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A new MMSE channel estimation algorithm for OFDM systems

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Abstract: In this report, we propose a new MMSE channel estimation algorithm for OFDM systems. It is well-known that time domain maximum likelihood estimators (MSEs) show highly accurate impulse response estimation by using the time domain long preamble of the OFDM frame. On the other hand, the impulse response estimation based on the minimum mean square error (MMSE) criterion achieves superior channel estimation in low SNR conditions; however, it requires prior statistical information such as delay profiles of channels. In addition, the computational complexity becomes large because of the inverse matrix calculation. In order to overcome these issues, we propose to estimate the power delay profile of the channel by using the MLE, and to suppress the increase of the computational complexity for the matrix inverse calculation by applying the conjugate gradient method without degradation of the PER performance.

Keywords: OFDM, channel estimation, ML, MMSE

Classification: Science and engineering for electronics

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1 Introduction

The OFDM (Orthogonal Frequency Division Multiplexing) systems have been applied to many wireless communication standards because of its superior frequency efficiency and robustness for frequency selective fading channels. In addition, recent semiconductor technologies make it possible to realize very high performance channel decoders such as Turbo decoders and LDPC decoders. These trends impose the receivers to operate in very low signal-to-noise ratios (SNRs), which results in the increase of the channel estimation error. Because of this phenomenon, the powerful channel decoders cannot give its maximum ability.

Generally, the frequency domain channel estimators have been applied to the OFDM receivers because of its simplicity. The scattered-pilots are the typical training signals for the frequency channel estimators [1]. On the other hand, a long preamble, that is attached on the head of the packet frame of the IEEE802.11a/g, makes it possible to carry out the channel estimation in the time domain [2]. By using the time domain long preamble, we can estimate the impulse response of the unknown channels and can obtain the frequency domain channel information by the FFT. It was shown that the time domain approach improved the performance by assuming to constrain the impulse response length within the guard intervals [3].

The maximum likelihood estimator (MLE) and minimum mean square error estimator (MMSEE) were proposed as the time domain channel estimation algorithms in [4]. These algorithms showed superior channel estimation performance. The MLE is the promising approach from the implementation point of view. On the other hand, the MMSEE shows superior BER performance in low SNRs compared with the MLE; however, the implementation of the MMSEE is difficult because it requires prior information of the power delay profile of the channel and the noise power in the receiving signals in addition to the calculation of the inverse matrix.

In this report, we propose a new MMSE algorithm for the OFDM systems. In order to overcome the difficulties mentioned above, we propose to estimate the power delay profiles of the channels based on the MLE. In addition, we show that we can restrain the increase of the computational complexity by applying the conjugate gradient method for the matrix inverse calculation while achieving the same performance with the ideal MMSEE.

2 ML and MMSE channel estimators

2.1 OFDM system structure

The transmitter and receiver structure of the OFDM system is based on the IEEE802.11a/g physical layer. The data from channel coding are interleaved and digital modulation such as the BPSK is applied. The modulated signals are transformed into the time domain signals by an IFFT and transmitted after attaching a long preamble on the head of the frame. The received signals are divided into the data part and preamble. The channel estimation for the coherent detection is carried out by using the received long preamble.

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2.2 Maximum likelihood estimator

We can carry out the impulse response estimation of unknown channels by using the time domain long preamble as shown in Fig. 1 [4]. The estimated impulse response is transformed into the frequency domain by the FFT to make it possible to apply the frequency domain equalizer.

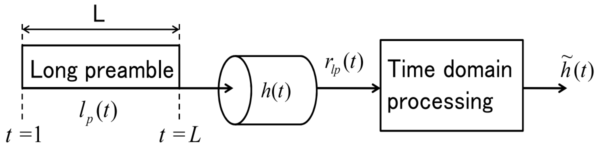

Fig. 1. An impulse response estimator.

