

ESSAYS ON LABOR HETEROGENEITY AND MACROECONOMICS

by

Brendan Epstein

A dissertation submitted in partial fulfillment
of the requirements of the degree of
Doctor of Philosophy
(Economics)
in The University of Michigan
2011

Doctoral Committee:

Professor Miles S. Kimball, Co-Chair

Professor Michael William Leamington Elsby, University of Edinburgh, Co-Chair

Professor Brian Aaron Jacob

Professor Jeffrey Andrew Smith

© Brendan Epstein 2011

To my parents, Margarita and Julio,
and Emilse, Abu, Esteban, Olga, Walter
Blanco y Marron.
Thank you for paving the road.

ACKNOWLEDGEMENTS

I am very grateful to my doctoral committee, Mike Elsby, Miles Kimball (my co-author on my second essay), Jeff Smith, and Brian Jacob for their invaluable help, support, and advise. I am also grateful to Ronda Abrigo, Mary Braun, and Olga Mustata, who have helped make everything so much easier for me throughout graduate school. Finally, I am very thankful to Dimitriy Masterov, Shanthi Ramnath (my co-author on my third essay), and Elena Spatoulas for their friendship and countless contributions to my work.

TABLE OF CONTENTS

DEDICATION	ii
ACKNOWLEDGEMENTS	iii
LIST OF FIGURES	vii
LIST OF TABLES	ix
CHAPTER	
I. Introduction	1
II. Heterogeneous Workers, Optimal Job Seeking, and Aggregate Unemployment	6
2.1 Introduction	6
2.2 The Model	10
2.2.1 Stocks and Flows	11
2.2.2 Workers	14
2.2.3 Firms	15
2.2.4 Surpluses, Wages, and Equilibrium	16
2.2.5 The Role of Worker-Side Heterogeneity	17
2.2.6 The Role Optimal Effective Search	20
2.2.7 Nested Models	24
2.3 Simulation Methodology and Calibration	25
2.3.1 Simulation Methodology	25
2.3.2 Parameter Values	26
2.4 Results	28
2.4.1 The V/U Ratio and Unemployment	28
2.4.2 Propagation	29
2.4.2.1 Propagation in the MA Model	29
2.4.2.1 Propagation in the MA-OS Model	31

2.4.3 The V/U Adjustment Process: Temporary Shocks	32
2.4.4 Match Quality	34
2.5 Related Literature	35
2.6 Conclusions	37
2.7 Figures	39
2.8 Tables	42
2.9 Proofs	43
2.10 References	47
III. The Paradox of Hard Work	50
3.1 Introduction	50
3.2 The General Framework	54
3.2.1 Households	55
3.2.1 Firms	58
3.3 Equilibrium	61
3.3.1 Analysis: Changes in Technology and Drudgery	63
3.3.2 Heterogeneity in the Labor Force	65
3.4 Heterogeneity in Production	67
3.4.1 Differences in Final Goods Producers	67
3.4.2 Industry-Level Differences	69
3.4.3 Firm-Level Incentives for Drudgery Declines	72
3.5 The Role of Amenities	75
3.6 Work Hours and Welfare	78
3.6.1 Equilibrium Work Hours	78
3.6.2 Welfare Under Additive Separability	79
3.6.3 Welfare Under Non-Separability	82
3.7 Conclusions	86
3.8 Figures	87
3.9 Derivations	93
3.9.1 Normalization	93
3.9.2 Details on Incentives for Drudgery Declines	94

3.9.3 Amenities	95
3.9.4 Welfare and Non-Separability	95
3.10 References	96
IV. Beyond Taxes: Understanding the Labor Wedge	98
4.1 Introduction	98
4.2 The Labor Wedge	102
4.2.1 The Standard Model	103
4.2.2 The Model with Taxes	107
4.3 The Role of the E/P Ratio	109
4.4 Heterogeneity	112
4.5 Applications of the Model	118
4.5.1 Hours per Worker	119
4.5.2 Net Employment Disutility	120
4.5.3 Comparative Statics	123
4.5.4 Average vs. Marginal Tax Rates	127
4.6 Related Literature	130
4.7 Conclusions	131
4.8 Data Sources and Summary Statistics	134
4.9 Derivations	135
4.10 Figures	140
4.11 Tables	150
4.12 References	151
V. Conclusions	152

LIST OF FIGURES

2.1	Response of V/U ratio to permanent increase in p	39
2.2	Response of u to permanent increase in p	39
2.3	Response of V/U ratio to permanent increase in p	39
2.4	Response of χ and u to permanent increase in p	39
2.5	Response of π_2^1 and Θ to permanent increase in p (MA model)	40
2.6	Response of π_2^1 and Θ to increase in p in (MA-OS model)	40
2.7	Response of effective search to permanent increase p (MA-OS model)	40
2.8	Response of χ and u to permanent increase in p (MA-OS model)	40
2.9	Response of V/U ratio to temporary increase in p (MA-OS model)	41
2.10	Temporary increase in p	41
2.11	Response of v and u to temporary increase in p (MA-OS model)	41
2.12	Response of χ/e to permanent increase in p	41
3.0.A	US consumption and work hours	87
3.0.B	Japan consumption and work hours	87
3.0.C	G-7 consumption and work hours (except US and Japan)	88
3.0.D	European aggregate consumption and work hours	88
3.1	Increase in W and increase in W and J	88
3.2	Increase in Y and increase in Y and J	88
3.3	Job utility	88
3.4	Labor supply	89
3.5	Partial equilibrium	89
3.6	General equilibrium under perfect competition	89
3.7	Determination of labor-earnings supply	89
3.8	Labor-earnings supply and demand	90
3.9	Effects of an increase in technology	90

3.10	Effects of a decline in drudgery	90
3.11	Solution under labor-force heterogeneity	90
3.12	Differences in equilibrium variables given differences in hungriness	91
3.13	Differences in final goods producers' job utility and technology, part 1	91
3.14	Differences in final goods producers' job utility and technology, part 2	91
3.15	Differences in final goods producers' job utility and technology, part 3	91
3.16	Differences in final goods producers' job utility and technology, part 4	92
3.17	Industry-specific firm-level optimality	92
3.18	Industry equilibrium variables given firm-level differences	92
3.19	Labor-earnings supply given indifference between jobs	92
3.20	Labor-earnings supply and demand given changes in relative prices	92
3.21	Labor-earnings supply and demand given changes in wealth	92
3.22	Monopolistic competition: impact of an increase in technology	93
3.23	Monopolistic competition: impact of a decline in drudgery	93
3.24	Firm-level optimality when amenities are a choice variable	93
3.25	Amenities: impact of an increase in wealth	93
4.1	Neoclassical model-generated H/P for $\varepsilon \in \{1, 4\}$	140
4.2	Actual H/P and neoclassical model-generated H/P	141
4.3	Labor wedge associated with neoclassical model-generated H/P	141
4.4	Actual H/P and H/P predicted by neoclassical model with taxes	142
4.5	Labor wedges associated with NM and NMT model-generated H/P	143
4.6	Contribution of H/E and E/P to changes in E/P	144
4.7	Actual H/E and predicted H/E using NMT alternative normalization	145
4.8	Actual H/P and hybrid H/P	146
4.9	Labor wedge associated with NMT and E/P	147
4.10	Actual H/E vs. model-predicted H/E , $\psi^E \in \{0.1, 0.5, 0.9\}$, Canada	147
4.11	Actual H/E vs. model-predicted H/E , $\psi^E \in \{0.1, 0.5, 0.9\}$, France	148
4.12	Actual H/E vs. model-predicted H/E , $\psi^E \in \{0.1, 0.5, 0.9\}$, Germany	148
4.13	Actual H/E vs. model-predicted H/E , $\psi^E \in \{0.1, 0.5, 0.9\}$, US	149
4.14	Actual H/E vs. model-predicted H/E using imputed weights	149

LIST OF TABLES

2.1	MA-OS model calibration (weekly frequency)	42
2.2	Elasticities with respect to output per worker	42
2.3	Standard Model calibration	42
2.4	Standard-OS and MA model calibrations	43
4.0.A	H/P summary statistics	134
4.0.B	H/E summary statistics	134
4.0.C	E/P summary statistics	134
4.0.D	Y/P summary statistics	134
4.0.E	C/P summary statistics	135
4.0.F	C/Y summary statistics	135
4.0.G	$(1 - \tau)$ summary statistics	135
4.1	Imputed optimal weights and correlations	150
4.2	Net employment disutility and the E/P ratio	150

CHAPTER I

INTRODUCTION

This dissertation is comprised of three essays, which examine various aspects of macroeconomics linked to labor economics. The major themes of these essays contribute to developing a better understanding of when heterogeneity matters for macroeconomic outcomes, of what factors can play an important role in the determination of the relative trend behavior of work hours, and also of what factors, in addition to taxes, can account for the labor wedge.¹ Each of these topics is representative of recent and ongoing research in the joint area of macro and labor economics.

The first essay develops an understanding of how, in addition to search frictions, labor heterogeneity can influence aggregate labor-market fluctuations, and, in particular, the cyclical behavior of aggregate unemployment. I capture heterogeneity by considering a labor force composed of individuals who have a comparative advantage in a particular job, yet are still able to work in jobs in which they are at a comparative disadvantage. I assume no worker has an absolute advantage in production. In addition, I endogenize the optimal job-seeking behavior across job opportunities of all searchers, both those unemployed and those searching on the job. On-the-job search results from workers who are employed in jobs in which they are at a comparative disadvantage, but search for comparative advantage employment.

