IEICE Electronics Express, Vol.8, No.13, 1089–1095

BER analysis of DS-UWB system employing a laplace distribution model

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Abstract: This letter takes a new approach to extract a closed-form expression for the bit error rate (BER) of direct-sequence ultra wideband (DS-UWB) system. In the analysis, the main signal is impaired by multi-user interference (MUI) and an external source of interference originated by simultanously transmitting multiband orthogonal frequency division multiplexing (MB-OFDM) systems which are located in the vicinity of the DS-UWB receiver. All the transmission channels are affected by Nakagami-m fading. A Laplacian distribution is considered for MUI to comply more with real statistical behaviors of this kind of interference.

Keywords: direct-sequence ultra wideband (DS-UWB), multiband orthogonal frequency division multiplexing (MB-OFDM), multi-user interference (MUI), bit error rate (BER), Laplacian distribution

Classification: Microwave and millimeter wave devices, circuits, and systems

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DOI: 10.1587/elex.8.1089
Received April 18, 2011
Accepted April 28, 2011
Published July 10, 2011

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1 Introduction

Direct-sequence ultra wideband (DS-UWB) [1] and multiband orthogonal frequency division multiplexing (MB-OFDM) UWB [2] are the two proposed UWB standards by IEEE 802.15.3a task group. These two technologies will be an important part of the future wireless industry because of the high transmission rate they provide, especially for short-range wireless applications. Research works evaluating the coexistence of the two afore-mentioned standards are a few. Even most works which analyze the performance of UWB systems with narrowband or other wideband wireless technologies, use a Gaussian approximation (GA) for the sources of interference especially multi-user interference (MUI). In the recent contributions, a GA assumption is proven to overestimate the system bit error rate (BER) [3]. In fact, in [4] it is proven that for low number of users, MUI fits a Laplacian distribution model better than the Gaussian model.

The work presented in [5] analyzes the effect of DS-UWB and time-hopping (TH) UWB interference on the performance of MB-OFDM UWB. This work is the reverse case of the analysis that we have performed in this work. In [6] a pulse-collision model is employed to evaluate the performance of DS-UWB when impaired by MB-OFDM interference. This work uses the IEEE 802.15.3 standard's channel model for the analysis and a Rake receiver for detection of the transmitted symbols. The approach and the scenario in [6] are completely different from the ones undertaken in this work.

In this letter, we present a performance analysis of the DS-UWB system in terms of BER. We consider two groups of interferers in the environment, the external interferers referred to as MB-OFDM and the MUI caused by other DS-UWB transmitters. To improve the analysis accuracy, we consider a Laplacian distribution model for MUI to calculate the BER. Later, the resultant BER expression which is conditioned on the channel fading coefficient, is finalized by averaging over the Nakagami-m distribution. The results for this new approach are compared for different numbers of MB-OFDM interferers and fading intensities.

The remainder of this letter is organized as follows. In Section II, the models for the channel and the signals under study are provided. Section III analyzes the interference terms affecting the DS-UWB system. Section IV

DOI: 10.1587/elex.8.1089
Received April 18, 2011
Accepted April 28, 2011
Published July 10, 2011

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derives the expressions for the system BER based on a Laplacian distribution for MUI. Section V presents the numerical results. Finally, concluding remarks are drawn in Section VI.

2 The signal and the channel models

The DS-UWB transmitted signal symbol of the kth user can be expressed as [5]:

$$P_{\mathrm{DS}}^{(k)}(t) = \sqrt{U_{\mathrm{p}}} \sum_{n=0}^{N_c-1} d_{n,k} c_{n,k} q(t - nT_c), \tag{1}$$

where t is the time index and $q(t)$ is the pulse waveform normalized such that $\int_{-\infty}^{+\infty} q^2(t)dt = 1$. U_{p} denotes the pulse energy, N_c indicates the number of chips per information bit (hence, the bit energy is given by $U_{\mathrm{b}} = N_c U_{\mathrm{p}}$), the sequence $\{c_{n,k}\}$ represents the spreading signature, T_c denotes the hop width, $T_f = N_c T_c$ is defined as frame width and $d_{n,k}$ represents the binary data transmitted.

