

Analysis of the stability of adiabatic reversible logic using the theory of normal modes in coupled oscillators

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Abstract: The stability of adiabatic stepwise charging reversible logic is discussed from the viewpoint of coupled oscillators. For adiabatic logic with asymmetric tank capacitors, we derive a matrix that connects the initial voltage with the voltage change after the charge-recycle process. This matrix is the same as the mechanical oscillator matrix of a string with equally spaced beads having different mass. From the theory of normal modes in coupled oscillators, it is proved that the eigenvalue of the matrix connecting the initial voltage with the final one is less than 1, which shows that a step waveform is spontaneously generated after many charge-recycle processes.

Keywords: adiabatic reversible logic, tank capacitor, mechanical oscillator, normal mode, eigenvalue

Classification: Integrated circuits

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1 Introduction

Recently, adiabatic logic using tank capacitors has been attracting much attention because it enables low-energy operation [1, 2]. Moreover, its quasi-static operation could offer a highly reliable SRAM [3] because it can avoid hot carrier effects and electromigration.

The stability of adiabatic logic with asymmetric tank capacitors was proved using a mathematical method in a previous paper [3]. However, the physical meaning of the stability is not yet clear. In this letter, we discuss a matrix connecting the initial voltage with the voltage change after the charge-recycle process from the viewpoint of coupled oscillators. It is shown that a matrix of asymmetric tank capacitors for describing a state as a function of time is the same as that of a string with equally spaced beads having different mass. We apply the theory of normal modes in coupled oscillators to the matrix equation derived from the asymmetric tank capacitor system. From the results, the eigenvalue of matrix connecting the initial voltage with the final voltage is physically understood in the case of asymmetric tank capacitors and is shown to be less than 1.

2 Stability of adiabatic logic with asymmetric capacitors

The switched capacitor regenerator with asymmetric capacitors is shown in Fig. 1 (a), where V is a power supply voltage, V_{Ci} is the voltage of the tank capacitor C_i , and C_L is load capacitance. Here, C_i is much larger than C_L . Fig. 1 (b) shows V_i , which is the voltage deviation of V_{Ci} from the i th step voltage ($V_i = V_{Ci} - i/N \cdot V$). N is the step number.

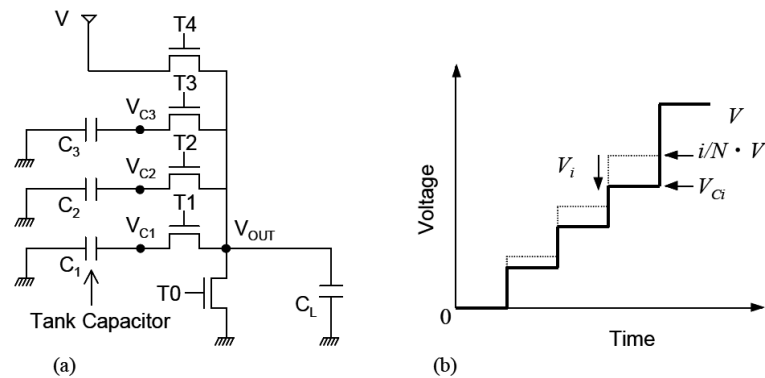


Fig. 1. Switched capacitor regenerator with asymmetric tank capacitors. (a) Circuit. (b) Explanation of V_i .

In a previous letter [3], we showed the relation between the initial voltage V_i and the voltage change ΔV_i after the charge-recycling process. The relation is

$$C_i \Delta V_i = C_L V_{i-1} - 2C_L V_i + C_L V_{i+1} \quad (1 \leq i \leq N-1). \quad (1)$$

V_0 and V_N are zero from the definition. ΔV_i is linear combination of V_{i-1} , V_i and V_{i+1} , which is due to the fact that charging and restoring process is

done between only adjacent tank capacitors. In other words, we only have to consider nearest-neighbor tank capacitors for describing the phenomenon. Formula (1) is rewritten as

$$\mathbf{A} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \vdots \\ \Delta V_{N-1} \end{bmatrix} + \mathbf{B} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_{N-1} \end{bmatrix} = 0, \quad \text{where} \quad \mathbf{A} = \begin{bmatrix} C_1 & & & 0 \\ & C_2 & & \\ & & \ddots & \\ 0 & & & C_{N-1} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 2C_L & -C_L & 0 & & 0 \\ -C_L & 2C_L & -C_L & \ddots & \\ 0 & -C_L & 2C_L & \ddots & 0 \\ & \ddots & \ddots & \ddots & -C_L \\ 0 & & 0 & -C_L & 2C_L \end{bmatrix}. \quad (2)$$

