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A small signal circuit model of two mode InAs/GaAs quantum dot laser

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Abstract: In this paper, for the first time, we present a small signal circuit model of quantum dot laser considering two photon modes, i.e., ground and first excited states lasing. By using the presented model, effect of temperature variations on modulation response of quantum dot laser is investigated. Simulation results of modulation response are in agreement with the numerical and experimental results reported by other researchers.

Keywords: circuit model, modulation response, quantum dot laser

Classification: Optoelectronics, Lasers and quantum electronics, Ultrafast optics, Silicon photonics, Planar lightwave circuits

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1 Introduction

Quantum dot lasers have attracted much attention in recent years because they exhibit excellent properties such as low threshold current, high modulation bandwidth, and low frequency chirp [1, 2].

It is necessary to determine the modulation response of quantum dot lasers. A numerical method has been used to study the modulation response of InAs/GaAs quantum dot laser [2].

The method used here is to transform the rate equations into a circuit model, which can be solved using circuit analysis techniques. Two small signal circuit models for quantum well laser have been already reported [3, 4]. Also a small signal circuit model for vertical-cavity surface emitting laser has been presented [5].

The main advantages of circuit modeling approach include that the circuit model gives an intuitive idea of the physics of the device, and helps in understanding of the modulation response behavior.

The agreement of the simulation results with the numerical and experimental results verifies the accuracy of our model. The paper is organized as follows:
In the next section, the circuit model implementation is described and results are discussed in section three, finally conclusion is presented in the last section.

2 Circuit model implementation

The rate equations for two state quantum dot laser are [2]:

$$\frac{dn_0}{dt} = \frac{n_1}{t_{10}}\left(1 - \frac{n_0}{2N_d}\right) - \frac{n_0}{t_{sp}} - \frac{n_0}{t_{01}}\left(1 - \frac{n_1}{4N_d}\right) - \frac{cg_0}{n_g WL} s_0 \left(\frac{n_0}{N_d} - 1\right) \quad (1)$$

$$\begin{aligned}\frac{dn_1}{dt} = {} & \frac{n_w}{t_{w1}}\left(1 - \frac{n_1}{4N_d}\right) + \frac{n_0}{t_{01}}\left(1 - \frac{n_1}{4N_d}\right) - \frac{n_1}{t_{sp}} - \frac{n_1}{t_{10}}\left(1 - \frac{n_0}{2N_d}\right) \\ & - \frac{n_1}{t_{1w}} - \frac{cg_1}{n_g WL} s_1 \left(\frac{n_1}{2N_d} - 1\right)\end{aligned} \quad (2)$$

$$\frac{dn_w}{dt} = \frac{\eta J}{q} + \frac{n_1}{t_{1w}} - \frac{n_w}{t_{sp}} - \frac{n_w}{t_{w1}}\left(1 - \frac{n_1}{4N_d}\right) \quad (3)$$

$$\frac{ds_0}{dt} = \frac{c}{n_g}\left[g_0\left(\frac{n_0}{N_d} - 1\right) - \alpha\right] s_0 + \beta \frac{n_0}{t_{sp}} WL \quad (4)$$

$$\frac{ds_1}{dt} = \frac{c}{n_g}\left[g_1\left(\frac{n_1}{2N_d} - 1\right) - \alpha\right] s_1 + \beta \frac{n_1}{t_{sp}} WL \quad (5)$$

where n_0, n_1, and n_w are the carrier densities in the ground state (GS), excited state (ES), and wetting layer (WL), respectively. s_0 and s_1 denote total

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photon population of GS and ES, respectively. g_0 and g_1 are the saturation gains for GS and ES, respectively.

The associated time constants are carrier relaxation from WL to ES (t_{w1}), carrier thermal escape from ES to WL (t_{1w}), carrier relaxation from ES to GS (t_{10}), carrier thermal escape form GS to ES (t_{01}), and spontaneous emission lifetime (t_{sp}). W and L are the width and length of the cavity, J is the injected current density, η is the carrier injecting efficiency, q is the electron's charge, c is the speed of light, n_g is the refractive index of the active region, α is the total loss of cavity, N_d is the areal dot density, and β is the spontaneous emission factor.

