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A SCN-TLM approach for the analysis of Lorentz dispersive media

Mohsine Khalladi[a)] and M. Iben Yaich

Electronic & Microwaves Group,

Faculty of Sciences, Abdelmalek Essaadi University,

P.O. Box: 2121, Tetuan 93000 Morocco

a) *khalladi@fst.ac.ma, M_khall@ieee.org*

Abstract: An approach for the analysis of dispersive media, based on a special transmission-line modelling method with symmetrical condensed node (TLM-SCN) with voltage sources, is proposed. It is used in the case of linear and isotropic Lorentz frequency dependence media. The scattering matrix of the proposed SCN is provided and the efficiency and the validity of this approach are proved by the computation of the reflection coefficients of air-Lorentz medium interfaces and the RCS of a dispersive sphere.

Keywords: Lorentz media, time-domain electromagnetics, SCN-TLM method

Classification: Electromagnetic theory

References

[1] C. Christopoulos, "The Transmission—Line modeling method TLM," *IEEE Series on Electromagnetic Wave Theory*, New York, 1995.

[2] P. B. Johns, "A symmetrical condensed node for the TLM method," *IEEE Trans. Microwave Theory Tech.*, vol. 35, no. 4, pp. 370–377, April 1987.

[3] M. Khalladi, M. Essaaidi, K. Bargach, and M. Iben Yaich, "A new junction model with two wires of different radius for the TLM method," *Microwave Opt. Tech. Lett.*, vol. 19, no. 2, pp. 103–107, Oct. 1998.

[4] K. Bargach, M. Khalladi, and M. Iben Yaich, "New Six-Wires-Junction Matrix Model for the TLM Method," *Microwave Opt. Tech. Lett.*, 20, vol. 29, no. 2, pp. 110–113, April 2001.

[5] M. Khalladi, J. A. Morentey, and J. A. Porti, "RCS of Arbitrary Shaped Targets with the TLM Method," *IEEE Trans. Antennas Propagat.*, vol. 42, no. 6, pp. 891–893, June 1994.

[6] L. R. A. X. de Menezes and W. J. R. Hoefer, "Modeling of general constitutive relationships in SCN TLM," *IEEE Trans. Microwave Theory Tech.*, vol. 44, no. 6, pp. 854–861, June 1996.

[7] I. El Babli, A. Sebak, and N. Simons, "Application of the TLM method to the interaction of the EM fields with dispersive dielectric Bodies," *IEE Proc. Microwave Antennas Propagat.*, vol. 147, no. 3, June 2000.

[8] J. Paul, C. Christopoulos, and D. W. P. Thomas, "Generalized material models in TLM-Part 1: materials with frequency-dependent properties,"

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IEEE Trans. Antennas Propagat., vol. 47, no. 10, pp. 1528–1534, Oct. 1999.
[9] M. I. Yaich and M. Khalladi, "The Far-zone scattering calculation of frequency-dependent materials objects using the TLM method," *IEEE Trans. Antennas Propagat.*, vol. 50, no. 11, pp. 1605–1608, Nov. 2002.
[10] R. J. Luebbers and F. Hunsberger, "FDTD for Nth-order dispersive media," *IEEE Trans. Antennas Propagat.*, vol. 40, no. 11, pp. 1297–1301, Nov. 1992.
[11] D. F. Kelley and R. J. Luebbers, "Piecewise linear recursive convolution for dispersive media using FDTD," *IEEE Trans. Antennas Propagat.*, vol. 44, no. 6, pp. 792–797, June 1996.
[12] D. F. Kelley and R. J. Luebbers, "A scattered field FDTD formulation for dispersive media," *1997 IEEE Dig. Int. Symp. Antennas and Propagat.*, vol. 1, pp. 360–363, 1997.