Matrices \mathbf{A} and \mathbf{B} are the same as that describing the motion of a string with equally spaced beads having different mass. We explain this point in more detail. The string model is shown in Fig. 2. We define the bead mass on the i th site as m_i , tension as T , and lattice distance as a . In this case, the motion equation of the string can be written as [4, 5]

$$\mathbf{C}\ddot{\mathbf{x}} + \mathbf{D}\mathbf{x} = 0, \quad \text{where} \quad \mathbf{C} = \begin{bmatrix} m_1 & & 0 \\ & m_2 & \\ & & \ddots \\ 0 & & & m_{N-1} \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} 2T/a & -T/a & & 0 \\ -T/a & 2T/a & -T/a & \ddots \\ & -T/a & 2T/a & \ddots \\ & \ddots & \ddots & \ddots & -T/a \\ 0 & & & -T/a & 2T/a \end{bmatrix}. \quad (3)$$

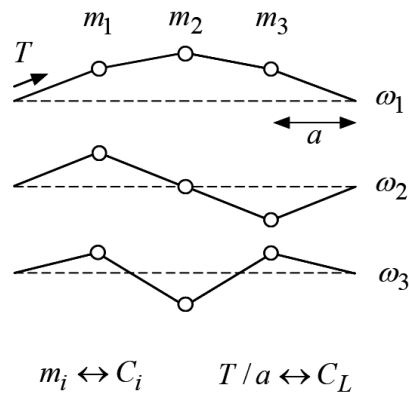


Fig. 2. String with bead mass m_i , tension T , and lattice distance a . The m_i and T/a correspond to C_i and C_L in \mathbf{A} and \mathbf{B} , respectively.

Comparing formulas (2) and (3), we see that the matrix structure is the same. The m_i and T/a correspond to C_i and C_L , respectively. Therefore, solving the normal mode of the string solves the switched capacitor regenerator phenomenon.

Matrix \mathbf{B} is also similar to the tight-binding model in solid-state physics, which is used for calculating energy bands [6].

Now, Let us consider the problem of general coupled oscillators. In general, it is known that the motion equation of coupled oscillators is written as follows [4, 7].

$$\begin{bmatrix} m_{11} & \cdots & \cdots & m_{1n} \\ & \ddots & & \\ & & \ddots & \\ m_{n1} & \cdots & \cdots & m_{nn} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \vdots \\ \vdots \\ \ddot{x}_n \end{bmatrix} + \begin{bmatrix} k_{11} & \cdots & \cdots & k_{1n} \\ & \ddots & & \\ & & \ddots & \\ k_{n1} & \cdots & \cdots & k_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \quad (4)$$

This is rewritten as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}, \quad (5)$$

where \mathbf{M} and \mathbf{K} are positive-definitive symmetric matrices from the physical definition. Now, we consider the transformation matrix \mathbf{P} , which changes $\mathbf{P}^t\mathbf{M}\mathbf{P}$ into a unit matrix and $\mathbf{P}^t\mathbf{K}\mathbf{P}$ into a diagonal matrix. We can realize this by calculating the eigenvalue and eigenvector of \mathbf{K} when \mathbf{M} is a metric matrix. In this case, the characteristic equation is

$$\phi(\lambda) = |\lambda\mathbf{M} - \mathbf{K}| = 0. \quad (6)$$

When the eigenvalue is λ_i , eigenvector \mathbf{p}_i is solved by

$$(\lambda_i\mathbf{M} - \mathbf{K})\mathbf{p}_i = \mathbf{0}. \quad (7)$$

In this case, it is known that \mathbf{p}_i is an orthogonal basis vector. The \mathbf{p}_i is normalized by

$$\mathbf{p}_i^t\mathbf{M}\mathbf{p}_i = 1. \quad (8)$$

Then, we define \mathbf{P} as

$$\mathbf{P} \equiv [\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_n]. \quad (9)$$

In this case, we have

$$\mathbf{P}^t\mathbf{M}\mathbf{P} = \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix}, \quad \mathbf{P}^t\mathbf{K}\mathbf{P} = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}. \quad (10)$$

Matrix \mathbf{K} is related to the potential energy so that $\mathbf{x}^t\mathbf{K}\mathbf{x}$ is positive. In other words, \mathbf{K} is a positive-definite symmetric matrix in linear algebra. Therefore, it is known that λ_i is positive. Then, we write λ_i as ω_i^2 .

