LETTER
IEICE Electronics Express, Vol.10, No.23, 1–7

Modelling the inhomogenity of the ferroelectric layer in a multilayered IDC

S. Ramezanpour[1a)], **S. Nikmehr**[1], **and L. Ghanbari**[2]

[1] *Department of Electrical and Computer Engineering, University of Tabriz, Tabriz, Iran*

[2] *Department of Electrical engineering, Shal Branch, Islamic Azad University, Shal, Iran*

a) *shahabramezanpour@yahoo.com*

Abstract: In this paper the average value between the electric fields, in the ferroelectric layer is assigned to the electric field of the ferroelectric layer. This method reveals good performance for low value of the ferroelectric thicknesses. Furthermore a more precise method is proposed in which the ferroelectric layer is divided into the one region at the gap between the strips and two regions below the strips and constant relative permittivities are assigned to these regions. Applying the method for the structures with different dimensions, approves the accuracy of the method.

Keywords: interdigital capacitor, ferroelectric

Classification: Electromagnetic theory

References

[1] I. Bahl: *Lumped Elements for RF and Microwave Circuits* (Artech House Inc., 2003).

[2] S. Gevorgian and A. Vorobiev: Proc. 37th European Microwave Conference (2007) 404.

[3] A. Tombak, J. P. Maria, F. Ayguavives, Z. Jin, G. T. Stauf, A. I. Kingon and A. Mortazawi: IEEE Microw. Wireless Compon. Lett. **12** [1] (2002) 3.

[4] J. Nath, D. Ghosh, J. Maria, A. I. Kingon, W. Fathelbab, P. D. Franzon and M. B. Steer: IEEE Trans. Microw. Theory Tech. **53** [9] (2005) 2707.

[5] S. Ramezanpour, S. Nikmehr and A. Poorziad: Progress In Electromagnetics Research B **32** (2011) 39.

[6] O. G. Vendik, S. V. Razumov, A. V. Tumarkin, M. A. Nikol'skii, M. M. Gaidukov and A. G. Gagarin: Appl. Phys. Lett. **86** (2005) 022902.

1 Introduction

Lumped elements are widely utilized in microwave integrated circuits (MICs) [1]. The tunable microwave components are such as semiconductor varactors, MEMs varactors and ferroelectric varactors [2]. Thin film ferro-

DOI: 10.1587/elex.10.20130180
Received March 16, 2013
Accepted April 04, 2013
Published December 10, 2013

electric can be beneficial in tunable RF and microwave components such as voltage controlled oscillators, tunable filters and phase shifters [3]. Tunable capacitors using ferroelectric thin film have two conventional structures; parallel plates and interdigital capacitors (IDC) [4].

In order to modelling the inhomogenity of the ferroelectric layer with thickness of h_2, in a multilayered IDC, [5] proposed to calculate the electric field at the middle of two strips and $0.25\,h_f$ away from them and relating this value of the electric field to the electric field of the whole ferroelectric layer based on the concept that the most effective region, to calculate the capacitance of an IDC, is the region between two strips and near of them. However, in this paper more developed methods have been represented and validated. In method 1, the average of the electric fields, between the strips which is almost constant is utilized to obtain the value of the ferroelectric permittivity. In method 2, the ferroelectric layer is replaced by the regions with equivalent permittivities of $\varepsilon_{eq,g}$ (for the region at the gap between the strips) and $\varepsilon_{eq,s}$ (for the regions below the strips).

2 Methods of the ferroelectric modelling

2.1 Method 1

Shematic of the periodical section of a multilayered IDC is shown in Fig. 1 (a). In this figure the ferroelectric and substrate layers are shown by relative permittivities of ε_2 and ε_1 while the strips and gap between them are shown by s and $2\,g$. Applying the voltage between the strips would create electric field distribution in the structure. The electric field components in the ferroelectric layer for the periodical section of an IDC (Fig. 1 (a)) is [5]:

$$E_2 \;=\; E_{2x} + jE_{2y} = \frac{1}{\varepsilon_2}((\varepsilon_1 - 1)E_{z1} + (\varepsilon_2 - \varepsilon_1)E_{z2} + E_{z3}) \qquad (1)$$

where E_{zi} is defined $(i = 1, 2, 3)$,

$$E_{zi} = \frac{V}{2K(k_i')} \frac{1/2\sqrt{t_2^{(i)} - t_4^{(i)}}\sqrt{t_3^{(i)} - t_1^{(i)}}}{\sqrt{(t - t_1^{(i)})(t - t_2^{(i)})(t - t_3^{(i)})(t - t_4^{(i)})}} \times \frac{\pi}{2h_i} \sinh\left(\frac{\pi z}{h_i}\right) \qquad (2)$$

