

# Enhanced pseudo Zernike moments in face recognition

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**Abstract:** This paper presents an approach to boost the performance of pseudo Zernike moments in face recognition. This approach is a hybrid of a kernel trick, discriminant function and pseudo Zernike moments (PZM), namely as Kernel-based Fisher Pseudo Zernike Moments (KFPZM). KFPZM maps the moment-based features into a high dimensional feature space via kernel function for disclosing the underlying variables which carry significant information about the image. Then, it performs discriminant analysis onto the mapped features to enhance the discrimination power via Fisher's Linear Discriminant (FLD). Experimental results show that the proposed method outperforms the sole PZM and the integrated FLD with PZM methods, achieving recognition rate of 98.11% and 93.03% in the face databases with facial expression variations and illumination variations, respectively.

**Keywords:** Face recognition, kernel, Fisher's Linear Discriminant, pseudo Zernike moments

**Classification:** Science and engineering for electronics

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## 1 Introduction

Moment functions model an image into a compact subspace represented by moment bases. Small number of moments is sufficient to describe most of the image structure. Therefore, Haddadnia et al. employed moment functions in face recognition [1]. [1] and [2] reported that pseudo Zernike moments show the recognition rate of 98.73% by using radial basis function neural network classifier, and 93.46% using Euclidean distance, respectively. Indeed, the performance of moment functions can be further improved by integrating a kernel and discriminant functions into moment analysis without resorting any complex classifier, likes neural network. Thus, in this paper, Kernel-based Fisher Pseudo Zernike Moments (KFPZM) method is proposed for face recognition.

Inspired by the ideas of Kernel PCA [3] and Kernel LDA [4], KFPZM combines a kernel trick with pseudo Zernike moments (PZM) that coupled with Fisher's Linear Discriminant (FLD). This integration not only can account for the variables that implicitly underlie on the image structure, but also enhances the discrimination power of the moments. Fig. 1 illustrates the overview of KFPZM. In this method, the image data is firstly transformed into a set of moment-based features via PZM computation. Then, these features are further mapped into a higher dimensional feature space via kernel function to disclose the underlying variables which carry more significant and expressive information about the image. Finally, the mapped features are input into FLD to further enhance the discrimination power of the features by maximizing the between-class scatter and minimizing the within-class scatter.

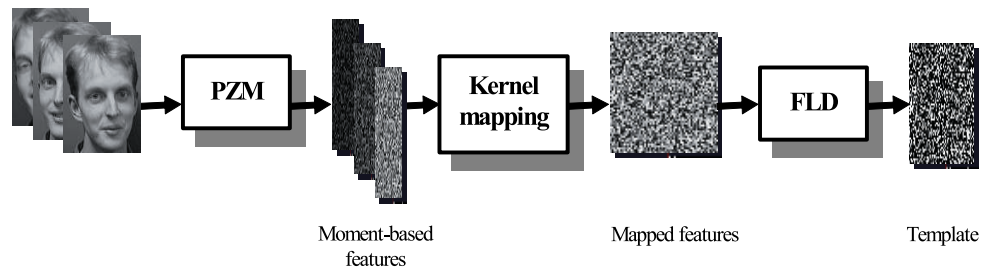


Fig. 1. Overview of KFPZM method.

## 2 Pseudo Zernike moments

The two-dimensional pseudo Zernike moments of order  $p$  with repetition  $q$  of image intensity function,  $f(r, \theta)$ , are defined as [5, 6]:

$$PZ_{pq} = \frac{p+1}{\pi} \int_0^{2\pi} \int_0^1 V_{pq}(r, \theta) f(r, \theta) r dr d\theta \quad (1)$$

where Zernike polynomials  $V_{pq}(r, \theta)$  are defined as:

$$V_{pq}(r, \theta) = R_{pq}(r) e^{-j\hat{q}\theta}; \quad \hat{j} = \sqrt{-1} \quad (2)$$

and  $r = \sqrt{x^2 + y^2}$ ,  $\theta = \tan^{-1}(\frac{y}{x})$ ,  $-1 < x, y < 1$

The real-valued radial polynomials are defined as:

$$R_{pq}(r) = \sum_{s=0}^{p-|q|} (-1)^s \frac{(2p+1-s)!}{s!(p+|q|+1-s)!(p-|q|-s)!} r^{p-s} \quad (3)$$

and  $0 \leq |q| \leq p$ ,  $p \geq 0$

In this study, the considered features are the magnitude of  $PZ_{pq}$ , abbreviated as  $PZ_{pq,mag}$ , due to their rotation invariant property; the phase information is omitted as the influence of phase information is rather insignificant especially when high order moments are included [7].

