

# Effectiveness of a correlated multiple sampling differential averager for reducing $1/f$ noise

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**Abstract:** Correlated multiple sampling differential averaging (CMS-DA) is useful for realizing low-noise column readout circuits in CMOS image sensors. CMSDA involves sampling the signal and reset levels multiple times and differentiating the averages. In other words, this method is like averaging of correlated double sampling (CDS) for the multiple sampling of signal and reset levels. This paper analyzes the effectiveness of CMSDA for reducing  $1/f$  noise and the analysis is in good agreement with time-domain simulations.

**Keywords:** correlated multiple sampling differential averaging (CMS-DA),  $1/f$  noise, low noise, column readout circuit, CMOS image sensor

**Classification:** Electron devices

## References

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## 1 Introduction

Noise reduction techniques such as correlated double sampling (CDS) are indispensable in image sensors as these techniques effectively reduce  $kTC$  noise, as well as fixed pattern noise (FPN) and  $1/f$  noise. Although CDS is widely used in all types of imagers, more advanced noise reduction techniques such as multiple sampling [1, 2] can be used in the column parallel noise reduction circuits of CMOS image sensors [3]. A technique of sampling the signal and reset levels multiple times and differentiating the averages is called correlated multiple sampling differential averaging (CMSDA) here. In this technique, since the averaging and differentiating can be done in digital domain for reducing the thermal and  $1/f$  noise in readout circuits as well as  $kTC$  noise and FPN from pixels, a wide digital dynamic range signal can be obtained. The effectiveness of differential averaging with continuous integration has been analyzed in [4]. CMSDA is expected to have a similar  $1/f$  noise reduction effect to the differential averaging with continuous integration. However, the practical effectiveness of CMSDA for reducing  $1/f$  noise has yet to be reported. In this paper, an analytical formula is derived to express the relationship between the noise reduction effect and the number of sampling times for CMSDA.

## 2 Principle of CMSDA

Figure 1 shows a block diagram of the readout circuit in a CMOS image sensor incorporating CMSDA at the column. The sampling timing of this method is shown in Fig. 2. The signal of the floating diffusion (FD) node in the pixel is sampled by the sample-and-hold (S/H) circuit, and then passed to an analog-to-digital converter (ADC). The reset level is sampled  $M$  times at a sampling period  $T_0$  and the sampled data are summed by a resistor. The charge transfer gate controlled by TX is then opened, and the charge is transferred to the FD node. The signal level at the FD node is sampled  $M$  times at sampling period  $T_0$ , and the sampled data are summed by another resistor. The averaged reset and signal levels are then differentiated in the digital domain to cancel  $kTC$  noise and FPN. This process also effectively reduces thermal noise in readout circuits consisting of a pixel buffer amplifier, S/H circuits and ADC by a factor of  $\sqrt{M}$  in amplitude. The noise reduction effect for  $1/f$  noise is analyzed as follows.

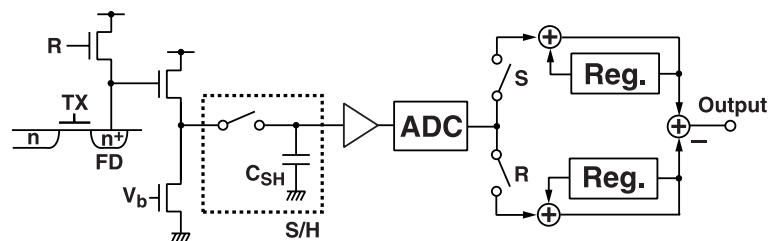
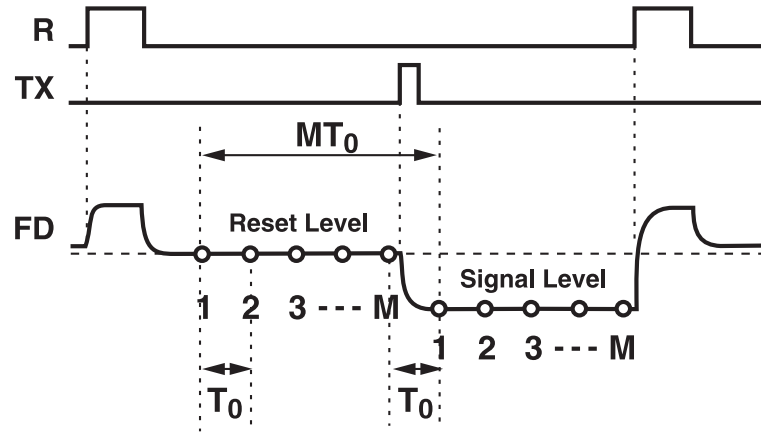


