf{G}}$ means the Moore-Penrose pseudo-inverse matrix of the new channel matrix $\tilde{\mathbf{H}}$. Thus, \mathbf{s} can be estimated by rounding up to integers.

$$\hat{\mathbf{s}} = Q(\tilde{\mathbf{s}}) = Q(\tilde{\mathbf{G}}\mathbf{y}) = Q(\mathbf{s} + \tilde{\mathbf{w}}). \quad (7)$$

After obtaining $\hat{\mathbf{s}}$, \mathbf{x} is selected by mapping to the appropriate constellation. Finally, the original symbols are estimated by $\hat{\mathbf{x}} = \mathbf{U}\hat{\mathbf{s}} = \mathbf{U}\mathbf{U}^{-1}\hat{\mathbf{x}}$.

4 Proposed detection scheme

In the proposed detection, a new parameter T is adopted to reduce the detection complexity. After the value of T is set, the QRD-M is executed for T stages ($T \leq N_t$), and then the complex LR-aided detection is executed from the next stage ($T+1$ -th stage). Among the decoded sequences, the most probable stream is selected by likelihood test. Fig. 1 shows tree diagram of the proposed scheme. The whole algorithm is described as follows.

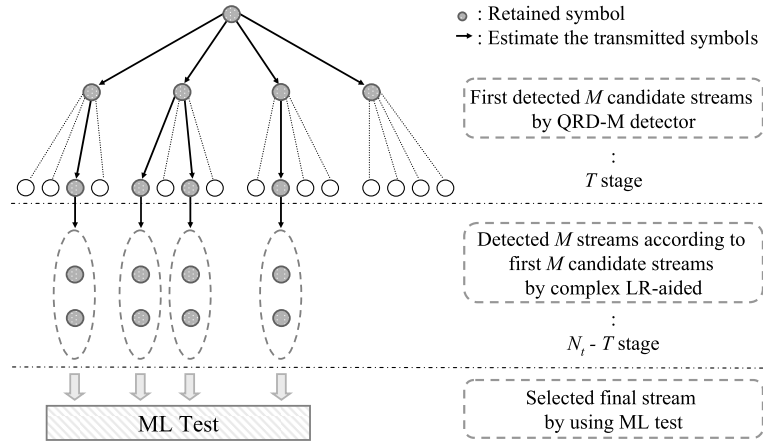


Fig. 1. Tree diagram of the proposed scheme with $N_t = N_r = 4$, $M = 4$, $T = 2$ and QPSK modulation.

Step 1: For accurate detection, the symbols through poor channel conditions are arranged in reverse order. The $\|\tilde{\mathbf{G}}_j\|^2$ is calculated and its elements are sorted from the largest to the smallest, where $\tilde{\mathbf{G}}_j$ is the j -th row of the $\tilde{\mathbf{G}}$. The sorted indexes are saved in sequence \mathbf{k} , $\mathbf{k} = [k_1 \ k_2 \ \cdots \ k_{N_t}]$. The columns of channel matrix \mathbf{H} are rearranged from best condition to worst channel, according to the sorted index sequence \mathbf{k} as $\tilde{\mathbf{H}} = [\mathbf{h}_{k_{N_t}}, \mathbf{h}_{k_{N_t-1}}, \cdots \mathbf{h}_{k_1}]$.

Step 2: T sorted symbols through poor channel condition are estimated. Generally, the QRD-M has better detection performance than the complex LR-aided detection. Therefore, the detection with the QRD-M of previous section is executed for the first T stages. After T stages, the surviving candidates of T -th stage can be written as

$$\hat{\mathbf{x}}_T = [\{\hat{x}_{k_1}^{(1)} \cdots \hat{x}_{k_T}^{(1)}\}, \{\hat{x}_{k_1}^{(2)} \cdots \hat{x}_{k_T}^{(2)}\}, \cdots, \{\hat{x}_{k_1}^{(M)} \cdots \hat{x}_{k_T}^{(M)}\}]^T. \quad (8)$$

Step 3: To detect residual symbols, $[\hat{x}_{k_{T+1}} \cdots \hat{x}_{k_{N_t}}]$, the complex LR-aided detection is executed from $T + 1$ -th stage to N_t -th stage. The largest sorted index denotes the best channel condition. Although the complex LR-aided detector is used, it has even weaker influence on overall performance. Thus, Eq. (2) is revised as follows.