For the kth MB-OFDM transmitter, the set of symbols $\{b_s^{(k)}, 0 \le s \le N-1\}$ are grouped into blocks which are modulated by an N-point IDFT (implemented with FFT) onto N subcarriers. After a guard interval, T_G, is inserted to reduce the interference between blocks, the general formula for the transmitted symbol of the kth MB-OFDM transmitter at baseband is expressed as [5]:

$$B^{(k)}(t) = \sqrt{U_{\mathrm{T}}^{(k)}} \sum_{s=0}^{N-1} b_s^{(n)} e^{\frac{j2\pi st}{T}}, -T_G \le t \le T, \tag{2}$$

where $U_{\mathrm{T}}^{(k)}$ is the transmit energy of the kth MB-OFDM transmitter and T is the OFDM symbol duration. $B^{(k)}(t)$ is assumed to be zero for $t < -T_G$ and $t \ge T$. At the RF block, a carrier is inserted and the signal is taken to the specified carrier frequency with respect to the frequency-hopping pattern of the MB-OFDM system:

$$P_{\mathrm{MB}}^{(k)}(t) = \mathbf{Re}\left\{B^{(k)}(t)e^{j2\pi(f_c+f_{\mathrm{MB}})t}\right\} = \mathbf{Re}\left\{\sum_{s=0}^{N-1} b_s^{(n)} e^{j2\pi(\frac{s}{T}+f_c+f_{\mathrm{MB}})t}\right\}, \tag{3}$$

where f_c is the constant carrier frequency offset for MB-OFDM and f_{MB} is the additive periodic value used to hop between the MB-OFDM frequency bands.

We consider the Nakagami-m distribution with parameters (m_k, Ω_k) and the probability density function (PDF) as [5]:

$$f_{h_k}(x) = \frac{2}{\Gamma(m_k)}\left(\frac{m_k}{\Omega_k}\right)^{m_k} x^{2m_k-1} e^{-\frac{m_k x^2}{\Omega_k}}, \tag{4}$$

where $\Gamma(m_k) = \int_0^{\infty} x^{m_k-1} e^{-x} dx$ is the complex Gamma function, m_k indicates the intensity of the fading where $m_k > \frac{1}{2}$, and $E\left[h_k^2\right] = \Omega_k$, with $\mathbf{E}[.]$ denoting the expectation. All the interferers experience the same fading condition.

DOI: 10.1587/elex.8.1089
Received April 18, 2011
Accepted April 28, 2011
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3 Interference analysis at the DS-UWB receiver

Considering perfect synchronization between the transmitter and the receiver, the signal after passing through the DS-UWB system correlation receiver becomes:

$$y = \tilde{h} S_{\mathrm{DS}} + I_{\mathrm{DS}} + I_{int} + n_{\mathrm{DS}}, \tag{5}$$

where $\tilde{h}$ is the Nakagami-m fading coefficient for the main signal with the parameters $\tilde{m}$ and $\tilde{\Omega}$, $S_{\mathrm{DS}} = \sqrt{U_b N_c} d_{0,1}$ is user 1's desired detected signal depending on its information bit, I_{DS} is the MUI, I_{int} is the contribution of the MB-OFDM interference and n_{DS} is the thermal noise with the variance $\sigma^2_{n_{\mathrm{DS}}} = N_0 N_c/2$. The variance of I_{DS} is given by

$$\sigma^2_{I_{\mathrm{DS}}} = \frac{U_b}{T_c}\left(\int_0^{T_c} \left(\bar{R}^2(\varepsilon) + R^2(\varepsilon)\right) d\varepsilon\right) \sum_{k=2}^{N_{\mathrm{DS}}} \Omega_k, \tag{6}$$

where N_{DS} is the total number of DS-UWB transmitters, ε is a random variable with a uniform distribution on $[0, T_c)$ and $\bar{R}$ and R are the partial autocorrelation functions of the DS-UWB pulse waveform given by [8]:

$$R(\varepsilon) = \int_0^{\varepsilon} q(t)q(t + T_c - \varepsilon)dt. \qquad 0 \leq \varepsilon \leq T_c, \tag{7}$$

$$\bar{R}(\varepsilon) = \int_{\varepsilon}^{T_c} q(t)q(t - \varepsilon)dt. \qquad 0 \leq \varepsilon \leq T_c, \tag{8}$$

