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Optimal allocation of sensing duration among multiple primary channels in cognitive radio

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Abstract: In cognitive radio networks, allocation of the sensing duration is of critical importance for efficient spectrum sensing. Conventional optimization of sensing duration is limited in two primary channels as well as same constant thresholds for all the primary channels. In this paper, we consider multiple primary channels and formulate the problem of sensing duration allocation as a convex optimization with respect to both the sensing durations and detection thresholds. Then, based on Penalty method and Newton's method, a searching algorithm is proposed to obtain the optimal solution. We show via simulation results that our scheme substantially improves the transmission opportunity of cognitive user compared with existing ones.

Keywords: cognitive radio, spectrum sensing, energy detection, transmission opportunity

Classification: Science and engineering for electronics

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1 Introduction

Cognitive radio (CR) [1] has been proposed to solve the spectrum scarcity problem [2] by allowing the secondary users (SUs) to opportunistically access the vacant spectrums licensed to primary users (PUs). In order to avoid interfering with primary users, spectrum sensing is a key problem in CR. Energy detection [3, 4] has been widely applied for spectrum sensing since it does not require any a priori knowledge of primary signals and has the lowest complexity.

Besides designing spectrum sensing algorithms, another important problem on spectrum sensing is the design of the sensing duration. The optimal sensing time for a single channel, which yields the maximum throughput of the CR system with energy detector, is discussed in [5]. In [6], optimal allocation of the sensing time between two channels, which maximizes the transmission opportunity, is investigated, when two primary channels are collocated and the total sensing time is given. In [6], the authors assume that the SU uses the same constant detection threshold for the two primary channels. This limitation imposed on the optimization parameters results in non-optimal solution.

In this paper, we focus on optimal allocation of the sensing duration among primary channels without the constraint that the thresholds of the detectors are the same constant. The problem is formulated as a joint optimization problem under linear constraints. The convexity of the problem is verified by proving that the Hessian matrix of the objective function is negative definite. Then, we propose a searching algorithm combining Penalty method and Newton's method to obtain the optimal solution. In the simulation results, we can observe that the maximum achievable transmission opportunity is improved.

2 System model

We consider a CR network with a SU and L primary channels. Given a fixed total sensing duration τ, the SU senses the primary channels sequentially. The time duration allocated for the lth channel is τ_l, then $\sum_{l=1}^{L} \tau_i = \tau$. We assume the states (idle or busy) of the primary channels are independent with each other. For the lth channel, the binary hypothesis test is formulated as follows:

$$r_l(n) = \begin{cases} \omega_l(n), & H_{0,l} \\ s_l(n) + \omega_l(n), & H_{1,l} \end{cases}, l = 1, 2, \cdots L, n = 1, 2, \cdots, N_l \tag{1}$$

where $H_{0,l}$ and $H_{1,l}$ denote the hypotheses corresponding to the idle state and the busy state of the lth channel, and N_l is the number of samples at

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the lth primary channel. $r_l(n)$, $s_l(n)$ and $\omega_l(n)$ are the nth sample of the received signal, the PU's signal and the noise at the lth channel, respectively. We assume that both $s_l(n)$ and $\omega_l(n)$ are independent identically distributed (i.i.d.) complex Gaussian random variables [4, 5], i.e. $s_l(n) \sim \mathcal{CN}(0, P_l)$, and $\omega_l(n) \sim \mathcal{CN}(0, \sigma^2), l = 1, 2, \cdots, L$. The decision rule of the energy detector at the lth channel is

$$Y_l = \frac{1}{N_l} \sum\nolimits_{n=1}^{N} |R_l\,(n)|^2 \underset{H_{0,l}}{\overset{H_{1,l}}{\gtrless}} \lambda_l, l = 1, 2, \cdots, L \tag{2}$$

where λ_l is the threshold. When N_l is large enough, Y_l can be considered asymptotically normally distributed with distribution $Y_l \sim \mathcal{N}(\sigma^2, \sigma^4/N_l)$ under hypothesis $H_{0,l}$ and $Y_l \sim \mathcal{N}(\sigma^2(1+\gamma_l), \sigma^4\ (1+2\gamma_l)/N_l)$ under $H_{1,l}$, where $\gamma_l = P_l/\sigma^2$ represents the received signal to noise ratio (SNR) at the lth channel. Assuming sampling rate is f_s, and the sensing duration for the lth channel is τ_l, then we have $N_l = \tau_l f_s$. The probability of false alarm and detection at the lth channel can be given by

$$P_{f,l}(\tau_l, \lambda_l) = Q\left(\frac{\lambda_l - \sigma^2}{\sigma^2}\sqrt{\tau_l f_s}\right) \tag{3}$$

$$P_{d,l}(\tau_l, \lambda_l) = Q\left(\frac{\lambda_l - \sigma^2(1+\gamma_l)}{(1+\gamma_l)\sigma^2}\sqrt{\tau_l f_s}\right) \tag{4}$$

where $Q(\cdot)$ denotes complementary distribution function of the standard Gaussian.