We assume that a time domain long preamble, $l_p(t)$, is received through the channel, $h(t)$, as shown in Fig. 1. Then, the received signal vector can be written as

$$\mathbf{r}_{lp} = \mathbf{L}_p\mathbf{h} + \mathbf{n}, \tag{1}$$

where $\mathbf{L}_p$, $\mathbf{h}$ and $\mathbf{n}$ denote the preamble matrix, the channel impulse response vector and the noise vector, respectively, that is,

$$\boldsymbol{r}_{lp} = [r_{lp}(\Delta) \quad r_{lp}(\Delta+1) \quad \cdots \quad r_{lp}(L)]^T, \tag{2}$$

$$\boldsymbol{h} = [h_1 \quad h_2 \quad \cdots \quad h_\Delta]^T, \tag{3}$$

$$\boldsymbol{L_p} = \begin{bmatrix} l_p(\Delta) & l_p(\Delta-1) & \cdots & l_p(1) \\ l_p(\Delta+1) & l_p(\Delta) & \cdots & l_p(2) \\ \vdots & \vdots & \vdots & \vdots \\ l_p(L) & l_p(L-1) & \cdots & l_p(L-\Delta+1) \end{bmatrix}, \tag{4}$$

$$\boldsymbol{n} = [n(\Delta) \quad n(\Delta+1) \quad \cdots \quad n(L)]^T, \tag{5}$$

where L and Δ are the long preamble length and impulse response length of the channel, respectively.

The MLE of the unknown channel is written as

$$\mathbf{h}_{ml} = (\mathbf{L}_p^H\mathbf{L}_p)^{-1}\mathbf{L}_p^H\mathbf{r}_{lp} = \mathbf{T}_{ml}\mathbf{r}_{lp}. \tag{6}$$

It is noticeable that we can obtain $\mathbf{T}_{ml}$ in advance at the receivers. This contributes to reduce the computational complexity of the MLE.

2.3 Minimum mean square error estimator

The MMSEE is obtained by

$$\mathbf{h}_{mmse} = (\sigma^2\mathbf{C}_h^{-1} + \mathbf{L}_p^H\mathbf{L}_p)^{-1}\mathbf{L}_p^H\mathbf{r}_{lp}, \tag{7}$$

where $\mathbf{C}_h$ denotes the covariance matrix of the channel, that becomes a diagonal matrix if fading coefficients are statistically independent each other, that is,

$$\mathbf{C}_h = diag\{\ E[|h_1|^2] \quad E[|h_2|^2] \quad \cdots \quad E[|h_\Delta|^2]\ \}. \tag{8}$$

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From Eq. (8) the diagonal component of $\mathbf{C}_h$ is equivalent to a power delay profile of the channel. As shown in Eq. (7), $\mathbf{h}_{mmse}$ requires the prior information of the power delay profile and the noise power, σ^2.

3 Proposed scheme

3.1 A new MMSE algorithm

From Eq. (7),

$$\boldsymbol{h}_{mmse} = \boldsymbol{R}^{-1}\boldsymbol{P}, \tag{9}$$

where

$$\boldsymbol{R} = (\sigma^2\boldsymbol{I} + \boldsymbol{X}), \quad \boldsymbol{P} = \boldsymbol{X}\boldsymbol{h}_{ml}, \quad \boldsymbol{X} = \boldsymbol{C}_h\boldsymbol{L_p}^H\boldsymbol{L_p}. \tag{10}$$

These equations suggest that the MMSEE can be produced by using the MLE and additional information. The proposed MMSE algorithm is shown in the Table I. Since $\boldsymbol{R}$ is the Hermitian matrix, we can solve the simultaneous equation by using the conjugate gradient method (CGM). Initially, we prepare a few matrices by using the preamble matrix, $\boldsymbol{L}_p$. $\boldsymbol{h}_{ml}$ is obtained by the ML scheme before estimating $\boldsymbol{C}_h$. $f_h(\mathbf{h}_{ml})$ denotes to execute the power delay profile estimation. By using these results $\boldsymbol{R}$ is constructed. After that we finally obtain $\boldsymbol{h}_{mmse}$ by using the CGM. From our experience, a few iterations of the CGM is enough to obtain sufficient PER performance. This contributes to restrain the increase of the computational complexity. The CGM is shown in Table II.

