

Inquiry into backscattering enhancement phenomenon in random media

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Abstract: Backscattering enhancement phenomenon has attracted considerable attention from radar engineering and remote sensing fields. This paper investigates the phenomenon by analyzing numerically the bistatic radar cross-section (RCS) of a conducting circular cylinder embedded in a continuous random medium. The RCSs of different size cylinders are depicted by changing the fluctuation intensity and thickness of the random medium. As a result, strong backscattering-enhancement and oscillation in the neighborhood of the backward direction are observed for a wavelength-size cylinder embedded in a strong random medium.

Keywords: random medium, backscattering enhancement, radar cross-section

Classification: Electromagnetic theory

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1 Introduction

The backscattering enhancement was found out in theory and experiment as a fundamental phenomenon in a random medium, which is produced by statistical coupling of incident and backscattered waves due to the effect of double passage [1, 2]. Moreover, it has been predicted [2] for a point scattering from the law of energy conservation that the scattering should be depressed in some other directions when the backscattering occurs.

We have investigated the phenomenon by numerically analyzing the radar cross-section (RCS) of a body embedded in a continuous random medium for more than ten years, and found some interesting behavior of the RCS as follows. The RCS may become nearly twice as large as that in free space [3, 4]. There is a depression outside the backscattering enhancement peak [5, 6]. The RCS normalized to that in free space becomes independent of the polarization of the incident wave when the body size is smaller enough than the wavelength [7]. Second enhancement in the neighborhood of the backward direction or backscattering depression occurs [8].

This paper presents new results about the RCSs for conducting circular cylinders embedded in random media.

2 Formulation

Consider a two-dimensional problem of electromagnetic wave scattering from a perfectly conducting circular cylinder of radius a , embedded in a continuous

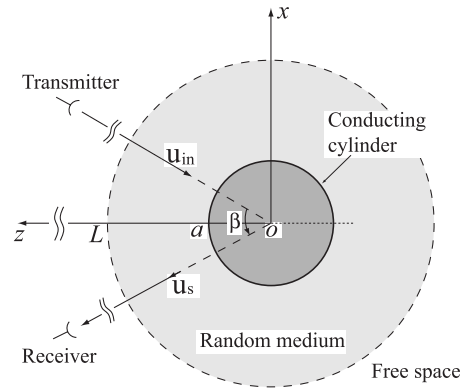


Fig. 1. Geometry of the scattering problem from a conducting cylinder surrounded by a random medium.

random medium, as shown in Fig. 1. Here L is the layer thickness of the random medium surrounding the cylinder and is assumed to be larger enough than the size of the cylinder cross-section. The random medium is assumed to be described by the dielectric constant ε , the magnetic permeability μ and the electric conductivity σ , which are expressed as

$$\varepsilon(\mathbf{r}) = \varepsilon_0[1 + \delta\varepsilon(\mathbf{r})], \quad \mu(\mathbf{r}) = \mu_0, \quad \sigma(\mathbf{r}) = 0, \quad (1)$$

where ε_0 , μ_0 are constant and $\delta\varepsilon(\mathbf{r})$ is a random function of spatial variable \mathbf{r} with

$$\langle \delta\varepsilon(\mathbf{r}) \rangle = 0, \quad (2)$$

$$\langle \delta\varepsilon(\mathbf{r}_1) \cdot \delta\varepsilon(\mathbf{r}_2) \rangle = B(\mathbf{r}_1 - \mathbf{r}_2). \quad (3)$$

Here the angular brackets denote the ensemble average and $B(\mathbf{r}_1 - \mathbf{r}_2)$ is the correlation function of the random function. For numerous cases, it can be approximated as

$$B(\mathbf{r}_1 - \mathbf{r}_2) = B_0 \exp \left[-\frac{|\mathbf{r}_1 - \mathbf{r}_2|^2}{l^2} \right], \quad (4)$$

$$B_0 \ll 1, \quad kl \gg 1, \quad (5)$$

where B_0 , l are the intensity and scale-size of the random medium fluctuation, respectively, and $k = \omega\sqrt{\varepsilon_0\mu_0}$ is the wavenumber in free space. Under the condition (5), depolarization of electromagnetic waves due to the medium fluctuation can be neglected; and the scalar approximation is valid. In addition, the forward multiple scattering approximation is valid, and hence the backscattering by the random medium becomes negligible. In the present analysis, consequently we do not need to consider the re-incidence of backscattered waves by the random medium on the cylinder [3, 9].