3 Problem formulation and optimization

The SU's transmission opportunity is defined as the sum of the probabilities that the primary channels are idle and no false alarm is generated by the SU. Let $P(H_{0,l})(l = 1, 2, \cdots, L)$ be the probability that the lth primary channel is not occupied, and the transmission opportunity can be represented by

$$p(\boldsymbol{\tau}, \boldsymbol{\lambda}) = \sum_{l=1}^{L} P(H_{0,l})(1 - P_{f,l}) \tag{5}$$

where $\boldsymbol{\tau} = [\tau_1, \tau_2 \cdots, \tau_L]$ and $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \cdots, \lambda_L]$. In CR networks, the higher the probability of detection, the better the PUs are protected. Thus, in order to sufficiently protect the PUs, we should maximize the transmission opportunity with the constraint that the detection probabilities are lower bounded. Therefore, the optimization problem can be formulated as follow

$$\begin{aligned} &\max_{\{\boldsymbol{\tau}, \boldsymbol{\lambda}\}} \quad p(\boldsymbol{\tau}, \boldsymbol{\lambda}) = \sum_{l=1}^{L} P(H_{0,l})(1 - P_{f,l}) \\ &s.t. \qquad \sum_{l=1}^{L} \tau_l = \tau \\ &\qquad\qquad P_{d,l} \geq P_{d0,l},\ \ for\ l = 1, 2, \cdots, L \\ &\qquad\qquad P_{f,l} \leq \alpha,\ \ for\ l = 1, 2, \cdots, L \end{aligned} \tag{6}$$

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where $P_{d0,l}$ denotes the lower bound of the detection probability at the lth primary channel. The third constraint $P_{f,l} \leq \alpha$ dictates that each channel should be able to achieve a minimum opportunistic spectrum utilization, generally $\alpha = 0.5$ [3, 6] in CR.

Proposition 1 For a given sensing duration $\tau_l\,(1 \leq l \leq L)$, transmission opportunity is a decreasing function with respect to $P_{d,l}\,(1 \leq l \leq L)$.

Proof Combining (3) and (4) and eliminating $\lambda_l\,(1 \leq l \leq L)$, we can obtain that

$$P_{f,l} = Q(Q^{-1}(P_{d,l})(1+\gamma_l) + \gamma_l\sqrt{\tau_l f_s}),\;\; 1 \leq l \leq L \tag{7}$$

Since $Q(x)$ and $Q^{-1}(x)$ are both decreasing functions, $P_{f,l}$ increases with the increase of $P_{d,l}$. Therefore, according to (5) and (7), the transmission opportunity is a decreasing function with respect to $P_{d,l}$.■

From proposition 1, we can obtain that the maximum transmission opportunity can be achieved when $P_{d,l} = P_{d0,l},\; for\; l = 1, 2, \cdots, L$. In this case, according to (4), the thresholds can be given by

$$\lambda_l = Q^{-1}(P_{d0,l})(1+\gamma_l)\sigma^2/\sqrt{\tau_l f_s} + (1+\gamma_l)\sigma^2 \tag{8}$$

here, we can observe that λ_l is a function with respect to τ_l, $P_{d0,l}$, and γ_l, rather than a constant as in [6]. We can obtain the optimal thresholds by finding the optimal $\tau_l\,(1 \leq l \leq L)$. Constituting (8) into (3), $P_{f,l}$ can be rewritten as

$$P_{f,l} = Q(Q^{-1}(P_{d0,l})(1+\gamma_l) + \gamma_l\sqrt{\tau_l f_s}) \tag{9}$$

Combining (9) and the third constraint in (6), we have

$$\tau_l \geq \tau_{\min,l}, (l = 1, 2, \cdots, L) \tag{10}$$

where $\tau_{\min,l} = \left[Q^{-1}(P_{d0,l})(1+\gamma_l)/(\gamma_l\sqrt{f_s})\right]^2$, which is the minimum sensing duration to satisfy the third constraint in (6). Thus, to guarantee the feasible set is nonempty, the value of τ should meet the following requirement

$$\tau \geq \sum_{l=1}^{L} \tau_{\min,l} \tag{11}$$

From the analysis above, we can rewrite (6) as

$$\begin{aligned} \max_{\tau} \quad & p(\boldsymbol{\tau}) = \sum_{l=1}^{L} P(H_{0,l})[1 - Q(Q^{-1}(P_{d0,l})(1+\gamma_l) + \gamma_l\sqrt{\tau_l f_s})] \\ s.t. \quad & \sum_{l=1}^{L} \tau_l = \tau \\ & \tau_l \geq \tau_{\min,l}, l = 1, 2, \cdots, L \end{aligned} \tag{12}$$

Proposition 2 The optimization problem in (12) is a convex optimization problem.

