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Robust carrier frequency offset estimation for OFDM systems in doppler shift channels

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Abstract: In this paper, a robust carrier frequency offset (CFO) estimation scheme is proposed for OFDM systems in Doppler shift channels. We first present a training symbol (TS) pattern that has the same value so as to provide a wide CFO estimation range and to ensure the accuracy of the CFO estimation. Using the TS, we then present maximum-Likelihood (ML) and averaging least square (LS) algorithms for performing acquisition and tracking steps of the CFO estimation. The performance of the proposed technique is evaluated with respect to a mean-square-error (MSE) and a bit-error-rate (BER) in a typical urban (TU) channel.

Keywords: Carrier frequency offset, frequency offset estimation, inter-carrier interference, orthogonal frequency division multiplexing

Classification: Wireless circuits and devices

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1 Introduction

In orthogonal frequency division multiplexing (OFDM) systems, a carrier frequency offset (CFO) causes the reduction of the desired signal amplitude and an inter-carrier interference (ICI), which deteriorates the system performance [1]. As a result, many researches have been done to minimize the CFO effects by designing an efficient training symbol (TS) pattern and corresponding acquisition and tracking algorithm for the CFO estimation [2, 3, 4]. In [2], the two TSs were employed to perform the CFO estimation, where the first TS is comprised of two identical halves, and the second one is comprised from the correlation relation of the first TS. In [4], the two TSs in which the second TS has inverse repetition of the first TS was proposed.

In this paper, methods are proposed for the robust CFO estimation for OFDM systems. First, we present a simple TS pattern that has all the same value, which avoids the extra overhead of using two TSs as in [2, 4]. Based on the TS pattern, we propose a maximum-Likelihood (ML) method for an integer part of the CFO and an averaging least square (LS) method for a fractional part of the CFO.

2 System model

We describe the method for producing the TS that has an identical signal in the time-domain. The TS pattern can be simply generated by a particular symbol pattern in the frequency-domain:

$$X_k = \begin{cases} \sqrt{N} c_k, & k = 0 \\ 0, & k = 1, \ldots, N-1 \end{cases} \tag{1}$$

where N denotes the block size of the discrete Fourier transform (DFT), X_k is the TS at the kth subcarrier for the CFO estimation, and c_k refers to a pseudonoise (PN) sequence other than 0. In (1), $\sqrt{N}$ value is inserted in order to keep TS power from being decreased due to zero insertion. After X_k passes the inverse DFT (IDFT), the identical TS is generated as follows

$$\mathbf{x} = [x_0, x_0, \ldots, x_0]^T \tag{2}$$

where $(\cdot)^T$ denotes the transpose operator, and x_0 is the sample in the time-domain. At a transmitter, the TS of (1) is inserted every frame before the IDFT. After passing the IDFT, a cyclic-prefix (CP) is appended.

3 Algorithms for acquisition and tracking

The algorithms for an acquisition and a tracking of the CFO are proposed based on the TS of (1). Removing the CP, the receive signal over a multipath channel considering the effect of the CFO and an additive white Gaussian noise (AWGN) is expressed as

$$\mathbf{y} = \mathbf{P}\mathbf{H}_c\mathbf{x} + \mathbf{w} \tag{3}$$

where $\mathbf{y} = [y_0 \; y_1 \; \ldots \; y_{N-1}]^T$ is the received TS, and $\mathbf{w} = [w_0 \; w_1 \; \ldots \; w_{N-1}]^T$ is the AWGN vector. In (3), a diagonal matrix $\mathbf{P} = \mathrm{diag}\left(1, \ldots, \exp(\frac{j2\pi\varepsilon(N-1)}{N})\right)$

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denotes the CFO effect, where ε is the normalized CFO. $\mathbf{H}_c$ denotes the circular channel matrix of a channel impulse response (CIR) $\mathbf{h} = [h_0\ h_1\ \ldots\ h_{L-1}]^T$, where L is the channel memory. For observing the relationship between corresponding samples in the received TS, the effect of AWGN is neglected. Correspondingly, the each received sample of (3) is expressed as

$$\begin{aligned} y_0 &= (h_0 + \cdots + h_{l-1})x_0 \\ y_1 &= (h_0 + \cdots + h_{l-1})x_0 e^{\frac{j2\pi\varepsilon}{N}} \\ &\vdots \\ y_{N-1} &= (h_0 + \cdots + h_{l-1})x_0 e^{\frac{j2\pi\varepsilon(N-1)}{N}}. \end{aligned} \tag{4}$$

In (4), it is found that the relationship between corresponding samples is

$$y_{n+d} = y_n e^{\frac{j2\pi\varepsilon d}{N}} \quad k \in [0, N-1] \tag{5}$$

where d is a time index in a window of N samples. From (5), we can see that multiplying the conjugate of y_n to both side of (5) cancels the effect of the CIR.

From the relationship of (5), we describe the acquisition and tracking algorithms to estimate an integer part and a fractional part of the CFO, respectively. For the acquisition step, we define the log-likelihood function of the CFO estimation as follows [4]

$$\begin{aligned} \Lambda(\varepsilon) &= \log\left(\prod_{n=0}^{N-d-1} f(y_n, y_{n+d})\right) \\ &= \sum_{n=0}^{N-d-1} \log(f(y_n, y_{n+d})). \end{aligned} \tag{6}$$

