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A Multi-stage Greedy Power Allocation scheme for average downlink system throughput maximization

Janghoon Yang[1] **and Dong Ku Kim**[2a)]

[1] *Newmedia department, Korean German Institute of Technology*
661, Deungchon-dong, Gangseo-gu, Seoul, 157–033, Korea
[2] *School of Electrical and Electronics Engineering, Yonsei University*
YERC 225D, Yonsei University, Seodaemun-Gu, Seoul, 120–749, Korea
a) *dkkim@yonsei.ac.kr*

Abstract: This paper considers a static downlink power allocation algorithm for a multi-cell system in order to maximize the average system throughput. Based on observation that at least one base station (BS) has to transmit at the maximum power for optimal power allocation, and the optimal binary power allocation (OBPA) may still cause excessive interference, we propose a multi-stage greedy power allocation (MSGPA) which allows some of BSs to have several discrete transmit power levels by introducing multi-stage power adjustment. The simulation results show that the MSGPA outperforms the OBPA while significantly reducing complexity and transmit power as well.

Keywords: multicell, system throughput maximization, downlink power allocation, greedy power allocation

Classification: Science and engineering for electronics

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1 Introduction

In an existing cellular system, more base stations (BSs) are installed to improve the spectral efficiency. However, increasing number of BSs may improve the performance marginally as the multi-cell system is over-populated with many BSs, where every BS transmitting at maximum power may be even harmful. To deal with this problem, an efficient BS transmit power allocation can be considered. To avoid possible huge complexity from power control in the continuous domain, several different types of the instantaneous binary power allocation (BPA) which allows each BS to transmit at either minimum or maximum power [1, 2]. A distributed nonbinary power control associated with orthogonal resource allocation maximization was also proposed to maximize the network utility [3]. There was also game theoretic approach for maximizing utility with power control for linear array two-cell code division multiple access (CDMA) data network [4].

However, many of existing power allocation algorithms are developed as dynamic ones which deal with instantaneous channel realization for a given serving BS assignment. In this paper, we formulate the static downlink power allocation problem which maximizes the average system throughput which is defined as sum of the average cell throughput. Instead of finding an optimal solution, we propose a multi-stage greedy power allocation (MSGPA) algorithm which outperforms the optimal BPA (OBPA) while having reduced transmit power.

2 System model

We consider a multi-cell downlink system consisting of B BSs and total K mobile stations (MSs) which are equipped with a single antenna. It is assumed that each BS transmits a single stream at a given time instance when its power is set to be nonzero. It is also assumed that MS estimates the signal to interference plus noise power ratio (SINR) perfectly, and every SINR information of all MSs is available to a centralized controller which decides system setup. The received signal r_k of the MS k for this system can be represented as

$$r_k = \sum_{b=1}^{B} \sqrt{P_b \xi_{b,k}} h_{b,k} x_b + n_k \tag{1}$$

where P_b is transmit power of the BS b, $\xi_{b,k}$ is long-term channel power, $h_{b,k}$ and n_k are short-term channel and thermal noise respectively which are circularly complex independent Gaussian random variables with mean zero and unit variance, and x_b is information symbol of the BS b. With this signal model, the short-term SINR $\gamma_{b,k}$ of the information symbol of the BS b measured at the MS k can be calculated as

$$\gamma_{b,k} = \frac{P_b \xi_{b,k} |h_{b,k}|^2}{\sum_{b'=1, b' \neq b}^{B} P_{b'} \xi_{b',k} |h_{b',k}|^2 + 1} \tag{2}$$

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To define the downlink power allocation explicitly, it is assumed that the serving BS is assigned based only on the average SINR, meaning that the serving BS of the user k can be determined as follows.

$$s(k) = \arg\max_b E\{\gamma_{b,k}\} \tag{3}$$

Based on the BS assignment rule, we can formulate BS power allocation problem which maximizes the average system throughput under round robin user scheduling policy as follows.

$$\mathbf{P}^* = \arg\max_{\mathbf{P}} \sum_{k=1}^{K} R_{s(k),k}(\mathbf{P}) \tag{4}$$

where $\mathbf{P} = [P_1, \cdots, P_B]$ is a power vector, and $R_{s(k),k}(\mathbf{P})$ is the average throughput of the user k which is defined as

$$R_{s(k),k}(\mathbf{P}) = \frac{1}{K_{s(k)}} E\{\log(1 + \gamma_{s(k),k})\} \tag{5}$$

where $K_{s(k)}$ is the number of users served by the BS $s(k)$.