The extent to which vacancies and unemployment coexist is summarized by the aggregate vacancy-unemployment (V/U) ratio. Empirically, in the US, the V/U ratio is strongly procyclical, and part of its adjustment in response to changes in productivity is sluggish. Under standard calibrations the benchmark, homogeneous-agent

¹The *labor wedge* is the name given to the percent difference between labor hours predicted by a standard neoclassical macroeconomic model and their empirical counterparts.

model of equilibrium unemployment theory can account for slightly less than half of the empirical elasticity of the V/U ratio with respect to productivity. However, contrary to the data, all adjustments occur instantaneously. Results from the model developed in the first essay suggest that the impact of worker-side heterogeneity and optimal job-seeking behavior can be substantial. Quantitative analysis reveals that the model can account for the majority of the empirical elasticity of the V/U ratio with respect to productivity in the United States. In addition, the theory uncovers a natural channel through which adjustments in the V/U ratio can be slow moving. Given heterogeneity, vacancy-posting decisions are based on firms' expectations regarding match quality. These expectations depend on worker-side optimal job-seeking behavior. In addition, they also depend on the (slow-moving) masses of unemployed and on-the-job searchers. Ultimately, this results in slow-moving expected gains from posting vacancies, which contributes to sluggish adjustment in the V/U ratio in response to changes in productivity.

Over the last centuries there has been a dramatic world-wide increase in real output, consumption, and wages. John Maynard Keynes predicted a large increase in leisure in his 1930 essay "Economic Possibilities for Our Grandchildren."² However, the leisure boom predicted by Keynes has not taken place; instead work hours have remained relatively constant compared to trends in other aggregate variables. The objective of the second essay, which is co-written with Miles S. Kimball,³ is to understand why people are still working so hard, and what the implications of this paradox of hard work are for the economy as a whole. In particular, we develop a theory that focuses on the long-run macroeconomic consequences of changes in on-the-job utility.

A typology of different sources of improvement in on-the-job utility elucidates how these improvements can be endogenized. Amenities, such as air-conditioning, increase due to both income effects and ordinary technological progress. Effort - which can be defined broadly as undesirable dimensions of a job that lead to higher output - behaves in a way qualitatively similar to hours per worker. However, unlike

²Printed in Vol. IX of *The Collected Writings of JM Keynes*, 1973. London: Macmillan for The Royal Economic Society.

³Miles is a Professor of Economics at the University of Michigan, Ann Arbor.

work hours, effort is unmeasured in standard data series. Unobserved movements in effort can act as substitutes for movements in hours per worker. A decline in drudgery represents an increase in on-the-job utility for a given level of effort and amenities. Such increases in on-the-job utility lead to firms gaining a competitive advantage. Therefore, profit-driven innovation is a natural channel through which declines in drudgery can occur. These declines, along with endogenous increases in amenities, tend to increase hours per worker and effort. This serves to counteract decreases in work hours and effort that would otherwise take place when income effects exceed substitution effects.

The third essay, which is joint with Shanthi P. Ramnath,⁴ relates to recent literature that has shown that the standard neoclassical macroeconomic model falls short of explaining the trend behavior of hours per population within countries. This is captured by the extent to which the model's first-order conditions for equilibrium hours per population fail to hold: the labor wedge. The literature argues that across countries a substantial fraction of the labor wedge can be explained when the standard model is enhanced to account for taxes. However, this improvement is limited to European countries. In particular, the model's predictions regarding the trend behavior of US and Canadian hours per population are for all purposes contrary to the data. While over the last several decades these two countries have exhibited an upward trend in hours per population, the standard model enhanced with taxes predicts that the opposite should have occurred. This suggests that the labor wedge amounts to more than just taxes. The aim of the third essay is to understand what factors, in addition to taxes, can account for the labor wedge.

The analysis we develop implies a surprising result, which is that the limitations of the standard model in accounting for the long-run behavior of hours per population in Canada and the US are actually evidence of the model's *overall* inability to explain the behavior of hours per population. The standard model implicitly assumes that all household members are employed and work the same amount of hours. Therefore, it has no channels through which adjustments in the employment-to-population

⁴Shanthi is an Economist at the US Treasury. The views and opinions expressed in this essay do not necessarily represent those of the US Treasury.

ratio can be captured. Because the labor wedge is defined as a residual, that is, the portion of the data that model-predicted hours per population cannot explain, the employment-to-population ratio automatically becomes part of the labor wedge. After identifying the shortcomings of the standard model, we develop a model that explicitly incorporates employment as a choice variable. In our model, a household planner maximizes household utility, which is a weighted sum of all employed and non-employed individuals' utility. Our model incorporates a time-varying fixed cost associated with employment, as well as a general non-employment disutility variable. The model nests the standard theory, and helps explain its shortcomings regarding the U.S. and Canada. We use our model to examine the macroeconomic implications of tax policy on hours per population through its disaggregate components: hours per worker and the employment-to-population ratio.

This dissertation's main contributions to the literature are as follows. The first essay establishes that determinants of match quality and aggregate labor-market fluctuations are tied together, sharing common roots in labor-force heterogeneity and individuals' optimal job-seeking behavior. In particular, worker-side heterogeneity can help explain why, empirically, the V/U ratio exhibits a stage of sluggish adjustment in response to changes in productivity. This is a key feature of the data that the standard, homogenous-agent model of equilibrium unemployment theory cannot account for. Many standard macro models are cast in representative agent frameworks given the notion that, at the aggregate level, heterogeneities average out, and therefore accounting for them is unnecessary for accounting for the average behavior of the aggregate economy. The first essay reveals that, in fact, labor-force heterogeneity can be an important factor behind the dynamic behavior of aggregate labor-market variables, and therefore, this type of heterogeneity is one that does not necessarily average out.

The second essay contributes to the labor economics literature by developing a theoretical framework through which an intertemporal understanding of the primitives that determine an economy's available trade-offs between output, wages, and job utility can be attained. Moreover it contributes to the macroeconomics literature

by showing that secular improvements in on-the-job utility are such that it is possible for work hours to remain relatively constant over time even if the income effect of higher wages on labor supply exceeds the substitution effect of higher wages. In turn, secular improvements in on-the-job utility can themselves be a substantial component of the welfare gains from technological progress. These two implications are connected by an identity: improvements in on-the-job utility that have a significant effect on labor supply tend to have large welfare effects.

The third essay contributes to the literature by providing substantial empirical- and theory-based evidence that implies that the standard neoclassical macroeconomic model lacks explanatory power regarding the extensive margin of labor supply. This limitation of the model leads to inaccurate predictions of hour per population when there are large changes in the employment-to-population ratio relative to those in hours per worker. Given this finding, the so-far puzzling fact that the standard neoclassical macroeconomic model extended to account for taxes is unable to match the trend behavior of hours per population in the US and Canada, even though it has been relatively successful in doing so for most European countries, can be rationalized. In contrast to the US and Canada, in most European countries the employment-to-population ratio has not changed much relative to hours per worker. Hence, in these countries the long-run behavior of hours per worker and hours per population is virtually indistinguishable. Moreover, the third essay provides an understanding of the labor wedge that complements previous studies. Earlier research successfully identifies that part of the existence of the labor wedge results from ignoring the role of taxes. The third essay complements this research by showing that another part of the labor wedge actually holds by construction, and stems from an inherent inability of the standard model to predict extensive-margin changes in labor supply.

CHAPTER II

HETEROGENEOUS WORKERS, OPTIMAL JOB SEEKING, AND AGGREGATE UNEMPLOYMENT

2.1 Introduction

Why do vacancies and unemployment coexist? The standard model of equilibrium unemployment theory is cast in a homogeneous-agent framework, and explains involuntary unemployment as the result of search frictions.¹ This essay develops an understanding of how heterogeneity, in addition to search frictions, can influence aggregate labor-market fluctuations, and, in particular, the cyclical behavior of aggregate unemployment. I capture heterogeneity by considering a labor force composed of individuals who have a comparative advantage in a particular job, yet are still able to work in jobs in which they are at a comparative disadvantage. I assume no worker has an absolute advantage in production. Within this context, two important questions emerge. Given changes in economic conditions, how does the search behavior of job seekers across employment opportunities affect the quality of employment matches that firms can expect to form (where quality refers to the relative productivity of a match)? In addition, how do cyclical changes in firm-side expected match quality affect vacancy-posting decisions, and therefore, the cyclical behavior of aggregate vacancies and unemployment?²

¹See, for example, Pissarides (1985), Mortensen and Pissarides (1994), and Pissarides (2000). In addition, Diamond (1982) and Mortensen (1982) represent key contributions to the search and matching framework.

²Bils, Chang, and Kim (2009) study worker-side heterogeneity in a different context, their focus being on unemployment and work hours. In their paper, “comparative advantage” refers to individuals who have high market productivity relative to their home productivity. Moreover, labor markets are segmented: although the labor force is heterogeneous, conditional on idiosyncratic characteristics individuals seek employment in only one production sector.

The extent to which vacancies and unemployment coexist is summarized by the aggregate vacancy-unemployment (V/U) ratio. Empirically, in the US, the V/U ratio is strongly procyclical, and part of its adjustment in response to changes in productivity is sluggish.³ Understanding the slow-moving behavior of the V/U ratio is important, since it captures the extent to which the labor market responds persistently, for example, in the wake of a recession. Under standard calibrations, the benchmark, homogeneous-agent model of equilibrium unemployment theory accounts for slightly less than half of the elasticity of the V/U ratio with respect to productivity. However, contrary to the data, in the standard model all adjustments in the V/U ratio occur instantaneously: the model has no channels through which the V/U ratio can exhibit sluggish adjustment.⁴ In this essay, I show that worker-side heterogeneity and optimal job-seeking behavior can have an important role in shaping aggregate labor-market fluctuations. Combined, these worker-side factors aid in accounting for the amplification of productivity shocks in the V/U ratio. Moreover, I show that slow-moving changes in expected vacancy-posting gains that stem from the presence of worker-side heterogeneity can help explain why, empirically, the V/U ratio exhibits a stage of sluggish adjustment in response to changes in productivity.

