A MODIFIED RATIO-CUM-PRODUCT ESTIMATOR OF FINITE POPULATION MEAN IN STRATIFIED RANDOM SAMPLING

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ABSTRACT

This paper suggests a ratio-cum-product estimator of finite population mean using a correlation coefficient between study variate and auxiliary variate in stratified random sampling. Bias and mean squared expressions of the suggested estimator are derived and compared with combined ratio estimator and several other estimators considered by Kadilar and Cingi (2003). An empirical study is also carried out to examine the performance of the proposed estimator.

Key words: Finite population mean, Correlation coefficient, Stratified random sampling, Bias, Mean squared error

1 INTRODUCTION

Auxiliary information is often used to improve the efficiency of estimators. Ratio, product, and regression methods of estimation are good examples of this context. When the correlation coefficient between the study variate and auxiliary variate is positive (high), ratio type estimators are used. On the other hand, if this correlation is negative, product type estimators are used. In the recent past, ratio-cum-product estimators have drawn the attention of researchers, see Singh and Ruiz Espejo (2003) and Singh and Tailor (2005). This encouraged the author to make an attempt to study the behavior of ratio-cum-product estimators. Sisodia and Dwivedi (1981) have used the coefficient of the variation of auxiliary variate in constructing a ratio type estimator in simple random sampling. Upadhyaya and Singh (1999) used information on the coefficient of kurtosis and coefficient of variation whereas Singh et al. (2004) used only the coefficient of kurtosis for estimating the population mean. Singh and Tailor (2003) utilized information on the correlation coefficient between study variate and auxiliary variate. Kadilar and Cingi (2003) defined various ratio type estimators in stratified random sampling.

Tailor and Singh (2005) proposed a ratio-cum-product estimator by using a coefficient of variation. This led the author to suggest a modified ratio-cum-product estimator for estimating the population mean using the correlation coefficient in stratified random sampling.

Let $U = (U_1, U_2, ..., U_N)$ be a finite population of size N, which is divided into k homogeneous strata of size $\mathrm{N_h}$ (h = 1,2…,k). A sample of size $\mathrm{n_h}$ is drawn from each stratum using simple random sampling without replacement.

Let y be the study variate taking values y_{hi} ($\mathrm{i^{th}}$ observation from $\mathrm{h^{th}}$ stratum), and similarly, let x_{hi} be the auxiliary variate taking values x_{hi}. Let $\bar{\mathrm{y}}_{\mathrm{st}} = \sum_{\mathrm{h=1}}^{\mathrm{k}} \mathrm{W_h} \bar{\mathrm{y}}_{\mathrm{h}}$ and $\bar{\mathrm{x}}_{\mathrm{st}} = \sum_{\mathrm{h=1}}^{\mathrm{k}} \mathrm{W_h} \bar{\mathrm{x}}_{\mathrm{h}}$ be the unbiased estimators of the population mean $\bar{\mathrm{Y}}$ and $\bar{\mathrm{X}}$ of the study variate and auxiliary variate respectively,

where

$\mathrm{W_h} = (\mathrm{N_h} / \mathrm{N})$ is the weight of $\mathrm{h^{th}}$ stratum,

$\bar{\mathrm{y}}_{\mathrm{h}} = (1/\mathrm{n_h}) \sum_{\mathrm{j=1}}^{\mathrm{n_h}} \mathrm{y_{hj}}$ is the sample mean of the study variate y for $\mathrm{h^{th}}$ stratum, and

$\bar{x}_h = (1/n_h)\sum_{j=1}^{n_h} x_{hj}$ is the sample mean of the auxiliary variate x for the h^{th} stratum.

The combined ratio and product estimators for population mean $\bar{Y}$ respectively are

$$\hat{\bar{Y}}_{RST} = \bar{y}_{st}(\bar{X}/\bar{x}_{st}) \tag{1}$$

$$\hat{\bar{Y}}_{PST} = \bar{y}_{st}(\bar{x}_{st}/\bar{X}) \tag{2}$$

The mean squared error (MSE) expressions of the combined ratio and product estimators up to the first degree of approximation are

$$MSE(\hat{\bar{Y}}_{RST}) = \sum_{h=1}^{k} W_h^2\,\gamma_h(S_{yh}^2 + R^2S_{xh}^2 - 2RS_{yxh}), \tag{3}$$