Assuming equal energy for the MB-OFDM interferers as $U_T^{(1)} = U_T^{(2)} = \ldots = U_T^{(N_{\mathrm{MB}})} = U_T$, the variance of I_{int} can be written as [8]:

$$\sigma^2_{I_{\mathrm{int}}} = U_T \sum_{k=1}^{N_{\mathrm{MB}}} \Omega_k \sum_{m=0}^{N_c-1} \sum_{n=0}^{N-1} Q^2(s_n)\sigma^2_{\tau}, \tag{9}$$

where N_{MB} is the number of MB-OFDM interferers, $Q(s_n) = \int_{-T_p/2}^{T_p/2} e^{j s_n x} q(x)dx$ is the Fourier transform of the left-shifted version of pulse $q(t)$ which gives us a symmetric pulse and the limits of the integral are changed to the pulse width of the signal, T_p. Also, $s_n = j2\pi(n\Delta_F + f_c + f_{\mathrm{MB}})$, $\sigma^2_{\tau} = \mathbf{E}_{\tau}[cos(\theta)]$ and θ is defined as:

$$\theta = j2\pi(n\Delta_F + f_c + f_{\mathrm{MB}})\left(mT_c + \frac{T_p}{2} - \tau\right), \tag{10}$$

where Δ_F is the subcarrier's bandwidth of MB-OFDM.

4 BER analysis based on a laplacian distribution assumption for MUI

The Laplacian PDF model for MUI is given by [4]:

$$f^L_{I_{\mathrm{DS}}}(I_{\mathrm{DS}}) = \frac{1}{2\tilde{\sigma}_{I_{\mathrm{DS}}}} e^{\left(-\frac{|I_{\mathrm{DS}}|}{\tilde{\sigma}_{I_{\mathrm{DS}}}}\right)}, \tag{11}$$

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where $2\tilde{\sigma}^2_{I_{\rm DS}} = \sigma^2_{I_{\rm DS}}$. We consider a Gaussian distribution for the mixture of thermal noise and the MB-OFDM interference with the PDF given by:

$$f^G_{I_{\rm INT}}(I_{\rm INT}) = \frac{1}{\sqrt{2\pi}\sigma_{I_{\rm INT}}} e^{\left(-\frac{I^2_{\rm INT}}{2\sigma^2_{I_{\rm INT}}}\right)}, \tag{12}$$

where $\sigma^2_{I_{\rm INT}} = \sigma^2_{n_{\rm DS}} + \sigma^2_{I_{\rm int}}$. Now the conditional BER can be computed by convolving $f^L(x)$ and $f^G(x)$ and deriving the cumulative distribution function (CDF):

$$\begin{aligned} P_{e|\tilde{h}} &= F^{GL}_x(x)\Big|_{x=-\tilde{h}\sqrt{U_b N_c}} = \int_{-\infty}^{-\tilde{h}\sqrt{U_b N_c}} f^G_x(x) \otimes f^L_x(x) dx \\ &= D\left[e^{\frac{\tilde{h}\sqrt{U_b N_c}}{\tilde{\sigma}_{I_{\rm DS}}}} Q\left(\frac{\tilde{h}\sqrt{U_b N_c}}{\sigma_{I_{\rm INT}}} + \frac{\sigma_{I_{\rm INT}}}{\tilde{\sigma}_{I_{\rm DS}}}\right) - e^{-\frac{\tilde{h}\sqrt{U_b N_c}}{\tilde{\sigma}_{I_{\rm DS}}}} Q\left(-\frac{\tilde{h}\sqrt{U_b N_c}}{\sigma_{I_{\rm INT}}} + \frac{\sigma_{I_{\rm INT}}}{\tilde{\sigma}_{I_{\rm DS}}}\right)\right] \\ &+ Q\left(-\frac{\tilde{h}\sqrt{U_b N_c}}{\sigma_{I_{\rm INT}}}\right), \end{aligned} \tag{13}$$

where $\otimes$ is the convolution sign and $D = (1/2)exp\ (\sigma^2_{I_{\rm INT}}/\tilde{\sigma}^2_{I_{\rm DS}})$.