Sections 2.2.1 through 2.2.4 develop the essay's theory. The framework consists of an economy with two types of workers and two production sectors.⁵ Each worker type has a comparative advantage in one of the two production sectors, and no worker type has an absolute advantage. Each production sector is composed of a continuum of firms. The production functions of all firms in a particular sector are identical. As in the standard model, there exists one job at each firm, which is filled through vacancy posting.

Employment in situations of comparative advantage (*skill match*) yields the high-

³See, for example, Shimer (2005) and Fujita and Ramey (2007) with regards to these facts, and also with regards to limitations of the standard/benchmark model in accounting for them. Empirically, on impact of a positive productivity shock the V/U ratio jumps, and thereafter continues to slowly increase for several months.

⁴A detailed explanation of the "standard/benchmark" (homogenous-agent) model/theory can be found in chapter 1 of Pissarides (2000).

⁵So is the case in similar work, although with different focus, such as Albrecht and Vroman (2002), and Gautier (2002).

est economic surplus. However, comparative disadvantage employment (*skill mismatch*) is still a valuable alternative for both workers and firms. For workers, it represents an additional channel through which they can exit unemployment. For firms, it represents a means through which vacancies can be filled faster, and therefore, through which expected vacancy-posting costs can be reduced. Hence, although both firms and workers prefer skill-matched employment, they are also willing to engage in skill-mismatched relationships; I assume this is the case for all states of the economy. In particular, unemployed individuals take advantage of all available job opportunities by searching across sectors.

Whenever a job seeker meets a firm with a vacant job a match is formed, wages are negotiated via Nash bargaining, and production begins. Nash bargaining implies that wages are fully flexible and are instantly renegotiated given any change in the state of the economy. When an individual is skill-mismatched, he or she engages in on-the-job (OTJ) search directed towards skill-matched employment. This results in endogenous job destruction. In addition, as in the standard theory, there is an exogenous job destruction component.

In Section 2.2.5 I focus on how heterogeneity in job seekers makes firms' vacancy-posting incentives depend on expected match quality (with some probability, the vacancy will be filled with a worker who has a comparative advantage in the sector to which the firm belongs).⁶ I show that firm-side match-quality expectations are slow moving, since they depend on the slow-moving masses of unemployed and OTJ searchers. Slow-moving match-quality expectations translate into slow-moving expected gains from posting vacancies. In particular, during an economic expansion, relative changes in the composition of searchers slowly increase the firm-side probability of skill-matched employment. This provides an incentive for vacancy-posting to remain higher than otherwise. The result: given an increase in productivity, the aggregate vacancy-unemployment (V/U) ratio will exhibit a stage of sluggish adjustment.

Initially, I treat workers' job-seeking decisions as being exogenously determined:

⁶Recall that "match quality" refers to the relative productivity of a match.

as in the standard theory, job seekers passively wait for job opportunities to arrive. However, in section 2.2.6 I extend the model to allow for job-seekers' search decisions to be endogenous. Search decisions depend on the state of the economy, worker idiosyncratic characteristics, and any given employment state an individual may be in. Optimal search enhances the model's channel for slow-moving changes in the V/U ratio. This owes to differences in endogenous search behaviors between unemployed and on-the-job searchers that result from differences in search incentives. In particular, the latter have a more attractive outside option (employment).

In Section 2.2.7 I show that the standard, homogeneous-agent model of equilibrium unemployment theory is nested in the theory I develop. Then, in order to quantify this essay's theoretical implications, I simulate the model numerically. Section 2.3.1 describes my methodology, and Section 2.3.2 details my selection of parameters for calibrating the model. Sections 2.4.1 through 2.4.3 present the essay's main results. The model can account for the majority of the elasticity of the V/U ratio with respect to productivity in the US, and also for the majority of the elasticity of US aggregate unemployment with respect to productivity. In particular, the model's ability to generate slow-moving adjustments in the V/U ratio is consistent with a much larger response of aggregate unemployment to changes in productivity than otherwise. Relative to the benchmark, homogeneous-agent model of equilibrium unemployment theory, the model's improvements in accounting for the data are substantial. Results imply that worker-side heterogeneity and optimal search can play an important role in shaping aggregate labor-market fluctuations.

Additionally, Section 2.4.4 shows that the theory has implications for cyclical changes in match quality, as measured by the fraction of skill-mismatched employment. I show that the dynamic response of the fraction of skill-mismatched employment to changes in productivity can involve both periods over which match quality is above average, and periods over which it is below average. This result can aid in reconciling research regarding the cyclical reallocation of resources. As noted in Barlevy (2002), while some research suggests that recessions are times in which resources are exclusively reallocated to better uses, other research implies the contrary.

Section 2.5 discusses related literature, and Section 2.6 concludes. This essay's main contribution lies in its finding that determinants of match quality and aggregate labor-market fluctuations are tied together, sharing common roots in labor-force heterogeneity and individuals' optimal job-seeking behavior. In particular, worker-side heterogeneity can help explain why, empirically, the V/U ratio exhibits a stage of sluggish adjustment in response to changes in productivity. This is a key feature of the data that the standard, homogenous-agent model of equilibrium unemployment theory cannot account for.

2.2 The Model

The model is cast in discrete time, which aids the mapping of theory to numerical simulation. I assume that each period of time is small enough so that discrete time represents a close approximation to continuous time.⁷ All economic agents discount the future at rate r , and $\beta = 1/(1+r)$ is the discount factor. In addition, all variables are normalized by the aggregate labor force.

As noted earlier, the framework consists of an economy with two types of workers and two production sectors. Each worker type has a comparative advantage in one of the two production sectors, and no worker type has an absolute advantage. Each production sector is composed of a continuum of firms, and the production functions of all firms in a particular sector are identical.

Workers and sectors/firms are indexed by $i, j \in \{1, 2\}$. In the notation subscripts refer to workers and superscripts to sectors/firms. Skill-matched employment occurs when the worker and firm type coincide. For example, a type-1 worker who is matched with a type-1 firm is skill-matched, and he or she is skill-mismatched when employed by a type-2 firm. For simplicity, I cast the model under full symmetry. Therefore, in all periods any type-specific worker/firm variable is equal to half of its aggregate counterpart, and all model parameters are symmetric across sectors. Given symmetry, whenever helpful I present the model from the point of view of type-1 economic agents. All statements carry over to type-2 agents by simple re-indexing.

⁷See Hagedorn and Manovskii (2008) for an analogous specification.

Let p_t denote an economy-wide (exogenous) productivity component that affects all firms. In steady state, $p_t = 1$. I assume that the output generated by a type- i individual employed by a type- i firm (skill-matched employment) is $y_{i,t}^i = p_t$. Moreover, for $j \neq i$ and $i, j \in \{1, 2\}$, the output generated by a type- j individual employed by a type- i firm (skill-mismatched employment) is $y_{j,t}^i = p_t(1 - \phi)$, where $\phi \in (0, 1)$ is a penalty parameter that captures the degree of comparative disadvantage. The higher ϕ is, the greater the degree of comparative disadvantage of a type- j worker employed in sector $i \neq j$. Unless noted otherwise, henceforth when i and j appear together in some expression, assume $i \neq j$.

2.2.1 Stocks and Flows

Let ψ_1 denote the fraction of type-1 workers. Then,

$$\psi_1 = u_{1,t} + \chi_{1,t} + n_{1,t} = 0.5, \quad (2.1)$$

where $u_{1,t}$, $\chi_{1,t}$ and $n_{1,t}$ are, respectively, the mass of type-1 individuals who are unemployed, skill-mismatch employed, and skill-match employed. Job seekers are in employment state $\mathbb{S} \in \{u, \chi\}$, where u means “unemployed” and χ means “skill-mismatch employed.” Unemployed individuals direct their search to both sector-1 and sector-2 firms.⁸ Given comparative advantage, skill-mismatched individuals direct their search towards the sector in which they are relatively more productive.⁹ Moreover, individuals who are skill-match employed do not search because they are already employed in their best possible match. Recall that, given symmetry, all statements regarding type-1 economic agents carry over to type-2 agents by simple re-indexing.

Each period, the number of matches formed in sector i is determined by the sectoral matching function $m_t^i = m(v_t^i, s_t^i)$, where v^i are sector- i vacancies, and s^i are sector- i searchers. Following related literature and empirical evidence, I assume m

⁸I assume that it is always optimal for them to do so.

⁹Since the production functions of all firms within a sector are identical, there are no gains in transitioning from skill-mismatched to skill-mismatched employment.

has constant returns to scale.¹⁰ In particular, let $m = A(v^i)^\alpha (s^i)^{1-\alpha}$, where $\alpha \in (0, 1)$ is the elasticity of sectoral matches with respect to sectoral vacancies, and A is the matching efficiency parameter.