3 Transmit power allocation in two cells

In this section, we make the system setup simple to capture the characteristics of the downlink power allocation for maximizing average system throughput. To do so, we consider a two cell model with considering distance dependent path loss only, not shadowing for long-term channel power. In this setup, the SINR of the user k can be rearranged as follows.

$$\gamma_{s(k),k} = SIR_{s(k),k} \frac{|h_{s(k),k}|^2}{|h_{-s(k),k}|^2 + (SNR_{-s(k),k})^{-1}} \tag{6}$$

where $-s(k)$ denotes an element in the set $\{1, 2\} - \{s(k)\}$, $SIR_{s(k),k} = SNR_{s(k),k}/SNR_{-s(k),k}$, and $SNR_{b,k} = P_b\xi_{b,k}$. It is noted that this SINR has the same form as one for random beamforming except scaling factor [5]. From this observation, the cumulative distribution function (CDF) of this random variable can be easily derived as

$$F_{s(k),k}(x) = 1 - \frac{e^{SNR_{s(k)}^{-1}}}{1 + SIR_{s(k)}^{-1}} \tag{7}$$

Finally, the average throughput $R_k([P_1, P_2])$ of the user under round robin scheduling policy can be calculated as

$$\begin{aligned} R_{s(k),k}([P_1, P_2]) &= \frac{1}{K_{s(k)}} \int \log(1 + x) f_{s(k),k}(x) dx \\ &= \frac{Q(SNR_{s(k),k}^{-1}) - Q(SNR_{-s(k),k}^{-1})}{\left(1 - SIR_{s(k),k}^{-1}\right) K_{s(k)}} \end{aligned} \tag{8}$$

where $f_{s(k),k}(x)$ is the probability density function (p.d.f.) of the user k's SINR, $Q(x) = e^x E_1(x)$, and $E_1(\cdot)$ is the well known exponential integral [6].

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To get some intuition from this sum rate expression, we introduce the following upper bound of the exponential integral [7].

$$E_1(x) < \mathrm{e}^{-x}\log(1+\frac{1}{x}) \text{ for } x > 0 \tag{9}$$

Replacing $E_1(x)$ with its upper bound in (9) approximates $R_{s(k),k}([P_1, P_2])$ as

$$\begin{aligned} &R_{s(k),k}([P_1, P_2]) \\ &\approx \frac{1}{1-SIR^{-1}_{s(k),k}}\frac{1}{K_{s(k)}}[\log(1+SNR_{s(k),k}) - \log(1+SNR_{-s(k),k})] \end{aligned} \tag{10}$$

From this approximation, it can be roughly said that the average data rate of the MS k in two cells environment is difference between two average single cell rates properly weighted by function of the SIR.

However, one can easily notice that inserting (8) to (4) is nonconvex problem. The BPA for a given serving BS assignment was proven to be optimal for instantaneous power allocation in two cells case [2]. However, since our problem is associated with power allocation in average sense and serving BS assignment can change with different power allocations, the BPA is not optimal any more. A direct optimization of this problem requires two-dimensional nonconvex optimization. Fortunately, it is turned out that this problem can be solved with two one-dimensional nonconvex optimizations.

Theorem 1 *One of two BSs or both BSs transmit at maximum power when the power is optimally allocated in average throughput sense for two cell environment regardless of the statistical property of the channel.*