$$MSE(\hat{\bar{Y}}_{PST}) = \sum_{h=1}^{k} W_h^2\,\gamma_h(S_{yh}^2 + R^2S_{xh}^2 + 2RS_{yxh}) \tag{4}$$

where

$$R = \bar{Y}/\bar{X},\ \gamma_h = \left(\frac{N_h - n_h}{N_h n_h}\right),\ S_{yh}^2 = (1/(N_h - 1))\sum_{j=1}^{N_h}(y_{hj} - \bar{Y}_h)^2,$$

$$S_{xh}^2 = (1/(N_h - 1))\sum_{j=1}^{N_h}(x_{hj} - \bar{X}_h)^2 \text{ and } S_{yxh} = (1/(N_h - 1))\sum_{j=1}^{N_h}(y_{hj} - \bar{Y}_h)(x_{hj} - \bar{X}_h).$$

Sisodia and Dwivedi (1981) suggested a ratio estimator of population mean $\bar{Y}$ using the coefficient of variation (C_x) of auxiliary variate as

$$\hat{\bar{Y}}_1 = \bar{y}\left[(\bar{X} + C_x)/(\bar{x} + C_x)\right] \tag{5}$$

Here, ($\bar{x}$, $\bar{y}$) are the sample means for (x, y).

Singh et al. (2004) proposed another ratio estimator for $\bar{Y}$, using the coefficient of kurtosis $\beta_2(x)$ of auxiliary variate *x* as

$$\hat{\bar{Y}}_2 = \bar{y}\left[(\bar{X} + \beta_2(x))/(\bar{x} + \beta_2(x))\right]. \tag{6}$$

Upadhyaya and Singh (1999) suggested two estimators using information on the coefficient of variation C_x and the coefficient of kurtosis $\beta_2(x)$ for $\bar{Y}$ as

$$\hat{\bar{Y}}_3 = \bar{y}\left[(\bar{X}\beta_2(x) + C_x)/(\bar{x}\beta_2(x) + C_x)\right], \tag{7}$$

and

$$\hat{\bar{Y}}_4 = \bar{y}\left[(\bar{X}C_x + \beta_2(x))/(\bar{x}C_x + \beta_2(x))\right]. \tag{8}$$

Singh and Tailor (2003) defined a modified ratio estimator of $\bar{Y}$ using the ρ_{xy}, correlation coefficient between study variate and auxiliary variate as

$$\hat{\bar{Y}}_5 = \bar{y}\left\{\frac{\bar{X} + \rho_{xy}}{\bar{x} + \rho_{xy}}\right\}. \tag{9}$$

Kadilar and Cingi (2003) defined $\hat{\bar{Y}}_1$, $\hat{\bar{Y}}_2$, $\hat{\bar{Y}}_3$, and $\hat{\bar{Y}}_4$ in a stratified random sampling respectively as

$$\hat{\bar{Y}}_{ST1} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h(\bar{X}_h + C_{xh}) / \sum_{h=1}^{k} W_h(\bar{x}_h + C_{xh})\right], \quad (10)$$

$$\hat{\bar{Y}}_{ST2} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h(\bar{X}_h + \beta_{2h}(x)) / \sum_{h=1}^{k} W_h(\bar{x}_h + \beta_{2h}(x))\right], \quad (11)$$

$$\hat{\bar{Y}}_{ST3} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h(\bar{X}_h\beta_{2h}(x) + C_{xh}) / \sum_{h=1}^{k} W_h(\bar{x}_h\beta_{2h}(x) + C_{xh})\right], \quad (12)$$

$$\hat{\bar{Y}}_{ST4} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h(\bar{X}_h C_{xh} + \beta_{2h}(x)) / \sum_{h=1}^{k} W_h(\bar{x}_h C_{xh} + \beta_{2h}(x))\right], \quad (13)$$

Tailor et al. (2008) have given $\hat{\bar{Y}}_5$ in stratified random sampling as

$$\hat{\bar{Y}}_{ST5} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h(\bar{X}_h + \rho_{yxh}) / \sum_{h=1}^{k} W_h(\bar{x}_h + \rho_{yxh})\right] \quad (14)$$

