

# Semi-analytical BER performance of a direct detection soliton transmission system

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**Abstract:** A semi-analytical approach is presented, to evaluate the bit error rate (BER) performance of a 10 Gb/s direct detection soliton transmission link in the presence of group velocity dispersion (GVD), self-phase modulation (SPM), and accumulated amplifiers' spontaneous emission (ASE). The results show that the BER performance is limited by accumulated ASE and inter-symbol interference due to pulse distortion caused by GVD and SPM. It is found that for transmission distance of 3000-km with cascaded EDFAs the receiver sensitivity corresponding to BER of  $10^{-9}$  are  $-25.2$  dBm,  $-20.7$  dBm and  $-13.2$  dBm for amplifier gain of 10 dB, 20 dB and 30 dB respectively.

**Keywords:** soliton, SPM, GVD, EDFA, intersymbol interference

**Classification:** Photonics devices, circuits, and systems

## References

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## 1 Introduction

High-speed long-haul transmission with optical fiber communication has been greatly increased by the development of solitons and Erbium Doped Fiber Amplifiers (EDFAs). The usefulness of such optical amplifiers with soliton has been experimentally investigated [1, 2]. However, these amplifiers also introduce amplified spontaneous emission which results in crosstalk due to beating of different channels during simultaneous amplification of dense WDM signals [3, 4]. The main factors limiting the transmission distance in soliton optical communication link are group velocity dispersion, self-phase modulation, and amplified spontaneous emission noise, which cause Inter-Symbol Interference (ISI) due to pulse broadening. The experimental results on the BER performance of a soliton transmission link were reported [5]. In this paper, we provide, a semi-analytical approach to evaluate the bit error performance of a point to point optical soliton transmission link with cascaded optical amplifiers using direct detection receiver incorporating the combined effect of GVD, SPM, ASE noise. The relative performance is determined in terms of the BER and receiver sensitivity degradation at a bit rate of 10 Gb/s.

## 2 System model

The system under consideration is depicted schematically in Fig. 1. In addition to transmitter and receiver, Fig. 1 shows multiple fiber sections consisting of EDFAs in cascade, which amplify the soliton pulse to minimize fiber loss, and adds accumulated ASE-noise. An optical filter follows each of the EDFAs to reduce the accumulated ASE-noise. The receiver consists of a photodetector, followed by a low-pass filter and an error rate estimator.

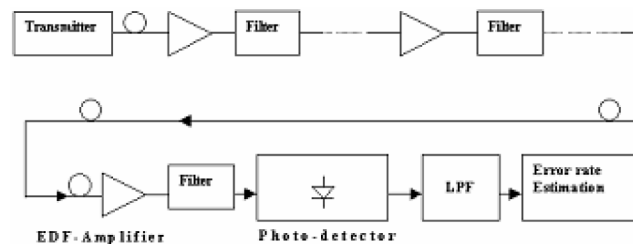


Fig. 1. Block diagram of IM-DD soliton receiver.

## 3 Theoretical analysis

The optical signal at the input of the fiber can be represented as

$$E_{in}(t) = \sqrt{2P_t} \sum_{k=-\infty}^{\infty} a_k p(t - kT) \exp(j\omega_c t) \quad (1)$$

where  $P_t$  is the power input to fiber with  $a_k = \pm 1$ , representing the  $k^{\text{th}}$  information bit,  $p(t)$  is a hyperbolic soliton pulse shape of Full Width Half

Maximum pulse width  $T_{FWHM}$ , related to the system bit rate  $B_r$  through,  $B_r = 1/T_{FWHM}$ ;  $\omega_c$  is the carrier angular frequency.