Proof Firstly we prove the objective function is concave. The Hessian matrix of $p(\boldsymbol{\tau})$ can be calculated as

$$\nabla^2 p(\boldsymbol{\tau}) = diag\left[\frac{\partial^2 p(\boldsymbol{\tau})}{\partial \tau_1^2}, \frac{\partial^2 p(\boldsymbol{\tau})}{\partial \tau_2^2}, \cdots, \frac{\partial^2 p(\boldsymbol{\tau})}{\partial \tau_L^2}\right] \tag{13}$$

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where $diag[\cdot]$ represents diagonal matrix, and $\frac{\partial^2 p(\boldsymbol{\tau})}{\partial \tau_l^2}$, $l = 1, 2, \cdots, L$ denotes the second order partial derivative of the objective function with respect to τ_l. In order to decide whether $\frac{\partial^2 p(\boldsymbol{\tau})}{\partial \tau_l^2}$ is positive or negative, we compute the partial derivative by

$$\frac{\partial p(\boldsymbol{\tau})}{\partial \tau_l} = \frac{P(H_{0,l})\gamma_l\sqrt{f_s}}{2\sqrt{2\pi\tau_l}} \exp\left[-\frac{1}{2}\left(Q^{-1}(P_{d0,l})(1+\gamma_l) + \gamma_l\sqrt{\tau_l f_s}\right)^2\right] \tag{14}$$

In (14), it is easy to see that $\frac{1}{\sqrt{\tau_l}}$ and $\exp\left[-\frac{1}{2}(Q^{-1}(P_{d0,l})(1+\gamma_l) + \gamma_l\sqrt{\tau_l f_s})^2\right]$ are both decreasing function of τ_l, so $\frac{\partial p(\boldsymbol{\tau})}{\partial \tau_l}$ decreases with an increase of τ_l, that is $\frac{\partial^2 p(\boldsymbol{\tau})}{\partial \tau_l^2} < 0$. Therefore, we can obtain that the Hessian matrix is negative definite, and the objective function is concave [7]. Furthermore, since the constraints are all linear, (12) is a convex optimization problem. ■

In the light of proposition 2, we can obtain that there are a unique group of $\tau_1, \tau_2, \cdots, \tau_L$ to maximize the transmission opportunity, and the optimal thresholds can be found by constituting $\tau_1, \tau_2, \cdots, \tau_L$ into (8). The optimization problem can be solved by many existing methods, such as Interior point method, Barrier method and Penalty method [7]. In the following, we propose a searching algorithm combining Penalty method and Newton's method.

Define the Penalty function $f(\boldsymbol{\tau}) = [\max\{0, \tau_{l\,\min} - \tau_l\}]^2 + \left(\sum\limits_{l=1}^{L} \tau_l - \tau\right)^2$, and the feasible set $S = \left\{\boldsymbol{\tau} \,\middle|\, \sum\limits_{l=1}^{L} \tau_1 = \tau, \tau_l \geq \tau_{\min,l}, l = 1, 2, \cdots, L\right\}$.

1. Set k=0, choose a starting point $\boldsymbol{\tau}^{(k)} = \left[\tau_1^{(k)}, \cdots, \tau_L^{(k)}\right] \in S$. Set n=0, and initialize penalty coefficient δ_n, multiplying factor c(> 1), and tolerance $\varepsilon\,(> 0)$.
2. Solve the optimization problem $\min F(\boldsymbol{\tau}) = -p(\boldsymbol{\tau}) + \delta_n f(\boldsymbol{\tau})$ by Newton's method:
   a) Compute the gradient $\nabla F(\boldsymbol{\tau}^{(k)})$ and the inverse of Hessian matrix $\nabla^2 F(\boldsymbol{\tau}^{(k)})^{-1}$;
   b) If $\left\|\nabla F(\boldsymbol{\tau}^{(k)})\right\| \leq \varepsilon$, break and go to step 3; else let $\boldsymbol{d} = -\nabla^2 F(\boldsymbol{\tau}^{(k)})^{-1} \nabla F(\boldsymbol{\tau}^{(k)})$, go to step (c);
   c) Update: $k = k+1, \boldsymbol{\tau}^{(k)} = \boldsymbol{\tau}^{(k-1)} + \mathbf{d}$, go to step (a).
3. If $\delta_n f(\boldsymbol{\tau}^{(k)}) < \varepsilon$, stop and the optimal solution is $\boldsymbol{\tau}^{(k)}$; else let $n = n+1$, $\delta_n = c\delta_{n-1}$, go to step 2.