The thermal escape times from GS and ES are given by [2]:

$$t_{01} = \frac{p_0}{p_1} t_{10} \exp\left(\frac{E_{10}}{K_B T}\right) \quad t_{1w} = \frac{p_1}{p_w} t_{w1} \exp\left(\frac{E_{w1}}{K_B T}\right) \tag{6}$$

where p_0, p_1, and p_w are the degeneracies of the GS, ES, and WL, respectively. E_{10} and E_{w1} represent the transition energies of electrons from the ES to the GS, and from the WL to the ES, respectively. K_B is the Boltzmann's constant and T is the temperature. The degeneracy of WL (p_w) is given by [2]:

$$p_w = \frac{m_{eff} K_B T}{N_d \pi \hbar^2} \tag{7}$$

Where m_{eff} is the electron's effective mass, and $\hbar$ is the Planck's constant divided by 2π.

We define the carrier population in the GS, ES, and WL using [6]:

$$n_0 = n_{0e} \exp\left(\frac{qV_0}{\eta_0 K_B T}\right) \quad n_1 = n_{1e} \exp\left(\frac{qV_1}{\eta_1 K_B T}\right) \quad n_w = n_{we} \exp\left(\frac{qV_w}{\eta_w K_B T}\right) \tag{8}$$

In this case, n_{0e}, n_{1e}, and n_{we} are the equilibrium carrier densities in the GS, ES, and WL, respectively, while η_0, η_1, and η_w are the corresponding diode ideality factors, typically set equal to 2 [5, 6]. V_0, V_1, and V_w are the voltages across the GS, ES, and WL, respectively.

With linearization of Eqs. (8), and substituting into linearized rate equations, one can write:

$$i + i_{ew} = C_w \frac{dv_w}{dt} + \frac{v_w}{R_w} + \frac{v_w}{R_{w1}} \tag{9}$$

$$\frac{v_w}{R_{w1}} - i_{ew} + i_{ge} = C_1 \frac{dv_1}{dt} + \frac{v_1}{R_1} + \frac{v_1}{R_{11}} + \frac{v_1}{R_{01}} + \frac{v_1}{R_{10}} + i_{L1} \tag{10}$$

$$\frac{v_1}{R_{01}} + \frac{v_1}{R_{10}} - i_{ge} = C_0 \frac{dv_0}{dt} + \frac{v_0}{R_0} + \frac{v_0}{R_{00}} + i_{L0} \tag{11}$$

$$v_1 = R_{s1}.i_{L1} + L_{s1} \frac{di_{L1}}{dt} \quad v_0 = R_{s0}.i_{L0} + L_{s0} \frac{di_{L0}}{dt} \tag{12}$$

$$i_{ew} = \left(\frac{q}{t_{1w}} + \frac{qn_{wdc}}{4N_d t_{w1}}\right) . \left(\frac{qWL.n_{1e}}{2KT}\right) .v_1 \tag{13}$$

$$i_{ge} = \left(\frac{q}{t_{01}}\left(1 - \frac{n_{1dc}}{4N_d}\right) + \frac{qn_{1dc}}{2N_d t_{10}}\right) . \left(\frac{qWL.n_{0e}}{2KT}\right) .v_0 \tag{14}$$