Suppose that the transmitter with the time factor $\exp(-j\omega t)$ suppressed throughout this paper is a line current source, located at \mathbf{r}_T , far from and parallel to the cylinder. Then the incident wave is expressed by Green's function in a medium containing the random medium and free space $G(\mathbf{r}, \mathbf{r}_T)$

whose dimension coefficient is understood. Using the current generator Y that transforms any incident wave into the surface current on the cylinder, we can give the average intensity of scattered waves u_s at the receiving point \mathbf{r} for E-wave incidence as follows [3, 9]:

$$\langle |u_s(\mathbf{r}, \mathbf{r}_T)|^2 \rangle = \int_S d\mathbf{r}_1 \int_S d\mathbf{r}_2 \int_S d\mathbf{r}'_1 \int_S d\mathbf{r}'_2 \left\{ Y_E(\mathbf{r}_1; \mathbf{r}'_1) Y_E^*(\mathbf{r}_2; \mathbf{r}'_2) \right. \\ \left. \langle G(\mathbf{r}; \mathbf{r}_1) G(\mathbf{r}'_1; \mathbf{r}_T) G^*(\mathbf{r}; \mathbf{r}_2) G^*(\mathbf{r}'_2; \mathbf{r}_T) \rangle \right\}, \quad (6)$$

where S denotes the cylinder surface, and the asterisk does the complex conjugate. The \mathbf{r}_i and \mathbf{r}'_i ($i = 1, 2$) are on S . The Y_E can be calculated for arbitrarily-shaped cylinders by Yasuura's method [3, 9] and expressed in an infinite series for a circular cylinder [10]:

$$Y_E(\mathbf{r}_i; \mathbf{r}'_i) = \frac{j}{\pi^2 a^2} \sum_{n=-\infty}^{\infty} \frac{\exp\{jn(\theta_i - \theta'_i)\}}{J_n(ka) H_n^{(1)}(ka)}, \quad (7)$$

where $\mathbf{r}_i = (a, \theta_i)$ and $\mathbf{r}'_i = (a, \theta'_i)$ in the cylindrical coordinates. The J_n is the Bessel function of order n and $J_n(ka) \neq 0$; that is, the internal resonance frequencies are excepted. The $H_n^{(1)}$ is the Hankel function of the first kind.

The fourth moment of Green's functions in (6) can be written as

$$\langle G(\mathbf{r}; \mathbf{r}_1) G(\mathbf{r}'_1; \mathbf{r}_T) G^*(\mathbf{r}; \mathbf{r}_2) G^*(\mathbf{r}'_2; \mathbf{r}_T) \rangle = \\ G_0(\mathbf{r}; \mathbf{r}_1) G_0^*(\mathbf{r}; \mathbf{r}_2) G_0(\mathbf{r}'_1; \mathbf{r}_T) G_0^*(\mathbf{r}'_2; \mathbf{r}_T) m_s(\mathbf{r}, \mathbf{r}_T), \quad (8)$$

where G_0 is Green's function in free space [11]. The m_s includes multiple scattering effects of the random medium and can be obtained by two-scale method [5, 6, 12, 13]; as a result, it is given by

$$m_s(\mathbf{r}, \mathbf{r}_T) = \frac{k}{2\pi z} \iint_{-\infty}^{\infty} d\eta d\rho \exp \left\{ -\frac{jk}{z} \eta [\rho - (x - x_T)] \right\} P(\rho, \eta), \quad (9)$$

where $\mathbf{r} = (x, z)$ and $\mathbf{r}_T = (x_T, z)$ in the Cartesian coordinates, and

$$P(\rho, \eta) = \exp \left\{ -\frac{\sqrt{\pi} k^2 l z}{8} \int_0^{L/z} dt \right. \\ \left(D[a(\sin \theta'_1 - \sin \theta'_2)t + \eta t] \right. \\ + D[a(\sin \theta_1 - \sin \theta_2)t + \eta t] \\ - D[a(\sin \theta'_1 - \sin \theta_1)t - \rho(1-t) + \eta t] \\ - D[a(\sin \theta'_2 - \sin \theta_2)t - \rho(1-t) - \eta t] \\ + D[a(\sin \theta'_1 - \sin \theta_2)t - \rho(1-t)] \\ \left. \left. + D[a(\sin \theta'_2 - \sin \theta_1)t - \rho(1-t)] \right) \right\}, \quad (10)$$

$$D(\xi) = 2B_0 \left[1 - \exp \left(-\frac{\xi^2}{l^2} \right) \right]. \quad (11)$$

Here $D(\xi)$ is called the structure function of the random medium.

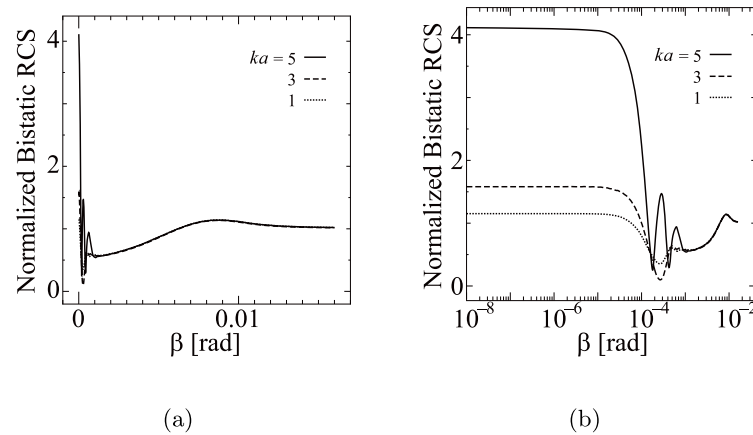


Fig. 2. The bistatic RCSs normalized to those in free space for different size cylinders. (a) Linear scale for β . (b) Logarithmic scale for β .