1 Introduction

The Transmission Line Matrix (TLM) method is a discrete implementation of Huygens's principle. Theoretically, it is based on existing similarities between the equations describing voltages and currents propagation in transmission-lines and Maxwell's equations describing electromagnetic (EM)-waves propagation. This method substitutes the problem of EM-waves propagation by an equivalent problem of voltage reflection and transmission through a lattice of transmission lines consisting of fundamental interconnected elements, namely, the nodes [1-4]. It is a powerful and versatile method for the time domain numerical analysis of EM field problems [3-9]. Recently, The TLM method has been successfully used for the analysis of dispersive media [6-9]. In this paper, we propose an approach for the modelling of Lorentz dispersive media using the TLM method with the SCN and voltage sources. In order to validate this model, the reflection coefficients at air-Lorentz medium interfaces and the RCS of a dispersive sphere are computed.

2 Formulation

Since its first introduction in 1987 by P. B. Johns [2], the SCN was a very significant advance in the TLM method. The basic SCN consists of 12 main lines modelling vacuum. Three open-circuit stubs (13, 14, 15) are added to this node to model permittivity, and three short-circuit stubs (16, 17, 18) are added to model permeability, other stubs (19, 20, 20) with voltage feeds V_{svu}, characterizing the physical properties of the linear isotropic nonmagnetic Lorentz dispersive media, can be added. At an instant $(n+1)\Delta t$, these voltage sources verify [9]:

$$\begin{aligned} {}_{n+1}V_{svu} + {}_{n}V_{svu} \\ = -4\chi_0 \cdot {}_{n}V_u - 4\sum_{m=0}^{n-1}\left(\chi^*_{m+1} - \chi^*_m\right){}_{n-m}V_u, \quad \mathrm{u} \in \{\mathrm{x}, \mathrm{y}, \mathrm{z}\}, \quad (1) \end{aligned}$$

Where χ_0 is the static dielectric susceptibility, χ^*_m is the general complex

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dielectric susceptibility of the Lorentz medium, which can be written in the following recursive form [10]:

$$\chi^*_{m+1} = e^{(-\delta + j\beta)\Delta t}\chi^*_m \tag{2}$$

In this equation, δ is the damping coefficient, $\beta = (\omega_0^2 - \delta^2)^{1/2}$ and ω_0 is the angular resonant frequency. The exponential nature of the generalized dielectric susceptibility permits to recursively evaluate, using two complex multiplications and a summation, the summation term in Eq. (1). This term, obtained from the convolution constant discretization, reduces to the following recursive expression:

$${}_nS^*_u = e^{(-\delta + j\beta)\Delta t}\,{}_{n-1}S^*_u + (\chi^*_1 - \chi^*_0)_n V_u \tag{3}$$

Eq. (1), giving the voltage sources, takes a simple form and becomes more practical for numerical computations. This can be written as follows:

$${}_{n+1}V_{svu} + {}_nV_{svu} = -4(\chi_0\ {}_nV_u - Re({}_nS^*_u)) \tag{4}$$

Thus, making use of charge and magnetic flux conservation principle through the transmission lines forming the SCN, and imposing the continuity conditions on the electric and magnetic fields [1], we obtain at time $n\Delta t$ the following scattering matrix of the SCN with voltage sources

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	a	b	d						b		-d	c	g					i	t_e		
2	b	a				d			c	-d		b	g				-i		t_e		
3	d		a	b				b			c	-d		g				-i		t_e	
4			b	a	d		-	c			b			g		i				t_e	
5				d	a	b	c	-		b					g	-i					t_e
6		d			b	a	b		-	c					g		i				t_e
7				-	c	b	a	d		b					g	i					t_e
8			b	c	-		d	a			b			g		-i				t_e	
9	b	c				-			a	d		b	g				i		t_e		
10		-			b	c	b		d	a					g		-i				t_e
11	-		c	b				b			a	d		g				i		t_e	
12	c	d	-						b		d	a	g					-i	t_e		
13	e	e							e			e	h						t_e		
14			e	e				e			e			h						t_e	
15					e	e	e			e					h						t_e
16				f	-f		f	-f								j					
17		-f				f			f	-f							j				
18	f		-f								f	-f						j			

Where :

a $= -0.5\text{Y}/(4+\text{Y}) + 0.5\text{Z}/(4+\text{Z})$, b $= 2/(4+\text{Y})$, c $= -0.5\text{Y}/(4+\text{Y}) - 0.5\text{Z}/(4+\text{Z})$, d $= 2/(4+\text{Z})$, e = b, f = Z.d, g = Y.b, h $= (\text{Y}-4)/(\text{Y}+4)$, i = d, J $= (\text{Z}-4)/(4+\text{Z})$, $t_e = {}_nV_{svu}/(4+\text{Y})$.

