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A THERMO-ELASTO-VISCOPLASTIC MODEL FOR SOFT SEDIMENTARY ROCK

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ABSTRACT

In this paper, a new thermo-elasto-viscoplastic model is proposed, which can characterize thermodynamic behaviours of soft sedimentary rocks. Firstly, as in the Cam-clay model, plastic volumetric strain which consists of two parts, stress-induced part and thermodynamic part, is used as hardening parameter. Both parts of the plastic volumetric strain can be derived from an extended e-ln p relation in which the thermodynamic part is deduced based on a concept of 'equivalent stress'. Secondly, regarding soft sedimentary rocks as a heavily overconsolidated soil in the same way as the model proposed by Zhang et al. (2005), an extended subloading yield surface (Hashiguchi and Ueno, 1977; Hashiguchi, 1980; Hashiguchi and Chen, 1998) and an extended void ratio difference are proposed based on the concept of the equivalent stress. Furthermore, a time-dependent evolution equation for the extended void ratio difference is formularized, which considers both the influences of temperature and stresses. Finally, it is proved that the proposed model satisfies thermodynamic theorems in the framework of non-equilibrium thermodynamics.

Key words: constitutive model, creep, equivalent stress, soft rock, temperature, thermodynamic (**IGC**: D6)

INTRODUCTION

Nuclear electricity power generation has drawn again a worldwide attention and may become much more important energy in the future. There always, however, exist some crucial problems in treating nuclear waste disposal. Deep burying of the nuclear waste in intact rock ground is considered to be a practical way. The heat emission due to the radioactivity of the nuclear waste disposal, however, will increase the temperature of surrounding ground and endanger the surrounding rock structures. It is known that for some geomaterials, e.g., soft sedimentary rock, temperature and its change may affect the mechanical behaviours of the rock, especially the long-term stability. Therefore, it is necessary to evaluate long-term stabilities of the ground subjected to heating process due to radioactivity of the nuclear waste disposal. In the analysis, a key important factor is to establish a simple and reasonable thermo-elasto-viscoplastic constitutive model to characterize the thermo-dynamic behaviour of soft rocks. Some experimental results can be found in literature, e.g., Okada (2005, 2006); Fujinuma et al. (2003). The influences on the behaviour of soft rocks induced by temperature and its change can be simply summarized as follows:

(1) The temperature and its change affect the stress-strain relation of soft rocks in a way that as the temperature decreases, the peak value of stress difference increases; meanwhile, the stress-strain relation changes from ductility to brittle, as shown in Fig. 1.
(2) The temperature and its change affect the creep behaviour of soft rocks greatly in a way that as temperature increases about 40 degrees, the creep failure time may decrease with $2 \sim 4$ orders, as is shown in Fig. 2.

In order to simulate thermodynamic behaviour of geomaterials, some thermo-elasto- viscoplastic models have been proposed, most of which are deduced using the thermodynamic theorems to establish a series of restricted relations for the variables involved in the models, e.g. stress tensor, strain tensor, hardening parameters and entropy at first, and then deduced the models using common concepts such as flow rule, yielding function, plastic potential, normality rule and etc. Detailed discussion on this issue can be found in the review by Kitagawa (1972), the work by Rojas et al. (2000) and the book by Lebon et al. (2008). In proposing a thermodynamic model, the most important but very difficult step is to formulize the thermodynamic functions, which satisfies the above-mentioned restricted relations for the variables, which always makes the model too complicated and difficult to understand.

A reversed research approach has been used in this paper. Unlike most of the models in literature that thermodynamic theorems are always used as restricted conditions in reasoning the formulation of the models, the thermodynamic theorems are only used to verify the logicality of the new model in thermodynamic meaning after the model is established. Therefore, in establishing the

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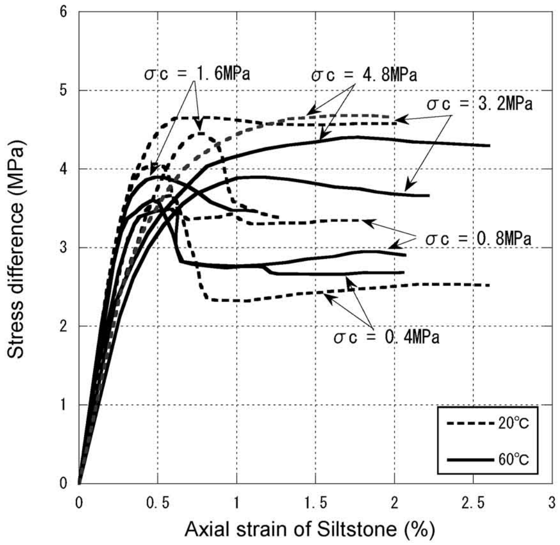
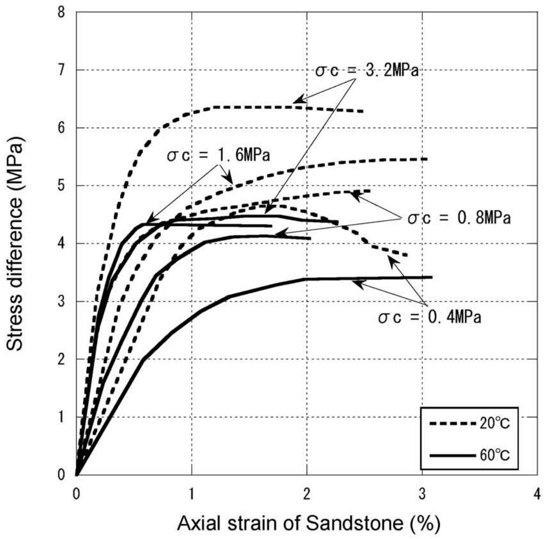
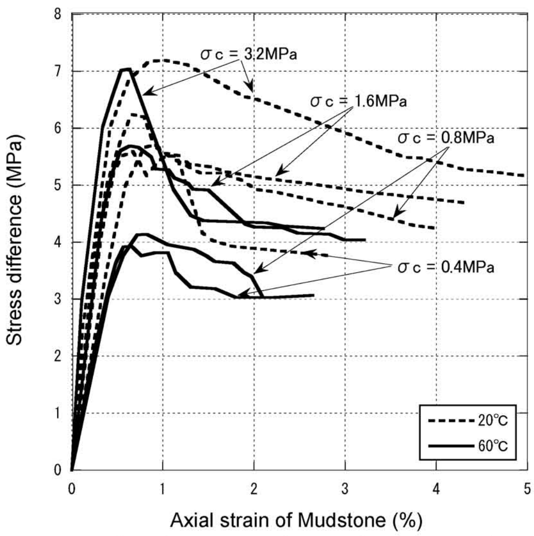


(a) Tests results of stress-strain relations for siltstone



(b) Tests results of stress-strain relations for sandstone



(c) Tests results of stress-strain relations for mudstone

Fig. 1. Test results of various sedimentary rocks under triaxial compression loading at normal and high constant temperatures (Okada, 2005)

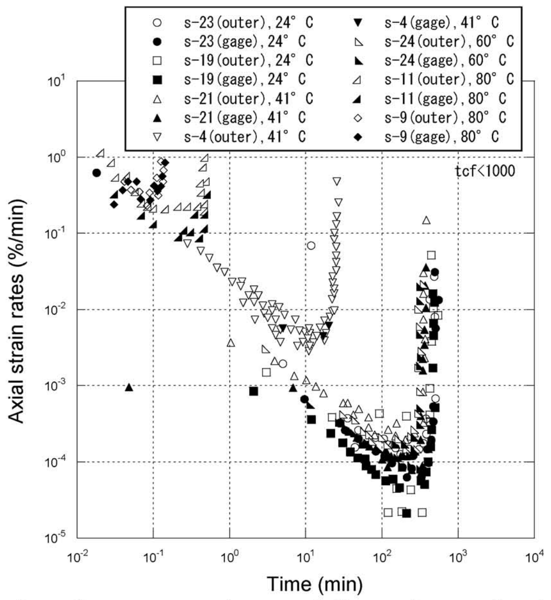
model, the flow rule, yielding function, plastic potential, evolution equation for subloading surface are chosen without considering the restrictions controlled by the thermodynamic theorems at the very beginning. It is therefore possible for us to choose reasonable formulations based on very simple physical meanings and practical equations in soil mechanics such as the e-ln p relation in consolidation tests, so that the establishment of the new model becomes much easier.

Generally speaking, within the framework of continuum mechanics, it is very difficult to formulize a constitutive model for rock due to discontinuous structures existing inside the rock mass. Some soft sedimentary soft rocks, however, can be considered as continuous media, and its mechanical behaviours can be described with elastoplastic/elasto-viscoplastic model. Based on the Endochronic theory (Valanis, 1971), Oka and Adachi (1985) proposed an elastoplastic model with strain softening for soft rock, which can describe not only the bifurcation problem of element tests but also has a feature of less dependency on mesh size in finite element analysis compared to the other models at that time. The model then was developed to an elasto-viscoplastic model for frozen sand by Adachi et al. (1994), which can describe both the strain softening and the time dependent behaviour of geomaterials, which consists of three aspects, namely, creep, strain rate effect and stress relaxation. By introducing the influence of intermediate stress with t_{ij} concept (Nakai and Mihara, 1984), Zhang et al. (2003a) proposed an elasto-viscoplastic model of soft rocks based on which a boundary value problem related to evaluation of remedial works for cracked tunnels in creep-behaved ground was analyzed (Zhang et al., 2003b).

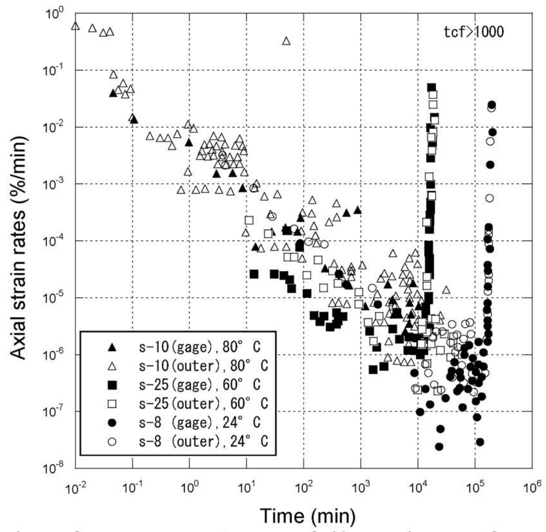
Regarding soft rock as a heavily overconsolidated soil, Zhang et al. (2005) proposed a simple elasto-viscoplastic model of soft rocks based on Cam-Clay model, using the concept of subloading yield surface (Hashiguchi and Ueno, 1977) and the t_{ij} concept (Nakai and Mihara, 1984). This model can not only describe the time dependent behaviours of soft rocks, but also can take into consideration the influence of intermediate stress properly.

