A recursive method for compensating ionospheric phase contamination based on multistage Taylor expansion

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Abstract: A recursive approach for the compensation of the ionospheric phase contamination is proposed based on multistage Taylor expansion and the extended Kalman filter. The proposed algorithm can dynamically compensate the ionospheric phase contamination. We show by numerical examples that the proposed algorithm outperformances the conventional methods such as the polynomial phase signal (PPS) and the singular value decomposition (SVD).

Keywords: OTHR, phase contamination, multistage Taylor expansion

Classification: Electron devices, circuits, and systems

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1 Introduction

High frequency (HF) sky-wave over the horizon radar (OTHR) can detect targets at ranges of 800–3200 km via ionospheric refraction. The frequency spectrum of an HF signal scattered by the sea surface is usually assumed to include a pair of spikes, called Bragg lines [1, 2, 3], located around $\pm f_B \approx \pm 0.102\sqrt{f_c}$, where f_c is the carrier frequency in MHz. Due to the non-stationarity of the ionosphere, the echoes may be subjected to regular and random phase-path variations over the coherent processing interval (CPI) [4]. These variations can significantly change the Bragg lines spectrum characteristics. Specifically, the Bragg lines spectrum in a particular range-azimuth cell may be subjected to a mean Doppler shift (frequency translation), due to the regular component of phase-path variation over the CPI, whereas any random component of phase-path variation results in smearing (frequency broadening) of the Bragg lines spectrum. In OTHR, the Doppler spreads and shifts of the Bragg lines can have a detrimental impact on useful signal detection, false-alarm rate, and target relative velocity estimation. Intuitively, the Doppler spreads of the Bragg lines may bury the returned signal of slow moving targets. A number of methods have been studied to compensate the ionospheric phase contamination, for example, the SVD-based method in [2, 5], the PPS-based method in [6], etc. In this letter, we propose a novel compensation algorithm which can more accurately compensate the ionospheric phase contamination. Notations: $(\cdot)^{\dagger}$, $(\cdot)^{H}$, $Diag(\cdot)$, $Phase(\cdot)$, $Im(\cdot)$, and $Re(\cdot)$ denote transpose, conjugate-transpose, block diagonal matrix and taking phase angle, real part, and imaginary part operators.

2 Signal model

The received signal (after matched filtering and beamforming) in the cell-under-test during a coherent processing interval (CPI) can be expressed as

$$r_k = \beta_k e^{j2\pi f_d kT} e^{j\varphi_k} + c_k e^{j\varphi_k} + v_k, \;\; 1 \le k \le K \tag{1}$$

where k denotes the slow time index, T is the pulse repetition interval (PRI), K is the number of pulses during the CPI, β_k is the target reflection coefficient, f_d is the Doppler shift, v_k denotes the zero-mean white Gaussian noise with covariance σ_v^2, and $\varphi_k = \varphi(kT)$ with $\varphi(t)$ denoting the time varying ionospheric phase contamination. The c_k in (1) represents the sea clutter return which can be modeled as [1]

$$c_k = b_k^+ e^{j2\pi f_B kT} + b_k^- e^{-j2\pi f_B kT} \tag{2}$$

where b_k^+ and b_k^- are the magnitudes of the positive and negative Bragg lines. The clutter return, which is usually much stronger than the target return, is extracted from the received signal and used to obtain an estimate $\hat{\varphi}_k$ of the contaminating phase φ_k. Thus, multiplying the received signal r_k by $e^{-j\hat{\varphi}_k}$, the phase contamination can be compensated. A bandpass filter (BPF) centered at $-f_B$ with bandwidth B is often adopted to extract the clutter return [6]. For simplicity, assume the BPF is ideal, then the filtered signal at time k can be expressed as

$$\tilde{r}_k = b_k^- e^{j(-2\pi f_B kT + \varphi_k)} + w_k \tag{3}$$

where w_k denotes the noise of the BPF output at time k, assumed to be zero-mean complex Gaussian distributed with unknown covariance $2\sigma_r^2$.