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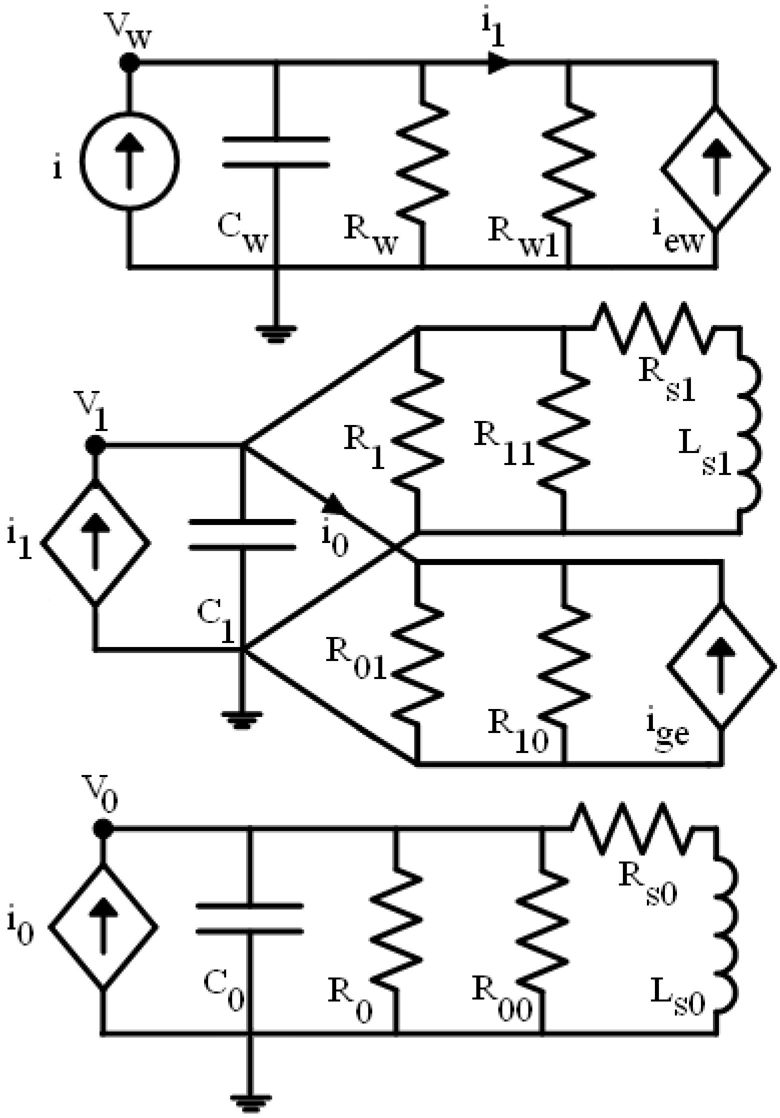

Fig. 1. Small signal circuit model for two state InAs/GaAs quantum dot Laser.

where v_w, v_0, and v_1 are the small signal junction voltages in the WL, GS, and ES, respectively. n_{0dc}, n_{1dc}, n_{wdc}, s_{0dc}, and s_{1dc} are the values obtained from the steady state solution of Eqs. (1)–(5). From Eqs. (9)–(14), the small signal circuit model for two mode quantum dot laser is illustrated in Fig. 1.

The circuit elements are given by:

$$C_w = \frac{q^2 n_{we} WL}{2KT} \quad R_w = \frac{2KTt_{sp}}{q^2 n_{we} WL} \quad R_{w1} = \frac{2KTt_{w1}}{q^2 n_{we}\left(1 - \frac{n_{1dc}}{4N_d}\right) WL}$$

$$C_1 = \frac{q^2 n_{1e} WL}{2KT} \quad R_{11} = \frac{4KTN_d}{q^2 n_{1e}(c/n_g) g_1 s_{1dc}} \quad R_{01} = \frac{4N_d t_{01}}{q n_{0dc} WL}\left(\frac{2KT}{q n_{1e}}\right)$$

$$R_{10} = \frac{t_{10}}{q\left(1 - \frac{n_{0dc}}{2N_d}\right) WL}\left(\frac{2KT}{q n_{1e}}\right) \quad C_0 = \frac{q^2 n_{0e} WL}{2KT} \quad R_1 = \frac{2KTt_{sp}}{q^2 n_{1e} WL}$$

$$R_{00} = \frac{2KTN_d}{q^2 n_{0e}(c/n_g) g_0 s_{0dc}} \quad R_0 = \frac{2KTt_{sp}}{q^2 n_{0e} WL}$$