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Y and Z are respectively the admittance and the impedance of the linear isotropic Lorentz dispersive medium. Columns 19, 20 and 21 of the scattering matrix are associated respectively with the electric field components E_x, E_y and E_z which affect the scattering without taking part in it and which characterize the dispersive medium. So for the modelling of other kind of dispersive nonmagnetic medium, only the elements t_e of this matrix need to be determined.

3 Numerical results

In order to prove the validity and the efficiency of the SCN-TLM method and voltage sources to model Lorentz media, the reflection coefficients of a z polarised plane Gaussian wave at air-Lorentz medium interfaces are computed. Both, second-order Lorentz medium and the two poles second order medium are considered. In the first example, the TLM mesh considered is (1000, 1, 1) cells, and the spatial step is $\Delta l = 250\,\mu$m. The second-order Lorentz medium is located between 50 and 1000 cells, the physical characteristics of this medium are: $\varepsilon_s = 3.0$, $\varepsilon_\infty = 1.5$ et $\omega_0 = (2.\pi)\ 20.0 \times 10^9$ rad/s and $\delta = 0.1\omega_0$. To calculate the reflection coefficient of the air-Lorentz medium interface, we store the reflected and incident fields just in front of this interface, and then these fields versus time data were transformed to the frequency domain via a simple fast Fourier transform (FFT). Fig. 1 depicts the reflection coefficient magnitude versus frequency, obtained using the proposed model.

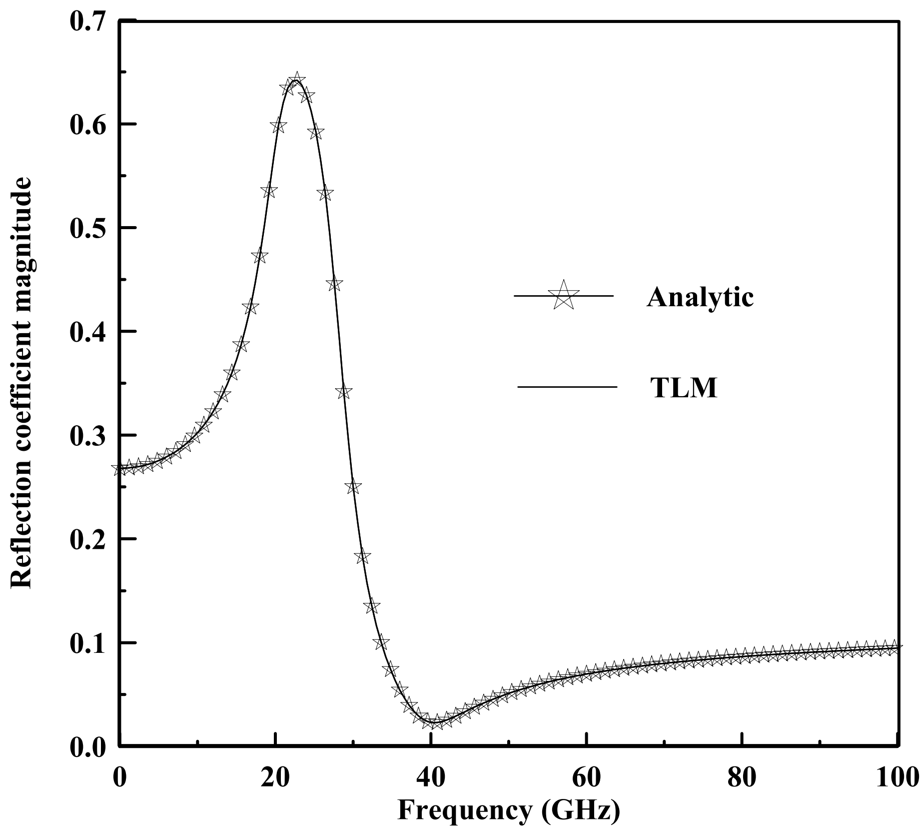

Fig. 1. Reflection coefficient magnitude at an air/Lorentz material interface.