Proof Suppose $(\alpha_1^* P_{\max,1}, \alpha_2^* P_{\max,2})$ with $\alpha_1^* < 1$ and $\alpha_2^* < 1$ be optimal power allocation in average throughput sense. However, $\sum_{k=1}^{K} R_{s(k),k}(\mathbf{P}^*) < \sum_{k=1}^{K} R_{s(k),k}(\beta\mathbf{P}^*)$ for $\beta > 1$, since $SIR_{s(k),k}\frac{|h_{s(k),k}|^2}{|h_{-s(k),k}|^2+(SNR_{-s(k),k})^{-1}} < SIR_{s(k),k}\frac{|h_{s(k),k}|^2}{|h_{-s(k),k}|^2+(\beta SNR_{-s(k),k})^{-1}}$ for all k. This is contradiction. This means $\alpha_1^* \geqslant 1$ or $\alpha_2^* \geqslant 1$. However, due to maximum power constraint, $\alpha_1^* \leqslant 1$ or $\alpha_2^* \leqslant 1$. Thus, $\alpha_1^* = 1$ or $\alpha_2^* = 1$, which implies that either one BS or both BS transmits at maximum power. Since (6) is an general expression for SINR regardless of the statistical property of the channel, maximum power condition holds regardless of statistical property of the channel.

This theorem has an analogy with the Lemma-1 in [2] which mandates at least one BS to transmit at maximum power for power allocation maximizing the instantaneous sum rate. This theorem can be readily generalized for any number of BSs.

4 Multi-Stage Greedy Power Allocation (MSGPA)

In this section, we propose a multi-stage greedy power allocation which can be considered as a generalization of the greedy BPA (GBPA) to reduce complexity. Greedy-type algorithm is usually used as an alternative for brute-force

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1. *Initialization*

$\mathbf{P}_1 = \mathbf{P}_{\max}$

2. *for* $p = 1 : N_{stage}$

$S_c = \{n \mid [\mathbf{P}_1]_n == P_{\max}\}, S'_c = S_c$

for $n = 1 : (|S_c|)$

$[r(n)\ \pi(n)] = \min_{b \in S'_c} \sum_{k=1}^{K} R_{s(k),k}(\mathbf{P}_n) I_b(s(k))$

$\mathbf{P}_{n+1} = \mathbf{P}_n$

$[\mathbf{P}_{n+1}]_{\pi(n)} = \begin{cases} 0, & p = 1 \\ P_{\max}/(P_{step})^{p-1}, & else \end{cases}$

$S'_c = S'_c - \{\pi(n)\}$

end

$[R(p)\ \overline{\pi}(n)] = \max_{n \in \{1,\cdots,|S_c|\}} r(n), \mathbf{P}^*_p = \mathbf{P}_{\overline{\pi}(n)}, \mathbf{P}_1 = \mathbf{P}^*_p$

end

$p^* = \arg\max_p R(p), \mathbf{P}_{MSGPA} = \mathbf{P}^*_{p^*}$

Fig. 1. Multi-stage greedy power allocation algorithm. (A and B in $[A\ B]$ refer to the resulting value and corresponding index resepctively from min or max).

search. It reduces the complexity by restricting candidate power allocations to several possibly good ones. Consequently, it may reduce the number of the BSs transmitting at maximum power. However, it is likely that excessive interference can be still present from the maximum power transmission of the GBPA. If this is the case, reducing the transmission power further can improve the system performance.

However, finding an optimal power level for each BS selected through the GBPA to improve the performance further is still too complicated. Thus, we propose a simple greedy-type power allocation scheme of refining the power allocation with multi-level, which we call a multi-stage greedy power allocation (MSGPA) algorithm. This is summarized in Fig. 1. It starts with maximum power allocation to all BSs. Basically it consists of an inner loop and an outer loop. The inner loop decides transmit power of each BS while the outer loop determines minimum nonzero transmit power level. For the first iteration of the outer loop, at each iteration of the inner loop, it turns off the BS having the smallest average transmit rate among the BSs with maximum power allocation. For the subsequent pth iteration of the outer loop, at each iteration of the inner loop, the power level of the BS having the smallest average transmit rate among the BSs with maximum power allocation is scaled by $(P_{step})^{p-1}$. After completing all iterations of the inner loop, it saves the maximum average system rate and corresponding power allocation, as which the power allocation for the next iteration of the outer loop, $\mathbf{P}_1$ is initialized. After finishing all iterations of the outer loop and inner loop, the power allocation which has the largest average system rate is selected as the multi-stage greedy power allocation, $\mathbf{P}_{MSGPA}$. It can be noted that MSGPA is redueced to GBPA when $N_{stage} = 1$.