In this paper, based on the works by Zhang et al. (2005), a new thermo-elasto-viscoplastic model is proposed to describe the thermodynamic behaviours of soft sedimentary rocks, in which, the plastic volumetric strain is adopted as the hardening parameter, as is the same as normal elastoplastic model like Cam-clay model. The plastic volumetric strain, however, consists of two parts, one is stress-induced and another is thermodynamic. The elastic volumetric strain is simply evaluated with thermo-elasticity, while the plastic volumetric strain is derived from an extended e-ln p relation commonly used in consolidation test, based on a new concept of 'equivalent stress', by which the plastic volumetric strain caused by the thermodynamic part can be properly evaluated. The aim of the present research is to propose a model which has the features as:

(1) It should be simple and reasonable. Not only the thermodynamic characteristics, but also the normal



(a) Results of creep test (creep failure time tcf < 1000 min)



(b) Results of creep test (creep failure time tcf > 1000 min)

Fig. 2. Tests results of creep rates history of marlstone at difference constant temperatures (Okada, 2006)

mechanical behaviours of soft rocks that have already been clarified in experiments and been modelled in the previous researches, can be described properly. The proposed model therefore, should be reasonable, sophisticated but comprehensive.

(2) The material parameters involved in the model can be determined definitely by conventional triaxial tests considering temperature effects.

(3) It should satisfy the thermodynamic theorems in spite of the fact that the theorems are not directly used for establishing the model itself.

CONCEPT OF EQUIVALENT STRESS

One of the important characteristics of the Cam-clay model is that the plastic volumetric strain is used as a hardening parameter for the elastoplastic model of soils. Furthermore, it is also shown in the work by Zhang et al. (2005) that the plastic volumetric strain can also be used as a hardening parameter in a constitutive model for soft rocks. Therefore, the plastic volumetric strain is also assumed as the hardening parameter in the newly proposed model.

Change of temperature may generate both elastic and plastic volumetric strains. It is reasonable to assume that the plastic volumetric strain of geomaterials is made up from two independent parts, that is, thermodynamic and stress-induced, and can be expressed as:

$$\varepsilon_v^p = \varepsilon_v^{p\sigma} + \varepsilon_v^{p\theta} \quad \text{or} \quad d\varepsilon_v^p = d\varepsilon_v^{p\sigma} + d\varepsilon_v^{p\theta} \tag{1}$$

where, ε_v^p is the total plastic volumetric strain. $\varepsilon_v^{p\sigma}$ is the stress-induced plastic volumetric strain and $\varepsilon_v^{p\theta}$ is the thermodynamic plastic volumetric strain. The second part of Eq. (1) expresses the incremental relation of the plastic

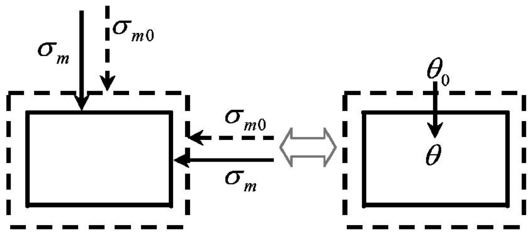


Fig. 3. Similarity of volumetric strains caused by real mean stress σ_m and equivalent stress $\tilde{\sigma}_m$ due to change of temperature

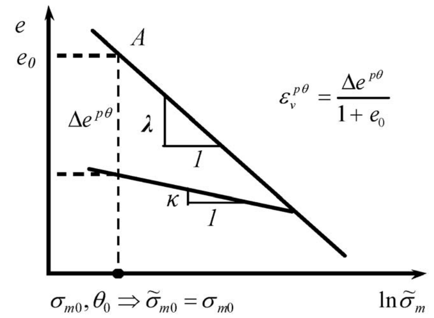


Fig. 4. Illustration of the relation between equivalent stress and void ratio difference

volumetric strains.

Firstly, it is assumed that the relation between the stress and the stress-induced plastic volumetric strain still can be expressed by the Cam-clay model as:

$$f_1(\sigma, \varepsilon_v^{p\sigma}) = \ln \frac{\sigma_m}{\sigma_{m0}} + \frac{\sqrt{3}\sqrt{J_2}}{M\sigma_m} - \frac{1}{C_p}\varepsilon_v^{p\sigma} = 0 \qquad (2)$$

where, σ_{m0} is a reference pressure and equals to 98 kPa, which is the standard atmospheric pressure. M is the ratio of shearing stress at critical state. $C_p = E_p/(1+e_0)$, where, e_0 is the reference void ratio at σ_{m0}; E_p is a plastic modulus and physically it equals to the value of λ-κ, where, λ is compression index and κ is swelling index.

In order to consider the effect of temperature and its change, it is necessary to formulize the relation between the temperature and the thermodynamic plastic volumetric strain, based on the concept of equivalent stress. It is known that under the condition of constant-stress state, change of temperature may also generate thermodynamic volumetric strain, including elastic volumetric strain $\varepsilon_v^{e\theta}$ and plastic volumetric strain $\varepsilon_v^{p\theta}$, as were the material subjected to a real mean stresses. Accordingly, it is assumed that thermodynamic volumetric strain is induced by an imaginary stress $\tilde{\sigma}_m$, namely, the equivalent stress. The similarity of volumetric strains caused by real mean stress and the incremental equivalent stress $d\tilde{\sigma}_m$ due to change of temperature is shown in Fig. 3.

In evaluating the stress-induced volumetric strain, it is necessary to define a reference pressure σ_{m0} at a reference temperature θ_0, which represents the global average absolute temperature and is assumed here to be 288 K. Considering the limitation of the variation range for temperature and the fact that θ should be larger or equal to 273 K, a linear relation between the change of temperature $\theta - \theta_0$ and the thermodynamic elastic volumetric strain $\varepsilon_v^{e\theta}$ is assumed as:

$$\varepsilon_v^{e\theta} = 3\alpha(\theta - \theta_0) \qquad (3)$$

where, α is linear thermo-expansion coefficient, and takes a negative value because a compressive volumetric strain is assumed as positive in geomechanics.

Based on the concept of equivalent stress, the relation between equivalent stress and thermo-dynamic elastic volumetric strain can then be evaluated with Hooke's law. Considering Eq. (3), this relation can be expressed as:

$$\begin{aligned}\tilde{\sigma}_m &= \sigma_{m0} + d\tilde{\sigma}_m = \sigma_{m0} + K\varepsilon_v^{e\theta} \\ &= \sigma_{m0} + 3K\alpha(\theta - \theta_0)\end{aligned} \qquad (4)$$

where, K is volume elastic modulus, and is equal to $E/3/(1\text{-}2\nu)$, in which E is the Young's modulus and ν is the Poisson's ratio.

On the other side, the relation between the equivalent stress and the thermodynamic plastic volumetric strain $\varepsilon_v^{p\theta}$ is evaluated by e-ln p relations in both compression and swelling processes based on the equivalent stress. Detailed process to evaluate the relation between the plastic thermodynamic void ratio $\Delta e^{p\theta}$ and the equivalent stress under the condition of constant-stress state is shown in Fig. 4. Considering Eq. (4), this relation can be expressed as:

$$\begin{aligned}\varepsilon_v^{p\theta} &= C_p \ln \frac{\tilde{\sigma}_m}{\tilde{\sigma}_{m0}} = C_p \ln \frac{\tilde{\sigma}_m}{\sigma_{m0} + 3K\alpha(\theta_0 - \theta_0)} \\ &= C_p \ln \frac{\sigma_{m0} + 3K\alpha(\theta_0 - \theta_0)}{\sigma_{m0}}\end{aligned} \qquad (5)$$

Similar to the Cam-clay model, the plastic potential function related to temperature is assumed in the following way based on Eq. (5):

$$f_2(\theta, \varepsilon_v^{p\theta}) = \ln \frac{\sigma_{m0} + 3K\alpha(\theta - \theta_0)}{\sigma_{m0}} - \frac{1}{C_p}\varepsilon_v^{p\theta} = 0 \qquad (6)$$

Considering both the effects of temperature and stress, the total plastic volumetric strain is made up from thermodynamic and stress-induced, and substituting Eqs. (5) and (6) into Eq. (1), a new thermoplastic potential function can be obtained as follows:

$$\begin{aligned}f(\sigma, \varepsilon_v^{p\sigma}, \theta, \varepsilon_v^{p\theta}) &= f_1(\sigma, \varepsilon_v^{p\sigma}) + f_2(\theta, \varepsilon_v^{p\theta}) \\ &= \left(\ln \frac{\sigma_m}{\sigma_{m0}} + \frac{\sqrt{3}\sqrt{J_2}}{M\sigma_m} - \frac{1}{C_p}\varepsilon_v^{p\sigma}\right) \\ &\quad + \left(\ln \frac{\sigma_{m0} + 3K\alpha(\theta - \theta_0)}{\sigma_{m0}} - \frac{1}{C_p}\varepsilon_v^{p\theta}\right) = 0\end{aligned} \qquad (7)$$

or

$$f(\sigma, \theta, \varepsilon_v^p) = \ln \frac{\sigma_m}{\sigma_{m0}} + \frac{\sqrt{3}\sqrt{J_2}}{M\sigma_m} + \ln \frac{\sigma_{m0} + 3K\alpha(\theta - \theta_0)}{\sigma_{m0}} - \frac{1}{C_p}\varepsilon_v^p = 0 \tag{8}$$

Because associated flow rule is adopted, the proposed thermoplastic potential is also the yield function. Consistency equation can then be obtained as:

$$df = 0 \Rightarrow \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \theta} d\theta + \frac{\partial f}{\partial \varepsilon_v^p}(d\varepsilon_v^{p\theta} + d\varepsilon_v^{p\sigma}) = 0 \tag{9}$$