In Eq. (2), $h_3 = \infty$ (related to air region), $z = x + jy$, V is the voltage between two strips and $K(k_i')$ is the complete elliptic integral with the modulus $k_i' = \sqrt{1 - k_i^2}$ with: $k_i = \sqrt{\frac{t_3^{(i)} - t_4^{(i)}}{t_3^{(i)} - t_1^{(i)}} \frac{t_2^{(i)} - t_1^{(i)}}{t_2^{(i)} - t_4^{(i)}}}$ and $t_1^{(i)} = \cosh^2(\frac{\pi g}{2h_i})$; $t_2^{(i)} = \cosh^2(\frac{\pi(s+g)}{2h_i}) t_3^{(i)} = 0$; $t_4^{(i)} = 1$. The capacitance of the periodical section with the overlapping length of the fingers, ℓ, can be written as:

$$C_n = \frac{\ell}{2}\varepsilon_0\left[(\varepsilon_1 - 1)\frac{K(k_1)}{K(k_1')} + (\varepsilon_2 - \varepsilon_1)\frac{K(k_2)}{K(k_2')} + 2\frac{K(k_3)}{K(k_3')}\right] \qquad (3)$$

Considering the Eq. (2), for a large value of ε_2, $E_2 \approx E_{z2}$ which is independent of ε_2. For $h_2 = 2\,\mu$m, $h_1 \gg h_2$, $s = 2\,g = 4\,\mu$m, $\varepsilon_1 = 9.8$, Fig. 1 (e) which shows the electric field components of the ferroelectric layer, $0.5\,\mu$m away from the strips, for large values of ε_2, also approves this concept. In

DOI: 10.1587/elex.10.20130180
Received March 16, 2013
Accepted April 04, 2013
Published December 10, 2013

ELectronics EXpress
IEICE Electronics Express, Vol.10, No.23, 1–7

the theory, using partial capacitance method, magnetic walls are assumed at the dielectric/dielectric interfaces, however assuming this wall below and near the strips is not exactly accurate and this is the cause of the discrepancy of the theoretical and simulation results, at these regions, in Fig. 1 (e). However, both theoretical and simulation results show that the electric fields in the ferroelectric layer are almost identical for various and large values of the ferroelectric permittivity. In Fig. 1 (e) the theoretical and simulation results are obtained by Eq. (1) and Maxwell 12 simulator. Therefore, initially the electric field, in the ferroelectric layer, is obtained for a large value of ε_2 and then it is substituted in a phenomenological model [5] in order to calculate the exact value of the ferroelectric layer permittivity. Fig. 1 (f) shows electric field components in the ferroelectric layer from various distances $(0.1, 0.5, 0.9, 1.3, 1.7\,\mu\text{m})$ away from the strips and their average value for the applied voltage $V = 1$. The average electric field is obtained by averaging of the electric fields from $0.1\,\mu\text{m}$ uto $h_2 - 0.1\,\mu\text{m}$ away from the strips with step of $0.1\,\mu\text{m}$. Fig. 1 (f) (right) shows that the average electric field is almost constant between two strips and it seems that this constant value can be a appropriate value for the equivalent electric field of the ferroelectric layer.

To investigate a high inhomogenity, $V = 100$ is assumed between two strips. On the other hand, due to the linear relation between applied voltage and electric field distribution, for a specified dimensions of a multilayered IDC (Eq. (2)), the electric field distribution for $V = 100$ is as Fig. 1 (f) which is multiplied by 100. Fig. 1 (g) shows its corresponding relative permittivity and their average for $V = 100$. This figure is utilized to calculate the capacitance of the inhomogenous structure.

The average value of the electric fields, is almost constant 1.7 kV/cm and can be assigned to the electric field of the whole ferroelectric layer. On the other word, in Eq. (3), ε_2 can be obtained by using 1.7 kV/cm, for the relation between electric field and applied voltage, in the phenomenological model and is called ε_{eq}. ε_{eq} is calculated for various h_2 and is depicted in Table I.

On the other hand, Vendik et al., in ref. [6] used an experimental formula for the relation between electric field of the ferroeelctric layer and applied voltage: $E = \frac{V}{2g+\xi h_2}$; $0.7 < \xi < 1$. Unfortunately, [6] has not mentioned that which values of ξ are suitable for different structures. Fig. 1 (h) shows close agreement between the average values which are obtained in this paper, and Vendik formula with $\xi = 0.88$. Therefore, it seems that the proposed method is suitable. However, the comparison of the capacitors of the inhomogenous and equivalent structures shows some deviations, for high values of the thickness of the ferroelectric layer.