Conditional on being in employment state $\mathbb{S} \in \{u, \chi\}$, the probability that a type-1 individual searching for a job in sector $i \in \{1, 2\}$ finds a job in that sector is $F_{1,\mathbb{S},t}^i = \ell_{1,\mathbb{S}}^i f_t^i$. For $i \in \{1, 2\}$ $f_t^i = m_t^i / s_t^i$ (sector- i matches per sector- i searchers). Given constant returns to scale, this can be stated as $f_t^i = f(\theta_t^i)$, where $\theta^i = v^i / s^i$ denotes *sectoral (market) tightness*, and $f' > 0$. Moreover, $\ell_{1,\mathbb{S}}^i$ is a worker-type/employment-state specific technological component in the job seeking process.¹¹ I refer to the search technologies $\ell_{1,\mathbb{S}}^i$ as *effective search*. Since $F_{1,\mathbb{S},t}^i = \ell_{1,\mathbb{S}}^i f_t^i$, it follows that $\ell_{1,\mathbb{S}}^i$ summarizes the effectiveness with which all of an individual's job-seeking activities lead to a job offer given his or her employment state and the sector in which the individual is searching. $\ell_{1,\mathbb{S}}^i$ includes different kinds of search activities and methods, the intensity with which search methods are used, etc.

It follows that sector-1 searchers are a weighted sum of individuals searching in that sector, where the weights are effective search:

$$s_t^1 = \ell_{1,u}^1 \cdot u_{1,t} + \ell_{2,u}^1 \cdot u_{2,t} + \ell_{1,\chi}^1 \cdot \chi_{1,t}. \quad (2.2)$$

Note that sector-1 searchers do not include the weighted mass of skill-mismatched type-2 individuals $\ell_{2,\chi}^2 \cdot \chi_{2,t}$, since type-2 individuals who are employed in sector-1 only search for sector-2 jobs.¹²

Using similar reasoning as earlier, the probability with which a sector- i vacant

¹⁰See, for example, Petrongolo and Pissarides (2001), Gautier (2002), and Krause and Lubik (2006).

¹¹This is based on the theory developed in Pissarides (2000), chapter 5, in which the standard, homogeneous-agent model features an endogenous job-seeking technological component. Later in the essay I endogenize the choice of idiosyncratic search technologies within the present multi-agent framework. For now, I assume that search technologies are fixed.

¹²Note that total matches in sector-1 are given by

$$\begin{aligned} F_{1,u,t}^1 u_{1,t} + F_{2,u,t}^1 u_{2,t} + F_{1,\chi,t}^1 \chi_{1,t} &= \ell_{1,u,t}^1 f_t^1 u_{1,t} + \ell_{2,u,t}^1 f_t^1 u_{2,t} + \ell_{1,\chi,t}^1 f_t^1 \chi_{1,t} \\ &= (\ell_{1,u,t}^1 u_{1,t} + \ell_{2,u,t}^1 u_{2,t} + \ell_{1,\chi,t}^1 \chi_{1,t}) f_t^1 = s_t^1 \cdot m_t^1 / s_t^1 = m_t^1. \end{aligned}$$

job is filled is given by $q_t^i = m_t^i/v_t^i$ (sector- i matches per sector- i vacancies). This can be stated as $q_t^i = q(\theta_t^i)$, where $q' < 0$. The probability that a sector-1 vacant job is filled with a worker who has a comparative advantage in that sector is $q_t^1(1 - \pi_{2,t}^1)$, and $q_t^1\pi_{2,t}^1$ otherwise. Given the earlier development,

$$\pi_{2,t}^1 = \frac{\ell_{2,u}^1 \cdot u_{2,t}}{\ell_{1,u}^1 \cdot u_{1,t} + \ell_{2,u}^1 \cdot u_{2,t} + \ell_{1,\chi}^1 \cdot \chi_{1,t}} = \frac{\ell_{2,u}^1 \cdot u_{2,t}}{s_t^1}. \quad (2.3)$$

That is, $\pi_{2,t}^1$ is the effective fraction of type-2 individuals looking for jobs in sector 1.

Let δ denote the economy-wide probability with which any employed individual becomes unemployed. Following related literature, for simplicity this probability is assumed to be time invariant.¹³ Then, the evolution of the mass of unemployed type-1 workers satisfies

$$u_{1,t+1} - u_{1,t} = \delta (n_{1,t} + \chi_{1,t}) - (F_{1,u,t}^1 + F_{1,u,t}^2)u_{1,t}. \quad (2.4)$$

This equation captures that with probability δ any employed type-1 individual, whether skill-matched ($n_{1,t}$) or -mismatched ($\chi_{1,t}$), enters unemployment. Moreover, because unemployed individuals search across sectors, they exit unemployment with probability $F_{1,u,t}^1 + F_{1,u,t}^2$. Note that the aggregate unemployment rate is $u_t = \sum_i u_{i,t}$.

The evolution of the mass of type-1 workers who are skill mismatched is given by

$$\chi_{1,t+1} - \chi_{1,t} = F_{1,u,t}^2 u_{1,t} - (\delta + F_{1,\chi,t}^1) \chi_{1,t}. \quad (2.5)$$

The first term in this equation captures that with probability $F_{1,u,t}^2$ a type-1 unemployed individual finds a job in sector-2, and therefore becomes skill-mismatched. The second term captures the two reasons for which a skill-mismatched job can end. With exogenous probability δ the job is destroyed, leading to unemployment. Moreover, given OTJ search, with endogenous probability $F_{1,\chi,t}^1$ a type-1 individual finds a job in sector-1; in this case he or she changes jobs, which implies that the skill-mismatched relationship is endogenously terminated. It follows that the aggregate

¹³In particular, this is also assumed in the standard, homogenous-agent model.

rate of skill-mismatch is $\chi_t = \sum_i \chi_{i,t}$. Moreover, let $e_t = (1 - u_t)$ denote the fraction of employed individuals. Then, χ_t/e_t is the aggregate fraction of skill-mismatched employment.

2.2.2 Workers

All unemployed individuals receive constant exogenous flow benefits $b > 0$. b includes unemployment benefits, the value of home production, etc. Denote net unemployment flow benefits by $z = b - C$, where C represents effective-search costs. The value of unemployment for a type-1 worker in period t is denoted by $U_{1,t}$, and his or her value of employment in sector $i \in \{1, 2\}$ is $W_{1,t}^i$. A type-1 unemployed individual searching for a job in sector $i \in \{1, 2\}$ during period t finds a job in that sector with probability $F_{1,u,t}^i$, and is employed in that job in the following period. Comparative advantage in production means that $W_{1,t}^1 > W_{1,t}^2 > U_{1,t}$ (I assume this last inequality always holds). Then,

$$U_{1,t} = z + \beta(F_{1,u,t}^1 \cdot \mathbb{E}_t W_{1,t+1}^1 + F_{1,u,t}^2 \cdot \mathbb{E}_t W_{1,t+1}^2 + (1 - F_{1,u,t}^1 - F_{1,u,t}^2) \cdot \mathbb{E}_t U_{1,t+1}), \quad (2.6)$$

where \mathbb{E}_t is the expectation operator, and from earlier β is the discount factor.

In period t a type-1 skill-mismatched individual receives the wage $w_{1,t}^2$, and engages in OTJ search directed towards sector 1. With probability $F_{1,\chi,t}^1$ he or she finds a job in that sector, in which case in period $t + 1$ the skill-mismatched job is endogenously destroyed, and the worker transitions to skill-matched employment. Moreover, with probability δ the job is destroyed exogenously, in which case in period $t + 1$ the individual in question is unemployed. Therefore, the value of skill-mismatched employment for a type-1 individual is

$$W_{1,t}^2 = w_{1,t}^2 + \beta((1 - \delta - F_{1,\chi,t}^1) \cdot \mathbb{E}_t W_{1,t+1}^2 + F_{1,\chi,t}^1 \cdot \mathbb{E}_t W_{1,t+1}^1 + \delta \cdot \mathbb{E}_t U_{1,t+1}). \quad (2.7)$$

Using analogous reasoning, it follows that the value of skill-matched employment for a type-1 individual is

$$W_{1,t}^1 = w_{1,t}^1 + \beta((1 - \delta) \cdot \mathbb{E}_t W_{1,t+1}^1 + \delta \cdot \mathbb{E}_t U_{1,t+1}). \quad (2.8)$$

2.2.3 Firms

Consider the value of a skill-matched job for a sector-1 firm: $J_{1,t}^1$. Recall that my notational convention assigns subscripts to workers and superscripts to firms. In period t , output $y_{1,t}^1$ is generated. With exogenous probability δ the job is destroyed, in which case the firm has a vacancy in period $t + 1$. I denote the period- t value of a vacancy for a sector-1 firm by V_t^1 . It follows that

$$J_{1,t}^1 = y_{1,t}^1 - w_{1,t}^1 + \beta((1 - \delta) \cdot \mathbb{E}_t J_{1,t+1}^1 + \delta \cdot \mathbb{E}_t V_{t+1}^1). \quad (2.9)$$

Now, consider the value to a sector-1 firm of a skill-mismatched job: $J_{2,t}^1$. Because skill-mismatched individuals engage in OTJ search, the job is destroyed endogenously with probability $F_{2,\chi,t}^2$. Hence, the value of skill-mismatched employment for a sector-1 firm is

$$J_{2,t}^1 = y_{2,t}^1 - w_{2,t}^1 + \beta((1 - \delta - F_{2,\chi,t}^2) \cdot \mathbb{E}_t J_{2,t+1}^1 + (\delta + F_{2,\chi,t}^2) \cdot \mathbb{E}_t V_{t+1}^1). \quad (2.10)$$

Comparative advantage in production implies that $J_{1,t}^1 > J_{2,t}^1$.