Considering the independency of thermodynamic plastic volumetric strain and stress-induced plastic volumetric strain, the following equation can be obtained:

$$\frac{\partial f}{\partial \sigma} = \frac{\partial f_1}{\partial \sigma}, \quad \frac{\partial f}{\partial \varepsilon_v^{p\sigma}} = \frac{\partial f_1}{\partial \varepsilon_v^{p\sigma}}, \quad \frac{\partial f}{\partial \theta} = \frac{\partial f_2}{\partial \theta}, \quad \frac{\partial f}{\partial \varepsilon_v^{p\theta}} = \frac{\partial f_2}{\partial \varepsilon_v^{p\theta}} \tag{10}$$

By which, the stress-induced plastic volumetric strain increment can be easily calculated as:

$$d\varepsilon_{ij}^{p\sigma} = \Lambda \frac{\partial f}{\partial \sigma_{ij}} \quad \text{and} \quad d\varepsilon_v^{p\sigma} = \Lambda \frac{\partial f}{\partial \sigma_{ii}} \tag{11}$$

Considering the factor that the change of temperature can only generate volumetric strain, the stress increment can be calculated by the Hooke's law as:

$$\begin{aligned} d\sigma_{ij} &= E_{ijkl} d\varepsilon_{kl}^{e\sigma} = E_{ijkl}(d\varepsilon_{kl} - d\varepsilon_{kl}^{p} - d\varepsilon_{kl}^{e\theta}) \\ &= E_{ijkl}\left(d\varepsilon_{kl} - d\varepsilon_{kl}^{p\sigma} - \frac{d\varepsilon_v^{p\theta}\delta_{kl}}{3} - \frac{d\varepsilon_v^{e\theta}\delta_{kl}}{3}\right) \end{aligned} \tag{12}$$

where, $\varepsilon_{kl}^{e\theta}$ is the thermoelastic strain tensor. Substituting Eqs. (3) and (11) into Eq. (12), it can be rewritten as:

$$d\sigma_{ij} = E_{ijkl} d\varepsilon_{kl} - E_{ijkl}\frac{\partial f}{\partial \sigma_{kl}}\Lambda - E_{ijkl}\delta_{kl}\frac{d\varepsilon_v^{p\theta}}{3} - E_{ijkl}\alpha\delta_{kl} d\theta$$

or

$$d\sigma_{ij} = E_{ijkl} d\varepsilon_{kl} - E_{ijkl}\frac{\partial f}{\partial \sigma_{kl}}\Lambda - K\delta_{ij} d\varepsilon_v^{p\theta} - 3K\alpha\delta_{ij} d\theta \tag{13}$$

where, δ_{ij} is the Kronecker tensor. From Eqs. (2) and (6), it is easy to obtain the following equations:

$$\begin{aligned} &\frac{\partial f}{\partial \varepsilon_v^p} = -\frac{1}{C_p}, \quad d\varepsilon_v^{p\theta} = C_p \frac{3K\alpha}{\sigma_{m0} + 3K\alpha(\theta - \theta_0)} d\theta \\ &\frac{\partial f}{\partial \theta} = \frac{\partial f_2}{\partial \theta} = \frac{3K\alpha}{\sigma_{m0} + 3K\alpha(\theta - \theta_0)} \end{aligned} \tag{14}$$

Furthermore, by substituting Eqs. (13), (14) and (11) into Eq. (9), Λ can be obtained:

$$\Lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} d\varepsilon_{kl}}{\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl}\frac{\partial f}{\partial \sigma_{kl}} + \frac{1}{C_p}\frac{\partial f}{\partial \sigma_{ii}}} - \frac{\frac{\partial f}{\partial \sigma_{ij}} K\delta_{ij}\left(C_p \frac{3K\alpha}{\sigma_{m0} + 3K\alpha(\theta - \theta_0)} + 3\alpha\right) d\theta}{\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl}\frac{\partial f}{\partial \sigma_{kl}} + \frac{1}{C_p}\frac{\partial f}{\partial \sigma_{ii}}} \tag{15}$$

By substituting the above equation into Eq. (13), incremental stress tensor can be obtained:

$$d\sigma_{ij} = (E_{ijkl} - E_{ijkl}^{p\sigma}) d\varepsilon_{kl} - (3K - 3Kh_\theta)\delta_{kl}\alpha d\theta \tag{16}$$

$$d\sigma_{ij} = (E_{ijkl} - E_{ijkl}^{p\sigma}) d\varepsilon_{kl} - (E_{ijkl} - E_{ijkl}^{p\theta})\delta_{kl}\alpha d\theta \tag{17}$$

$$\begin{aligned} d\sigma_{ij} &= (E_{ijkl} - E_{ijkl}^{p\sigma}) d\varepsilon_{kl} - (E_{ijkl} - E_{ijkl}^{p\theta}) d\varepsilon_{kl}^{e\theta} \\ &= (E_{ijkl} - E_{ijkl}^{p\sigma}) d\varepsilon_{kl} - (E_{ijkl} - E_{ijkl}^{p\theta})\frac{d\varepsilon_v^{e\theta}}{3}\delta_{kl} \end{aligned} \tag{18}$$

Where,

$$h_p = \frac{1}{C_p}\frac{\partial f}{\partial \sigma_{mm}} + \frac{\partial f}{\partial \sigma_{mn}} E_{mnpq}\frac{\partial f}{\partial \sigma_{pq}} \tag{19}$$

$$h_\theta = \left(E_{ijmn}\frac{\partial f}{\partial \sigma_{mn}}\frac{\partial f}{\partial \sigma_{ij}}\frac{1}{h_p} - 1\right)\frac{C_p \cdot K}{\sigma_{m0} + 3K\alpha(\theta - \theta_0)} + E_{ijmn}\frac{\partial f}{\partial \sigma_{mn}}\frac{\partial f}{\partial \sigma_{ij}}\frac{1}{h_p} \tag{20}$$

$$E_{ijkl}^{p\theta} = h_\theta E_{ijkl} \tag{21}$$

$$E_{ijkl}^{p\sigma} = E_{mnkl}\frac{\partial f}{\partial \sigma_{pq}}\frac{\partial f}{\partial \sigma_{mn}} E_{ijpq}\frac{1}{h_p} \tag{22}$$

The above deduced thermo-elastoplastic model can only be applied to normally- consolidated geomaterials and not for overconsolidated soil which will be discussed in the next section.

THERMO-ELASTOPLASTIC MODEL OF SOFT ROCKS BASED ON THE EXTENDED SUBLOADING YIELD SURFACE

In the works by Zhang et al. (2005), one important feature of the model is that soft sedimentary rock can be regarded as a heavily overconsolidated soil. Based on this assumption, the concept of subloading yield surface proposed by Hashiguchi and Ueno (1977) is introduced into the constitutive model for soft rocks.

The subloading yield surface is also introduced in this paper. A brief introduction to the subloading yield surface is given in Fig. 5, in which void ratio difference ρ^σ due to stress and overconsolidated ratio OCR^σ can be expressed as:

$$\rho^\sigma = C_p(1 + e_0)\ln\frac{\sigma_{N1e}}{\sigma_{N1}} = C_p(1 + e_0)\ln OCR^\sigma, \quad OCR^\sigma = \frac{\sigma_{N1e}}{\sigma_{N1}} \tag{23}$$

In Fig. 5, the point P^σ represents the present stress state and σ_{N1} is the value of cross point of σ_m axis with the sub-

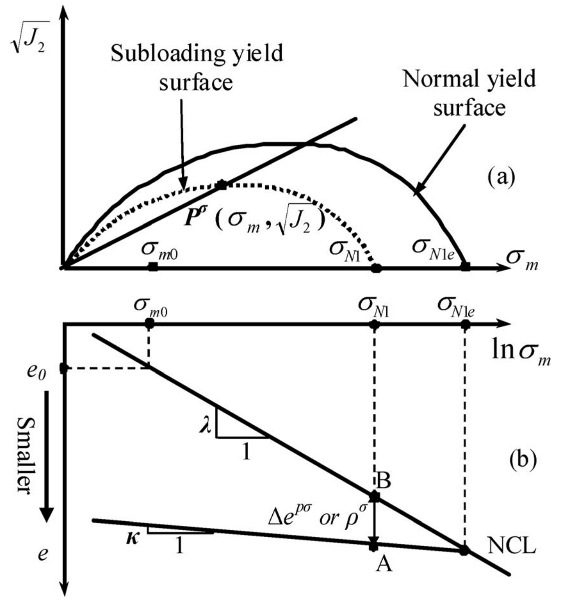


Fig. 5. **Normal yield surface and subloading failure surface formed due to the change of stress state**

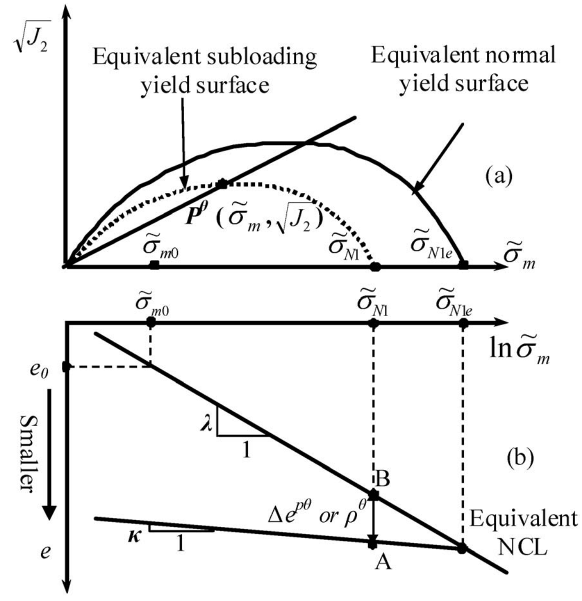


Fig. 6. **Equivalent normal yield surface and equivalent subloading failure surface formed due to the change of equivalent stress state correspond to temperatures**

loading yield surface that passes through the present stress state, while σ_{N1e} is the value of cross point of σ_{m} axis with the normal yield surface.