$$\begin{bmatrix} \tilde{z}_{N_t} \\ \tilde{z}_{N_t-1} \\ \vdots \\ \tilde{z}_{T+1} \end{bmatrix} = \begin{bmatrix} r_{N_t, N_t} & r_{N_t, N_t-1} & \cdots & r_{N_t, T+1} \\ 0 & r_{N_t-1, N_t} & \cdots & r_{N_t-1, T+1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_{1, T+1} \end{bmatrix} \begin{bmatrix} x_{k_{N_t}} \\ x_{k_{N_t-1}} \\ \vdots \\ x_{k_{T+1}} \end{bmatrix} + \begin{bmatrix} \bar{w}_{N_t} \\ \bar{w}_{N_t-1} \\ \vdots \\ \bar{w}_{T+1} \end{bmatrix}, \quad (9)$$

where $\tilde{z}_n = z_n - \sum_{i=1}^T r_{n,i} \cdot x_{k_i}$, (where $T < n \leq N_t$).

The complex LR-aided detection scheme executes M times (M surviving candidate streams are considered). The all detected signals can be written as

$$\hat{\mathbf{x}} = [\hat{\mathbf{x}}^{(1)} \cdots \hat{\mathbf{x}}^{(m)} \cdots \hat{\mathbf{x}}^{(M)}]; \quad \hat{\mathbf{x}}^{(m)} = [\hat{x}_{k_1}^{(m)} \hat{x}_{k_2}^{(m)} \cdots \hat{x}_{k_{N_t}}^{(m)}] \quad (10)$$

Step 4: The last stream maximizing the likelihood is selected among M streams. Thus, the final decision value can be obtained as

$$\hat{\mathbf{x}}_{final} = \arg \min_{\hat{\mathbf{x}}^{(m)}} \|\mathbf{y} - \mathbf{H} \cdot \hat{\mathbf{x}}^{(m)}\|^2. \quad (11)$$

The last detected symbol, $\hat{\mathbf{x}}_{final} = [\hat{x}_{k_1} \hat{x}_{k_2} \cdots \hat{x}_{k_{N_t}}]$, is rearranged according to the order of transmit antenna by using the index sequence \mathbf{k} .

5 Computational complexity

For computational complexity comparison, the multiplications are considered as real multiplications. Therefore, the total number of one complex multiplication is counted as four. To calculate the complexity of the complex LR-aided detection, we obtain the average complexity of the processing part of CLLL algorithm by means of computer simulation [3]. As the Table I shows, the complexity of the proposed detection scheme with $M = 16$, $M = 24$ are decreased by about 1/2 in comparison with that of the QRD-M with $N = 4$, $L = 16$, $T = 2$, respectively.

Table I. Number of Multiplications for Each Detection.

Detection scheme	Required number of multiplications per subcarrier ($N = N_r = N_t$)			Computational complexity($L=16$)		
Complex LR-aided ZF	CLLL algorithm	ZF	Generation	1821		
	$\tilde{\mathbf{H}}: 4 \cdot N^3$ CLLL algorithm: $N = 4 \rightarrow 669$	$12 \cdot N^3 + 4 \cdot N^2$	$\hat{\mathbf{x}} = \mathbf{U}\hat{\mathbf{s}} : 4 \cdot N^2$			
QRD-M	QRD	Generation	Euclidian distances	$M=16$	$M=24$	
	$4 \cdot N^3 + 4 \cdot N^2$	$4 \cdot L + \sum_{n=2}^N 4 \cdot M \cdot L \cdot n$	$4 \cdot L + \sum_{n=2}^N 4 \cdot M \cdot L$	12736	18880	
Proposed scheme	Reverse ordering	QRD-M (T stage)		$T=1$	3620	4731
	G matrix: $12 \cdot N^3$	QRD-M: $4 \cdot (N^3 + N^2 + 2 \cdot L + \sum_{n=2}^T M \cdot L \cdot (n+1))$				
	G matrix power: $4 \cdot N^2$	Revise $\tilde{\mathbf{Z}}: 4 \cdot (N - T) \cdot T \cdot M$		$T=2$	6112	8416
	CLLL ($N - T$ stage)		ML test			
	$N - T = 1 \rightarrow 0, N - T = 2 \rightarrow 80, N - T = 3 \rightarrow 295$		$4 \cdot M \cdot N^2$	$T=3$	9828	13972
$16 \cdot (N - T)^3 + 4 \cdot (N - T)^2 \cdot (1 + M)$						

6 Simulation results

In this section, the bit error rate (BER) performances of the proposed detection scheme are compared with the QRD-M and the complex LR-aided detection. To evaluate the BER performance, we consider the V-BLAST system and suppose that the receiver acquires the perfect channel state information.