Here, we define $\mathbf{y} = \mathbf{P}^{-1}\mathbf{x}$. Motion equation (5) is transformed as

$$\mathbf{P}^t(\mathbf{M}\mathbf{P}\ddot{\mathbf{y}} + \mathbf{K}\mathbf{P}\mathbf{y}) = \mathbf{0}. \quad (11)$$

Using formula (10), we have

$$\begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \begin{bmatrix} \ddot{y}_1 \\ \vdots \\ \ddot{y}_n \end{bmatrix} + \begin{bmatrix} \omega_1^2 & & 0 \\ & \ddots & \\ 0 & & \omega_n^2 \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (12)$$

where ω_i is called normal frequency.

Similarly, we consider formula (2). \mathbf{A} and \mathbf{B} are positive-definite symmetric matrices as follows:

$$\begin{aligned} \mathbf{x}^t \mathbf{A} \mathbf{x} &= C_1 x_1^2 + C_2 x_2^2 + \cdots + C_{N-1} x_{N-1}^2 \\ \mathbf{x}^t \mathbf{B} \mathbf{x} &= C_L [x_1^2 + (x_1 - x_2)^2 + \cdots + (x_{N-2} - x_{N-1})^2 + x_{N-1}^2]. \end{aligned} \quad (13)$$

Therefore, the previous discussion is applicable. Here, we define $\mathbf{W} = \mathbf{P}^{-1} \mathbf{V}$. Then, we have

$$\mathbf{P}^t (\mathbf{A} \mathbf{P} \Delta \mathbf{W} + \mathbf{B} \mathbf{P} \mathbf{W}) = 0. \quad (14)$$

Then,

$$\begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix} \begin{bmatrix} \Delta W_1 \\ \vdots \\ \Delta W_{N-1} \end{bmatrix} + \begin{bmatrix} \omega_1^2 & & 0 \\ & \ddots & \\ 0 & & \omega_{N-1}^2 \end{bmatrix} \begin{bmatrix} W_1 \\ \vdots \\ W_{N-1} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \quad (15)$$

Here, we define $\mathbf{W}' = \mathbf{W} + \Delta \mathbf{W}$. Then,

$$\begin{bmatrix} W_1' \\ \vdots \\ W_{N-1}' \end{bmatrix} = \mathbf{F} \begin{bmatrix} W_1 \\ \vdots \\ W_{N-1} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} 1 - \omega_1^2 & & 0 \\ & \ddots & \\ 0 & & 1 - \omega_{N-1}^2 \end{bmatrix} \quad (16)$$

Now, let us consider the value of $1 - \omega_i^2$ in (16). First, we define the smallest tank capacitance among the asymmetric tank capacitors as C_S . Then, we consider the symmetric tank capacitors, whose value is C_S . This situation corresponds to the mechanical string oscillator with beads having the same mass. In this case, ω_i is written as

$$\omega_i = 2\sqrt{K} \sin \frac{i\pi}{2N}, \quad i = 1, 2, \dots, N-1. \quad (17)$$

where $K = C_L/C_S$ [3, 4, 5]. We thus have $\omega_i \ll 1$ owing to $K \ll 1$.

Here, let us consider Rayleigh's theorem [4]. The theorem is explained as follows. We assume that there are coupled oscillators that interact each other. We assume that normal frequency is $\omega_1, \omega_2, \dots, \omega_{N-1}$, which has the relation

$$0 < \omega_1 \leq \omega_2 \leq \cdots \leq \omega_{N-1}. \quad (18)$$

Here, we make the mass of the k th site heavier than the initial mass. Then, all normal oscillation frequencies become smaller than the initial value. In this situation, the new normal oscillation ω_i' has the relation

$$0 < \omega_1' \leq \omega_2' \leq \cdots \leq \omega_{N-1}' \quad (19)$$

We apply the theorem to equation (2). In this case, the capacitance corresponding to the k th site becomes larger than C_S . Then, ω_i in (15) is smaller than that in the symmetric case. This means that the eigenvalue $1 - \omega_i^2$ becomes closer to 1, but is still smaller than 1. Therefore, \mathbf{F}^n becomes zero when $n \rightarrow \infty$, although it takes longer for the state to converge. Then, \mathbf{W} approaches zero when $n \rightarrow \infty$, so that \mathbf{V} (or V_i) approaches zero. This means V_{Ci} becomes $i/N \cdot V$ spontaneously after many charge-recycle operations.

3 Conclusion

In summary, using coupled oscillation theory, we showed that adiabatic logic using asymmetric tank capacitors is stable. The matrix connecting the initial voltage with voltage change after the charge recycling process is the same as the mechanical string oscillator matrix. Matrix characteristics were analyzed using the theory of normal modes in coupled oscillators. Making the mass heavier is consistent with making the capacitance larger. The eigenvalue of the matrix connecting the initial voltage with the final one after recycling process is smaller than 1. This means a step waveform is generated spontaneously.

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