The signal at the output of the optical filter of the first fiber section can be represented by

$$E_{in}(t) = \sqrt{2P_r} \sum_{k=-\infty}^{\infty} a_k p'(t - kT) \exp(j\omega_c t + \phi(t)) \quad (2)$$

where  $P_r$  is the average received power,  $p'(t)$  represents the received soliton pulse shape, and  $\phi(t)$  is a random phase noise. The optical signal at the output of the fiber is obtained by solving numerically the nonlinear Schrödinger equation (NLSE) using beam propagation method. The NLS equation describes the propagation in a dispersive-nonlinear fiber, where, the slowly varying envelope  $E(t, z)$  obeys the following nonlinear wave equation:

$$j \frac{\partial E}{\partial z} = -\frac{1}{2} \beta_2 \frac{\partial^2 E}{\partial t^2} + \frac{j}{6} \beta_3 \frac{\partial^3 E}{\partial t^3} - \gamma |E|^2 E \quad (3)$$

where  $E$  is the input signal envelope,  $z$  the longitudinal coordinate of the fiber,  $\beta_2$  is the group velocity dispersion, and  $\beta_3$  represents the second order dispersion. The parameter  $\gamma$  is the nonlinear coefficient. The signal at the output of the optical filter of the first fiber section can be represented by:

$$E_{out}(t) = (E_{in}(t) + E_{sp}(t)) \otimes h_f(t) \otimes h_{filter}(t) \quad (4)$$

where  $h_f(t)$  is the complex impulse response of the fiber,  $E_{sp}(t)$  the spurious signal known as the amplified spontaneous emission, and  $h_{filter}(t)$  the impulse response of the filter. The amplified spontaneous emission is an additive white Gaussian noise with zero mean and variance given by:

$$\sigma_{sp}^2 = hf \eta_{sp} (G - 1) B_o \quad (5)$$

where,  $B_o$  represents the optical filter bandwidth. The parameter  $\eta_{sp}$  represents the spontaneous emission factor,  $h$  is the Planck's constant,  $f$  the optical frequency,  $G$  is the optical amplifier gain, and  $h_{filter}(t)$  the filter impulse response of a third order Butterworth filter with the time-bandwidth product assumed to be  $B_o T_{FWHM} = 1$ . To include the effect of in-line amplifier noise, the real and imaginary components of the amplified spontaneous emission are modeled as zero mean white Gaussian noise and added to the signal field after each amplifier. At the output of the photodetector the photocurrent is given by:

$$i(t) = R_d |E_{out}(t)|^2 \quad (6)$$

Where  $R_d$  is the responsivity of the photodiode. Hence, the current at the output of the lowpass filter is given by:

$$i(t) = 2R_d P_r \cos(\theta(t)) \times \left[ |a_0 p'_0(t)|^2 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} a_n a_m p'_n(t - nT) p'^*_m(t - mT) \right] + n(t) \quad (7)$$

where the first term in the brackets represents the signal component, the second term accounts for inter-symbol interference (ISI),  $n(t)$  is the receiver noise contributed by the quantum shot noise and the receiver thermal noise.

The signal photocurrent is given by:

$$i_s(t) = 2R_d P_r \cos(\Delta\theta) |a_o p_o(t)|^2 \quad (8)$$

Further, the detector current which result in ISI can be expressed as:

$$i_{isi}(t) = 2R_d P_r \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} a_n a_m p'_n(t - nT) \cdot p'^*_m(t - mT) \quad (9)$$

The total noise variance can be written as:

$$\sigma_n^2 = \sigma_{sh}^2 + \sigma_{th}^2 + \sigma_{s-sp}^2 + \sigma_{sp-sp}^2 \quad (10)$$

where  $\sigma_{sh}^2$  and  $\sigma_{th}^2$  are the variances of shot noise and thermal noise respectively,  $\sigma_{s-sp}^2$  and  $\sigma_{sp-sp}^2$  are the variances of signal-spontaneous and spontaneous-spontaneous beat noise respectively and are given by:

$$\begin{aligned} \sigma_{s-sp}^2 &= 2I_{sp} I_s \frac{B_e}{B_o} \\ \sigma_{sp-sp}^2 &= I_{sp}^2 B_e \frac{(2B_o - B_e)}{(GB_o)^2} \end{aligned} \quad (11)$$