As mentioned, by applying the voltage between the strips, inhomogenous permittivity would create in the ferroelectric layer. Therefore, in Fig. 1 (b), in order to obtain a simulation model, the ferrolectric layer is divided into the cells and the corresponding permittivities are applied to the each cells, by considering the location of the cells. In order to calculate the capacitance, the

DOI: 10.1587/elex.10.20130180
Received March 16, 2013
Accepted April 04, 2013
Published December 10, 2013

IEICE Electronics Express, Vol.10, No.23, 1–7

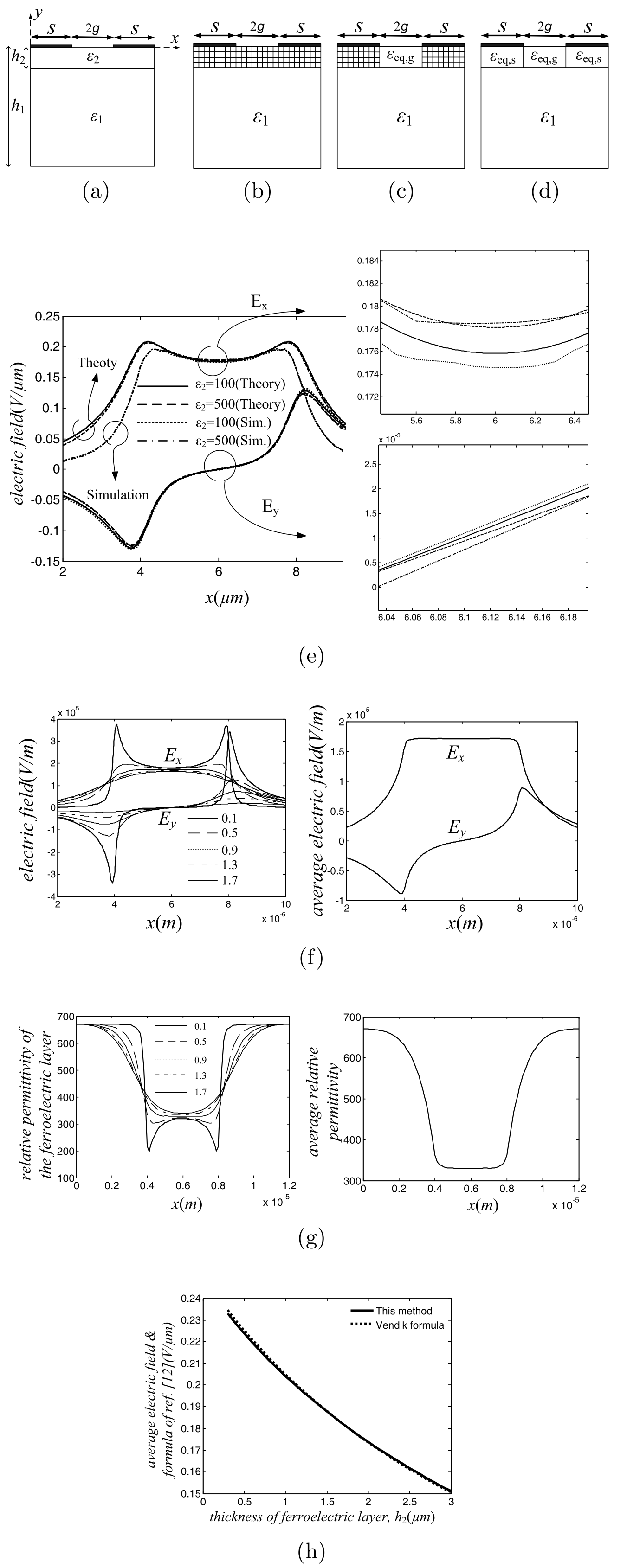

Fig. 1. (a)&(b)&(c)&(d) Shematic of the periodical section with inhomogenous and equivalent permittivities. (e) Electric field components. (f)&(g) Electric field and relative permittivity distribution and their average. (h) Comparison of the average electric field values and Vendik formula.

DOI: 10.1587/elex.10.20130180
Received March 16, 2013
Accepted April 04, 2013
Published December 10, 2013

IEICE Electronics Express, Vol.10, No.23, 1–7

Table I. Equivalent permittivities.