The expression of the subloading yield surface passing through the present stress state P^{σ} is:

$$f_{s\sigma} = C_{\mathrm{p}} \ln \frac{\sigma_{\mathrm{m}}}{\sigma_{\mathrm{N1}}} + C_{\mathrm{p}} \frac{\sqrt{3}\sqrt{J_2}}{M\sigma_{\mathrm{m}}} = 0 \tag{24}$$

Using the concept of equivalent stress, it is easy to define an equivalent void ratio difference ρ^{θ} and an equivalent overconsolidated ratio OCR^{θ}, taking after those induced by real stresses shown in Eq. (23). Both ρ^{θ} and OCR^{θ} are caused by the change of equivalent stress due to change of temperature and can be expressed as:

$$\begin{aligned}\rho^{\theta} &= C_{\mathrm{p}}(1+e_0) \ln \frac{\tilde{\sigma}_{\mathrm{N1e}}}{\tilde{\sigma}_{\mathrm{N1}}} \\ &= C_{\mathrm{p}}(1+e_0) \ln \frac{\sigma_{\mathrm{m0}} + 3K\alpha(\theta_{\mathrm{N1e}} - \theta_0)}{\sigma_{\mathrm{m0}} + 3K\alpha(\theta_{\mathrm{N1}} - \theta_0)}\end{aligned} \tag{25}$$

and

$$\mathrm{OCR}^{\theta} = \frac{\tilde{\sigma}_{\mathrm{N1e}}}{\tilde{\sigma}_{\mathrm{N1}}} = \frac{\sigma_{\mathrm{m0}} + 3K\alpha(\theta_{\mathrm{N1e}} - \theta_0)}{\sigma_{\mathrm{m0}} + 3K\alpha(\theta_{\mathrm{N1}} - \theta_0)} \tag{26}$$

where, the point P^{θ} represents the present equivalent stress state and $\tilde{\sigma}_{\mathrm{N1}}$ is the value of cross point of $\tilde{\sigma}_{\mathrm{m}}$ axis with the subloading yield surface that passes through the present equivalent stress state, while $\tilde{\sigma}_{\mathrm{N1e}}$ is the value of cross point of $\tilde{\sigma}_{\mathrm{m}}$ axis with the normal yield surface, as shown in Fig. 6 $\tilde{\sigma}_{\mathrm{N1}}$ and $\tilde{\sigma}_{\mathrm{N1e}}$ correspond to temperatures of θ_{N1} and θ_{N1e} respectively.

Accordingly, both the extended void ratio difference ρ and the extended overconsolidated ratio OCR include the thermodynamic and the stress-induced parts and can be expressed as:

$$\begin{aligned}\rho = \rho^{\sigma} + \rho^{\theta} &= C_{\mathrm{p}}(1+e_0) \ln \frac{\sigma_{\mathrm{N1e}}}{\sigma_{\mathrm{N1}}} \\ &\quad + C_{\mathrm{p}}(1+e_0) \ln \frac{\sigma_{\mathrm{m0}} + 3K\alpha(\theta_{\mathrm{N1e}} - \theta_0)}{\sigma_{\mathrm{m0}} + 3K\alpha(\theta_{\mathrm{N1}} - \theta_0)} \\ &= C_{\mathrm{p}}(1+e_0)(\ln \mathrm{OCR}^{\sigma} + \ln \mathrm{OCR}^{\theta}) \\ &= C_{\mathrm{p}}(1+e_0) \ln \mathrm{OCR}\end{aligned} \tag{27}$$

where

$$\mathrm{OCR} = \mathrm{OCR}^{\sigma}\mathrm{OCR}^{\theta}$$

In fact, the equivalent stress is a stress state that includes the influence of temperature. Therefore, in representing a real present stress state in stress space (σ_{m}, $\sqrt{J_2}$), both present stress state and present temperature state are considered simultaneously. That is, point P^{σ} and point P^{θ} is the same in stress state but different in temperature. Therefore, in Fig. 7, point P at present state includes three independent state variables: θ, σ_{m}, $\sqrt{J_2}$. The expression for the extended subloading yield surface passing through the present stress and temperature state P can then be given by the following:

$$\begin{aligned}f_{\mathrm{s}} &= C_{\mathrm{p}} \ln \frac{\sigma_{\mathrm{m}}}{\sigma_{\mathrm{N1}}} + C_{\mathrm{p}} \ln \frac{\sigma_{\mathrm{m0}} + 3K\alpha(\theta - \theta_0)}{\sigma_{\mathrm{m0}} + 3K\alpha(\theta_{\mathrm{N1}} - \theta_0)} \\ &\quad + C_{\mathrm{p}} \frac{\sqrt{3}\sqrt{J_2}}{M\sigma_{\mathrm{m}}} = 0\end{aligned} \tag{28}$$

as shown in Fig. 7. Equation (28) can also be written as:

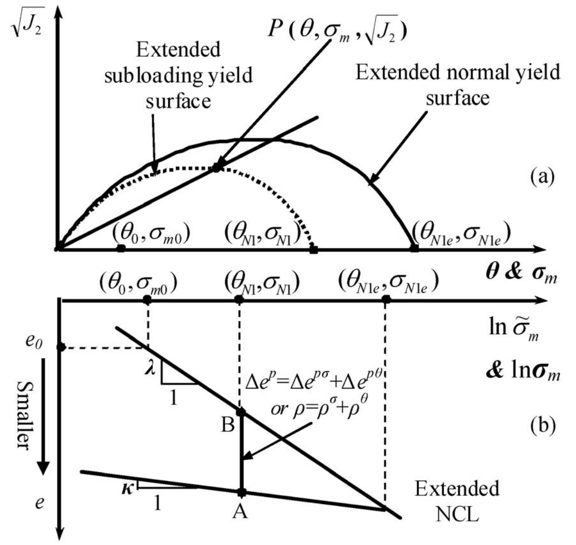


Fig. 7. Extended normal yield surface and extended subloading yield surface

$$f_s = C_p \left[\ln \frac{\sigma_m}{\sigma_{m0}} - \left(\ln \frac{\sigma_{N1e}}{\sigma_{m0}} - \ln \frac{\sigma_{N1e}}{\sigma_{N1}} \right) \right] + C_p \ln \frac{\sigma_{m0} + 3K\alpha(\theta - \theta_0)}{\sigma_{m0} + 3K\alpha(\theta_0 - \theta_0)} - C_p \ln \frac{\sigma_{m0} + 3K\alpha(\theta_{N1e} - \theta_0)}{\sigma_{m0} + 3K\alpha(\theta_0 - \theta_0)} + C_p \ln \frac{\sigma_{m0} + 3K\alpha(\theta_{N1e} - \theta_0)}{\sigma_{m0} + 3K\alpha(\theta_{N1} - \theta_0)} + C_p \frac{\sqrt{3}\sqrt{J_2}}{M^* \sigma_m} = 0 \quad (29)$$

The stress-induced plastic volumetric strain due to the change of stress from σ_{m0} to σ_{N1e} can be evaluated as:

$$\varepsilon_v^{p\sigma} = C_p \ln \frac{\sigma_{N1e}}{\sigma_{m0}} \quad (30)$$

Meanwhile, the thermodynamic plastic volumetric strain due to the change of temperature from θ_0 to θ_{N1e} can be evaluated as:

$$\varepsilon_v^{p\theta} = C_p \ln \frac{\sigma_{m0} + 3K\alpha(\theta_{N1e} - \theta_0)}{\sigma_{m0} + 3K\alpha(\theta_0 - \theta_0)} = C_p \ln \frac{\sigma_{m0} + 3K\alpha(\theta_{N1e} - \theta_0)}{\sigma_{m0}} \quad (31)$$

By substituting Eqs. (27) and (30), (31) into Eq. (29), the extended subloading yield surface can be written as:

$$f_s(\sigma, \theta, \varepsilon_v^{p\sigma}, \varepsilon_v^{p\theta}) = \ln \frac{\sigma_m}{\sigma_{m0}} + \ln \frac{\sigma_{m0} + 3K\alpha(\theta - \theta_0)}{\sigma_{m0}} - \frac{1}{C_p}(\varepsilon_v^{p\sigma} + \varepsilon_v^{p\theta}) + \frac{1}{C_p} \frac{\rho^\sigma + \rho^\theta}{1 + e_0} + \frac{\sqrt{3}\sqrt{J_2}}{M\sigma_m} = 0 \quad (32)$$

or

$$f(\sigma, \theta, \varepsilon_v^p) = f_s(\sigma, \theta, \varepsilon_v^p) = \ln \frac{\sigma_m}{\sigma_{m0}} + \frac{\sqrt{3}\sqrt{J_2}}{M\sigma_m} + \ln \frac{\sigma_{m0} + 3K\alpha(\theta - \theta_0)}{\sigma_{m0}} - \frac{1}{C_p}\left(\varepsilon_v^p - \frac{\rho}{1 + e_0} \right) = 0 \quad (33)$$

In a constitutive model, consistency equation must be obeyed, by which the value of Λ can be determined:

$$df = 0 \Rightarrow df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} + \frac{\partial f}{\partial \theta} d\theta - \frac{1}{C_p}\left(d\varepsilon_v^p - \frac{d\rho}{1 + e_0} \right) = 0 \quad (34)$$

In the works by Zhang et al. (2005), it is assumed that the evolution equation of the stress-induced ρ^σ is dependent on the present state variables ρ^σ and σ_m, and is proportional to the positive variable Λ in such a way that:

$$-\frac{1}{1 + e_0} d\rho^\sigma = \frac{a\rho^{\sigma 2}}{\sigma_m} \Lambda \quad (35)$$

where, a is a material parameter which controls the evolution rate of the stress-induced ρ^σ. Therefore, similar to Eq. (35), the evolution equation for the extended void ratio difference ρ can be expressed by the sum of actual stress σ_m and the equivalent stress increment $(\tilde{\sigma}_m - \sigma_{m0})$ in the following way:

$$-\frac{1}{1 + e_0} d\rho = \frac{a\rho^2}{\sigma_m + (\tilde{\sigma}_m - \sigma_{m0})} \Lambda = \frac{a\rho^2}{\sigma_m + 3K\alpha(\theta - \theta_0)} \Lambda \quad (36)$$

By substituting this evolution equation into Eq. (34), it is easy to obtain the relation:

$$\Lambda = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} \Big/ \left(\frac{h_{psub}}{C_p} \right), \quad h_{psub} = \frac{\partial f}{\partial \sigma_{ii}} + \frac{a\rho^2}{\sigma_m + 3K\alpha(\theta - \theta_0)} \quad (37)$$

Based on the Hooke's law, the variable Λ also can be obtained as:

$$\Lambda = \frac{\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} d\varepsilon_{kl}}{\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} + \frac{h_{psub}}{C_p}} - \frac{\frac{\partial f}{\partial \sigma_{ij}} K\delta_{ij} \left(C_p \frac{3K\alpha}{\sigma_{m0} + 3K\alpha(\theta - \theta_0)} + 3\alpha \right) d\theta}{\frac{\partial f}{\partial \sigma_{ij}} E_{ijkl} \frac{\partial f}{\partial \sigma_{kl}} + \frac{h_{psub}}{C_p}} \quad (38)$$

The loading criteria are given in the same way as in the previous work by Zhang et al. (2005) as:

$$\|d\varepsilon_{ij}^{p\sigma}\| > 0 \text{ if } \Lambda > 0 \text{ and } \begin{cases} \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} > 0 \ \textit{hardening} \\ \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} < 0 \ \textit{softening} \end{cases}$$

$$\|d\varepsilon_{ij}^{p\sigma}\| = 0 \text{ if } \Lambda \leq 0 \text{ elastic} \quad (39)$$

THERMO-ELASTO-VISCOPLASTIC MODEL OF SOFT ROCKS

In establishing a thermo-elasto-viscoplastic model of soft rocks, it is necessary to add an ability of describing time dependent behaviours of materials into the above deduced thermo-elastoplastic model. The main work is to formulize the time dependent evolution equation for the extended void ratio difference ρ due to the state valuables of stress and temperature.