To the first degree of approximation, the mean squared errors of $\hat{\bar{Y}}_{ST1}$, $\hat{\bar{Y}}_{ST2}$, $\hat{\bar{Y}}_{ST3}$, $\hat{\bar{Y}}_{ST4}$ and $\hat{\bar{Y}}_{ST5}$ respectively are

$$MSE(\hat{\bar{Y}}_{ST1}) = \sum_{h=1}^{k} W_h^2\gamma_h(S_{yh}^2 + R_{SD}^2 S_{xh}^2 - 2R_{SD}S_{yxh}), \quad (15)$$

$$MSE(\hat{\bar{Y}}_{ST2}) = \sum_{h=1}^{k} W_h^2\gamma_h(S_{yh}^2 + R_{SK}^2 S_{xh}^2 - 2R_{SK}S_{yxh}), \quad (16)$$

$$MSE(\hat{\bar{Y}}_{ST3}) = \sum_{h=1}^{k} W_h^2\gamma_h(S_{yh}^2 + R_{US1}^2\beta_{2h}^2(x) S_{xh}^2 - 2R_{US1}S_{yxh}), \quad (17)$$

$$MSE(\hat{\bar{Y}}_{ST4}) = \sum_{h=1}^{k} W_h^2\gamma_h(S_{yh}^2 + R_{US2}^2 C_{xh}^2 S_{xh}^2 - 2R_{US2}C_{xh}S_{yxh}), \quad (18)$$

$$MSE(\hat{\bar{Y}}_{ST5}) = \sum_{h=1}^{k} W_h^2\gamma_h(S_{yh}^2 + R_T^2 S_{xh}^2 - 2R_T S_{yxh}) \quad (19)$$

where

$$R_{SD} = (\sum_{h=1}^{k} W_h\bar{Y}_h) / \sum_{h=1}^{k} W_h(\bar{X}_h + C_{xh}),\ R_{SK} = (\sum_{h=1}^{k} W_h\bar{Y}_h) / \sum_{h=1}^{k} W_h(\bar{X}_h + \beta_{2h}(x)),$$

$$R_{US1} = (\sum_{h=1}^{k} W_h\bar{Y}_h\beta_{2h}(x)) / \sum_{h=1}^{k} W_h(\bar{X}_h\beta_{2h}(x) + C_{xh})$$

$$R_{US2} = (\sum_{h=1}^{k} W_h\bar{Y}_h C_{xh}) / \sum_{h=1}^{k} W_h(\bar{X}_h C_{xh} + \beta_{2h}(x)),\ R_T = (\sum_{h=1}^{k} W_h\bar{Y}_h) / \sum_{h=1}^{k} W_h(\bar{X}_h + \rho_{yxh})$$

2 PROPOSED RATIO ESTIMATOR

Assuming that the correlation coefficient ρ_{yxh} between y and x in the h^{th} stratum is known for all strata, the proposed ratio-cum-product estimator is

$$\hat{\bar{Y}}_T = \bar{y}_{st}\left[\alpha\left\{\frac{\sum_{h=1}^{k} W_h(\bar{X}_h + \rho_{yxh})}{\sum_{h=1}^{k} W_h(\bar{x}_h + \rho_{yxh})}\right\} + (1-\alpha)\left\{\frac{\sum_{h=1}^{k} W_h(\bar{x}_h + \rho_{yxh})}{\sum_{h=1}^{k} W_h(\bar{X}_h + \rho_{yxh})}\right\}\right]. \quad (20)$$

$$\hat{\bar{Y}}_T = \bar{y}_{st}\left[\alpha\hat{\bar{Y}}_{ST5} + (1-\alpha)\hat{\bar{Y}}_{ST6}\right]$$

Where $\hat{\bar{Y}}_{ST6} = \bar{y}_{st}\left[\sum_{h=1}^{k} W_h(\bar{x}_h + \rho_{yxh}) / \sum_{h=1}^{k} W_h(\bar{X}_h + \rho_{yxh})\right]$ is proposed by Tailor et al. (2008).

Here α is a suitably chosen scalar. We note that for α=1, $\hat{\bar{Y}}_T$ reduces to the estimator $\hat{\bar{Y}}_{ST5}$ while for α=0, it reduces to the estimator $\hat{\bar{Y}}_{ST5}$.

It is to be mentioned that if the scalar α closes to unity, the ratio estimator $\hat{\bar{Y}}_{ST5}$ is to be used, whereas $\hat{\bar{Y}}_{ST6}$ is used when α is closer to 'zero'.