Fig. 1. Block diagram of readout circuit with CMSDA.



**Fig. 2.** Timing diagram for multiple sampling of image sensor outputs.

The process of CMSDA is expressed as

$$y = \frac{1}{M} \sum_{i=1}^M \{V_R(i) - V_S(i)\} \quad (1)$$

where  $y$  is the output, and  $V_R(i)$  and  $V_S(i)$  are the reset and signal levels of the  $i$ -th sample. The transfer function is expressed in the  $z$  domain as

$$H_{DA}(z) = \frac{1}{M} \frac{(1 - z^{-M})^2}{1 - z^{-1}}. \quad (2)$$

It is assumed that the readout circuit has a transfer function characterized by a first-order low-pass filter given by

$$|H_{LPF}(x)|^2 = \frac{1}{1 + (x/x_c)^2}. \quad (3)$$

Here,  $x = \omega MT_0/2$  and  $x_c = \omega_c MT_0/2$ , where  $\omega_c$  is the cutoff angular frequency. The cutoff frequency of the readout circuits is mainly determined by the S/H capacitor and the impedance of the buffer amplifier of the pixel. As  $M$ -times higher sampling rate requires an  $M$ -times faster circuit response, it is reasonable to assume that  $\omega_c$  increased in proportion to  $M$ . However,  $MT_0$  remains unchanged with  $M$ , that is,  $T_0$  decreases in inverse proportion to  $M$ . From Eq. (2) with  $z = \exp(j\omega T_0)$  and  $x = \omega MT_0/2$ , the noise power transfer function for CMSDA is given by

$$|H_{DA}(x)|^2 = \frac{1}{M^2} \frac{4 \sin^4 x}{\sin^2(x/M)}. \quad (4)$$

From these equations, the noise power  $P(M)$  of CMSDA with  $M$  times sampling is given by

$$P(M) = \frac{1}{M^2} \int_0^\infty S_n(x) \frac{1}{1 + (x/x_c)^2} \frac{4 \sin^4 x}{\sin^2(x/M)} dx \quad (5)$$

where  $S_n(x) = N_w + N_f/x$ , and  $N_w$  and  $N_f$  are the coefficients giving the relative magnitudes of white noise and  $1/f$  noise. In Eq. (5), the quantization

noise of the ADC is neglected. In the special cases of  $M = 1$  and  $M \rightarrow \infty$ , the  $1/f$  noise power is respectively given by

$$P(1) = \int_0^\infty S_n(x) \frac{4 \sin^2 x}{1 + (x/x_c)^2} dx \quad (6)$$

and

$$P(\infty) = \int_0^\infty S_n(x) \frac{4 \sin^4 x}{x^2} dx \quad (7)$$

Equations (6) and (7) correspond to the “double delta” [4, 5] and a special case of the “differential averager” [4], respectively. For  $1/f$  noise,  $P(1) \cong 2N_f \{C + \ln(\omega_c T_0)\}$  and  $P(\infty) = 2N_f \ln(4)$ , where  $C$  is Euler’s Constant ( $= 0.577215\dots$ ).