Based on the works by Nakai and Hinokio (2004), Zhang et al. (2005) proposed an elasto-viscoplastic model for soft rocks, in which, the time dependent evolution equation for the void ratio difference ρ^{σ} is written as:

$$\frac{\dot{\rho}^{\sigma}}{1+e_0} = -\Lambda \frac{G^{\sigma}(\rho^{\sigma}, t)}{\sigma_m} + h^{\sigma}(t) \tag{40}$$

where,

$$\begin{cases} h^{\sigma}(t) = \dot{\varepsilon}_{v0}^{\sigma}\,(1+t/t_1) - \bar{\alpha} \\ G^{\sigma}(\rho^{\sigma}, t) = a\rho^{\sigma 1 + C_n \ln(1+t/t_1)} \end{cases} \tag{41}$$

$\dot{\varepsilon}_{v0}^{\sigma}$ is an initial volumetric strain rate at the time $t=0$ which represents the time when shearing begins. t_1 is a unit time used to standardize the time and always takes the value of 1.0. $\bar{\alpha}$ is a time dependent parameter that controls the gradient of strain rate vs. time in logarithmic axes during a creep test. C_n controls the strain rate dependency of soft rocks. It should be pointed out that the values of the time dependent parameters C_n and $\bar{\alpha}$ are not objective and are dependent on the unit of time. In its application to boundary value problem, however, if the unit of time used in numerical analysis is the same as the one used in determining the parameters based on laboratory tests, then there is no problem in using the model.

Similar to Eqs. (40) and (41), the time dependent evolution equation for the extended void ratio difference ρ, which includes the thermodynamic part and stress-induced part, can be given as:

$$\begin{aligned} \frac{\dot{\rho}}{1+e_0} &= -\Lambda \frac{G(\rho, t)}{\sigma_m + (\tilde{\sigma}_m - \sigma_{m0})} + h(t) \\ &= -\Lambda \frac{G(\rho, t)}{\sigma_m + 3K\alpha(\theta-\theta_0)} + h(t) \end{aligned} \tag{42}$$

where,

$$\begin{cases} h(t) = \dot{\varepsilon}_v^0[1+t/t_1]^{-\bar{\alpha}} = (\dot{\varepsilon}_{v0}^{\theta} + \dot{\varepsilon}_{v0}^{\sigma})[1+t/t_1]^{-\bar{\alpha}} \\ G(\rho, t) = a\rho^{1+C_n \ln(1+t/t_1)} = a(\rho^{\theta}+\rho^{\sigma})^{1+C_n \ln(1+t/t_1)} \end{cases} \tag{43}$$

where, $\dot{\varepsilon}_v^0$ is the total volumetric strain rate, which includes the thermodynamic part and the stress-induced part, in the same way as the extended void ratio difference ρ.

Total plastic volumetric strain rate is expressed as:

$$\dot{\varepsilon}_v^p = \dot{\varepsilon}_v^{p\sigma} + \dot{\varepsilon}_v^{p\theta} \tag{44}$$

Associated flow rule is adopted and therefore the viscoplastic potential is expressed as:

$$\dot{\varepsilon}_{ij}^{p\sigma} = \Lambda \frac{\partial f}{\partial \sigma_{ij}} \quad \text{and} \quad \dot{\varepsilon}_v^{p\sigma} = \Lambda \frac{\partial f}{\partial \sigma_{kk}} \tag{45}$$

Based on Eq. (6), thermodynamic plastic strain rate can then be expressed as:

$$\dot{\varepsilon}_{ij}^{p\theta} = C_p \frac{3K\alpha}{\sigma_{m0} + 3K\alpha(\theta-\theta_0)}\,\dot{\theta}\,\frac{\sigma_{ij}}{3},$$

$$\dot{\varepsilon}_v^{p\theta} = C_p \frac{3K\alpha}{\sigma_{m0} + 3K\alpha(\theta-\theta_0)}\,\dot{\theta} \tag{46}$$

The consistency equation can be written as:

$$\dot{f} = \frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij} + \frac{\partial f}{\partial \theta}\dot{\theta} - \frac{1}{C_p}\left(\dot{\varepsilon}_v^p - \frac{\dot{\rho}}{1+e_0}\right) = 0 \tag{47}$$

Substituting Eqs. (42) ~ (46) into the consistency Eq. (47), the variable Λ can be obtained as:

$$\Lambda = \left(\dot{f}_{\sigma} + \frac{h(t)}{C_p}\right) \Big/ \frac{h_{sib}^p}{C_p} \tag{48}$$

where,

$$\dot{f}_{\sigma} = \frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}_{ij}, \quad h_{sub}^p = \frac{\partial f}{\partial \sigma_{kk}} + \frac{G(\rho, t)}{\sigma_m + 3K\alpha(\theta-\theta_0)} \tag{49}$$

The stress rate can be calculated by Hooke's law as:

$$\begin{aligned} \dot{\sigma}_{ij} &= E_{ijkl}\dot{\varepsilon}_{kl}^{e\sigma} = E_{ijkl}(\dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^p - \dot{\varepsilon}_{kl}^{e\theta}) \\ &= E_{ijkl}\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}_{kl}^{p\sigma} - \dot{\varepsilon}_v^{p\theta}\frac{\delta_{kl}}{3} - \dot{\varepsilon}_v^{e\theta}\frac{\delta_{kl}}{3}\right) \end{aligned} \tag{50}$$

$$\text{where } \dot{\varepsilon}_v^{e\theta} = 3\alpha\dot{\theta} \tag{51}$$

Substituting Eqs. (42), (45), (46), (50), (51) into the consistency Eq. (47), the variable Λ can also be obtained as:

$$\Lambda = \frac{\dfrac{\partial f}{\partial \sigma_{ij}}E_{ijkl}\dot{\varepsilon}_{kl} + \dfrac{h(t)}{C_p}}{\dfrac{\partial f}{\partial \sigma_{ij}}E_{ijkl}\dfrac{\partial f}{\partial \sigma_{kl}} + \dfrac{h_{sub}^p}{C_p}} - \frac{\dfrac{\partial f}{\partial \sigma_{ij}}K\delta_{ij}\left(C_p\dfrac{3K\alpha}{\sigma_{m0}+3K\alpha(\theta-\theta_0)} + 3\alpha\right)\dot{\theta}}{\dfrac{\partial f}{\partial \sigma_{ij}}E_{ijkl}\dfrac{\partial f}{\partial \sigma_{kl}} + \dfrac{h_{sub}^p}{C_p}} \tag{52}$$

Substituting Eqs. (45), (46), (48) into Eq. (44), it is easy to obtain the following equations:

$$\begin{cases} \dot{\varepsilon}_v^p = \dfrac{\dot{f}_{\sigma} + h(t)/C_p}{h_{sub}^p/C_p}\dfrac{\partial f}{\partial \sigma_{kk}} + C_p\dfrac{3K\alpha}{\sigma_{m0}+3K\alpha(\theta-\theta_0)}\dot{\theta} \\ \dot{\varepsilon}_{ij}^p = \dfrac{\dot{f}_{\sigma} + h(t)/C_p}{h_{sub}^p/C_p}\dfrac{\partial f}{\partial \sigma_{ij}} + C_p\dfrac{3K\alpha}{\sigma_{m0}+3K\alpha(\theta-\theta_0)}\dot{\theta}\dfrac{\delta_{ij}}{3} \end{cases} \tag{53}$$

The loading criteria are given as:

$$\|d\varepsilon_{ij}^{p\sigma}\| > 0 \text{ if } \Lambda > 0 \text{ and } \begin{cases} \dot{f}_{\sigma} > 0 \; \textit{hardening} \\ \dot{f}_{\sigma} < 0 \; \textit{softening} \\ \dot{f}_{\sigma} = 0 \; \textit{pure creep} \end{cases} \tag{54}$$

$$\|d\varepsilon_{ij}^{p\sigma}\| = 0 \text{ if } \Lambda \le 0 \text{ elastic}$$

As is the same as the model proposed by Zhang et al. (2005), it is necessary to define a pure creep state when $\dot{f}_{\sigma} = 0$. The plastic strain rate in the pure creep state can then be evaluated as:

$$\begin{cases} \dot{\varepsilon}_{\mathrm{v}}^{\mathrm{p}} = \dfrac{h(t)}{h_{\mathrm{sub}}^{\mathrm{p}}} \dfrac{\partial f}{\partial \sigma_{\mathrm{kk}}} + C_{\mathrm{p}} \dfrac{3\mathrm{K}\alpha}{\sigma_{\mathrm{m0}} + 3K\alpha(\theta - \theta_0)} \dot{\theta} \\ \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}} = \dfrac{h(t)}{h_{\mathrm{sub}}^{\mathrm{p}}} \dfrac{\partial f}{\partial \sigma_{\mathrm{ij}}} + C_{\mathrm{p}} \dfrac{3\mathrm{K}\alpha}{\sigma_{\mathrm{m0}} + 3K\alpha(\theta - \theta_0)} \dot{\theta} \dfrac{\delta_{\mathrm{ij}}}{3} \end{cases} \tag{55}$$

Compared with the model proposed by Zhang et al. (2005), the newly proposed model only adds one parameter, the linear thermo-expansion coefficient α, which can be definitely determined with clear physical meanings. All other parameters are determined in the same way as in the previous model. A detailed discussion on this issue can be found in the corresponding reference (Zhang et al., 2005).