Following the literature, while a firm has a vacancy it incurs the time-invariant flow cost $c > 0$. Therefore the value of a vacancy for a sector-1 firm is

$$V_t^1 = -c + \beta(q_t^1(1 - \pi_{2,t}^1) \cdot \mathbb{E}_t J_{1,t+1}^1 + q_t^1 \pi_{2,t}^1 \cdot \mathbb{E}_t J_{2,t+1}^1) + (1 - q_t^1) \mathbb{E}_t V_{t+1}^1. \quad (2.11)$$

Free entry into vacancy creation implies the zero-profit condition $V_t^1 = 0 \forall t$. Using this condition in equation (2.11) and rearranging yields the vacancy/job-creation condition

$$\beta((1 - \pi_{2,t}^1) \cdot \mathbb{E}_t J_{1,t+1}^1 + \pi_{2,t}^1 \cdot \mathbb{E}_t J_{2,t+1}^1) = c/q(\theta_t^1). \quad (2.12)$$

This equation's left-hand side represents the expected gains from vacancy posting, and its right-hand side the expected costs.

2.2.4 Surpluses, Wages, and Equilibrium

For $i \in \{1, 2\}$, $S_{1,t}^i = W_{1,t}^i - U_{1,t} + J_{1,t}^i$ is the surplus generated by an employment match between a type-1 worker and a sector- i firm. Wages are negotiated via Nash bargaining, and instantly renegotiated given any changes in the state of the economy. In particular, for $i \in \{1, 2\}$ the wage maximizes the Nash product $(W_{1,t}^i - U_{1,t})^\eta (J_{1,t}^i)^{1-\eta}$, where $\eta \in (0, 1)$ is the bargaining power of workers. This results in the surplus-sharing rule

$$(W_{1,t}^i - U_{1,t})/\eta = S_{1,t}^i = J_{1,t}^i / (1 - \eta). \quad (2.13)$$

Since $S_{1,t}^1 > S_{1,t}^2$ (and $S_{1,t}^1 > S_{2,t}^1$), given Nash bargaining $w_{1,t}^1 > w_{1,t}^2$ (and $w_{1,t}^1 > w_{2,t}^1$). Therefore, the wage of an individual who is skill-mismatched is lower than his or her skill-matched wage. Moreover, the wage of an individual who is skill-mismatched in a sector is lower than that of individuals who are skill-matched in that same sector.

Using the definition of surplus, the job-creation condition in equation (2.12) can be stated, for $i \neq j$, as

$$\beta (1 - \eta) ((1 - \pi_{j,t}^i) \cdot \mathbb{E}_t S_{i,t+1}^i + \pi_{j,t}^i \cdot \mathbb{E}_t S_{j,t+1}^i) = c/q(\theta_t^i). \quad (2.14)$$

This is, in fact, the model's fundamental equilibrium equation. Changes in the expected gains from vacancy posting require changes in expected costs, which occur through changes in q^i . In turn, q^i is a (decreasing) function of θ^i . Therefore, θ^1 and θ^2 are the model's fundamental equilibrium variables. Once searchers s^i are determined, sectoral vacancies can be backed out from θ^i . Given knowledge of the masses of unemployed and skill-mismatched individuals it is straightforward to derive s^i , and therefore, π_j^i , where $i \neq j$. Hence, for $i, j \in \{1, 2\}$ and $i \neq j$ knowledge of the key endogenous variables π_j^i , u_i , χ_i , and θ^i (eight variables total) is sufficient to derive all of the model's endogenous variables. There are four employment values: J_1^1 , J_2^1 ,

J_2^2 , and J_1^2 . Using the surplus-sharing rule in equation (2.13) these can be stated in terms of employment surpluses, and solved for using the four job values implicit in equations (2.9) and (2.10). As noted before, θ^1 and θ^2 are defined by the two corresponding job creation conditions defined by equation (2.14). Finally, the remaining six key variables π_1^2 , π_2^1 , u_1 , u_2 , χ_1 , and χ_2 , are defined through the 6 expressions implicit in equations (2.3), (2.4), and (2.5): recall the environment is symmetric. Thus, the model reduces to 8 equations in 8 unknowns.

2.2.5 The Role of Worker Heterogeneity

I refer to the model developed above as the *multi-agent (MA) model*. For simplicity, unless noted otherwise, assume that in this model all effective search is fixed at unity: for $i, j \in \{1, 2\}$ $\ell_{i,u}^j = 1$ and $\ell_{i,\chi}^i = 1$. This simply means that the number of searchers in any sector are one and the same with the number of individuals searching in that sector. The MA model is not solvable analytically. However, it is still possible to gauge the role of worker heterogeneity.

In the multi-agent model, expected vacancy-posting gains are directly dependent on the key variables p (through the value of output) and π_j^i , where $i \neq j$. Consider a permanent increase in p . On impact, the expected gains from vacancy posting (the left-hand side of equation (2.14)) rise, so firms respond by posting more vacancies. This results in an instantaneous increase in sectoral market tightness θ^i , which increases the expected costs of posting vacancies (the right-hand side of equation (2.14)). Hence, the job-creation condition in equation (2.14) continues to hold. However, all adjustments do not end there. This is because the probabilities π_j^i consist of the slow-moving masses of unemployed and OTJ searchers (see equation (2.3)), and are therefore slow moving themselves. Slow adjustments in these probabilities will lead to post-shock slow-moving changes in the expected gains from vacancy posting, which for equilibrium to hold will be matched by slow-moving changes in θ^i .

Given symmetry, to understand the adjustment process in π_j^i , I focus on the firm-side probability of skill-mismatched employment in sector 1: π_2^1 . Consider the pool of sector-1 searchers: u_2 , u_1 , and χ_1 . When productivity increases, firms post more

vacancies. This immediately translates into an increase in all job-finding probabilities (θ^i rises). In particular, the increase in the probability of exiting unemployment triggers a slow-moving reduction in u_1 and u_2 (see equation (2.4)). As the pool of unemployed individuals declines, all type-2 individuals who exit unemployment also exit the pool of sector-1 searchers: they either become skill-matched, and therefore exit search entirely, or they become skill-mismatched, in which case they direct OTJ search towards sector-2 only. At the same time, as the pool of unemployed individuals declines, not all type-1 individuals who exit unemployment also exit the pool of sector-1 searchers: while some of them become skill-matched, and therefore exit search entirely, others become skill-mismatched. Type-1 individuals who exit unemployment by becoming skill-mismatched continue to form part of the pool of sector-1 searchers: they engage in OTJ search directed towards this sector. It follows that in an expansion, as unemployment declines, because type-1 individuals take longer to exit the pool of sector-1 searchers than type-2 individuals, π_2^1 will decline.

Proposition 1. *In the multi-agent model the firm-side probability of skill mismatched employment, π_j^i , where $i \neq j$, is countercyclical.*

Proof. In Section 2.9.¹⁴

Because π_j^i is a function of the slow-moving masses u_i , u_j , and χ_i , it follows that a positive shock to productivity will induce a slow-moving decline in π_j^i . Given the job-creation conditions in equation (2.14), this will lead to slow-moving increases in the expected gains from vacancy posting, and therefore to slow-moving increases in sectoral market tightness θ^i : the availability of vacancies per searchers will continue to rise after its initial post-shock jump (*propagation* in sectoral market tightness). By extension, after an increase in productivity, the availability of vacancies per unemployed searchers will experience a period of slow-moving increase. Hence, the aggregate vacancy-unemployment (V/U) ratio will exhibit propagation as well. I denote the V/U ratio by $\Theta = v/u$.

¹⁴As noted in Shimer (2005) and Mortensen and Nagypal (2007), the cyclical properties of models of this sort are well assessed by considering differences between steady states. Therefore, in the present essay propositions and proofs referring to the cyclicity of variables are based on steady-state to steady-state changes.

Proposition 2. *In the multi-agent model, the rate of skill-mismatch, χ , is countercyclical.*

Proof. In Section 2.9.

Intuitively, note from equations (2.4) and (2.5) that given an increase in productivity, while the probability of exiting unemployment increases, *both* the probability of entering and exiting skill-mismatched employment rise. In an expansion, the decline in the pool of unemployed individuals serves to ultimately decrease inflows into skill-mismatched employment relative to outflows. In particular, given a permanent increase in productivity, this results in a decrease in the rate of skill-mismatch. Proposition 3 and Corollary 1, which follow below, highlight the importance of OTJ search for the multi-agent model's propagation channel: π_j^i .

Proposition 3. *In the absence of OTJ search, in the multi-agent model π_j^i is always constant.*

Proof. Given symmetry, for $i \neq j$ $u_{i,t} = u_{j,t} = 0.5u_t$, and $\chi_{i,t} = \chi_{j,t} = 0.5\chi_t$. Then, rearranging equation (2.3) implies that

$$\pi_{j,t}^i = \left(1 + \frac{\ell_{i,u}^i}{\ell_{j,u}^i} + \frac{\ell_{i,\chi}^i}{\ell_{j,u}^i} \frac{\chi_t}{u_t} \right)^{-1}. \quad (2.15)$$

Assuming OTJ search is not possible is equivalent to setting $\ell_{i,\chi}^i = 0$, in which case π_j^i is equal to the constant $(1 + \ell_{i,u}^i/\ell_{j,u}^i)^{-1}$. In particular, under the assumption that $\ell_{i,u}^i = \ell_{j,u}^i = 1$, where $i \neq j$, in the absence of OTJ search $\pi_{j,t}^i = 0.5 \forall t$. \square

Corollary 1. *In the multi-agent model skill-mismatched employment is necessary, but not sufficient, for propagation in the V/U ratio to occur in response to productivity shocks.*

Corollary 1 is straightforward to understand. Eliminating the possibility of skill-mismatch implies that for $i \neq j$ $\pi_{j,t}^i = 0 \forall t$ (see equation (2.3)). Since slow-moving changes in π_j^i are what generates propagation in the V/U ratio, $\pi_{j,t}^i = 0 \forall t$ shuts down the MA model's propagation channel. Moreover, given Proposition 3, even if skill-mismatched employment does exist, in the absence of OTJ search π_j^i is always constant as well; this also results in the MA model's propagation channel being shut

down.