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$$L_{s1} = \frac{N_d}{(c/n_g)^2 g_1^2 s_{1dc} q \left(\frac{n_{1dc}}{2N_d} - 1\right)} \left(\frac{2KT}{qn_{1e}}\right) \quad R_{s1} = \frac{L_{s1}\beta n_{1dc} WL}{t_{sp} s_{1dc}}$$

$$L_{s0} = \frac{N_d}{(c/n_g)^2 g_0^2 s_{0dc} q \left(\frac{n_{0dc}}{N_d} - 1\right)} \left(\frac{2KT}{qn_{0e}}\right) \quad R_{s0} = \frac{L_{s0}\beta n_{0dc} WL}{t_{sp} s_{0dc}} \tag{15}$$

In this circuit model, all of the elements have physical meaning and describe the phenomena that occurs in the active region of quantum dot laser.

The carrier loss in the WL is modeled by R_w and R_{w1}. The carrier storage in the WL is modeled by C_w. The carrier loss in quantum dots is modeled by R_1, R_0, R_{11}, and R_{00}. The carrier storage in quantum dots is modeled by C_0 and C_1. The carrier emission from ES to GS is modeled by R_{10}. The carrier emission from GS to ES, is modeled by R_{01}. The effects of stimulated and spontaneous emission are modeled by L_{s1}, L_{s0}, R_{s0}, and R_{s1}. Current sources i_{ew} and i_{ge}, represent carrier capture by the WL and ES, respectively.

This circuit model is compatible with general purpose circuit simulators such as spice.

3 Results

The quantum dot laser used in our modeling is grown by molecular beam epitaxy. The active region is formed by five layers of InAs quantum dots, which are covered by 4 nm $In_{0.17}Ga_{0.83}As$ strain reducing layer and separated from each other by 40 nm GaAs spacer layer. The laser cavity is clad by 1.4 μm of $Al_{0.4}Ga_{0.6}As$ which is n-type on the substrate side and p-doped on the top side [2]. The values of parameters used in our calculation are listed in Table I. here, we simulate the modulation response of two state quantum dot laser by using the equivalent circuit of Fig. 1.

Table I. Parameters used in quantum dot laser circuit model [2].

Symbol	Description	value
N_d	Areal dot density	25×10^{10} cm^{-2}
g_0	GS saturation gain	20 cm^{-1}
g_1	ES saturation gain	40 cm^{-1}
t_{sp}	Spontaneous emission lifetime	1ns
n_g	Refractive index	3.6
W	Cavity width	4μm
L	Cavity length	1000μm
β	Spontaneous emission factor	10^{-5}
η	Carrier injection efficiency	0.6
q	Electron's charge	1.6×10^{-19} C
α	Total loss of cavity	5.4 cm^{-1}
c	Speed of light	3×10^{8} m/s
m_{eff}	Electron's effective mass	4.5×10^{-32} kg
h	Planck's constant	6.63×10^{-34} J.s
K_B	Boltzmann's constant	8.617×10^{-5} eV/K
P_0	Degeneracy of the GS	2
P_1	Degeneracy of the ES	4
E_{10}	Transition energy of electrons from ES to GS	46meV
E_{w1}	Transition energy of electrons from WL to ES	200meV
t_{w1}	Carrier relaxation lifetime from WL to ES	1ps
t_{10}	Carrier relaxation lifetime from ES to GS	2ps