THERMODYNAMIC BEHAVIOR OF PROPOSED MODEL

Unlike most existing thermo-elasto-viscoplastic models where thermodynamic theorems are used to establish a series of restricted relations for the variables involved in the models, these thermodynamic theorems are not discussed in formulating the newly proposed model at the beginning. Therefore, it is necessary to verify if the new model satisfies the thermodynamic theorems, especially the 1st and 2nd thermodynamic theorems, so that the rationality of the new model can be assured.

When field equations of thermodynamics for a body are considered, state variables such as stresses, strains and temperature are in general inhomogeneous and change constantly. It is necessary to describe properly the energy exchange that has happened between an arbitrary element and its 'external system'. It is however, very difficult and at times even impossible to define the external system. For instance, when we consider the heating effect of nuclear radiation, it is related to mass energy conversion of the element itself which is not taken into consideration within the framework of this research. It is necessary to use non-equilibrium thermodynamics (Kittel, 2000; Lebon et al., 2008) to describe the thermodynamic behaviours of any arbitrary element in the body.

In the 1st thermodynamic theorem, it is stated that the total energy flowing in/out of the element is equal to the energy store/lost of the element, namely, internal energy U. The total energy flow in/out of the element is made up from two parts: the work W done by external forces and the heat Q. The heat includes external heat and internal heat. The former one is the heat flux h_{i} that generates from external heat source and flows in/out through the surfaces surrounding the element; while the latter one is generated from the internal heat source such as radioactivity of nuclear waste disposal within the element. Because of the conservation of energy in the element, the following equation can be obtained:

$$U = W + Q \tag{56}$$

It can also be expressed in rate form as:

$$\dot{U} = \dot{W} + \dot{Q} \Leftrightarrow D\dot{u} = D\dot{w} + D\dot{q} \tag{57}$$

where, D is the density of the element, u is the internal energy per unit mass, w is the work per unit mass and q is the heat energy per unit mass. The relation between θ and h_{i} obeys the Fourier's law, that is:

$$h_{\mathrm{i}} = -k\, \partial\theta / \partial x_{\mathrm{i}} \tag{58}$$

where, k is heat conductivity coefficient. Then, $\dot{W}$ and $\dot{Q}$ can be expressed as:

$$\begin{aligned} \dot{W} &= D\dot{w} = \sigma_{\mathrm{ij}}\dot{\varepsilon}_{\mathrm{ij}} = \sigma_{\mathrm{ij}}(\dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{e}} + \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}}) \\ &= \sigma_{\mathrm{ij}}(\dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{e}\sigma} + \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{e}\theta}) + \sigma_{\mathrm{ij}}(\dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}\sigma} + \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}\theta}) \end{aligned} \tag{59}$$

$$\begin{aligned} \dot{Q} &= D\dot{q} = -\frac{\partial h_{\mathrm{i}}}{\partial x_{\mathrm{i}}} + rD = -\frac{\partial(-k \cdot \partial\theta/\partial x_{\mathrm{i}})}{\partial x_{\mathrm{i}}} + rD \\ &= k\frac{\partial^2\theta}{\partial x_{\mathrm{i}}\partial x_{\mathrm{i}}} + rD \end{aligned} \tag{60}$$

where, r is the internal heat supply per unit time per unit mass.

The changing rate of internal energy $\dot{U}$ includes reversible work rate $\dot{W}^{\circ}$ and the corresponding changing rate of thermal energy $\dot{Q}^{\circ}$. In the element, $\dot{W}^{\circ}$ is stored in the form of elastic potential energy:

$$\dot{W}^{\circ} = \sigma_{\mathrm{ij}} \cdot \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{e}} = \sigma_{\mathrm{ij}} \cdot (\dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{e}\sigma} + \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{e}\theta}) \tag{61}$$

The changing rate of thermal energy $\dot{Q}^{\circ}$ can then be evaluated by the following equation:

$$\begin{aligned} &\dot{Q}^{\circ} + \dot{W}^{\circ} = \dot{W} + \dot{Q} \Rightarrow \\ &\dot{Q}^{\circ} = \sigma_{\mathrm{ij}}(\dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{e}} + \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}}) + \left(k\frac{\partial^2\theta}{\partial x_{\mathrm{i}}\partial x_{\mathrm{i}}} + rD\right) - \sigma_{\mathrm{ij}}\dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{e}} \\ &\quad = \sigma_{\mathrm{ij}}(\dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}\sigma} + \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}\theta}) + \left(k\frac{\partial^2\theta}{\partial x_{\mathrm{i}}\partial x_{\mathrm{i}}} + rD\right) \end{aligned} \tag{62}$$

The changing rate of thermal energy per unit mass $\dot{q}^{\circ}$ can then be expressed as:

$$\dot{q}^{\circ} = \frac{\sigma_{\mathrm{ij}}(\dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}\sigma} + \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}\theta})}{D} + \left(\frac{k}{D}\frac{\partial^2\theta}{\partial x_{\mathrm{i}}\partial x_{\mathrm{i}}} + r\right) \tag{63}$$

Based on the definition of entropy, the material time derivative of the entropy $\dot{\eta}$ can be calculated:

$$\dot{\eta} = \frac{\dot{q}^{\circ}}{\theta} = \left(\frac{\sigma_{\mathrm{ij}}(\dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}\sigma} + \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}\theta})}{D} + \frac{k}{D}\frac{\partial^2\theta}{\partial x_{\mathrm{i}}\partial x_{\mathrm{i}}} + r\right)\frac{1}{\theta} \tag{64}$$

In non-equilibrium thermodynamics, the changing rate of entropy density is made up from three parts: the first part comes from the irreversible course in the element, the second part comes from the heat flux and the third part comes from inner heat source such as radioactivity of nuclear waste disposal within the element. The relation among them can be shown as:

$$D\dot{\eta} = \dot{\gamma} + \left[-\left(\frac{h_{\mathrm{i}}}{\theta}\right)_{,\mathrm{i}} + D\frac{r}{\theta}\right] \quad \text{or}$$

$$\dot{\gamma} = D\dot{\eta} - \left[-\left(\frac{h_{\mathrm{i}}}{\theta}\right)_{,\mathrm{i}} + D\frac{r}{\theta}\right] \tag{65}$$

where, $\dot{\gamma}$ is called as entropy production in the irreversible course of the element and must be greater than or equal to zero (Clausius-Duhem inequality). Substituting the Eqs. (58) and (64) into Eq. (65), $\dot{\gamma}$ can be expressed as:

$$\begin{aligned}\dot{\gamma} &= D\left(\frac{\sigma_{ij}(\dot{\varepsilon}_{ij}^{p\sigma}+\dot{\varepsilon}_{ij}^{p\theta})}{D}+\frac{k}{D}\frac{\partial^2\theta}{\partial x_i \partial x_i}+r\right)\frac{1}{\theta} \\ &\quad -k\frac{1}{\theta}\frac{\partial^2\theta}{\partial x_i^2}+k\frac{1}{\theta^2}\left(\frac{\partial\theta}{\partial x_i}\right)\left(\frac{\partial\theta}{\partial x_i}\right)-D\frac{r}{\theta} \\ &= \frac{1}{\theta}\sigma_{ij}(\dot{\varepsilon}_{ij}^{p\sigma}+\dot{\varepsilon}_{ij}^{p\theta})+k\frac{1}{\theta^2}\left(\frac{\partial\theta}{\partial x_i}\right)\left(\frac{\partial\theta}{\partial x_i}\right)\end{aligned} \tag{66}$$

It also can be written as:

$$\dot{\gamma}=\frac{1}{\theta}\dot{W}^{p}+k\frac{1}{\theta^2}\left(\frac{\partial\theta}{\partial x_i}\right)\left(\frac{\partial\theta}{\partial x_i}\right) \tag{67}$$

where, $\dot{W}^p$ is the rate of plastic work that can be proved to be positive. Detail demonstration of this statement is given in APPENDIX. Because both θ and k are positive, it is therefore easy to conclude that:

$$\dot{\gamma}\geq 0 \tag{68}$$

This inequality states clearly that the irreversible course in the element can convert work to heat automatically, but the reverse conversion cannot happen automatically.

PERFORMANCE OF THE NEWLY PROPOSED MODEL

Calculated Results of Proposed Thermo-Elastoplastic Model

In order to check the performance of the proposed model, drained conventional triaxial compression tests of soft rock under constant shear strain but with different temperatures during shearing, is simulated. The physical properties of the soft rock and the material parameters involved in the model are listed in Table 1. Because viscosity is not considered here, time dependent parameters C_n and $\bar{\alpha}$ take the values of zero.

Figure 8 shows the stress-strain relations of the soft rocks at different constant temperatures at which the shearing is carried. It is seen from the figure that the peak value of stress difference increases as temperature decreases in drained conventional triaxial compression tests at different constant temperatures. It is also known that the stress-strain relation changes from ductility to brittle as temperature decreases, which is coincident with the experimental results of the thermodynamic behaviours of soft rocks obtained from the works by Okada (2005, 2006).