To understand the benchmark, homogeneous-agent model's lack of propagation in the V/U ratio, note that assuming identical economic agents is equivalent to assuming that in the MA model $\phi = 0$. Such homogeneity means that no skill-mismatch is possible to begin with, which implies that $\chi = 0$, and hence, for $i \neq j$, $\pi_{j,t}^i = 0 \forall t$. Since there is no skill-mismatch, there are no incentives for OTJ search, and the only job seekers are individuals who are unemployed. Thus, matches are a function of aggregate vacancies and unemployment: $m_t = m(v_t, u_t)$. It follows that the expected costs of posting a vacancy can be stated as $c/q(\Theta_t)$, and the (only) key variable affecting expected vacancy-posting gains is economy-wide productivity p . Given the absence of the slow-moving variables π_j^i in the expected gains from vacancy posting, in the standard model all adjustments in these gains occur instantaneously. This instantaneous adjustment must be matched by an instantaneous adjustment in $c/q(\Theta_t)$, and therefore, in Θ (no propagation).

2.2.6 The Role of Optimal Effective Search

In the multi-agent model, workers have different job opportunities available. Therefore, a natural step that follows in understanding the economic implications of worker-side heterogeneity is to endogenize the choice of effective search. Since job-finding probabilities are linear in effective search, in order to bound effective-search choices, search costs must be introduced. The costs of effective search directed towards a sector are simply the costs of generating job offers in that sector. As noted in Krueger and Mueller (2008), the time that unemployed individuals spend searching is small, which suggests that time constraints are not binding in optimal search decisions.¹⁵ Given this, an intuitive reason for which unemployed individuals might limit the effective search they devote to any given type of job opportunity is that search costs are sector specific. In turn, sector-specific search costs are a natural motivation for individuals to broaden their search to include jobs in which they do not have a

¹⁵In their cross-country investigation, Krueger and Mueller (2008) find that, conditional on searching, the average search time ranges from 40 minutes per week in Slovenia, to slightly less than 4 hours per week in Canada (which is a small amount more than in the U.S.).

comparative advantage. Hence, I assume that individuals bear the effective-search disutility function

$$C(\ell_{1,S,t}^1, \ell_{1,S,t}^2) = \Gamma \left((\ell_{1,S,t}^1)^{(1+\varepsilon)/\varepsilon} + (\ell_{1,S,t}^2)^{(1+\varepsilon)/\varepsilon} \right), \quad (2.16)$$

where $\Gamma > 0$ is a scaling parameter and $\varepsilon > 0$. Since skill-mismatched individuals direct their search to the sector in which they have a comparative advantage,

$$C(\ell_{1,X,t}^1) = \Gamma (\ell_{1,X,t}^1)^{(1+\varepsilon)/\varepsilon}. \quad (2.17)$$

Within this context, the only value functions that must be updated are the worker's value of skill-mismatched employment and the value of unemployment. These now become, respectively,

$$\begin{aligned} W_{1,t}^2 = \max_{\ell_{1,X,t}^1} \{ & w_{1,t}^2 - C(\ell_{1,X,t}^1) + \beta((1 - \delta - F_{1,X,t}^1) \cdot \mathbb{E}_t W_{1,t+1}^2 \\ & + F_{1,X,t}^1 \cdot \mathbb{E}_t W_{1,t+1}^1 + \delta \cdot \mathbb{E}_t U_{1,t+1}) \}, \end{aligned} \quad (2.18)$$

and

$$\begin{aligned} U_{1,t} = \max_{\ell_{1,u,t}^1, \ell_{1,u,t}^2} \{ & z_t + \beta(F_{1,u,t}^1 \cdot \mathbb{E}_t W_{1,t+1}^1 + F_{1,u,t}^2 \cdot \mathbb{E}_t W_{1,t+1}^2 \\ & + (1 - F_{1,u,t}^1 - F_{1,u,t}^2) \cdot \mathbb{E}_t U_{1,t+1}) \}. \end{aligned} \quad (2.19)$$

As before, z_t is net unemployment flow benefits; in particular, for $i \neq j$, $z_t = b - C(\ell_{i,t,u}^i, \ell_{i,t,u}^j)$.

I continue to assume that in all states of the economy it is optimal for unemployed individuals to search for jobs across sectors. Given the surplus-sharing rule in equation (2.13), the first-order conditions for optimal search can be stated as

$$\Gamma \frac{1+\varepsilon}{\varepsilon} (\ell_{1,X,t}^1)^{1/\varepsilon} = f_t^1 \beta \eta \mathbb{E}_t (S_{1,t+1}^1 - S_{1,t+1}^2) \quad (2.20)$$

when skill-mismatched, and for $i \in \{1, 2\}$

$$\Gamma \frac{1+\varepsilon}{\varepsilon} (\ell_{1,u,t}^i)^{1/\varepsilon} = f_t^i \beta \eta \mathbb{E}_t S_{1,t+1}^i \quad (2.21)$$

when unemployed.¹⁶ In each of these first-order conditions the right-hand side represents the expected gains from search. Therefore, ε is the elasticity of effective search with respect to any given sector's expected-search gains. In terms of equilibrium, the first-order conditions above define optimal effective search.

The first-order condition for unemployed effective search, in particular, highlights the intuitive nature of the chosen cost function. Under symmetry, $f^1 = f^2 = f$. Therefore, since $S_1^1 > S_1^2$, given equation (2.21) unemployed individuals will always devote greater effective search towards skill-matched employment: $\ell_{1,u,t}^1 > \ell_{1,u,t}^2$. This implies self selection. Furthermore, if non-symmetric environments were considered, the chosen cost function provides an additional and natural motivation for skill-mismatched employment to exist. Suppose $f^1 = 0$. Then, it is optimal to set $\ell_{1,u,t}^1 = 0$, but as long as the expected gains from skill-mismatched search are positive $\ell_{1,u,t}^2 > 0$.

Although the model is not solvable analytically, the impact of effective search can still be gauged. In the model, procyclical expected gains from search imply that effective search is procyclical. Intuitively, in an expansion, jobs are easier to find and employment surpluses are higher. This means that, all else constant (in particular, gross unemployment flow benefits), the opportunity cost of not having a job increases. Therefore, individuals react to above-average economic conditions by supplying above-average effective search. In a recession, the opposite occurs. For instance, think of discouraged workers as an extreme example of this: these are individuals who have set effective search equal to zero.¹⁷

Procyclical effective search provides a channel for the amplification of aggregate

¹⁶Individuals choose effective search taking market conditions as given (in particular, they take θ^i as given). In addition, note that it is endogenously optimal to set $\ell_{1,\chi,t}^2 = 0$.

¹⁷Christensen et al. (2005) provide empirical analysis on the procyclicality of effective search. The procyclicality of effective search is also a feature of the standard, homogenous-agent model enhanced to account for endogenous effective search: see Pissarides (2000), chapter 5.

shocks.¹⁸ On impact of a positive shock, firms post more vacancies. This increases job-finding probabilities, which on its own provides an incentive for job seekers to increase effective search (see the expected gains from search in the relevant first-order conditions). Higher effective search decreases expected vacancy-posting costs: all else equal, the job-filling probability increases. These lower costs induce firms to further increase vacancy posting, which induces greater effective search, and so on and so forth. This feedback between firm- and worker-side decisions will generate amplification of productivity shocks in the V/U ratio in excess of that which would occur in the absence of endogenous effective search.¹⁹

In addition to contributing towards amplification, procyclical effective search will enhance the model's propagation channel. Employed skill-mismatched job seekers have a more attractive outside option than unemployed job seekers. Indeed, note from equation (2.20) that OTJ effective search is a function of the difference between employment surpluses. In contrast, note from equation (2.21) that unemployed effective search is a function of the employment surplus associated with a particular job. Given procyclical employment surpluses, it follows that the effective search of unemployed individuals will be more procyclical than that of OTJ searchers. In particular, this means that on impact of a positive productivity shock entries into skill-mismatched employment will exceed exits. Combined with the fact that unemployment will be decreasing, an increasing rate of skill-mismatch will tend to further enlarge the pool of searchers who are directing their search exclusively towards the sector in which they have a comparative advantage. This will occur beyond what would happen in the absence of endogenous effective search, and will therefore induce further post-shock reductions in the firm-side probability of skill-mismatched employment, π_2^1 , than otherwise.

¹⁸See Krause and Lubik (2006) and Nagypal (2006) for an analysis of amplification induced by procyclical effective search in other theoretical contexts.

¹⁹The structure of the model implies that after an increase in productivity, in spite of increases in effective search, the job-filling probability ultimately declines. This countercyclicality of job-filling probabilities is also a feature of the standard and MA models, and is in line with the data. See, for example, Davis, Faberman, and Haltiwanger (2009).