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3 Proposed method

We model the contaminating phase φ_k at time k using a $(q-1)$th order Taylor expansion around φ_{k-1} as follows [7]

$$\varphi_k = \varphi_{k-1} + T\varphi_{k-1}^{(1)} + \cdots + \frac{1}{(q-1)!}T^{q-1}\varphi_{k-1}^{(q-1)} + \Delta_\xi \tag{4}$$

where $\varphi_{k-1}^{(q)} = \varphi^{(q)}((k-1)T)$ denotes the qth derivative of the phase modulation function $\varphi(t)$ sampled at time $(k-1)T$, $\Delta_\xi = \frac{1}{q!}T^q\varphi^{(q)}(\xi)$ with $\xi \in ((k-1)T, kT)$ denoting the Lagrange remainder and $q!$ the factorial of q. Since the closed-form ξ is unavailable, we assume

$$\varphi^{(q)}(\xi) = \varphi_{k-1}^{(q)} + w_\varphi \tag{5}$$

where w_φ is a zero-mean Gaussian distributed random variable. Further approximate the sampled ith order derivative $\varphi_k^{(i)}$, $i = 1, \cdots, q$, using a qth order Taylor expansion as follows

$$\begin{aligned}
\varphi_k^{(1)} &= \varphi_{k-1}^{(1)} + T\varphi_{k-1}^{(2)} + \cdots + \frac{1}{(q-1)!}T^{q-1}(\varphi_{k-1}^{(q)} + w_\varphi)\\
&\cdots\\
\varphi_k^{(q-1)} &= \varphi_{k-1}^{(q-1)} + T(\varphi_{k-1}^{(q)} + w_\varphi)\\
\varphi_k^{(q)} &= \varphi_{k-1}^{(q)} + w_\varphi.
\end{aligned} \tag{6}$$

The unknown clutter magnitude b_k^- is modeled using a $(p-1)$th order Taylor expansion similar to the calculations from (4) to (6). Stacking the unknown contaminating phase φ_k and clutter magnitude b_k^- and their derivatives in a state vector

$$\mathbf{x}_k = \left[\varphi_k, \cdots, \varphi_k^{(q)}, b_k^-, \cdots, (b_k^-)^{(p)}\right]^\dagger \tag{7}$$

and taking the real and imaginary parts of the BPF output $\tilde{r}_k$ in (3) as the measurements, a state-space model can be formulated as follows

$$\begin{aligned}
\mathbf{x}_k &= \mathbf{F}\mathbf{x}_{k-1} + \mathbf{G}_w\\
\mathbf{y}_k &= [Im(\tilde{r}_k), Re(\tilde{r}_k)]^\dagger
\end{aligned} \tag{8}$$

where $\mathbf{F} = Diag(\mathbf{F}_\varphi, \mathbf{F}_b)$, $\mathbf{G}_w = [w_\varphi \mathbf{G}_\varphi^\dagger, w_b \mathbf{G}_b^\dagger]^\dagger$, w_φ and w_b are zero-mean Gaussian distributed with unknown variances σ_φ^2 and σ_b^2,

$$\mathbf{F}_\varphi = \begin{bmatrix} 1 & T & \cdots & \frac{1}{q!}T^q \\ 0 & 1 & \cdots & \frac{1}{(q-1)!}T^{q-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, \quad \mathbf{G}_\varphi = \begin{bmatrix} \frac{1}{q!}T^q \\ \frac{1}{(q-1)!}T^{q-1} \\ \vdots \\ 1 \end{bmatrix}, \tag{9}$$

and $\mathbf{F}_b$ and $\mathbf{G}_b$ can be obtained by replacing q with p in $\mathbf{F}_\varphi$ and $\mathbf{G}_\varphi$, respectively. Then, the extended Kalman filter (EKF) is employed to dynamically estimate the state vector as follows

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}\hat{\mathbf{x}}_{k-1|k-1} \tag{10}$$