In (6), the 2-D complex-valued Gaussian distribution $f(y_n, y_{n+d})$ is given by [3]

$$f(y_n, y_{n+d}) = \frac{e^{-\frac{|y_n|^2 - 2\rho Re\{e^{j2\pi\varepsilon/N} y_n y_{n+d}^*\} + |y_{n+d}|^2}{(\sigma_s^2+\sigma_n^2)(1-\rho^2)}}}{\pi^2(\sigma_s^2+\sigma_n^2)^2(1-\rho^2)} \tag{7}$$

where σ_s and σ_n are the signal and the noise powers, respectively, $(\cdot)^*$ is a complex conjugate, and ρ is defined as

$$\rho = \left|\frac{E\{y_n y_{n+d}^*\}}{\sqrt{E\{|y_n|^2\}E\{|y_{n+d}|^2\}}}\right|. \tag{8}$$

From (6), taking the partial derivative of $\Lambda(\varepsilon)$ with respect to ε and setting the result to zero, we obtain the maximum-likelihood (ML) estimate of ε as

$$\hat{\varepsilon} = -\frac{N\sum_{n=0}^{N-d-1} |y_n y_{n+d}^*| \cdot \angle(y_n y_{n+d}^*)}{2\pi \sum_{n=0}^{N-d-1} |y_n y_{n+d}^*|}. \tag{9}$$

From (9), it is noticed that as d gets smaller, a phase rotation due to the noise has more effects on the angle of $y_n y_{n+d}^*$ than the CFO. Hence, to mitigate the problem, we take only integer value of $\hat{\varepsilon}$, i.e., $\hat{\varepsilon}_i$, as follows

$$\hat{\varepsilon}_i = \lfloor \hat{\varepsilon} \rfloor \tag{10}$$

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where $\lfloor c \rfloor$ denotes an integer value that is less than or equal to c. From (5) and (9), it is noticed that the angle of $\exp(j2\pi\varepsilon d/N)$ should be less than π for the proper acquisition. This reveals that the acquisition range is within $\pm N/2$ subcarrier spacings, which is equal to the overall signal bandwidth when $d = 1$.

Next, we present an averaging least-squares (LS) algorithm to estimate the fractional part of the CFO. In order to eliminate the effect of the integer part of the CFO, we first multiply $e^{-j2\pi\hat{\varepsilon}_i l/N}, l \in [0, N-1]$ to $\mathbf{y}$ of (3). Then, the averaging LS algorithm is accomplished over all possible consecutive symbol in the TS, which is given by

$$\hat{\varepsilon}_f = \frac{2}{N(N-1)} \sum_{n=0}^{N-2} \left(\frac{\angle \left(\sum_{m=1}^{N-n-1} y_n^* y_{n+m} \right)}{2\pi m} \right) \tag{11}$$

where we note that the average approach allows for mitigating the drawback such as the noise enhancement of the LS algorithm.

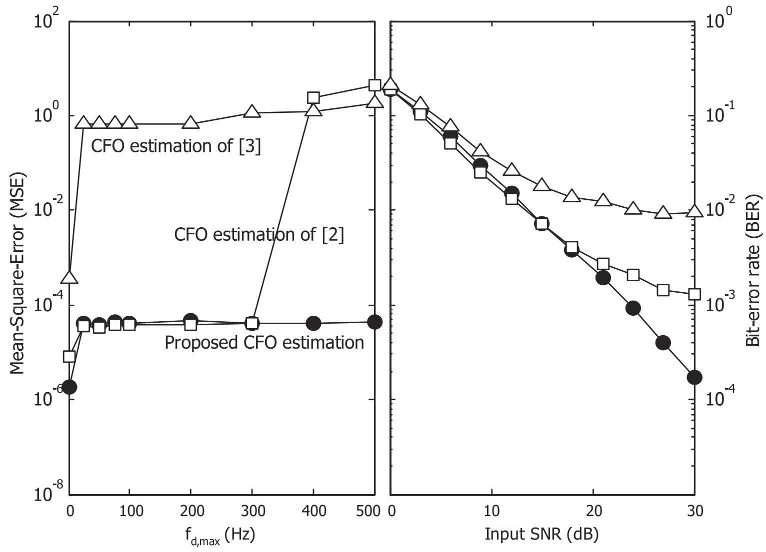

Fig. 1. (a) MSE curves versus maximum Doppler frequency shift (input SNR=20 dB, $\varepsilon = 0.3$), (b) BER curves versus input SNR ($f_{d,max} = 500$ Hz, $\varepsilon = 0.3$)

4 Performance evaluation

The performance of the proposed scheme is compared with that of the schmidl's [2] and zhang's [4] schemes in the typical urban (TU) channel [5]. In the simulation, a BPSK modulation was considered, and it was assumed that one frame consists of 20 OFDM symbols and $N = 64$. It was also assumed that channel varies on block-by-block basis.

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Under the assumption of the perfect CIR estimation and timing synchronization, Fig. 1 depicts the mean-square-error (MSE) and bit-error-rate (BER) curves when $\varepsilon = 0.3$ and an input signal-to-noise ratio (SNR) is 20 dB. In Fig. 1. (a), the MSE curves show performance variation of each scheme as a function of a maximum Doppler frequency shift, $f_{d,max}$. The BER curves are given as a function of the input SNR when $f_{d,max} = 500$ Hz in Fig. 1. (b). In the MSE curves, it is shown that the performance of the Schdmidl's and zhang's schemes worsen as Doppler frequency shift increases because the employed two TSs lose their synchronization relationship. On the other hand, the proposed scheme preserves a low MSE since it utilizes one TS. The robustness of the proposed scheme is also can be found in the BER performance. In the BER curves, the proposed scheme does not suffer from an error floor problem while the other schemes do.

5 Conclusions

In this paper, we presented simple TS pattern, which is appropriate for use in OFDM systems over block-by-block varying Doppler shift channels. The proposed acquisition and tracking algorithms are the ML and averaging LS estimation, respectively. The proposed acquisition range is as large as overall signal bandwidth. The performance evaluation in the TU channel showed the robustness of the proposed scheme with respect to the MSE and BER at the high Doppler frequency shift.

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