### 3 Kernel-based Fisher Pseudo Zernike Moments

PZM transforms face images into moment-based feature vectors,  $PZ_{pq,mag}$ , in input space,  $R$ . Then, this input data is projected into a higher dimensional implicit feature space,  $F$ , via nonlinear mapping,  $\Phi : PZ_{pq,mag} \in R \rightarrow f \in F$ . Next, FLD is performed onto the mapped features in order to maximize the between-class scatter and minimize the within-class scatter. It is no need to compute  $\Phi$  explicitly but just compute the inner product of the two vectors,  $f$ , in  $F$  with a kernel function:

$$k(PZ_{pq,mag j}, PZ_{pq,mag k}) = (\Phi(PZ_{pq,mag j}) \cdot \Phi(PZ_{pq,mag k})) \quad (4)$$

In feature space,  $F$ , the between- and within-class scatter,  $\hat{S}_b$  and  $\hat{S}_w$  respectively, are defined as:

$$\hat{S}_b = \sum_{i=1}^C n_i (\mu_i - \mu) (\mu_i - \mu)^T \quad (5)$$

$$\hat{S}_w = \sum_{i=1}^C \sum_{k=1}^{n_i} (\Phi(PZ_{pq,mag ik}) - \mu_i) (\Phi(PZ_{pq,mag ik}) - \mu_i)^T \quad (6)$$

where

$$\text{class mean, } \mu_i = \frac{1}{n_i} \sum_{k=1}^{n_i} \Phi(PZ_{pq,mag k}) \quad (7)$$

$$\text{grand mean, } \mu = \frac{1}{n} \sum_{j=1}^n \Phi(PZ_{pq,mag j}) \quad (8)$$

Note that  $C$  is the number of classes,  $n$  is the total number of mapped samples,  $n_i$  is the number of mapped samples in  $i$ th class and  $PZ_{pq,mag ik}$  is the  $k$ th moment-based feature vector in  $i$ th class.

To perform FLD in  $F$ , it is equal to maximize expression [4],

$$J(w) = \frac{w^T \hat{S}_b w}{w^T \hat{S}_w w} \quad (9)$$

Because any solution  $w \in F$  must lie in the span of all the samples in  $F$ , there exist coefficients  $\alpha_j$ ,  $j = 1, 2, \dots, n$ , such that

$$w = \sum_{j=1}^n \alpha_j \Phi(PZ_{pq,mag j}) \quad (10)$$

By expression (10), the projection of each class mean  $\mu_i$  onto  $w$  can be written

$$\begin{aligned}
 w^T \mu_i &= \alpha^T \sum_{j=1}^n \Phi(PZ_{pq, mag j}) \frac{1}{n_i} \sum_{k=1}^{n_i} \Phi(PZ_{pq, mag k}) \\
 &= \alpha^T \begin{bmatrix} \frac{1}{n_i} \sum_{k=1}^{n_i} \Phi(PZ_{pq, mag 1}) \cdot \Phi(PZ_{pq, mag k}) \\ \frac{1}{n_i} \sum_{k=1}^{n_i} \Phi(PZ_{pq, mag 2}) \cdot \Phi(PZ_{pq, mag k}) \\ \vdots \\ \frac{1}{n_i} \sum_{k=1}^{n_i} \Phi(PZ_{pq, mag n}) \cdot \Phi(PZ_{pq, mag k}) \end{bmatrix} \\
 &= \alpha^T \begin{bmatrix} \frac{1}{n_i} \sum_{k=1}^{n_i} k(PZ_{pq, mag 1}, PZ_{pq, mag k}) \\ \frac{1}{n_i} \sum_{k=1}^{n_i} k(PZ_{pq, mag 2}, PZ_{pq, mag k}) \\ \vdots \\ \frac{1}{n_i} \sum_{k=1}^{n_i} k(PZ_{pq, mag n}, PZ_{pq, mag k}) \end{bmatrix} \\
 &= \alpha^T M_i
 \end{aligned} \tag{11}$$

It follows that  $w^T \hat{S}_b w = \alpha^T K_b \alpha$ , where  $K_b = \sum_{i=1}^C n_i (M_i - M)(M_i - M)^T$ ,

$$\text{and } M = \begin{bmatrix} \frac{1}{n} \sum_{k=1}^n k(PZ_{pq, mag 1}, PZ_{pq, mag k}) \\ \vdots \\ \frac{1}{n} \sum_{k=1}^n k(PZ_{pq, mag n}, PZ_{pq, mag k}) \end{bmatrix}; \text{ similar to within-class scat-}$$

ter,  $w^T \hat{S}_w w = \alpha^T K_w \alpha$ , where  $K_w = \sum_{i=1}^C \sum_{k=1}^{n_i} (\xi_k - M_i)(\xi_k - M_i)^T$ , and  $\xi_k = (k(PZ_{pq, mag 1}, PZ_{pq, mag k}), k(PZ_{pq, mag 2}, PZ_{pq, mag k}), \dots, k(PZ_{pq, mag n}, PZ_{pq, mag k}))^T$ . The kernel trick used in this study is Gaussian function:

$$k(x, y) = \exp(-||x - y||^2 / \sigma) \tag{12}$$

where  $\sigma$  is the Gaussian parameter.