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$$\mathbf{P}_{k|k-1} = \mathbf{F}\mathbf{P}_{k-1|k-1}\mathbf{F}^{\dagger} + \mathbf{Q}_{k-1} \tag{11}$$

$$\mathbf{L}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^{\dagger}(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^{\dagger} + \mathbf{R}_k)^{-1} \tag{12}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{L}_k\big(\mathbf{y}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1})\big) \tag{13}$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{L}_k\mathbf{H}_k)\mathbf{P}_{k|k-1} \tag{14}$$

where $\mathbf{H}_k = \partial\mathbf{h}(\mathbf{x}_k)/\partial\mathbf{x}_k|_{\mathbf{x}_k=\hat{\mathbf{x}}_{k|k-1}}$, $\mathbf{h}(\mathbf{x}_k) = [b_k^- sin(\varphi_k - 2\pi f_B kT), b_k^- cos(\varphi_k - 2\pi f_B kT)]^{\dagger}$, and $\mathbf{Q}_k$ and $\mathbf{R}_k$ the unknown covariance matrices of the state and measurement noise, updated by

$$\mathbf{Q}_k = \eta\mathbf{Q}_{k-1} + (1-\eta)(\hat{\mathbf{x}}_{k|k} - \mathbf{F}\hat{\mathbf{x}}_{k-1|k-1})(\hat{\mathbf{x}}_{k|k} - \mathbf{F}\hat{\mathbf{x}}_{k-1|k-1})^H, \tag{15}$$

$$\mathbf{R}_k = \eta\mathbf{R}_{k-1} + (1-\eta)(\mathbf{y}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k})(\mathbf{y}_k - \mathbf{H}_k\hat{\mathbf{x}}_{k|k})^H \tag{16}$$

in which $0 \le \eta \le$ denotes the forgetting factor.

After obtaining the estimate $\hat{\mathbf{x}}_{k|k}$ of the state vector and hence the estimated contaminating phase $\hat{\varphi}_k$, the received signal can be decontaminated as $r_k^d = r_k e^{-j\hat{\varphi}_k}$.

4 Numerical results

In the simulations, assume that the ionospheric phase contamination can be approximated by [2, 5, 6]

$$\varphi_k = \pi sin(2\pi\gamma kT) \tag{17}$$

where γ is the modulation frequency. The clutter-to-noise ratio (CNR) is defined as $CNR = 10\log((b_k^-)^2/\sigma_v^2)$. The parameter settings and the initial values of the covariance matrices $\mathbf{Q}_0$ and $\mathbf{R}_0$ are given in Table I.

Table I. Simulation parameter settings for the numerical examples

$f_c = 15\,\text{MHz}$	$\beta_k = 0.2$	$\eta = 0.98$
$T = 0.2\,\text{s}$	$p = 2$	$\mathbf{R}_0 = \mathbf{I}$
$K = 512$	$q = 1$	$\mathbf{Q}_0 = Diag(\mathbf{G}_\varphi\mathbf{G}_\varphi^{\dagger}, \mathbf{G}_b\mathbf{G}_b^{\dagger})$
$f_d = 0.5\,\text{Hz}$	$b_k^- = 6$	$\hat{\mathbf{x}}_{0\|0} = [Phase(\tilde{r}_1), 0, 0, \|\tilde{r}_1\|, 0]^{\dagger}$
$B = 0.6\,\text{Hz}$	$b_k^+ = 3$	$\mathbf{P}_{0\|0} = \mathbf{I}$