3 Numerical results

By using (6), we can calculate the bistatic RCS of a conducting circular cylinder embedded in the random medium modeled by (1)~(5). To clear the effect of the medium, we normalize the RCS in the random medium to that in free space.

Figure 2 shows the normalized bistatic RCSs versus the angle β in Fig. 1 for three conducting circular cylinders ($ka = 1, 3, 5$), where $k^2 B_0 l L = 2\pi^2 \times 10^3 \gg 1$ which means that the incident wave becomes incoherent about the cylinder. We find that there are backscattering enhancement peaks at $\beta = 0$ and depressions outside the peaks where the normalized RCS is less than one. For all the cases, the normalized RCS tends to one if β is large enough, and the integral value of the normalized RCS with respect to β becomes almost one. This fact means that the results agree with the law of energy conservation.

As ka increases, the backscattering enhancement becomes larger, and the oscillation of the RCS about the backward direction occurs complicatedly because of interference between incident and scattered waves. In the case of $ka = 5$, the backscattering peak value is about 4.1, much larger than the well-known level that is nearly twice as large as that in free space.

Then it is interesting to know what happens to the RCS if the fluctuation intensity or the layer thickness increases in the case of $ka = 5$. The RCSs for four cases of B_0 : $B_0 = 10^{-6}, 3 \times 10^{-6}, 5 \times 10^{-6}, 10^{-5}$ where $k^2 l L = 4\pi^2 \times 10^8$, and two cases of kL : $kL = 2000, 3000$ where $klB_0 = 4\pi \times 10^{-3}$ are plotted in Fig. 3. The interference between incident and scattered waves becomes stronger, as B_0 or L increases. In the case of $B_0 = 10^{-5}$, the RCS oscillates violently in the neighborhood of the backward direction, and we find that the RCS becomes more than one hundred times as large as that in free space. Similar result but for monostatic RCS was indicated in [14] where the reason has been described.

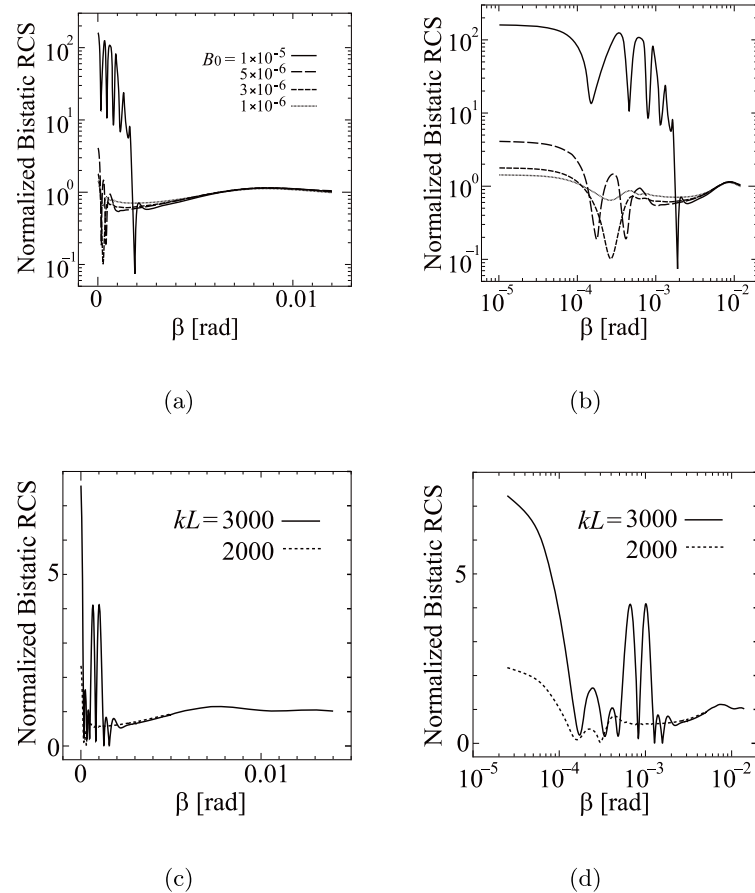


Fig. 3. The effect of fluctuation intensity and layer thickness of the random medium on the RCS. (a) The bistatic RCS for different cases of B_0 . (b) As in (a), but for change in the scale of β . (c) The bistatic RCS for different cases of kL . (d) As in (c), but for change in the scale of β .

4 Conclusion

We discussed the scattering characteristics of a conducting circular cylinder embedded in a random medium by changing the cylinder size and the fluctuation intensity and thickness of the medium. The numerical results of bistatic radar cross-section (RCS) show that sometimes the RCS in the neighborhood of backward direction plays a violent oscillation and becomes much larger than that in free space. The complicated oscillation of the RCS is considered to be caused by statistical interference between incident and scattered waves.