To obtain the bias and mean squared error of $\hat{\bar{Y}}_T$ let $\bar{y}_{st} = \bar{Y}(1+e_0)$ and $\bar{x}_{st} = \bar{Y}(1+e_1)$,

such that $E(e_0) = E(e_1) = 0$ and $E(e_0^2) = \frac{1}{\bar{Y}^2}\sum_{h=1}^{k} W_h^2\gamma_h S_{yh}^2$, $E(e_1^2) = \frac{1}{\bar{X}^2}\sum_{h=1}^{k} W_h^2\gamma_h S_{xh}^2$

and $E(e_0 e_1) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{k} W_h^2\gamma_h S_{yxh}$.

Expressing (20) in terms of e_i

$$\hat{\bar{Y}}_T = \bar{Y}(1+e_0)\left\{\alpha(1+\lambda e_1)^{-1} + (1-\alpha)(1+\lambda e_1)\right\}, \tag{21}$$

where $\lambda = \dfrac{\bar{X}}{\bar{X} + \sum_{h=1}^{k} W_h \rho_{xh}}$.

We now assume that $|\lambda e_1|$<1 so that we may expand $(1+\lambda e_1)^{-1}$ as a series in powers of λe_1. To the first degree of approximation, the bias and mean squared error of the proposed estimator $\hat{\bar{Y}}_T$ are

$$\mathrm{Bias}(\hat{\bar{Y}}_T) = (\lambda/\bar{X})\sum_{h=1}^{k} W_h^2\gamma_h\left[(1-2\alpha)S_{yxh} + \alpha\lambda R\, S_{xh}^2\right], \tag{22}$$

$$\mathrm{MSE}(\hat{\bar{Y}}_T) = \sum_{h=1}^{k} W_h^2\gamma_h\left[S_{yh}^2 + R_T^2(1-2\alpha)^2 S_{xh}^2 + 2R_T(1-2\alpha)S_{yxh}\right], \tag{23}$$

2.1 Estimator at optimum α

The mean squared error of $\hat{\bar{Y}}_T$ is minimized for $\alpha = \frac{1}{2}\left[1 + \dfrac{\sum_{h=1}^{k} W_h^2\gamma_h S_{yxh}}{R_T\sum_{h=1}^{k} W_h^2\gamma_h S_{xh}^2}\right] = \frac{1}{2}\left(1 + \frac{\beta}{R_T}\right)$

where $\beta = \dfrac{\sum_{h=1}^{k} W_h^2\gamma_h S_{yxh}}{\sum_{h=1}^{k} W_h^2\gamma_h S_{xh}^2}$.

By the substitution of α in (20) we get the asymptotically optimum estimator (AOE) for $\overline{Y}$ as

$$\hat{\overline{Y}}_T^{(opt)} = \overline{y}_{st}\left[\frac{1}{2}\left(1+\frac{\beta}{R_T}\right)\left(\frac{\sum_{h=1}^{k} W_h(\overline{X}_h+\rho_{yxh})}{\sum_{h=1}^{k} W_h(\overline{x}_h+\rho_{yxh})}\right)+\left\{1-\frac{1}{2}\left(1+\frac{\beta}{R_T}\right)\right\}\left(\frac{\sum_{h=1}^{k} W_h(\overline{x}_h+\rho_{yxh})}{\sum_{h=1}^{k} W_h(\overline{X}_h+\rho_{yxh})}\right)\right]$$

Substituting the value of α in (23), the minimum mean squared error of $\hat{\overline{Y}}_T$ (MSE of $\hat{\overline{Y}}_T^{(opt)}$) is $\sum_{h=1}^{k} W_h^2\gamma_h S_{yh}^2 - \beta$, which is equivalent to the variance of the regression estimator in stratified random sampling. When α is not known, then it is advisable to estimate $\alpha_{(opt)}$ from the sample data at hand.

3 EFFICIENCY COMPARISONS

The variance of the unbiased estimator $\overline{y}_{st}$ in stratified random sampling is

$$V(\overline{y}_{st}) = \sum_{h=1}^{k} W_h^2\gamma_h S_{yh}^2 \tag{24}$$

From (3), (15), (16), (17), (18), (19), (23), and (24)

(i) $MSE(\hat{\overline{Y}}_T) < MSE(\overline{y}_{st})$ if

$$\left.\begin{array}{ll} \textit{either} & \frac{1}{2} < \alpha < \frac{1}{2}\left[1+\frac{2B}{R_T A}\right] \\ \textit{or} & \frac{1}{2}\left[1+\frac{2B}{R_T A}\right] < \alpha < \frac{1}{2} \end{array}\right\} \tag{25}$$

where $A = \sum_{h=1}^{L} W_h^2\gamma_h S_{xh}^2$ and $B = \sum_{h=1}^{L} W_h^2\gamma_h S_{yxh}$