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For the sake of comparison these results are compared with those computed analytically [8, 11].

In the second example, we compute the reflection coefficient of the interface between air and a two poles second-order Lorentz medium. The spatial TLM Lattice considered is (1500, 1, 1) cells, with a spatial step $\Delta l = 37.5\mu\,\text{m}$. Lorentz medium spans the space comprised between 500 and 1500 cells. The physical characteristics of this medium are: $\varepsilon_s = 3.0$, $\varepsilon_\infty = 1.5$, $\omega_1 = (2\pi)\ 20.0 \times 10^9\,\text{rad/s}$, $\omega_2 = (2\pi)\ 50.0 \times 10^9\,\text{rad/s}$, $\delta_1 = 0.1\omega_1$, $\delta_2 = 0.1\omega_2$, $G_1 = 0.4$ and $G_2 = 0.6$. Where ω_p is the p^{th} resonant frequency, $p \in \{1, 2\}$, δ_p is the p^{th} damping coefficient and G_p is a constant value such that $G_1 + G_2 = 1$. Fig. 2 shows the reflection coefficient magnitude for the air-two poles second-order Lorentz medium interface, versus frequency. A very good agreement is obtained between our results and those of the analytical ones [10].

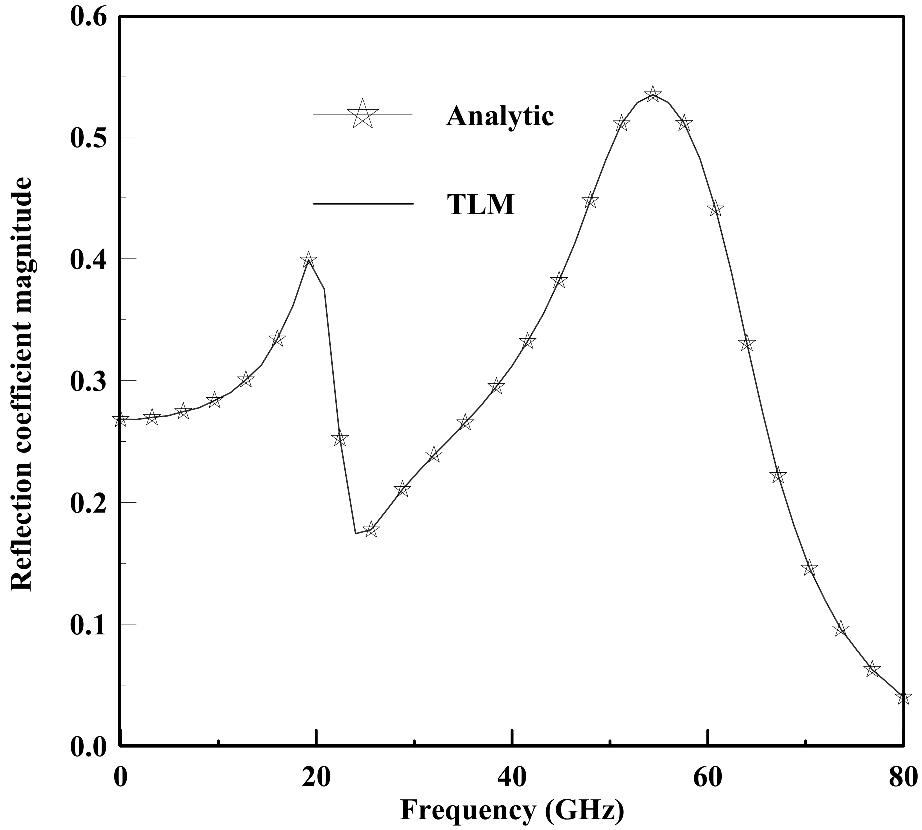

Fig. 2. Reflection coefficient magnitude at an air/two poles second-order Lorentz medium interface.