Figure 9 shows the comparison of the calculated results at constant temperatures during shearing, and under the conditions that temperature changes from low to high, or from high to lower during shearing. Figure 10 shows the comparison of calculated results between the test with constant temperature during shearing and the test with changing temperature during shearing. It is known from these figures that change of temperature during shear may affect the stress-strain relation a little but is so much larger as the influence of the initial temperature when the

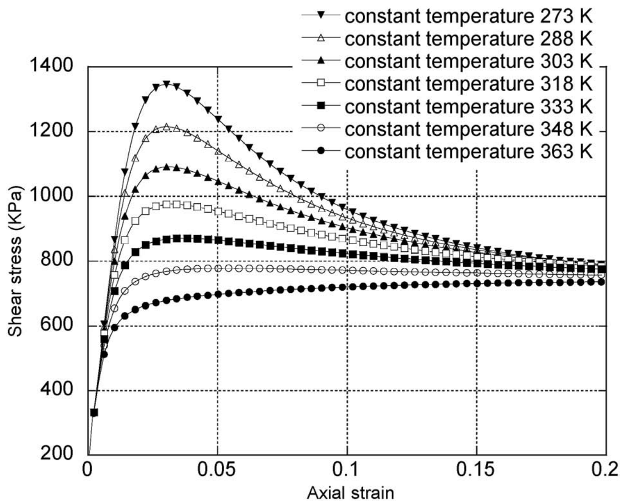


Fig. 8. Stress-strain relations at the different constant temperatures

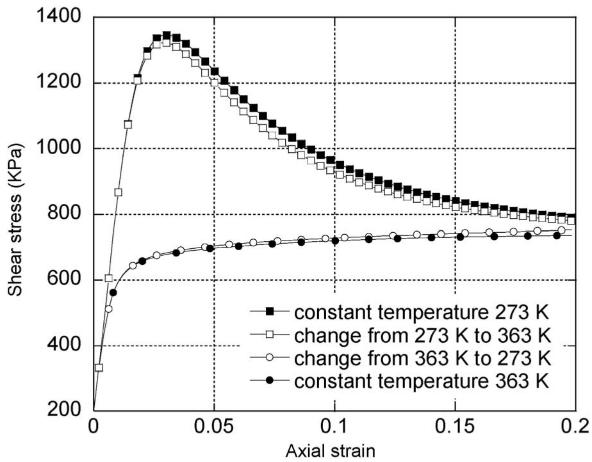


Fig. 9. Stress-strain relations at the changing temperature during shearing

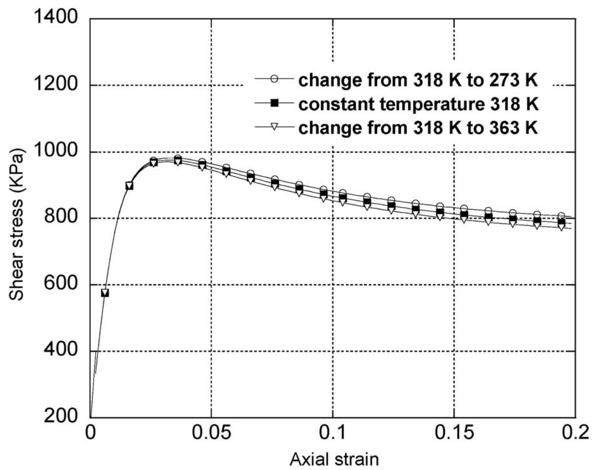


Fig. 10. Stress-strain relations at the changing temperature during shearing

Table 1. Physical properties of soft rock and material parameters involved in thermo-elastoplastic model

α(1/K)	8.0×10^{-6}	E (MPa)	900.0
β	1.50	E_p (λ-κ)	0.005
a	500.0	$\bar{\alpha}$	—
v	0.0864	C_n	—
R_f	11.0		
OCR	16.0	Void ratio at reference state e_0	0.72
σ'_{30} (MPa)	0.098	Initial yielding stress of consolidation p'_c (MPa)	15.0

Table 2. Physical properties of soft rock and material parameters involved in thermo-elasto-viscoplastic model

α(1/K)	8.0×10^{-6}	E (MPa)	900.0
β	1.50	E_p (λ-κ)	0.040
a	500.0	$\bar{\alpha}$	0.70
v	0.0864	C_n	0.025
R_f	11.0		
OCR	150.0	Void ratio at reference state e_0	0.72
σ'_{30} (MPa)	0.098	Initial yielding stress of consolidation p'_c (MPa)	15.0

shearing is started.

Calculated Results of Proposed Thermo-Elasto-Viscoplastic Model

In order to check the viscoplastic behaviours of the proposed model, drained conventional triaxial compression tests controlled with constant strain rate and consequent drained triaxial pure creep tests, are simulated with the newly proposed model under the condition that temperatures are kept in constant with different values during shearing while the shear rate is $\dot{\varepsilon}=0.10\%/\text{min}$. Physical properties of soft rock and material parameters involved in thermo-elasto-viscoplastic model are listed in Table 2. Compared to Table 1, the difference between Tables 1 and 2 is that because viscosity is considered here, time dependent parameters C_n and $\bar{\alpha}$ are no longer to be zero.

Figure 11 shows the relations between the stress difference and strain of soft rocks in drained conventional triaxial compression tests under constant strain rate ($\dot{\varepsilon}=0.10\%/\text{min}$) at different constant temperatures. The calculated results also can describe the thermodynamic characteristics of soft rocks, which are observed in the experiments by Okada (2005, 2006).

In simulating drained creep tests, a shear stress of $q=3.5$ MPa is chosen as the creep stress which is not applied to the specimen abruptly but loaded with drained shearing at the same strain rate ($\dot{\varepsilon}=0.10\%/\text{min}$) as is used in the simulation of the drained compression tests shown in Fig. 11. Table 3 lists the creep stress ratio (q_{creep}/q_u) in the creep test under different constant temperatures.

Figure 12 shows the simulated time histories of creep rates of the soft rocks at different constant temperatures. It is seen from this figure that the general characteristics of creep behaviour, such as initial creep rate, steady creep and creep rupture, can be simulated properly. Moreover, the calculated results can describe properly the factors

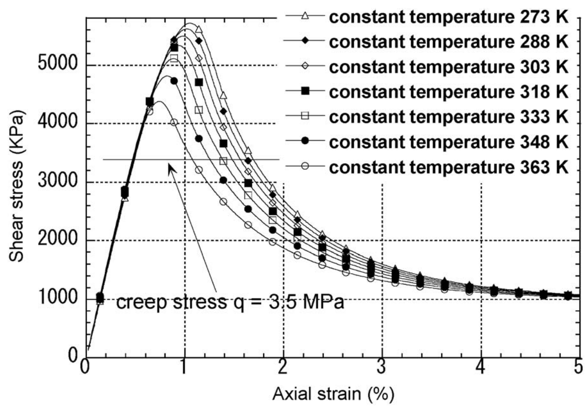


Fig. 11. Simulated stress-strain relations at different constant temperatures

Table 3. Creep stress ratio (q_{creep}/q_u) in the creep test under different constant temperatures

Temperature (K)	q_{creep} (MPa)	q_u (MPa)	q_{creep}/q_u
273	3.5	5.713	0.6126
288	3.5	5.617	0.6231
303	3.5	5.493	0.6372
318	3.5	5.331	0.6565
333	3.5	5.114	0.6844
348	3.5	4.815	0.7269
363	3.5	4.380	0.7991

Table 4. Physical properties of soft rock and material parameters in the new model for comparing between theoretical and experimental results

α(1/K)	6.0×10^{-6}	E (MPa)	1200.0
β	1.50	E_p (λ-κ)	0.040
a	500.0	$\bar{\alpha}$	0.70
v	0.0864	C_n	0.025
R_f	11.5		
OCR	150.0	Void ratio at reference state e_0	0.72
σ'_{30} (MPa)	0.098	Initial yielding stress of consolidation p'_c (MPa)	15.0

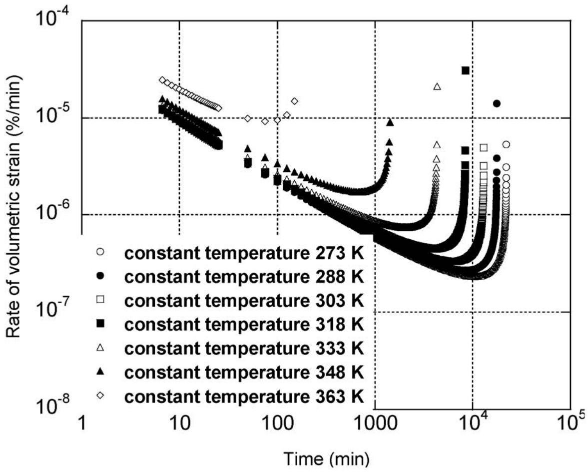


Fig. 12. Time histories of creep rates at different constant temperatures

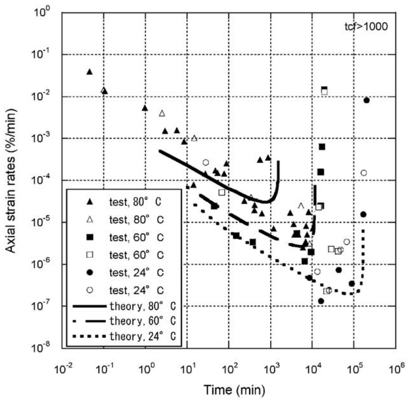


Fig. 13. Comparison between theoretical and experimental results for creep stain rates at different constant temperatures (tests by Okada, 2006)

that the creep failure time is largely dependent on temperature, and that the higher the temperature is, the faster the creep rupture will be, which have already been confirmed in the experiments by Okada (2005, 2006).

CONCLUSIONS

In this paper, a thermo-elasto-viscoplastic model for soft sedimentary rocks is proposed. Concluding remarks are listed as below:

(1) Based on the fact that the temperature and its change can generate volumetric strain of geomaterials, like those generated by hydrostatic pressure acted on the materials, a concept called as equivalent stress, is proposed in this paper. Change of the equivalent stress can not only generate elastic volumetric strain but also plastic volumetric strain. For elastic thermal strain, the relationship between the change of the equivalent stress and the change of temperature is derived by thermo-elastic theory and Hooke's law. The relationship between the change of temperature and the thermo-plastic strain is simply evaluated with e-$\ln p$ relations in both compression and swelling processes based on the concept of equivalent stress.

(2) By assuming that total plastic volumetric strain is made up from two independent parts: thermodynamic and stress-induced, a thermo-elastoplastic model based on Cam-clay model is established at first, using an associate flow rule, which considers both the effects of stresses and the change of temperature. Then, based on the fact that change of void ratio, or in other words, change of density or change of overconsolidation ratio, is brought about by changes of temperature and stresses, a new evolution equation for the change of total void ratio difference, is proposed based on subloading concept. Similar to the evolution equation for the subloading surface related to real stresses, the evolution equation for the subloading surface related to the equivalent stress due to the change of temperature is simply formularized, by which the newly proposed thermo-elastoplastic model is able to consider properly the thermodynamic behaviors of soft rocks in drained conventional triaxial compression tests under different constant and changing temperatures during shearing.