### 3 Simulation results

Figure 3 shows the calculation results for the noise power due to  $1/f$  noise component as a function of  $\omega_c T_0$  using Eq. (5) for  $M = 1, 2, 8, 64$  and  $\infty$ , normalized by  $2N_f$ . The curve for  $M = 1$  corresponds to the response of CDS to  $1/f$  noise. As the number of samples increases, the CMSDA suppresses  $1/f$  noise more effectively than CDS. The curve for  $M \rightarrow \infty$  corresponds to a special case of the differential averager using continuous-time integration. In this case, the noise power takes a constant value of  $\ln(4) \cong 1.39$ , independent of  $\omega_c T_0$ .

In noise-cancelling circuits for CMOS image sensors, the value of  $\omega_c T_0$  is chosen so as to achieve precise FPN cancelling. For example, with  $\omega_c T_0 = 6.9$ , the settling error is 0.1%, and at this value the noise power of the CMSDA with  $M = 8$  is 40% lower than that achieved by CDS ( $M = 1$ ). The lower limit of noise reduction effect is 44% with  $M \rightarrow \infty$ . It is revealed that the

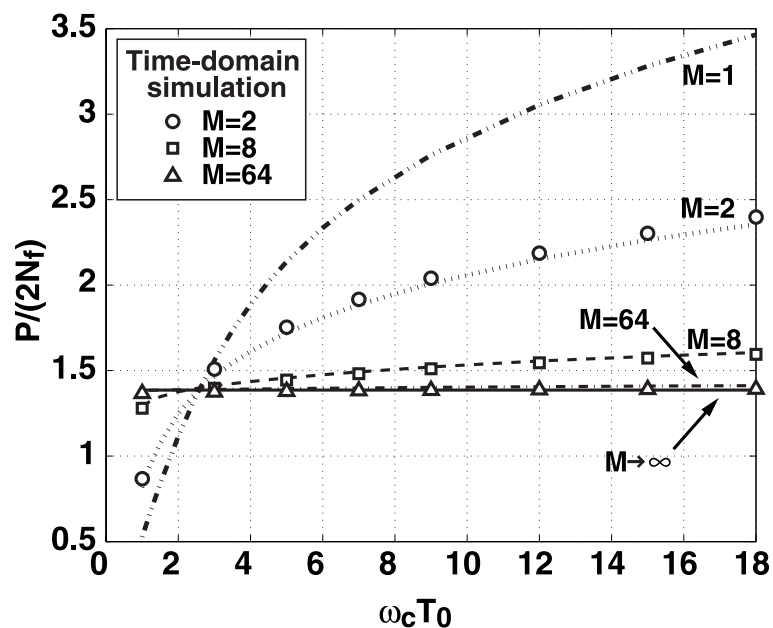


Fig. 3. Estimated  $1/f$  noise power as a function of  $\omega_c T_0$ .

large noise reduction effect for  $1/f$  noise can be expected with relatively small number of samples.

The corresponding time-domain simulation results for the  $1/f$  noise component are also shown in Fig. 3. In the time-domain simulation,  $1/f$  noise is generated by the summation of Lorentzians [6]. The markers of circles, squares and triangles correspond to the results of  $M = 2$ ,  $M = 8$  and  $M = 64$ , respectively. The analytical formula using a transfer function is in good agreement with the results of time-domain simulation.

Another interesting property of Fig. 3 is that the noise reduction effect becomes larger as  $M$  decrease in the case of  $\omega_c T_0 < 3$ . This is because a band-limiting effect of the S/H circuit exceeds the averaging effect of multiple sampling. However, the region of  $\omega_c T_0 < 3$  is not useful because of the insufficient settling.

#### 4 Conclusion

An effectiveness of CMSDA for reducing  $1/f$  noise was described. An analytical formula of CMSDA for calculating noise power was derived and was estimated for  $1/f$  noise. The CMSDA can suppress  $1/f$  noise more effectively than CDS as the number of samples increases. The transfer function-based analysis is in good agreement with the results of time-domain simulation.

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