where  $B_o$  is the bandwidth of the optical filter placed after each amplifier and  $I_{sp}$  represents the time average of ASE-noise current. For soliton detection, one bit is sent and the receiver makes decision every  $T_{FWHM}$  second. Assuming that the receiver noise is Gaussian, the conditional BER conditioned on a given value of  $i_{isi}$ , is given by:

$$\begin{aligned} P(e/i_{isi}) &= P_m E \left[ Q \left( \frac{i_{s,m} - i_{isi,m} - i_{th}}{\sigma_m} \right) \right] \\ &+ P_s E \left[ Q \left( \frac{i_{th} - i_{s,s} - i_{isi,s}}{\sigma_s} \right) \right] \end{aligned} \quad (12)$$

where the letters “m” and “s” stand for ‘mark’ and ‘space’ respectively. The average bit error is obtained by averaging the bit error rate  $P(e/i_{isi})$  over the probability density function (pdf) of  $i_{isi}$ :

$$P_b = 0.5 \int_{-\infty}^{\infty} P(e/i_{isi}) p(i_{isi}) d(i_{isi}) \quad (13)$$

where  $p(i_{isi})$  is the probability density function (pdf) of  $i_{isi}$ . Using the Gauss-quadrature rule, the  $P_b$  can be numerically computed as:

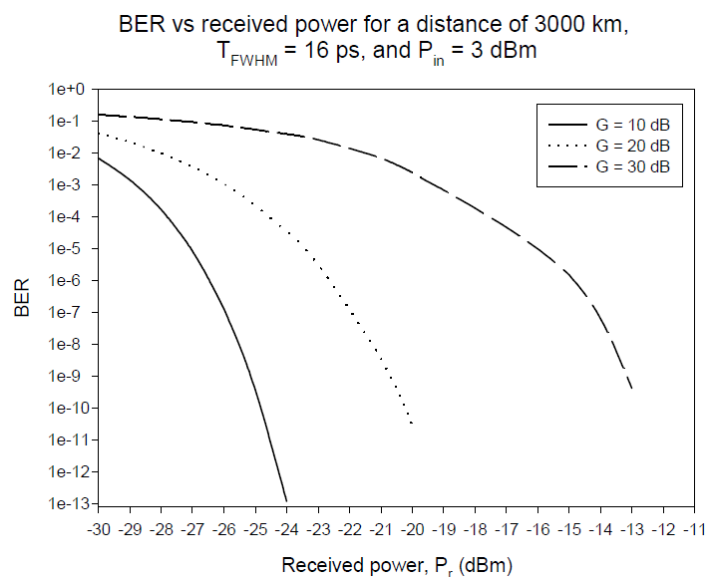
$$P_b = 0.5 \sum_{h=1}^N w_h \operatorname{erfc} \left[ \frac{2R_d P_t - 2x_h}{\sqrt{2(\sigma_s^2 + \sigma_n^2)}} \right] \quad (14)$$

where the weight  $w_h$  and nodes  $x_h = i_{isi}$  are evaluated from the knowledge of the moments of the random process  $i_{isi}$ . To evaluate the soliton performance,