(ii) $MSE(\hat{\overline{Y}}_T) < MSE(\hat{\overline{Y}}_{RST})$ if

$$\left.\begin{array}{ll} \text{either} & \frac{1}{2}\left\{1+\frac{R}{R_T}\right\} < \alpha < \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}-R\right\}\right] \\ \text{or} & \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}-R\right\}\right] < \alpha < \frac{1}{2}\left\{1+\frac{R}{R_T}\right\} \end{array}\right\} \tag{26}$$

(iii) $MSE(\hat{\overline{Y}}_T) < MSE(\hat{\overline{Y}}_{PST})$ if

$$\left.\begin{array}{ll} \text{either} & \frac{1}{2}\left\{1-\frac{R}{R_T}\right\} < \alpha < \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}+R\right\}\right] \\ \text{or} & \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}+R\right\}\right] < \alpha < \frac{1}{2}\left\{1-\frac{R}{R_T}\right\} \end{array}\right\} \tag{27}$$

(iv) $\quad MSE(\hat{\bar{Y}}_T) < MSE(\hat{\bar{Y}}_{ST1})$ if

$$\left.\begin{array}{ll} \text{either} & \frac{1}{2}\left\{1+\frac{R_{SD}}{R_T}\right\} < \alpha < \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}-R_{SD}\right\}\right] \\ \text{or} & \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}-R_{SD}\right\}\right] < \alpha < \frac{1}{2}\left\{1+\frac{R_{SD}}{R_T}\right\} \end{array}\right\} \quad (28)$$

(v) $\quad MSE(\hat{\bar{Y}}_T) < MSE(\hat{\bar{Y}}_{ST2})$ if

$$\left.\begin{array}{ll} \text{either} & \frac{1}{2}\left\{1+\frac{R_{SK}}{R_T}\right\} < \alpha < \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}-R_{SK}\right\}\right] \\ \text{or} & \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}-R_{SK}\right\}\right] < \alpha < \frac{1}{2}\left\{1+\frac{R_{SK}}{R_T}\right\} \end{array}\right\} \quad (29)$$

(vi) $\quad MSE(\hat{\bar{Y}}_T) < MSE(\hat{\bar{Y}}_{ST3})$ if

$$\left.\begin{array}{ll} \text{either} & \frac{1}{2}\left\{1+\frac{R_{US1}}{R_T}\right\} < \alpha < \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}-R_{US1}\right\}\right] \\ \text{or} & \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}-R_{US1}\right\}\right] < \alpha < \frac{1}{2}\left\{1+\frac{R_{US1}}{R_T}\right\} \end{array}\right\} \quad (30)$$

(vii) $\quad MSE(\hat{\bar{Y}}_T) < MSE(\hat{\bar{Y}}_{ST4})$ if

$$\left.\begin{array}{ll} \text{either} & \frac{1}{2}\left\{1+\frac{R_{US2}}{R_T}\right\} < \alpha < \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}-R_{US2}\right\}\right] \\ \text{or} & \frac{1}{2}\left[1+\frac{1}{R_T}\left\{\frac{2B}{A}-R_{US2}\right\}\right] < \alpha < \frac{1}{2}\left\{1+\frac{R_{US2}}{R_T}\right\} \end{array}\right\} \quad (31)$$

(viii) $\quad MSE(\hat{\bar{Y}}_T) < MSE(\hat{\bar{Y}}_{ST5})$ if

$$\left.\begin{array}{ll} \textit{either} & \frac{1}{2} < \alpha < \left[\frac{2B}{AR_T}\right] \\ \textit{or} & \left[\frac{2B}{AR_T}\right] < \alpha < \frac{1}{2} \end{array}\right\} \quad (32)$$

4 EMPIRICAL STUDY

For empirical study, we used the data given in Kadilar and Cingi (2003).