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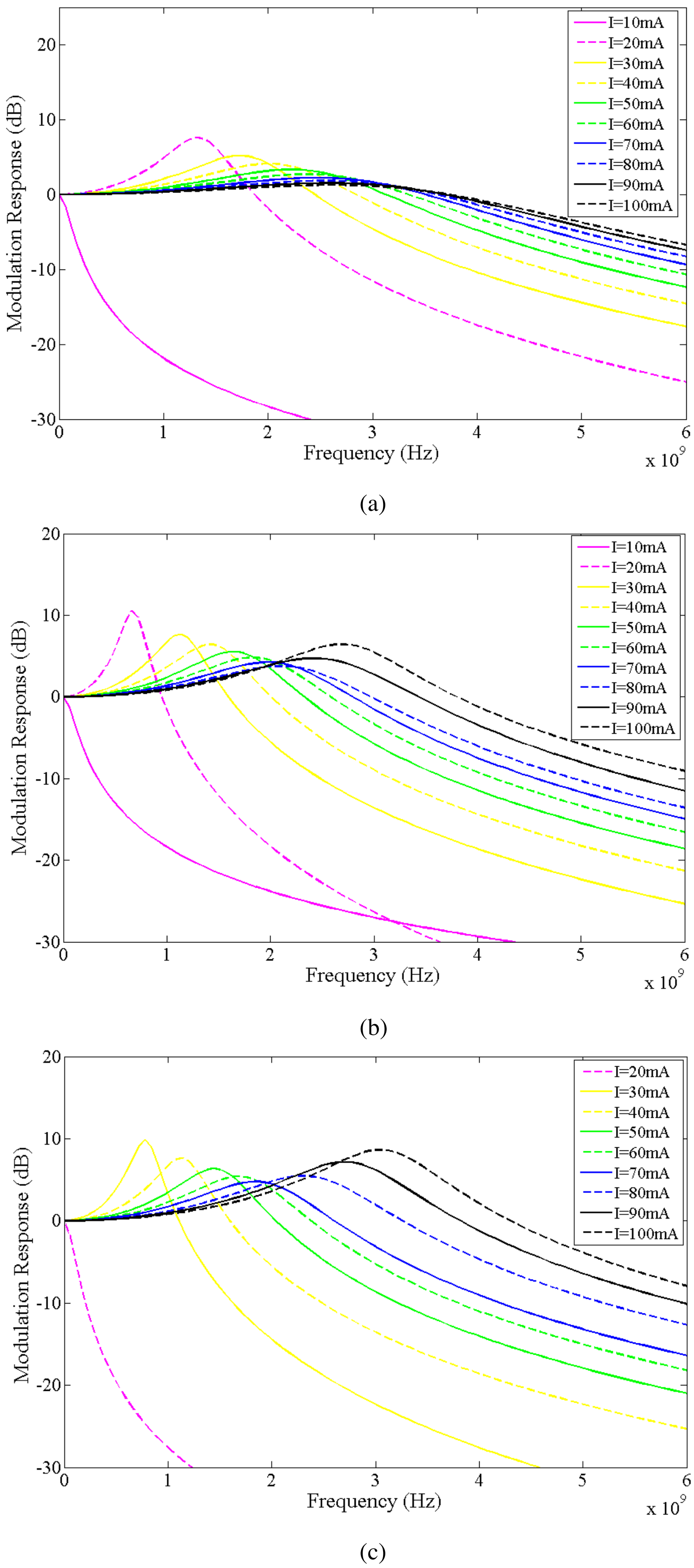

Fig. 2. Simulated small signal modulation response under various bias currents at (a) 20°C, (b) 50°C, (c) 60°C.

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Fig. 2 (a) shows the modeled small signal modulation response at 20°C, when the bias current increases from 10 to 100 mA. It can be found that the increase of bias current from 10 to 60 mA, leads to the higher modulation bandwidth. However, by increasing the bias current from 70 to 100 mA, the modulation bandwidth reaches saturation.

Fig. 2 (b) shows the modeled small signal modulation response at 50°C, when the bias current increases from 10 to 100 mA. As shown in the Fig. 2 (b), when the bias current is under 90 mA, the modulation response shows the same rule as that at 20°C. But for the bias currents of 90 and 100 mA, the modulation bandwidth significantly increases.

In Fig. 2 (c), the bias current for this significant increase in the modulation bandwidth, decreases to 80 mA.

Indeed, the ES photons do not contribute to the modulation response at 20°C. But, at temperature of 50 and 60°C, the ES begins to the lasing at the bias current of 90 and 80 mA, respectively.

Therefore, the modulation bandwidth increases when ES lasing emerges at high temperature.

It is noteworthy that because of using Eq. (7), there are a few differences between the circuit model results and the numerical ones.

4 Conclusion

A small signal circuit model for two state quantum dot laser has been derived from the rate equations. Simulation results show that ES lasing can increase modulation bandwidth of quantum dot laser at a high temperature. Our results have good agreement with the numerical and experimental results.

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