Replacing the variable $\tilde{h}$ by x in (13), the final expression for the average probability of error can be written by removing the condition on $P_{e|\tilde{h}}$ as:

$$\begin{aligned} P_e =& \frac{2}{\Gamma(\tilde{m})}\left(\frac{\tilde{m}}{\tilde{\Omega}}\right)^{\tilde{m}} \int_0^{\infty} x^{2\tilde{m}-1} e^{-\frac{\tilde{m}x^2}{\tilde{\Omega}}} P_{e|x}(x)dx \\ =& S_1(\gamma,\beta) + S_2(\gamma,\beta) + S_3, \end{aligned} \tag{14}$$

where $S_1(\gamma,\beta)$, $S_2(\gamma,\beta)$ and S_3 are functions which are calculated and defined in the following. $S_1(\gamma,\beta)$ is given by:

$$\begin{aligned} S_1(\gamma,\beta) &= D' \int_0^{\infty} e^{-\alpha x^2 - \gamma x} x^{b-1} Q(-(\beta x - \sigma'))dx \\ &= \frac{D'}{\sqrt{2\pi}} \int_0^{\infty} e^{-\alpha x^2 - \gamma x} x^{b-1} \int_{\beta x - \sigma'}^{\infty} e^{-u^2/2} du dx \\ &= \frac{D'}{\sqrt{2\pi}} \int_0^{\infty} e^{-u^2/2} \int_0^{\frac{u+\sigma'}{\beta}} e^{-\alpha x^2 - \gamma x} x^{b-1} dx du, \end{aligned} \tag{15}$$

where $\alpha = (\tilde{m}/\tilde{\Omega})$, $\gamma = \sqrt{U_b N_c}/\tilde{\sigma}_{I_{\rm DS}}$ is defined as the signal-to-MUI ratio, $b = 2\tilde{m}$, $\beta = \sqrt{U_b N_c}/\sigma_{I_{\rm INT}}$ is defined as the signal-to-MB-OFDM interference ratio, $\sigma' = \sigma_{I_{\rm INT}}/\tilde{\sigma}_{I_{\rm DS}}$ is the MB-OFDM interference-to-MUI power ratio and

$$D' = \frac{2D}{\Gamma(\tilde{m})}\left(\frac{\tilde{m}}{\tilde{\Omega}}\right)^{\tilde{m}}. \tag{16}$$

To solve the integral in (15), we approximate the integral by the two first terms of the integral's taylor expansion as:

$$S_1(\gamma,\beta) = \frac{D'}{\sqrt{2\pi}} \int_0^{\infty} e^{-u^2/2}\left(\frac{1}{b}\left(\frac{u+\sigma'}{\beta}\right)^b - \frac{\gamma}{b+1}\left(\frac{u+\sigma'}{\beta}\right)^{b+1}\right)du. \tag{17}$$

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It is easy to show that $S_2(\gamma,\beta) = -S_1(-\gamma,-\beta)$ and

$$S_3 = \sqrt{\frac{\lambda}{\lambda+1}}\frac{(\lambda+1)^{-\tilde{m}}\Gamma(\tilde{m}+\frac{1}{2})}{2\sqrt{\pi}\,\Gamma(\tilde{m}+1)}\ {}_2F_1\left(1,\tilde{m}+\frac{1}{2};\tilde{m}+1;\frac{1}{\lambda+1}\right), \quad (18)$$

where $\lambda = \gamma^2/2\alpha$. For integer values of $\tilde{m}$ we have [7]:

$$S_3 = \frac{1}{2}\left[1 - U(\lambda)\sum_{k=0}^{\tilde{m}}\binom{2k}{k}\left(\frac{1-U^2(\lambda)}{4}\right)^k\right], \quad (19)$$

where $U(\lambda) = \sqrt{\lambda/(\lambda+1)}$.

