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An Investigation on the Effect of the Coupled and Uncoupled Formulation on Transient Seepage by the Finite Element Method

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Abstract: 2-D transient seepage beneath a dam was investigated by the finite element method. Governing equations were considered in coupled and uncoupled methods. At first stage, the fluid continuity equation for compressible porous media considered as uncoupled. Because of the occurrence of seepage forces, and their interaction with the fluid pore pressure, particularly in high compressible soils, uncoupled methods seem to be far from real conditions. Therefore force equilibrium equations were taken into account by coupling with the fluid continuity equation. Finite element formulation based on Galerkin method. Results of two mentioned methods were compared with steady-state seepage results. Comparing the results of coupled and uncoupled models showed that the time require to reach steady-state condition in coupled model considerably shorter than the uncoupled model. Coupled analyses shows that the effective stress due to seepage forces were smaller than ones calculated by the uncoupled method.

Key Words: Transient seepage, couple, uncouple, F.E.M

INTRODUCTION

Seepage problem is one of the most important issues in the design and construction of dams and hydraulic structures. Seepage forces change stress state, deformation and change permeability of soil elements which would affect the amount of the seepage rate and stability of structure. In routine analyses, steady-state conditions commonly are considered ignoring the time required to reach the steady-state condition. But actually, moving water trough soil voids is a time consuming procedure. Increase of the water table height in a dam reservoir is gradual and time dependent, so transient analysis is essential to yield reliable results.

In addition, in uncouple systems there is only one degree of freedom in the governing equation which is the hydraulic potential. So in order to determine deformations of the structure due to seepage forces it is essential to consider element equilibrium equations besides the fluid continuity equation. In this case a set of partial differential equations including the continuity and equilibrium equations must be solved at the same time.

Considering seepage problem in the coupled form is similar to the coupled consolidation problem except in loading which contains hydraulic potential only. In this research the seepage problem is investigated by two coupled and uncoupled methods. Biot's coupled consolidation equations are adopted to investigate the seepage problem along with the traditional uncoupled transient seepage equation. Galerkin method is used to finite element formulation.

F.E.M formulation for uncoupled system: Fluid flow through porous media is governed by hydrodynamic equations considering the interaction of the fluid in motion with the porous media ensuring the continuity of the fluid. Continuity is ensured by requiring that the net volume of water flowing per unit of time into or out of an element of soil be equal to change per unit of time of the volume of water in that element. Difference of the quantity of the water that leaves or inters to an element is equal to change of the element volume. Therefore the uncoupled fluid continuity equation in the transient state would be as follows[1]:

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=-\frac{\partial}{\partial t}(ndv) \qquad (1)$$

Where; v_x and v_y are flow velocities in horizontal and vertical directions respectively and, n, is the porosity. The above equation can be rewritten in the following form:

$$k_x\frac{\partial^2 u_e}{\partial x^2}+k_y\frac{\partial^2 u_e}{\partial y^2}=\gamma_w m_v\frac{\partial u_e}{\partial t} \qquad (2)$$

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Where; k_x and k_y are permeability coefficients in horizontal and vertical directions, u_e is the water pressure and m_v is the compressibility coefficient of the soil.

Equation 2, is a 2-dimensional consolidation P.D.E which can be solved by the finite element method.

Using linear shape functions with rectangular elements, the final form of the Equation 2 discretized by the F.E.M would be as follows[2]:

$$KPu_e + PM\frac{du_e}{dt} = 0 \tag{3}$$

Where;

$$KP_{ij} = \iint (k_x \frac{\partial N_i}{\partial x}\cdot\frac{\partial N_j}{\partial x} + k_y \frac{\partial N_i}{\partial y}\cdot\frac{\partial N_j}{\partial y})dx.dy \tag{4}$$

And:

$$PM_{ij} = \iint N_i N_j dx.dy \tag{5}$$

In the above equations, N_i is the linear shape function.

Using the Crank-Nicolson's method, time integration of equation 3 involving linear interpolation and fixed time steps Δt would yield unconditionally stable and converge results according to Equation 6 [3]:

$$\left(PM + \frac{\Delta t}{2}KP\right)u_{e(t+\Delta t)} = \left(PM - \frac{\Delta t}{2}KP\right)u_{e(t)} \tag{6}$$

After finding governing matrix equations for a single element, the assembled matrices for all the elements can be obtained and boundary conditions can be introduced.

Therefore values of unknown variables can be calculated at time $t = t_0 + \Delta t$ based on known parameters at time $t = t_0$.

The initial conditions at time $t = 0$ all are known.

F.E.M formulation for coupled system: Biot formulated the theory of coupled solid-fluid interaction where the soil skeleton is treated as porous elastic solid and the laminar pore fluid is coupled to the solid by the conditions of compressibility and of continuity. Thus Biot's governing equations are combination of Equation 1 and element equilibrium equations.

For two-dimensional equilibrium in the absence of body forces considering seepage forces, the gradient of effective stress must be augmented by the gradients of the fluid pressure as follows[2]:

$$\begin{aligned}\frac{\partial \sigma'_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial y} + \frac{\partial u_e}{\partial x} = 0\\ \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial u_e}{\partial y} = 0\end{aligned} \tag{7}$$

The stress-strain relations based on generalized Hook's law for plane strain condition can be written as follows [4]:

$$\sigma_{ij} = 2G\varepsilon_{ij} + \left(K - \frac{2G}{3}\right)\delta_{ij}(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) \tag{8}$$

Where, K and G are bulk and shear modulus respectively.

Continuity equation can be obtained using Equations 1 and 2 where the volume change of the element is written in terms of displacement components:

$$\frac{k_x}{\gamma_w}\cdot\frac{\partial^2 u_e}{\partial x^2} + \frac{k_y}{\gamma_w}\frac{\partial^2 u_e}{\partial y^2} = -\frac{d}{dt}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{9}$$

Where u_e is defined before, u and v are horizontal and vertical components of displacements.

As is usual in the displacement method in solid mechanics, stress and strain are replaced with displacement components so, final coupled variables are the pore water pressure, and horizontal and vertical displacements.

Same as the uncoupled method, using the linear shape functions with rectangular elements for the solid body and the pore fluid, the final form of the set of P.D.E which is combining Equations 7 and 9 discretized by the F.E.M can be as follows[2]:

$$\begin{aligned}KMr + Cu_e = 0\\ C^T\frac{dr}{dt} - KPu_e = 0\end{aligned} \tag{10}$$

Where: *KP* is already defined and, r is the displacement vector [u,v]; and also *KM* and *C* are as follows:

$$KM = \begin{bmatrix} KMxx & KMxy \\ KMxy^T & KMyy \end{bmatrix} \tag{11}$$

Where:

$$\begin{aligned}KMxx_{ij} &= \iint (R_{11}\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + R_{33}\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y})dxdy\\ KMxy_{ij} &= \iint (R_{12}\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial y} + R_{33}\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial x})dxdy\\ KMyy_{ij} &= \iint (R_{22}\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y} + R_{33}\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x})dxdy\end{aligned} \tag{12}$$