(3) In order to describe the viscoplastic behavior of soft sedimentary rock based on an elasto-viscoplastic model proposed by Zhang et al. (2005), a new time-dependent evolution equation for the total void ratio difference is formularized, by adding the time effect into the evolution equation adopted for the subloading surface of the newly proposed thermo-elastoplastic model. Based on the evolution equation, a new thermo-elasto-viscoplastic model is established, which can describe the thermodynamic behaviors of soft rocks not only in drained conventional triaxial compression tests but also drained triaxial creep tests.

(4) Being different from most existing thermo-elasto-viscoplastic models in which the thermodynamic theorems were used to establish a series of restricted relations for the variables involved in the models, the thermodynamic theorems are not discussed in formulating the new model at the beginning. In order to verify if the new model satisfies the thermodynamic theorems, especially the 1st and 2nd thermodynamic theorems, non-equilibrium thermodynamics is used. Firstly, it is illustrated that the 1st thermodynamic theorem can be satisfied by the model. Secondly, it was proved that entropy production of the element is always greater or equal to zero, that is, the second thermodynamic theorem is satisfied. The newly proposed model is developed based on very simple physical meaning and the requirement for satisfying thermodynamic theorems is verified after the model is established, which makes it possible to propose the model in a reasonable, sophisticated but comprehensive way.

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NOTATION

$\varepsilon_{\mathrm{v}}^{\mathrm{p}}$: Total plastic volumetric strain
$\varepsilon_{\mathrm{v}}^{\mathrm{p}\sigma}$: Stress-induced plastic volumetric strain
$\varepsilon_{\mathrm{v}}^{\mathrm{e}\theta}$: Thermodynamic elastic volumetric strain
$\varepsilon_{\mathrm{v}}^{\mathrm{p}\theta}$: Thermodynamic plastic volumetric strain
σ_{m0}: Reference pressure
M: Ratio of shearing stress at critical state
e_0: Reference void ratio at reference pressure σ_{m0}
E_{p}: Plastic modulus
λ: Compression index
κ: Swelling index
$\tilde{\sigma}_{\mathrm{m}}$: Equivalent stress
θ: Temperature
θ_0: Reference temperature

α: Linear thermo-expansion coefficient
K: Elastic volume modulus
E: Young's modulus
ν: Poisson's ratio
E_{ijkl}: Fourth order stiffness tensor
ρ^{σ}: Void ratio difference due to stress
ρ^{θ}: Equivalent void ratio difference
OCR^{σ}: Overconsolidated ratio due to stress
OCR^{θ}: Equivalent overconsolidated ratio
a: Material parameter controls the evolution rate of void ratio difference
t_1: Unit time
$\bar{\alpha}$: Material parameter controls the gradient of creep rate vs. time in logarithmic axes
C_{n}: Material parameter controls the strain rate dependency of soft rocks
U: Internal energy
W: Work done by external forces
$\dot{W}^{\circ}$: Reversible work rate
Q: Heat energy
$\dot{Q}^{\circ}$: Changing rate of thermal energy corresponding to $\dot{W}^{\circ}$
h_{i}: Heat flux
D: Density of element
u: Internal energy per unit mass
w: Work per unit mass
q: Heat energy per unit mass
k: Heat conductivity coefficient
r: Internal heat supply per unit time per unit mass
$\dot{\eta}$: Material time derivative of the entropy
$\dot{\gamma}$: Entropy production

REFERENCES

1) Adachi, T., Oka, F. and Zhang, F. (1994): An elasto-viscoplastic constitutive model with strain softening and its application to the progressive failure of a cut slope, AMD-Vol. 183/MD-Vol. 50, *Material Instabilities, Theory and Applications*, ASME: 203–217.
2) Fujinuma, S., Okada, T., Hibino, S. and Yokokura, T. (2003): Consolidated undrained triaxial compression test of sedimentary soft rock at a high temperature, *Proc. 32nd Symposium on Rock Mechanics*, Japan, 125–130 (in Japanese).
3) Hashiguchi, K. and Ueno, M. (1977): Elastoplastic constitutive laws of granular materials, *Constitutive Equations of Soils (Proc. 9th ICFSME, Spec. Session 9)*, JSSMFE, Tokyo, 73–82.
4) Hashiguchi, K. (1980): Constitutive equations of elastoplastic materials with elastic-plastic transition, *J. Appl. Mech. (ASME)*, **47**, 266–272.
5) Hashiguchi, K. and Chen, Z.-P. (1998): Elastoplastic constitutive equations of soils with the subloading surface and the rotational hardening, *Int. J. Numer. Anal. Meth. Geomech*, **22**, 197–227.
6) Kitagawa, H. (1972): Thermodynamic constitutive theories of elastic-plastic material, *Journal of the Society of Materials Science*, **21**(230), 969–977 (in Japanese).
7) Kittel, C. (2000): *Thermal Physics*, Twenty-first Century Printing.
8) Lebon, G., Jou, D. and Casas-Vazquez, J. (2008): *Understanding Non-equilibrium Thermodynamics*, Springer.
9) Nakai, T. and Mihara, Y. (1984): A new mechanical quantity for soils and its application to elastoplastic constitutive models, *Soils and Foundations*, **24**(2), 82–94.
10) Nakai, T. and Hinoko, M. (2004): A simple elastoplastic model for normally and overconsolidated soils with unified materials parameters, *Soils and Foundations*, **44**(2), 53–70.
11) Oka, F. and Adachi, T. (1985): A constitutive equation of geologic materials with memory, *Proc. 5th Int. Conf. on Numerical Method in Geomechanics*, (1), 293–300.
12) Okada, T. (2005): Mechanical properties of sedimentary soft rock at high temperatures (Part1)-Evaluation of temperature dependency based on triaxial compression test, *Civil Engineering Research Laboratory Rep. No. N04026* (in Japanese).
13) Okada, T. (2006): Mechanical properties of sedimentary soft rock at high temperatures (Part2)-Evaluation of temperature dependency of creep behaviour based on unconfined compression test, *Civil Engineering Research Laboratory Rep. No. N05057* (in Japanese).
14) Rojas, E. and Garnica, P. (2000): Thermomechanical anisotropic model for soils, *Soils and Foundations*, **40**(2), 61–75.
15) Valanis, K. C. (1971): A theory of viscoplasticity without a yield surface, *Archiwum Mechaniki Stosowanej*, **23**(4), 517–533.
16) Zhang, F., Yashima, A., Nakai, T., Ye, G. L., Adachi, T. and Oka, F. (2003a): An elastoplastic strain-hardening and strain-softening constitutive model for soft rock considering the influence of intermediate stress, *Soils and Foundations*, **43**(5), 107–117.
17) Zhang, F., Yashima, A., Matsuda, S., Sekine, Y. and Hyodo, H. (2003b): Evaluation of the remedial works for cracked tunnels in creep-behaved ground, (eds.), *Proc. 12th Asian Regional Conference of ISSMGE*, Singapore 895–898.
18) Zhang, F., Yashima, A., Nakai, T., Ye, G. L. and Aung, H. (2005): An elasto-viscoplastic model for soft sedimentary rock based on t_{ij} concept and subloading yield surface, *Soils and Foundations*, **45**(1), 65–73.

APPENDIX

Rate of plastic work $\dot{W}^{\mathrm{p}}$ is made up from thermodynamic part and stress-induced part. Therefore, substituting Eqs. (45) and (53) into $\dot{W}^{\mathrm{p}}$, the following equation can be obtained:

$$\begin{aligned}\dot{W}^{\mathrm{p}} &= \sigma_{\mathrm{ij}}(\dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}\sigma} + \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}\theta}) = \Lambda \frac{\partial f}{\partial \sigma_{\mathrm{ij}}} \sigma_{\mathrm{ij}} + \sigma_{\mathrm{ij}} \dot{\varepsilon}_{\mathrm{ij}}^{\mathrm{p}\theta} \\ &= \Lambda \frac{\partial f}{\partial \sigma_{\mathrm{ij}}} (s_{\mathrm{ij}} + \sigma_{\mathrm{m}} \cdot \delta_{\mathrm{ij}}) \\ &\quad + \sigma_{\mathrm{ij}} C_{\mathrm{p}} \frac{3K\alpha}{\sigma_{\mathrm{m0}} + 3K\alpha(\theta - \theta_0)} \dot{\theta} \frac{\delta_{\mathrm{ij}}}{3} \end{aligned} \tag{A–1}$$

where, s_{ij} is deviatory stress tensor.

Furthermore, the following equation can be obtained as:

$$\frac{\partial f}{\partial \sigma_{\mathrm{ij}}} = \left(\frac{1}{\sigma_{\mathrm{m}}} - \frac{\sqrt{3}}{M^*} \frac{\sqrt{J_2}}{\sigma_{\mathrm{m}}^2} \right) \frac{\delta_{\mathrm{ij}}}{3} + \frac{\sqrt{3}}{M} \frac{s_{\mathrm{ij}}}{2\sqrt{J_2}} \frac{1}{\sigma_{\mathrm{m}}} \tag{A–2}$$

By substituting Eq. (A–2) into Eq. (A–1), it is easy to obtain the following relation:

$$\dot{W}^{\mathrm{p}} = \Lambda + \sigma_{\mathrm{m}} C_{\mathrm{p}} \frac{3K\alpha}{\sigma_{\mathrm{m0}} + 3K\alpha(\theta - \theta_0)} \dot{\theta} \tag{A–3}$$

In the Eq. (A–3), according to the loading criteria, plastic strain only develops in the case of $\Lambda \geq 0$. Therefore, the first term in the right side of Eq. (A–3) is positive. For the second term in the right side of Eq. (A–3), according to the fact that plastic strain due to the change of temperature can only develops in the case of $\dot{\theta} \leq 0$, and the fact that $\alpha < 0$, the second term is always positive, that is,

$$\sigma_{\mathrm{m}} C_{\mathrm{p}} \frac{3K\alpha}{\sigma_{\mathrm{m0}} + 3K\alpha(\theta - \theta_0)} \dot{\theta} \geq 0 \tag{A–4}$$

Therefore, $\dot{W}^{\mathrm{p}} \geq 0$ is always valid.