2.2.7 Nested Models

I refer to the multi-agent model enhanced with optimal effective search as the *multi-agent optimal-search (MA-OS) model*. The MA-OS model nests three models. These are the multi-agent model, the standard (homogeneous-agent) model, and a version of the standard model in which effective search is endogenous (*standard optimal search (standard-OS) model*). Taking the MA-OS model and fixing all effective search at unity recovers the multi-agent model. Alternatively, taking the MA-OS model and setting $\phi = 0$ (that is, imposing that everyone is identical and therefore eliminating skill-mismatch), but allowing unemployed individuals to choose their effective search endogenously, recovers the standard-OS model. In the homogenous environment of the standard-OS model, denote the effective search of unemployed individuals (the only searchers) by ℓ_t . Taking the standard-OS model and setting $\ell_t = 1 \ \forall t$ recovers the standard model.²⁰ Moreover, as noted earlier, taking the multi-agent model and setting $\phi = 0$ recovers the standard model as well. Therefore, the MA-OS model nests the multi-agent and standard-OS models, and through them, the standard, homogenous-agent model of equilibrium unemployment theory.

Since the standard-OS model is a special case of the standard model, the key variable influencing expected vacancy-posting gains is limited to economy-wide productivity, p . Hence, as is the case in the standard model, the standard-OS model has no channels through which propagation in the V/U ratio can occur.

Given its relative tractability, some points discussed intuitively with regards to the MA-OS model can be shown analytically when effective search is endogenized in a homogeneous-agent framework. In particular, Propositions 4 and 5 address, respectively, the cyclical behavior of effective search in the standard-OS model, and the impact of endogenous effective search on the amplification of shocks in the V/U ratio.

Proposition 4. *In the standard-OS model effective search is procyclical.*

Proof. In Section 2.9.

²⁰Recall that further details on the standard and standard-OS models can be found in Pissarides (2000), chapters 1 and 5.

Proposition 5. *If effective search is endogenized in the standard model, then the elasticity of the V/U ratio with respect to productivity is greater than otherwise.*

Proof. In Section 2.9.

2.3 Simulation Methodology and Calibration

Although I have made reference to several models, this essay’s central model of interest is the MA-OS model. Since the MA-OS model is not solvable analytically, I will further the analysis by making use of the model’s linearized representation to generate impulse response functions, and also to calculate elasticities based on model-generated data. Below, I describe general aspects of my analysis methodology, and also the choice of parameter values for the MA-OS model.²¹

2.3.1 Simulation Methodology

I assume the time period is equal to one week. Using the model’s linearized representation, following the literature, model-generated data is obtained by using the exogenous productivity process $\ln p_t = \rho \ln p_{t-1} + \xi_t$, where $\rho \in (0, 1)$, and $\xi \sim N(0, \sigma_\xi^2)$. Empirical productivity data is only available at quarterly frequency. Hence, I present statistics of interest at this frequency. Since the model’s time period is defined to be one week, I obtain a series of data by simulating 4000 weeks worth of data. To control for initial conditions I discard the first 1000 observations. Then, I average the remaining data to obtain 250 quarters.²² The logarithm of the data is detrended using a Hodrick-Prescott filter with, following Shimer (2005), smoothing parameter equal to 10^5 . This data is used to obtain any desired statistics, and results are stored. I repeat this process 10,000 times, and then present averages over stored results.

²¹Whenever appropriate, later in the paper other models are referenced and their calibrations noted.

²²Empirical counterparts to model-generated data are available for approximately 250 quarters.

2.3.2 Parameter Values

As noted in Table 2.1, twelve parameters require that values be assigned to them.²³ Given that the time period is one week, I set the discount factor, β , equal to 0.999. This is consistent with a quarterly interest rate of 0.012. Moreover, given the research in Hall and Milgrom (2008), I choose gross unemployment flow benefits, b , so that equilibrium net unemployment flow benefits, z , are equal to 0.71. I use the bargaining power of workers, η , to set the equilibrium expected sectoral cost of posting a vacancy, c/q^i , equal to 14% of equilibrium (sectoral) average quarterly wages. This cost is in line with the research in Silva and Toledo (2006), in which the Saratoga Institute’s (2004) estimate of vacancy-posting costs is used. The parameters for the economy’s exogenous productivity process, ρ and σ_ξ , are chosen so that the model-generated output per worker (OPW) matches that of the US, which at quarterly frequency has standard deviation 0.02 and autocorrelation 0.88.²⁴

Petrongolo and Pissarides (2001) find that the elasticity with respect to the V/U ratio of aggregate matches formed with individuals exiting unemployment is approximately 0.5. To target this elasticity, I use the elasticity of sectoral matches with respect to sectoral vacancies, α . The elasticity of effective search, ε , determines the extent to which effective search changes in response to changes in the expected gains from search. In particular, this means that given a change in productivity, ε will impact the change in unemployment relative to vacancies. Hence, I use ε to target the slope of the Beveridge curve, which is the empirical negative relationship between aggregate vacancies and unemployment. Using quarterly data, I find the slope of the Beveridge curve approximately equal to -1 .²⁵

Although I have used the term “sectors” to structure the notion of skill-mismatch,

²³All tables are available in Section 2.8.

²⁴Data spans 1948:Q1 through 2009:Q4, and uses the Bureau of Labor Statistic’s (BLS) measure of non-farm business output per person. Statistics are based on the data’s log deviations from trend, obtained using a Hodrick-Prescott filter with, following Shimer (2005), smoothing parameter equal to 10^5 .

²⁵Data spans 1951:Q1 through 2006:Q2. As is standard in the literature, I use the Conference Board’s Help-Wanted Advertising Index for vacancies (data span is limited to the availability of this index). Data on unemployment is available from the BLS. Statistics are based on the data’s log deviations from trend, obtained using a Hodrick-Prescott filter with, following Shimer (2005), smoothing parameter equal to 10^5 .

in reality skill-mismatch can exist over a wide range of situations. These can relate to industry affiliation, education, job-specific requirements, etc. McLaughlin and Bils (2001) argue that average within-industry wage differentials between individuals who remain in an industry and those who switch can be interpreted as the result of equilibrium self-selection. McLaughlin and Bils show that, empirically, the wages of industry switchers are on average 16% lower than those of industry non-switchers. I take this number as a reference point. Therefore, I use the skill-mismatch penalty parameter, ϕ , to set the equilibrium ratio of wages of skill-mismatched individuals to average wages in a sector equal to 0.84; I denote this ratio by ω .

I use the matching function efficiency parameter, A , and the flow cost of vacancy posting, c , to target, respectively, the equilibrium V/U ratio $\Theta = 0.72$ and the equilibrium aggregate unemployment rate $u = 0.057$.²⁶ Note that an increase in the search-cost parameter, Γ , makes search more costly; all else equal, this decreases optimal effective search and increases unemployment. Hence, for example, increasing Γ means that in order to attain any given target for the unemployment rate A must be increased as well. Numerical analysis reveals that for each Γ there is a value of A that will hit the target equilibrium unemployment rate, but nothing else in the model will change. For concreteness I follow Lise, Seitz, and Smith (2003) and set $\Gamma = 0.0526$.²⁷

Using US unemployment data, I obtain the job-finding probability of the (average) unemployed worker using the methodology described in Shimer (2005). At monthly frequency, the average of this is equal to 0.44.²⁸ The implied job-finding probability at weekly frequency is given by $1 - (1 - 0.44)^{1/4}$, which is equal to 0.135; I take this as the relevant steady-state value. Using this and the target equilibrium unemployment rate, solving for the exogenous job-destruction probability implies $\delta = 0.0082$. The

²⁶The unemployment rate corresponds to the average at monthly frequency spanning 1948:M1 through 2010:M5 using data from the BLS. The value of Θ is based on findings noted in Pissarides (2009).

²⁷Lise, Seitz, and Smith (2003) focus on program evaluation and endogenize the effective search of unemployed individuals in a theoretical framework in which search is non-directed. In the present model, recall that the value of skill-matched employment is normalized to unity. Intuitively, Γ must necessarily be reasonable relative to this so that b is as well, and results are unaffected by the choice of Γ .

²⁸I construct this variable using the same unemployment data noted earlier.

full choice of parameter values for analysis of the MA-OS model is available in Table 2.1.²⁹

2.4 Results

2.4.1 The V/U Ratio and Unemployment

Figures 2.1 and 2.2 show the dynamic adjustment of Θ and u in response to a 1% permanent increase in economy-wide productivity, p , in the MA-OS and standard models.³⁰ Note from Figure 2.1 that while in the standard model on impact of the shock Θ instantaneously jumps to its new equilibrium value, in the MA-OS model Θ jumps, but thereafter continues to increase for a period of approximately 4 months (propagation). That is, in the MA-OS model the availability of vacancies per unemployed individual continues to rise after the shock. As shown in Figure 2.2, relative to the standard model, the slow-moving increase in the V/U ratio that characterizes adjustment in the MA-OS model is consistent with a much larger decline in aggregate unemployment than in the standard, homogeneous-agent model.

As in Pissarides (2009), I turn attention to the elasticities of the V/U ratio and the aggregate unemployment rate with respect to output per worker (OPW). As shown in Table 2.2, in the US the empirical elasticities of Θ and u with respect to OPW are, respectively, 7.261 and -3.580. Table 2.2 also shows the multi-agent optimal-search (MA-OS) model's counterpart to these elasticities based on model-generated data. In the MA-OS model the elasticity of Θ is 5.201, and the elasticity of u is -2.611 (in both cases, around 72% of the data). This is reasonable, since for simplicity I have assumed that the probability of entering unemployment, δ , is constant. In fact, Elsby, Michaels, and Solon (2009) show that in a recession approximately 30%

²⁹The choice of parameters are such that, in equilibrium, output per worker is equal to 0.996 and the fraction of skill-mismatched employment is 0.03. These values are not targeted. Moreover, the fact that $z = 0.71$ in equilibrium implies that net unemployment flow benefits are almost exactly 71% of equilibrium output per worker.