Finally, using the near-to-far-field transformation based on the electromagnetic equivalence principle described in references [5, 9], we compute the radar cross section RCS of a second-order Lorentz sphere of diameter 4 mm, placed in free space. The physical parameters of this sphere are: $\varepsilon_s = 3.0$ et $\varepsilon_\infty = 1.5$, $\omega_0 = (2\pi)\ 20.0 \times 10^9\,\text{rad/s}$ and $\delta = (2\pi)\ 2.0 \times 10^9\,\text{rad/s}$. The TLM space domain dimensions are (40, 40, 40) cells and the sphere's diameter is $27\,\Delta l$. The imaginary surfaces surrounding this sphere and which are used

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to compute the equivalent currents according to the equivalence principle are placed $5\,\Delta l$ away from the TLM lattice. Fig. 3 shows the backscattered RCS versus frequency corresponding to this sphere where a fairly good agreement can be seen between the TLM results and those of the analytical solutions [12].

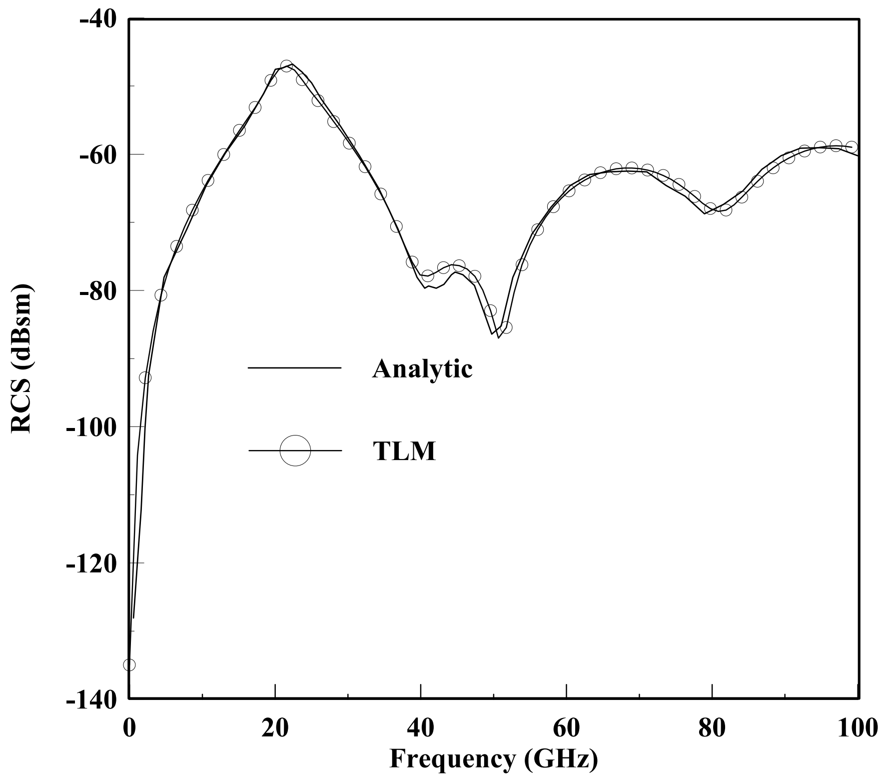

Fig. 3. Normalized monostatic RCS of a Lorentz sphere scatterer.

4 Conclusion

A time domain procedure allowing to model Lorentz dispersive media using the TLM with special symmetrical condensed node (SCN) and voltage sources is proposed. Dispersion effects of the SCN are modified adding voltage sources in three new ports controlling dispersive properties of the Lorentz material. The scattering matrix of the proposed node has been presented and the obtained results prove the capabilities of the TLM with this node to analyse electromagnetic-waves propagation and scattering by Lorentz dispersive media.

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