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5 Numerical results and conclusions

In this section, we present several numerical evaluations of the proposed algorithms to prove its efficiency compared to the OBPA with the minimum power level of zero. Unless otherwise stated, the simulation assumption follows the system model description in the section-2. Users are distributed randomly with the uniform distribution over the square of 3 km by 3 km. For each link, independent shadowing log normal distributed with standard deviation of 7 dB was multiplied to each path loss which was calculated as d^3 with d being in the unit of km. For each drop of MSs and BSs, 200 independent short-term channels were generated to calculate the average system throughput for given distribution. To average out the effect of the random position, 200 uniformly distributed independent drops of MS and BSs were made.

To find out the proper parameters for the MSGPA, the average system throughputs were evaluated for several different P_{step}s and N_{stage}s in Fig. 2. BSs are distributed randomly with the uniform distribution over the square of 3 km by 3 km, and the number of users in the system is 100. The normalized system throughput is defined as the average system throughput divided by its maximum average system throughput over P_{step}s and N_{stage}s for a fixed number of the BSs and a given maximum transmit power level. For wide range of P_{step}, $N_{stage} = 4$ provides almost identical performance to that with larger N_{stag} regardless of the number of BSs and the maximum transmit power level. However, it can be observed that optimal P_{step} tends to be proportional to the maximum transmit power. Based on this simulation result, we roughly set the N_{stage} and P_{step} to be 4 and $10 \times \max(1, \sqrt{P_{\max}/10})$ respectively for subsequent simulations. We compare the performance of the MSGPA with those of the MPA, OBPA, and GBPA in Fig. 3. The number of users in the system is 200, and we consider two cases of BS distributions which are a fixed uniform location with equal distance between the adjacent BSs and random locations of the BSs following the uniform distribution. It can be observed that the MSGPA outperforms all other power allocation schemes regardless of the statistical properties of the locations of the BSs. For large maximum transmit power, the MSGPA can effectively reduce the excessive interference so that the average system throughput can be even larger than

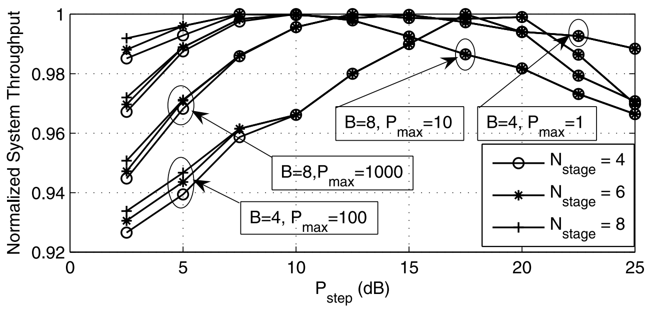

Fig. 2. The average system throughputs for various P_{step}s and N_{stage}s.

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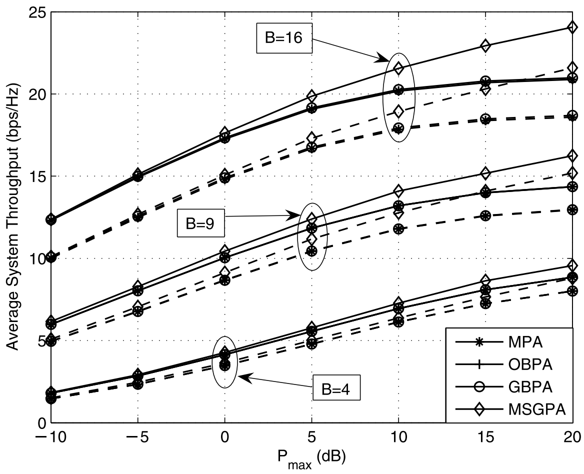

Fig. 3. Comparison of the performances of the proposed algorithms with the OBPA and the MPA for the average system throughput (dotted line: random distribution of the BSs, solid line: a fixed uniform distribution with equal distance between adjacent BSs).

that of the OBPA while its complexity is significantly lower than that of the OBPA. It is also noted that the fixed locations of the BSs with equal distance achieves the better performance than the random locations, which naturally follows from the well defined BS locations for uniformly distributed MSs.

In this paper, we proposed the multi-stage greedy power allocation for maximizing average system throughput based on observation that the OBPA might still cause excessive other cell interference. It was numerically shown that the proposed MSGPA outperforms the existing power allocation schemes while having reduced transmit power.

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