Fig. 1 illuminates the power spectrum density (PSD) of the contaminated signal r_k and the decontaminated signal r_k^d using the proposed EKF-based method. It is observed that, for the contaminated signal (dashed curve), two peaks appear around $\pm 0.102\sqrt{f_c} = \pm 0.4\,\text{Hz}$ which are the frequency centers of the Bragg lines caused by sea clutter. The mainlobes of the clutter return completely bury the target return, which shows the Doppler spread effect of the ionospheric phase contamination. For the decontaminated signal (solid curve), three peaks are observed. The two peaks appearing at $\pm 0.4\,\text{Hz}$ indicate the clutter returns which have narrower mainlobes compared with the contaminated signal. Thanks to the reduction of the clutter mainlobes, another peak becomes visible at $f_d = 0.5\,\text{Hz}$ in this case, which comes from the target return, making it possible for further signal processing. In Figs. 2 and 3, the root mean square error (RMSE), averaged over noise realizations and over time, for the estimates of the contaminating phase $\hat{\varphi}_k$ and the corresponding frequency $\hat{\varphi}_k^{(1)}$ are plotted versus CNR, respectively. In the simulations, the column number of the Hankel matrix is set to 4 for the SVD-based method [5], and the data

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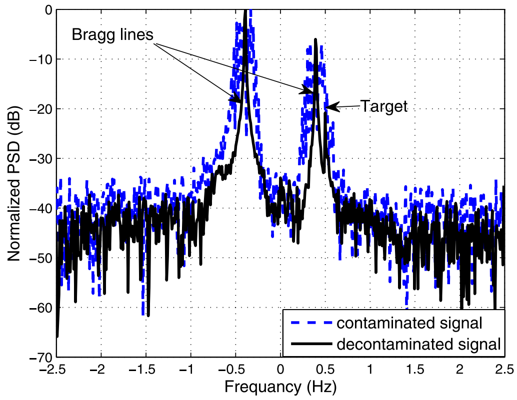

Fig. 1. Compensation using the proposed method, $CNR = 25\,\text{dB}$, $\gamma = 0.02$.

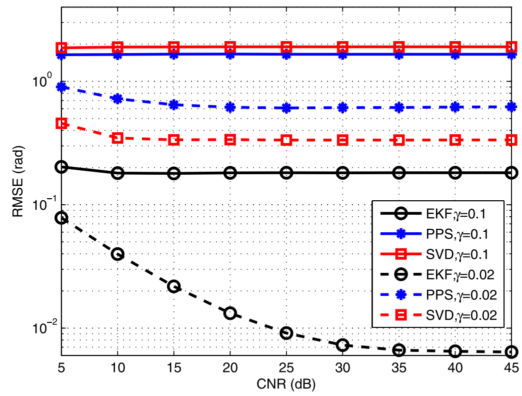

Fig. 2. RMSE for the estimate of contaminating phase.

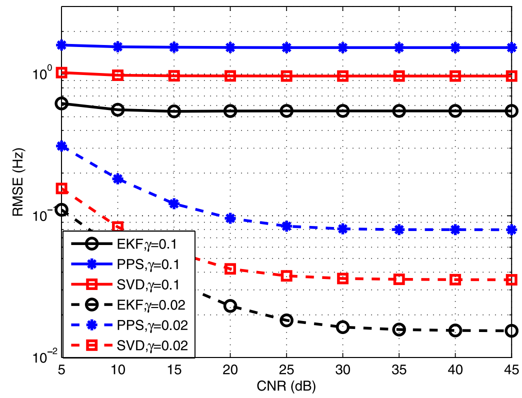

Fig. 3. RMSE for the estimate of contaminating frequency.

size and number of overlapping points are set to 16 and 15 for every segment for the PPS-based method [6]. It is seen that the proposed method leads to smaller RMSEs than the PPS and SVD methods for the tested values of the CNRs and γ's, which means that our method can better compensate the ionospheric phase contamination in terms of RMSE performance. In our model in (8), we considers not only the phase change, but also the magnitude change in CPI. Our model can better match with the data characteristics that are actually received. However, the echo magnitude is considered to be constant in the SVD and PPS methods.

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5 Conclusions

This letter proposes to model the ionospheric phase contamination in a recursive manner based on multistage Taylor expansion. The estimate of the contaminating phase obtained using EKF is used to compensate the contaminated signal dynamically. Numerical examples show that the proposed method can compensate the phase contamination effectively and has superior RMSE performance in estimating the contaminating phase.

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