Table 1. Data Statistics

$N = 854$	$N_1 = 106$	$N_2 = 106$	$N_3 = 94$	$N_4 = 171$	$N_5 = 204$	$N_6 = 173$
$n = 140$	$n_1 = 9$	$n_2 = 17$	$n_3 = 38$	$n_4 = 67$	$n_5 = 7$	$n_6 = 2$
$\bar{X} = 37600$	$\bar{X}_1 = 24375$	$\bar{X}_2 = 27421$	$\bar{X}_3 = 72409$	$\bar{X}_4 = 74365$	$\bar{X}_5 = 26441$	$\bar{X}_6 = 9844$
$\bar{Y} = 2930$	$\bar{Y}_1 = 1536$	$\bar{Y}_2 = 2212$	$\bar{Y}_3 = 9384$	$\bar{Y}_4 = 5588$	$\bar{Y}_5 = 967$	$\bar{Y}_6 = 404$

$\beta_x = 312.07$	$\beta_{x1} = 25.71$	$\beta_{x2} = 34.57$	$\beta_{x3} = 26.14$	$\beta_{x4} = 97.60$	$\beta_{x5} = 27.47$	$\beta_{x6} = 28.10$
$\beta_y = 195.84$	$C_{x1} = 2.02$	$C_{x2} = 2.10$	$C_{x3} = 2.22$	$C_{x4} = 3.84$	$C_{x5} = 1.72$	$C_{x6} = 1.91$
$C_x = 3.85$	$C_{y1} = 4.18$	$C_{y2} = 5.22$	$C_{y3} = 3.19$	$C_{y4} = 5.13$	$C_{y5} = 2.47$	$C_{y6} = 2.34$
$C_y = 5.84$	$S_{x1} = 49189$	$S_{x2} = 57461$	$S_{x3} = 160757$	$S_{x4} = 285603$	$S_{x5} = 45403$	$S_{x6} = 18794$
$S_x = 144794$	$S_{y1} = 6425$	$S_{y2} = 11552$	$S_{y3} = 29907$	$S_{y4} = 28643$	$S_{y5} = 2390$	$S_{y6} = 946$
$S_y = 17106$	$\rho_1 = 0.82$	$\rho_2 = 0.86$	$\rho_3 = 0.90$	$\rho_4 = 0.99$	$\rho_5 = 0.71$	$\rho_6 = 0.89$
$\rho = 0.92$	$\gamma_1 = 0.102$	$\gamma_2 = 0.049$	$\gamma_3 = 0.016$	$\gamma_4 = 0.009$	$\gamma_5 = 0.138$	$\gamma_6 = 0.006$
$\chi = 0.975$	$\omega_1^2 = 0.015$	$\omega_2^2 = 0.015$	$\omega_3^2 = 0.012$	$\omega_4^2 = 0.04$	$\omega_5^2 = 0.057$	$\omega_6^2 = 0.041$

Table 2. Mean Squared Errors of $\bar{y}_{st}$, $\hat{\bar{Y}}_{RST}$, $\hat{\bar{Y}}_{PST}$, $\hat{\bar{Y}}_{ST1}$, $\hat{\bar{Y}}_{ST2}$, $\hat{\bar{Y}}_{ST3}$, $\hat{\bar{Y}}_{ST4}$, $\hat{\bar{Y}}_{ST5}$, $\hat{\bar{Y}}_{ST6}$ and $\hat{\bar{Y}}_T$.

Estimators	**Mean Squared Errors**
$\bar{y}_{st}$	673477.70
$\hat{\bar{Y}}_{RST}$	212047.28
$\hat{\bar{Y}}_{PST}$	1833176.29
$\hat{\bar{Y}}_{ST1}$	212082.02
$\hat{\bar{Y}}_{ST2}$	212206.41
$\hat{\bar{Y}}_{ST3}$	289586.67
$\hat{\bar{Y}}_{ST4}$	206287.38
$\hat{\bar{Y}}_{ST5}$	212077.45
$\hat{\bar{Y}}_{ST6}$	182349.98
$\hat{\bar{Y}}_T$ **at optimum** $\alpha = \mathbf{1.085}$	***202122.07***

Table 2 shows that the suggested estimator $\hat{\bar{Y}}_T$ has the lowest mean squared error, i.e., it is more efficient (with substantial gain) than the usual unbiased estimator $\bar{y}_{st}$, the combined ratio estimator $\hat{\bar{Y}}_{RST}$, the combined product estimator $\hat{\bar{Y}}_{PST}$, the estimators proposed by Kadilar and Cingi (2003) $\hat{\bar{Y}}_{STi}$ $(i = 1 \text{ to } 4)$, and the Tailor et al. (2008) estimators $\hat{\bar{Y}}_{ST5}$ and $\hat{\bar{Y}}_{ST6}$. Thus the proposed estimator $\hat{\bar{Y}}_T$ is recommended for use in practice.

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