The integration pertaining to $(S_1(\gamma,\beta)+S_2(\gamma,\beta))$ is computable for integer values of b, e.g. for $b=1$ or $\tilde{m}=1/2$ the summation $(S_1(\gamma,\beta)+S_2(\gamma,\beta))$ reduces to:

$$S_1 + S_2 = D'\left(\frac{\sigma'(\sigma'^2+1) - 2\sigma'}{2\beta}\right). \quad (20)$$

For $b=2$ or $\tilde{m}=1$ we have

$$S_1 + S_2 = \frac{2D'}{3\sqrt{2\pi}}\left(\frac{\sigma'(3\sigma'^2+2) - 3\sigma'}{\beta^2}\right). \quad (21)$$

For $b=4$ or $\tilde{m}=2$ we have

$$S_1 + S_2 = \frac{2D'}{5\sqrt{2\pi}}\left(\frac{\sigma'(5\sigma'^4+20\sigma'^2+8) - 5\sigma'(\sigma'^2+2)}{\beta^4}\right). \quad (22)$$

5 Numerical results

We have considered typical MB-OFDM and DS-UWB systems based on the standard specifications [1, 2] to study our approach in analyzing the DS-UWB system performance. Number of multipath for all users is considered to be $L = 20$, with $\Omega_k = \tilde{\Omega} = 5\,\text{dB}$. We have fixed a value of $\sigma' = 2$ for the MB-OFDM interference-to-MUI power ratio and $N_c = 128$ for the number of chips per information bits. We have investigated the effect of different number of MB-OFDM interferers, N_{MB}, and different fading intensity values, m, on the DS-UWB receiver. Fig. 1 shows the results for the average BER versus U_b/N_0 of the DS-UWB system for $\sigma' = 2$, and compared for different N_{MB} and three fading intensity values, m. As expected, system performance deteriorates as the fading intensity increases. We also observe that an increase in the number of MB-OFDM interferers results in system degradation. However, in average ratios of σ', the DS-UWB is less sensitive to the change in the number of MB-OFDM interferers compared to the change in fading intensity.

6 Conclusion

In this letter a new method was proposed to study the performance of DS-UWB receiver while exposed to multi-user interference (MUI) and MB-OFDM interference. Based on the fact that Gaussian approximation in not precise enough to model the interference, we have used the Laplacian distribution to model the MUI in this environment. Laplacian distribution is

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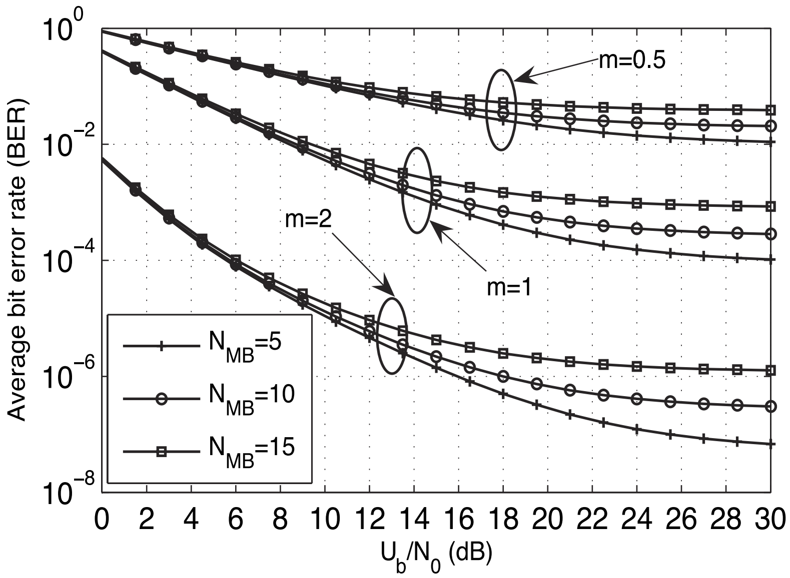

Fig. 1. Average BER versus U_b/N_0 of the DS-UWB system for $\sigma' = 2$.

proven to have a better accuracy for average number of multi-user interferers. The analytical results based on this new approach show that for moderate ratios of MB-OFDM interference-to-MUI, the DS-UWB system is less sensitive to the increase in the number of MB-OFDM interferers.

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Received April 18, 2011
Accepted April 28, 2011
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