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4 Simulation results

In this section, Simulation results will be presented. Firstly, in figure 1, we plot the optimal transmission opportunity versus L. The total sensing time τ is 15 ms. For each primary channel l, we set $\gamma_l = -12\,\mathrm{dB}$, $P(H_{0,l}) = 0.8$, and $P_{d0,l} = 0.9$. It can be seen that the optimal transmission opportunity increases with L. However, the improvement reduces when the number of primary channels is getting larger due to the decreasing of the value range of sensing duration for each primary channel.

In order to make comparison with Kim's work, we consider $L = 2$ primary channels, with the total sensing time as $\tau = 15\,\mathrm{ms}$. two scenarios are taken into account: 1) $\gamma_1 = -8\,\mathrm{dB}$, $\gamma_2 = -12\,\mathrm{dB}$, $P(H_{0,l}) = 0.8(l = 1, 2)$, and $P_{d0,l} = 0.9(l = 1, 2)$; 2) $\gamma_l = -12\,\mathrm{dB}(l = 1, 2)$, $P(H_{0,1}) = 0.2$, $P(H_{0,2}) = 0.8$, and $P_{d0,l} = 0.9(l = 1, 2)$. Figure 2 plots the transmission opportunity against τ_1. From figure 2, we can see that the optimal transmission opportunities are 1.599 and 1.277 in scenario 1 and 2, respectively, which are larger than

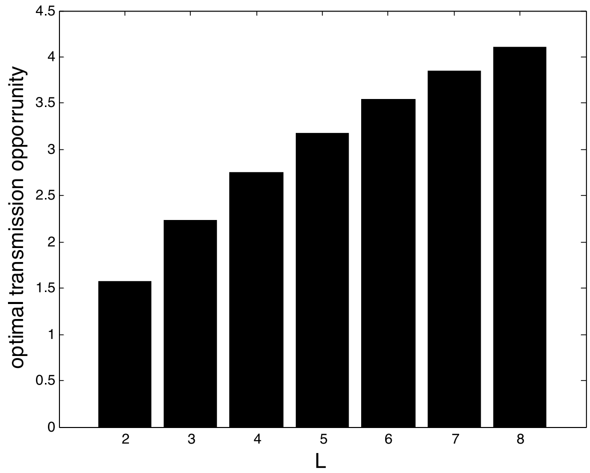

Fig. 1. Optimal transmission opportunity versus L : $\gamma_l = -12\,\mathrm{dB}$, $P(H_{0,l}) = 0.8$, and $P_{d0,l} = 0.9$

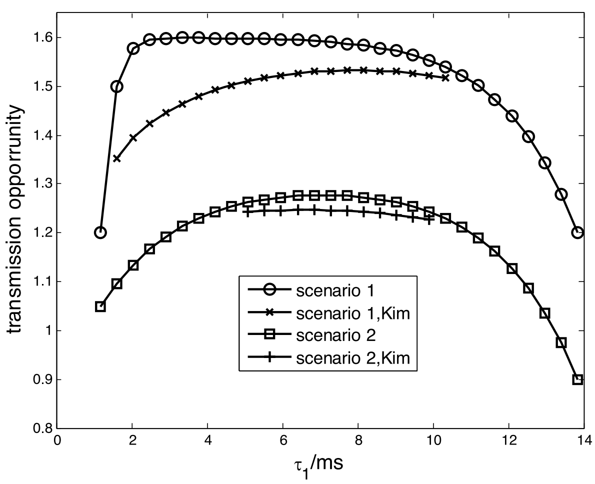

Fig. 2. Transmission opportunity versus sensing time for the first primary channel

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that Kim obtains in [6]. Moreover, as indicated in (8), the thresholds at the two channels vary with τ_1. The optimal thresholds are $\lambda_1 = 1.1575\sigma^2$ and $\lambda_2 = 1.0627\sigma^2$ in scenario 1, and $\lambda_1 = 1.0631\sigma^2$ and $\lambda_2 = 1.0626\sigma^2$ in scenario 2, while Kim sets $\lambda_1 = \lambda_2 = 1.0315\sigma^2$ all the time. In addition, since τ_1 and τ_2 have lower bounds, the value range of τ_1 is smaller than the interval [0, 15].

5 Conclusion

Given a fixed total sensing duration, the optimal allocation of the sensing time for maximizing the transmission opportunities of a CR network with multiple primary channels is obtained in this paper. The simulation results show that the achievable optimal transmission opportunity of our allocation is apparently larger than that in the existing literature.

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