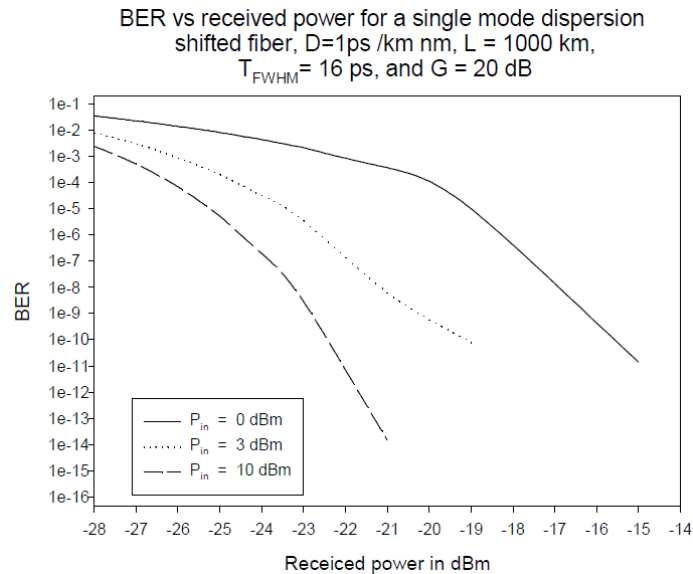
average soliton propagation, which propagates in dispersion compensating fibre and dispersion shifted fibre with lumped amplifier is studied by solving numerically the shrödinger equation using the beam propagation method. The pulse full width at half maximum is considered to be 16 ps and the other parameter values used in the simulation are:  $n_2 = 3.2 \times 10^{-26}$  km/W,  $A_{eff} = 50 \mu\text{m}^2$ ,  $\lambda = 1.55 \mu\text{m}$ ,  $\beta_2 = -1.27 \text{ ps}^2/\text{km}$ ,  $\beta_3 = 0.08 \text{ ps}^3/\text{km}$ , the fibre loss is 0.2 dB/km with an average dispersion coefficient of 0.108 ps/km/nm, and the time-bandwidth product is assumed to be  $B_0 T_{FWHM} = 1$ .

#### 4 Results and discussions

Following the semi-analytical approach, we determine the bit error rate performance of a soliton transmission link with cascaded EDFAs in the presence of higher order nonlinear effects of fiber such as group velocity dispersion (GVD), self-phase modulation (SPM) and the effects of accumulated optical amplifiers' spontaneous emission (ASE) noise. The results are evaluated at a bit rate of 10 Gb/s considering the effects of inter-symbol interference (ISI) introduced due to pulse broadening by to GVD and SPM. The beam propagation method (BPM) is used to find the pulse shape at the output of the multi-segment optical soliton link with cascaded optical amplifiers. The input soliton pulse is considered to be hyperbolic with full width half-maximum pulse width as the reciprocal of bit rate. The ISI samples at the output of the receiver are determined from the received pulse shape. The plots of BER versus received power are shown in Fig. 2 with amplifier gain  $G$  as a parameter. The results show that there is a significant increase in BER due to the effect of accumulated ASE. The degradation is more significant at higher amplifier gains due to increased accumulated ASE. The receiver sensitivities at BER of  $10^{-9}$  are found to be  $-25.2$  dBm,  $-20.7$  dBm and  $-13.2$  dBm corresponding



**Fig. 2.** The Bit Error Rate vs. received power for fiber length of 3000 km and FWHM pulse width of 16 ps for amplifier gain of 10, 20, and 30 dB.



**Fig. 3.** The Bit Error Rate vs. received power for Fiber length of 1000 km, FWHM pulse width of 16 ps, and amplifier gain of 20 dB for input power of 0, 3, 10 dBm.

to EDFA gain of 10 dB, 20 dB and 30 dB respectively.

Figure 3 depicts the impact of input power variation on the BER performance as a function of the average optical power of the received soliton pulse for three different input power levels. It is found that higher input power can provide lower value of the minimum achievable BER and ensure less received power which is required to attain the same BER. When the input power is increased properly in such a way that self-phase modulation (SPM) effect is adjusted to cancel the effect of dispersion the BER will be improved. However in some cases when the power is very high, SPM may become more significant and may cause additional broadening which will cause degradation of system performance and thus high values of the BER.

## 5 Conclusion

A semi-analytical approach is presented, to find the bit error rate performance of a soliton transmission link with cascaded optical amplifiers using a direct detection receiver. The analysis includes the effects of accumulated optical amplifiers' spontaneous emission, the influences of GVD and GVD-induced SPM. The results show that the BER performance of the receiver is highly sensitive to the amplifier gain. The system suffers a power penalty due to accumulated ASE with increased amplifier gain. The analytical approach can be extended to include Gordon-Hauss jitter and to evaluate the performance of WDM transmission systems with solitons.