$s = 2g = 4\,\mu$m	$h_2\,(\mu\text{m})$	1	1.5	2	2.5	3
$V = 50\,V$	$\varepsilon_{eq,g}$	445.46	455.12	470.57	480.73	490.21
	$\varepsilon_{eq,s}$	617.93	626.20	625.17	627.74	630.76
$V = 100\,V$	ε_{eq}	290.98	310.94	330.92	348.64	365.96
	$\varepsilon_{eq,g}$	290.18	307.42	319.41	327.50	335.59
	$\varepsilon_{eq,s}$	578.61	558.75	541.31	537.93	538.37
$V = 200\,V$	$\varepsilon_{eq,g}$	191.27	193.03	200.51	205.62	210.88
	$\varepsilon_{eq,s}$	471.93	440.30	409.39	400.16	397.78
$g = 2s = 4\,\mu$m	$h_2\,(\mu\text{m})$	1	2	3	4	5
$V = 100\,V$	ε_{eq}	411.75	448.38	480.52	511.73	540.60
	$\varepsilon_{eq,g}$	415.69	448.69	475.50	497.15	511.02
	$\varepsilon_{eq,s}$	497.47	504.12	532.28	558.95	581.91

inhomogenous layer of a multilayered IDC is divided into the $0.5 \times 0.5\,\mu\text{m}^2$ cells (Fig. 1 (b)) and the permittivity at the location near to the middle of each cell is assigned to the permittivity of the whole cell from Fig. 1 (g) (left). Fig. 2 (a) shows the comparison of the capacitors which are obtained from the simulation of the inhomogenous structure (Fig. 1 (b)), the simulation of the equivalent structure (Fig. 1 (a), with $\varepsilon_2 = \varepsilon_{eq}$) and from Eq. (3) (with $\varepsilon_2 = \varepsilon_{eq}$), for the structure with $s = 2g = 4\,\mu$m while ℓ is assumed $16\,\mu$m. The simulation results are obtained by HFSS 11 simulator. The results are also obtained by simulation of the structure by cells with smaller cross sections ($0.2 \times 0.4\,\mu\text{m}^2$, $0.25 \times 0.4\,\mu\text{m}^2$ and $0.3 \times 0.4\,\mu\text{m}^2$ for the structures with $h_2 = 1, 2$ and $3\,\mu$m). Fig. 2 (a) shows good accordance for the low ferroelectric layer thicknesses. This method is also incorporated for the structure with $2s = g = 4$. The results are shown in Fig. 2 (b) which shows excellent accordance however the deviation of the theoretical results, for the high values of h_2 is due to the fact that the theoretical expressions are valid for $\frac{s}{h_2} > 1$.

2.2 Method 2

In Fig. 2 (a), the simulation results are validated by theory and also by decreasing the dimension of the cells. Therefore, In the following, another method is proposed based on the simulation results which is more precise and also has more generality. For this purpose, firstly, the regions below the strips are kept inhomogenous and the constant value, $\varepsilon_{eq,g}$, is assigned to the relative permittivity of the region at the gap between two strips (Fig. 1 (c)). Fig. 1 (c) is as Fig. 1 (b) in which the region at the gap between two strips, is replaced by the region with equivalent permittivity. Then, the inhomogenous regions below the strips are also replaced by the regions with the equivalent permittivity, $\varepsilon_{eq,s}$ (Fig. 1 (d)). In Fig. 1 (d) the whole of the ferroelectric layer is divided into the three regions with constant equivalent permittivities. It is observed that $\varepsilon_{eq,g}$ is very close to the reletive permittivity of the inhomogenous structure at the middle of the strips and just below them (Fig. 1 (g) (left) at $x = 6\,\mu$m and $0.1\,\mu$m away from the strips). For $s = 4\,\mu$m, the value

DOI: 10.1587/elex.10.20130180
Received March 16, 2013
Accepted April 04, 2013
Published December 10, 2013