Fig. 2 and 3 show the BER performances of the ZF, the complex LR-aided, the QRD-M and the proposed detection schemes, when 16-QAM modulation ($L = 16$) is used. As expected, the proposed scheme has better BER performance than the ZF and the complex LR-aided. In the proposed detection, we acquire the comparable performance of the QRD-M. Although the performance is degraded as the value of T decreases, the degradation value is not high. In the case of $N_t = N_r = 4$ with $T = 2$, the performance degradation of the proposed detection is about 1.5 dB compared to the QRD-M and the complexity is about 50% of the QRD-M for a BER of 10^{-4} . Although there are some performance degradations, the complexity of proposed detection can be reduced less than half. Therefore, T can be controlled according to the required system performance and complexity.

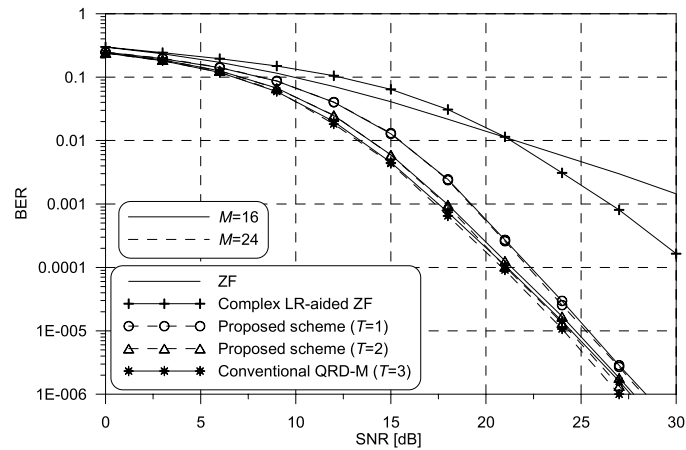


Fig. 2. BER performance of the proposed scheme with $N_t = N_r = 3$ according to the value of T .

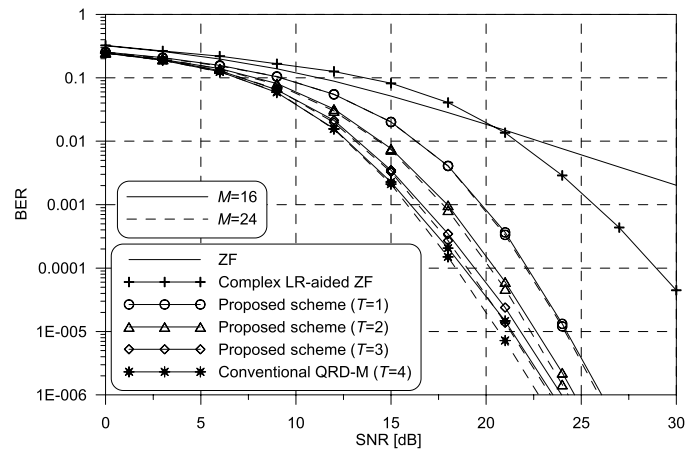


Fig. 3. BER performance of the proposed scheme with $N_t = N_r = 4$ according to the value of T .

7 Conclusion

The basic idea of the proposed detection is to apply the complex LR-aided detection to the QRD-M according to T . Simulation results show the suitable BER performance and complexity of the proposed scheme. Therefore, if the QRD-M is difficult to implement because of its high complexity, the proposed scheme can be efficiently used.

Acknowledgments

This research is supported by the Ubiquitous Computing and Network (UCN) Project, Knowledge and Economy Frontier R&D Program of the Ministry of Knowledge Economy (MKE) in Korea as a result of UCN's subproject 11C3-C2-10M and this research is supported by Seoul R&BD Program (SS100009).