Table I. MMSE channel estimation algorithm.

1.Initialization	$\boldsymbol{L}_p \leftarrow$ $\boldsymbol{A} = \boldsymbol{L}_p^H\boldsymbol{L}_p$ $\boldsymbol{B} = \boldsymbol{A}^{-1}$
2.Long preamble	$\boldsymbol{r}_{lp} \leftarrow$
3.MLE	$\boldsymbol{h}_{ml} = \boldsymbol{B}\boldsymbol{L}_p^H\boldsymbol{r}_{lp}$
4.MMSE parameters	σ^2 is to be constant. $\boldsymbol{C}_h = f_h(\boldsymbol{h}_{ml})$
5.CGM initialization	$\boldsymbol{X} = \boldsymbol{C}_h\boldsymbol{A}$ $\boldsymbol{R} = (\sigma^2\boldsymbol{I} + \boldsymbol{X})$ $\boldsymbol{P} = \boldsymbol{X}\boldsymbol{h}_{ml}$ $ITE \leftarrow$
6.CGM	$\boldsymbol{h}_{mmse} = CGM(\boldsymbol{R}, \boldsymbol{P}, ITE)$

3.2 Handling of the σ^2 and C_h

As shown in Eq. (7) the noise power, σ^2, and the covariance matrix, $\boldsymbol{C}_h$ are required for the MMSE channel estimation.

Concerning the noise power we give a constant value that is based on the operating point of the SNR. According to [5], giving a constant SNR value has resulted in negligible estimation error. We refer to this result.

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Table II. CGM.

1.Input parameters	$\mathbf{R}, \mathbf{P}, ITE$
2.Initialization	$\mathbf{h}_{mmse} = \mathbf{0}$
3.Initial setting	$\mathbf{r} = \mathbf{P},\ \mathbf{p} = \mathbf{r}$
4.Start of loop	for $LOOP = 1 : ITE$
5.	$\mathbf{R}_p = \mathbf{R} \cdot \mathbf{p}$
6.	$D = \mathbf{p}^H \cdot \mathbf{R}_p$
7.	$\alpha = \frac{\mathbf{r}^H \cdot \mathbf{R}_p}{D}$
8.	$\mathbf{h}_{mmse} = \mathbf{h}_{mmse} + \alpha \mathbf{p}$
9.	$\mathbf{r}_p = \mathbf{r}$
10.	$\mathbf{r} = \mathbf{r} - \alpha \mathbf{R}_p$
11.	$\beta = \frac{\mathbf{r}^H \mathbf{r}}{\mathbf{r}_p^H \mathbf{r}_p}$
12.	$\mathbf{p} = \mathbf{r} + \beta \mathbf{p}$
13.End of loop	end

In contrast to the noise power, we need to estimate $\boldsymbol{C}_h$ as properly as possible. When each ray of the impulse response is independently fluctuated based on the Rayleigh fading principle, $\boldsymbol{C}_h$ becomes a diagonal matrix whose diagonal elements are given by

$$\boldsymbol{h}_d = \ [E[|h_1|^2] \quad E[|h_2|^2] \quad \cdots \quad E[|h_\Delta|^2]]^T. \tag{11}$$

In order to obtain Eq. (11) we use the MLE, $\boldsymbol{h}_{ml}$, in Table I. When an MLE at k^{th} receiving frame is defined by $\boldsymbol{h}_{ml}(k)$, the mean value of the MLE, $\tilde{\boldsymbol{h}}_{ml}(k)$, is written as a low pass filter output which is given by

$$\tilde{\boldsymbol{h}}_{ml}(k) = \mu \tilde{\boldsymbol{h}}_{ml}(k-1) + (1-\mu)\boldsymbol{h}_{ml}(k). \tag{12}$$