IEICE Electronics Express, Vol.10, No.23, 1–7

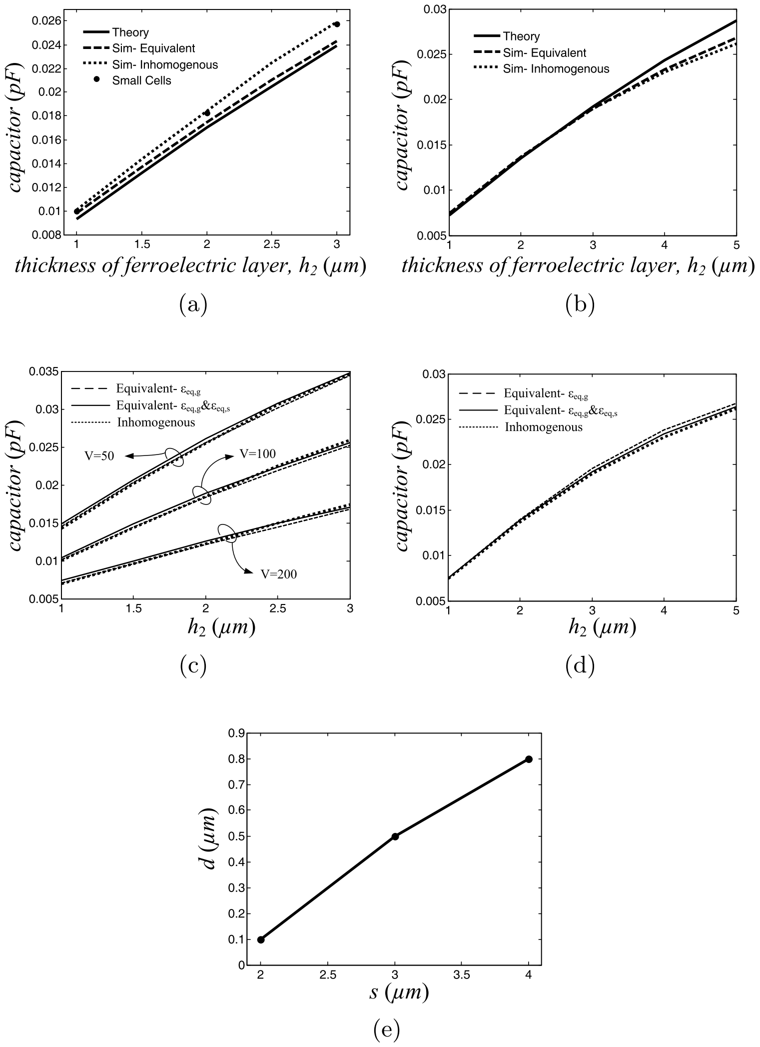

Fig. 2. Comparison of the capacitors (method 1): (a) $s = 2g = 4\,\mu$m. (b) $g = 2s = 4\,\mu$m; Comparison of the capacitors (method 2): (c) $s = 2g = 4\,\mu$m. (d) $g = 2s = 4\,\mu$m; (e) Desired horizontal distance versus strip size.

of the average permittivity at the horizontal distance (d) of $0.8\,\mu$m away from the edge of the strips shows a appropriate value for $\varepsilon_{eq,s}$ (Fig. 1 (g) (right) at $x = 3.2\,\mu$m). It is seen that d is related to the dimension of the strips (s) (Fig. 2 (e)). To obtain the curve of Fig. 2 (e), another structures with $g = 2s = 4\,\mu$m, for various h_2 and $g = s = 3\,\mu$m, for a typical value of $h_2 = 3\,\mu$m are also investigated.

Figs. 2 (c) and 2(d) show the comparison of the capacitors which are obtained from simulation of the inhomogenous and equivalent structures for $s = 2g = 4\,\mu$m and $g = 2s = 4\,\mu$m which show perfect accordance. Fig. 2 (c) is also contained the capacitors for other applied voltages $V = 50\,V$ and $V = 200\,V$.

The values of the ε_{eq}, $\varepsilon_{eq,g}$ and $\varepsilon_{eq,s}$, for different structures, applied voltages and ferroelectric layer thicknesses are depicted in Table I (for the structure of $s = g = h_2 = 3\,\mu$m, $\varepsilon_{eq,g} = 414.48$, $\varepsilon_{eq,s} = 553.25$ and the ca-

DOI: 10.1587/elex.10.20130180
Received March 16, 2013
Accepted April 04, 2013
Published December 10, 2013

pacitors for both inhomogenous and equivalent stuctures are obtained almost 0.022 pF).

3 Conclusion

In the first method, the average electric field in the periodical section was obtained and its value between the strips was assigned to the electric field of the ferroelectric layer. Furthermore, another method was proposed in which the ferroelectric layer was divided into the regions with equivalent permittivites of $\varepsilon_{eq,g}$ and $\varepsilon_{eq,s}$. The comparison of the capacitors of the inhomogenous and equivalent structures revealed excellent accordance for the second method. It is also showed good accordance for the structure with low values of the ferroelectric thicknesses, for method 1.

DOI: 10.1587/elex.10.20130180
Received March 16, 2013
Accepted April 04, 2013
Published December 10, 2013