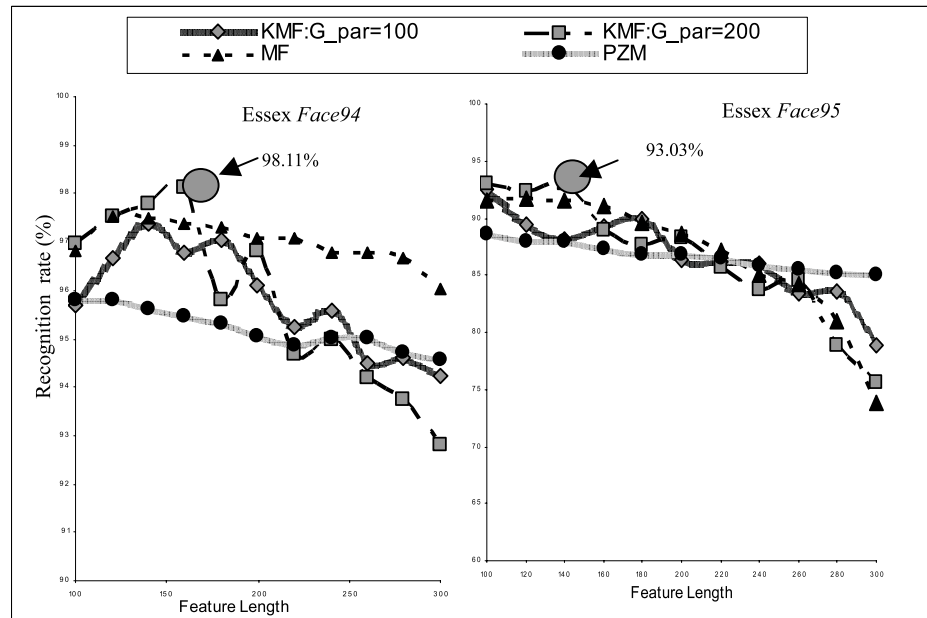
## 4 Experiment

We evaluate the performance of KFPZM method by using two face databases: Essex *Face94* with facial expression variations and Essex *Face95* with illumination variations. These databases are made publicly available at URL <http://cswwww.essex.ac.uk/mv/allfaces/index.html>. In both databases, 10 face samples are used for training and another 10 for testing for each subject. There are 100 and 72 face classes in *Face94* and *Face95* databases. Comparative analysis is carried out among PZM, integrated FLD and PZM (MF) and KFPZM methods by using Euclidean distance as classifier.

Excessive higher order moments comprise redundant and unwanted information, such as noise, and these high frequency variables may influence the discrimination power. Thus, the performance of KFPZM method with different moment orders is evaluated to justify the statement.

## 5 Results and discussions

Fig. 2 shows the recognition rates of the PZM, MF and KFPZM methods in both face databases. From Fig. 2, we can see that KFPZM with kernel parameter,  $\sigma = 200$  (represented by KMF : G\_par = 200) obtain the highest recognition rate of 98.11% (with moment length 160) in *Face94* database and 93.03% (with moment length 140) in *Face95* database. This demonstrate that the integration of kernel and FLD with moments helps enhance the discrimination power by disclosing the underlying significant image features and maximizing the between-class scatter while minimizing the within-class scatter.



**Fig. 2.** Recognition rates of PZM, MF and KFPZM methods.

However, KFPZM obtains poorer result when higher order moments are included. This is because higher order moments bring in high frequency information and redundant data that affect the separability of KFPZM. The statement is further justified in Table I that KFPZM method with lower order moments performs better than that with higher order moments.

## 6 Conclusion

A new approach by using pseudo Zernike moments integrated with kernel and discriminant functions has been presented in this paper in order to boost the recognition performance of pseudo Zernike moments on face images. This

**Table I.** Performance on different order moments in KF-PZM method.

Face database	Kernel parameter, $\sigma$	Moments in order	FAR (%)	FRR (%)	TSR (%)
<i>Face94</i>	100	1 to 100	4.30	4.30	95.70
		100 to 200	10.43	10.40	89.57
		200 to 300	17.93	18.10	82.06
		300 to 400	21.98	21.70	78.03
		400 to 500	24.61	24.40	75.40
	200	1 to 100	3.02	3.00	96.98
		100 to 200	12.08	12.10	87.92
		200 to 300	16.33	16.4	83.67
		300 to 400	23.55	23.40	76.46
		400 to 500	24.30	24.20	75.70
<i>Face95</i>	100	1 to 100	7.50	7.50	92.50
		100 to 200	17.14	17.50	82.84
		200 to 300	26.16	26.25	73.84
		300 to 400	27.72	27.78	72.28
		400 to 500	29.61	29.44	70.40
	200	1 to 100	6.94	6.95	93.04
		100 to 200	14.88	14.86	85.12
		200 to 300	20.63	20.70	79.37
		300 to 400	31.06	31.25	68.93
		400 to 500	32.51	32.36	67.50

method is called Kernel-based Fisher Pseudo Zernike Moments which hybrids the Gaussian function and Fisher's Linear Discriminant with pseudo Zernike moments. In this approach, the kernel function maps the moment-based features into a high dimensional space so that the underlying informative features can be revealed. Fisher's Linear Discriminant is then performed onto the mapped features to maximize the between-class scatter and minimize the within-class scatter for increasing the separability of the features. Experimental results show that the proposed method achieves superior performance than the PZM and MF methods.

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