Here we assume that the difference of the fading coefficients between neighbor frames is negligible. The power average of $\tilde{\boldsymbol{h}}_{ml}(k)$ is given by

$$\begin{aligned} P_{h,i}(k) &= \xi P_{h,i}(k-1) + (1-\xi)|\tilde{h}_{ml,i}(k)|^2, \\ i &= 1, 2, \cdots, \Delta. \end{aligned} \tag{13}$$

Finally, the estimator of $\boldsymbol{C}_h$ at k^{th} frame becomes as

$$\tilde{\boldsymbol{C}}_h(k) = \ diag\{P_{h,1}(k) \quad P_{h,2}(k) \quad \cdots \quad P_{h,\Delta}(k)\}. \tag{14}$$

4 Simulation results

4.1 Simulation scenarios

In order to consider the performance of MMSE estimators at low SNR conditions, we have assumed the IEEE802.11a PHY and applied BPSK modulation with the convolutional coding of $K = 7$ and $R = \frac{1}{2}$. The packet length was 100 octets. An exponentially attenuated delay profile with delay spread of 100 ns was given for the channel. The normalized Doppler frequency, $\bar{f}_d = f_d \times T_f$, was 3.3×10^{-3}. T_f denotes the frame length, which was 148 micro-seconds. To relax the fading effect the receiving antenna diversity was given, and no correlations between receiving antennas were assumed.

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4.2 The estimation of C_h

The optimum values concerning the low pass filter coefficients, μ and ξ, have been evaluated by simulations. The results have shown that 0.97 and 0.9 are proper for μ and ξ, respectively.

4.3 Packet error rates

The PER curves of the Ideal MMSE estimator and the proposed scheme are shown in Fig. 2. The number of iterations of the CGM was five because the number was enough to obtain the proper solution while it required 16 iterations for the complete solution. The results confirm that the MMSE channel estimator show superior PER performance compared with that of the MLE, and proposed scheme achieves almost the same performance with that of the ideal MMSE estimator.

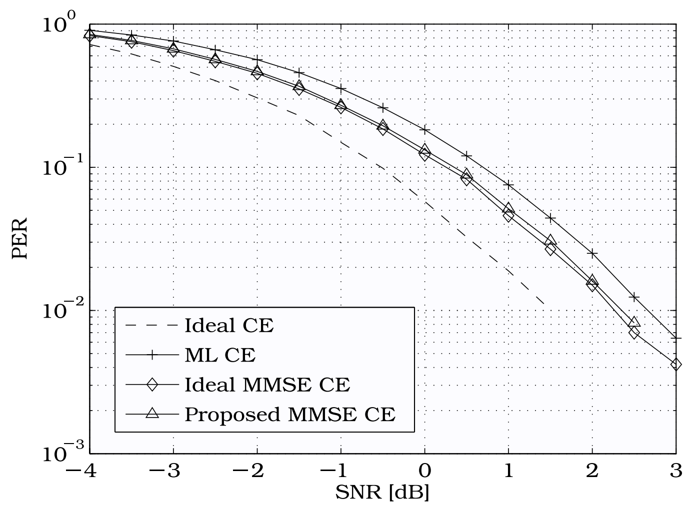

Fig. 2. PER curves.

5 Conclusion

In this report, we have proposed a new MMSE channel estimation algorithm by using the time domain long preamble for the OFDM systems. The ML channel estimation was executed before carrying out the MMSE channel estimation in order to produce the channel covariance matrix. By using the channel covariance matrix the MMSE channel estimation was executed, and the PER performance was equivalent to that of the ideal MMSE estimator. In addition, the CGM, which was applied to solve the simultaneous equations, played an important role in restraining the computational complexity.

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