³⁰Dynamic-adjustment results are best understood in the context of permanent productivity shocks. In explaining and contrasting results, this avoids having to keep track of mean-reverting changes in p . Later in the essay I return to the issue of temporary shocks. All figures can be found in Section 2.7. The calibration for the standard model is available in Table 2.3; all applicable calibration targets are as described earlier for the MA-OS model.

of the increase in unemployment stems from an increase in unemployment inflows, while the remaining 70% owes to a decrease in the outflows from unemployment. In the present model, it is the probability of exiting unemployment that is endogenous, while the entry probability is kept constant. For reference, the last column of Table 2.2 shows elasticities based on data generated using the standard, homogenous-agent model. The standard model predicts an elasticity of Θ equal to 3.462 (48% of the data), and an elasticity of u equal to -1.537 (43% of the data). The MA-OS model yields substantial gains in explanatory power relative to the standard model. Results suggest that worker-side heterogeneity and optimal search can have an important role in aggregate labor-market fluctuations.

2.4.2 Propagation

For the purposes of comparison, Figure 2.3 once again shows the dynamic response of Θ in the MA-OS model given a 1% permanent increase in p , and also in the three models it nests: 1) the MA model (relative to the MA-OS model, endogenous effective search is shut down) 2) the standard-OS model (relative to the MA-OS model, there is no heterogeneity) and 3) the standard model.³¹ As suggested by the earlier analysis, the response of Θ in the MA-OS model is a combination of that in the MA and standard-OS models. In particular, the MA model is the key behind propagation, which neither the standard-OS nor standard models feature. In addition, amplification (the total change in Θ) is enhanced by endogenous effective search. This is highlighted by contrasting the response of Θ in the standard-OS model relative to the standard model. Moreover, note that the MA-OS model's combination of endogenous effective search and heterogeneity yields greater amplification and propagation than in, respectively, the standard-OS and MA models on their own.

2.4.2.1 Propagation in the MA Model To understand propagation, I focus initially on the MA model. Recall from the earlier analysis that propagation will result

³¹The response of Θ in the MA-OS and standard models was already shown in Figure 2.1 and is reproduced for the purposes of comparison. The calibration for the MA and standard-OS models is available in Table 2.4.

from slow-moving changes in the firm-side probability of skill-mismatched employment $\pi_2^1 = u_2/se^1$.³² For $i \in \{1, 2\}$ symmetry implies that the percent changes in u_i and χ_i are the same as those in their aggregate counterparts u and χ . The relative rate of adjustment between χ and u is key for the direction of change in π_2^1 (see equation (2.15), and recall that in the MA model all effective search is fixed at unity). In particular, given an increase in productivity, as long as χ/u is increasing, π_2^1 will be decreasing. Intuitively, relatively more individuals directing their search towards skill-matched employment only (OTJ searchers) imply a lower firm-side probability of skill mismatch.

An increase in productivity increases the probability of exiting unemployment (see equation (2.4)), but increases *both* the probability of entering and exiting skill-mismatched employment (see equation (2.5)). These opposing forces on skill mismatched employment imply that unemployment will adjust relatively faster. Figure 2.4 shows the dynamic response of χ and u to a 1% increase in economy-wide productivity in the MA model. The fact that unemployment adjusts faster than skill-mismatched employment has two effects. Immediately after the shock, the rate of adjustment in χ is slower than that in u : this tends to decrease π_2^1 . Because u adjusts faster than χ , then it converges towards its new equilibrium value faster (while χ continues to decline): this tends to increase π_2^1 . Therefore, although from Proposition 1 after an increase in productivity π_2^1 will ultimately decrease relative to its initial value, its adjustment will be U-shaped.

The U-shaped adjustment in the firm-side probability of skill-mismatched employment, π_2^1 , is shown in Figure 2.5. This figure depicts the dynamic adjustment of π_2^1 in the MA model given a 1% permanent increase in productivity p . The corresponding dynamic adjustment in Θ is also shown for reference. While π_2^1 decreases incentives for vacancy-posting remain relatively high and Θ increases. Thereafter, Θ decreases and converges to its new (higher) equilibrium value.

³²For simplicity, whenever convenient I continue to exploit the model's symmetry, developing analysis from the point of view of type-1 economic agents.

2.4.2.2 Propagation in the MA-OS Model Now, consider the MA-OS model. Figure 2.6 shows the response of π_2^1 and Θ to a 1% permanent increase in p . Note that after the shock, the dynamic response of π_2^1 is similar to that in the MA model. Therefore, the reasons for propagation in the MA-OS model are analogous to those in the MA model. Note, however, that on impact of the shock π_2^1 rises. This implies that the jump (effective search) components of π_2^1 dominate its on-impact response, but thereafter its behavior is dominated by its slow-moving components. Proposition 1 can be thought of carrying over as follows: on impact of a shock π_2^1 can jump, but relative to its position after this initial jump, π_2^1 will decrease. Figure 2.6 highlights that what matters for propagation is the slow-moving post-shock direction of change in π_2^1 , and not its initial jump.

To understand the jump in π_2^1 shown in Figure 2.6, consider Figure 2.7, which shows the response of effective search in the MA-OS model given a 1% permanent increase in p . Note that the effective search that unemployed individuals devote to skill-mismatched jobs ($U \rightarrow$ skill-mismatch, $\ell_{j,u,t}^i$, $i \neq j$) experiences the greatest percentage-wise increase, while unemployed and OTJ effective search devoted to skill-matched search ($U \rightarrow$ skill-match, $\ell_{i,u,t}^i$, and OTJ search, $\ell_{i,x,t}^i$) follow in magnitude. In particular, the relatively greater percent increase in effective search devoted towards skill-mismatched employment is what makes π_2^1 increase on impact of the shock (recall that in the MA-OS model $\pi_2^1 = \ell_{2,u,t}^1 u_{2,t} / se_t^1$). This relatively greater percent increase in $\ell_{2,u}^1$ will always occur, and stems from the expected gains from search for skill-mismatched employment always being relatively lower.³³ To further understand this, consider an extreme example. Suppose the expected gains from skill-mismatched search were zero; then, the first-order conditions for optimal effective search imply that $\ell_{2,u}^1$ would be zero as well. Given a positive productivity shock, assume these expected gains increase by an arbitrarily small amount. Then, so will $\ell_{2,u}^1$. However, because $\ell_{2,u}^1$ was originally zero, the percent change in $\ell_{2,u}^1$ would technically be infinity.³⁴

³³However, recall from earlier that by comparative advantage $\ell_{1,u,t}^1 > \ell_{2,u,t}^1$ for all states of the economy.

³⁴Note from Figure 7 that although small in magnitude, effective search does exhibit propagation.

Having analyzed the jump components of π_2^1 in the MA-OS model, Figure 2.8 turns to its slow-moving components. As before, given symmetry, I focus on the aggregate rates of unemployment and skill mismatch. Note that the dynamic response of unemployment is similar to that in the MA model. However, while in the MA model χ is always decreasing after the shock, in the MA-OS model after the shock χ initially rises, and thereafter decreases. The initial rise in χ owes to the greater procyclicality of unemployed effective search relative to OTJ effective search (which results in entries into skill-mismatch initially exceeding outflows), and serves to magnify the slow-moving decrease in π_2^1 . This induces greater propagation in Θ in the MA-OS model than in the MA model.

Note from comparison of Figures 2.6 and 2.8 that changes in the mass of skill-mismatched employment are not a sufficient statistic for changes in vacancy-posting incentives. In particular, in response to an increase in productivity, there are stages of adjustment over which at the same time that the rate of skill-mismatched employment is increasing, the firm-side probability of skill-mismatch is decreasing. Given the earlier analysis the intuition for this is straightforward: the relatively more skill-mismatched individuals there are, the more job seekers there are who direct their search exclusively towards skill-matched employment; this tends to decrease π_2^1 .

2.4.3 The V/U Adjustment Process: Temporary Shocks

In their detailed analysis, Fujita and Ramey (2007) show that, empirically, in the US, in response to a temporary increase in productivity: 1.a) on impact the V/U ratio jumps; however, around 60% of the total increase in the V/U ratio occurs during its post-shock stage of slow-moving adjustment, 1.b) the peak of the response of the V/U ratio in response to a temporary increase in productivity occurs approximately 12 months after the increase in productivity takes place, 2) the aggregate unemployment rate declines for approximately 15 months before mean reversion begins, and 3) vacancies exhibit propagation as well; on impact of the shock vacancies jump, and

After the shock, all effective search continues to increase for approximately 5 months. Given equation (2.15), if OTJ search is not possible, then even when effective search is endogenous all adjustments will be instantaneous. It is the the slow-moving ratio χ/u in π_j^i that triggers propagation.

thereafter they continue to increase for approximately 12 months before mean reversion begins. It follows that, empirically, propagation in vacancies is a contributing factor to propagation in the V/U ratio.

It is of interest to gauge the extent to which worker-side heterogeneity and optimal search contribute to the dynamic adjustments noted in Fujita and Ramey (2007). Hence, in the MA-OS model, I consider the response of the V/U ratio, aggregate vacancies, and aggregate unemployment to a temporary shock in productivity, p , similar in magnitude and persistence to the (empirical) one shown in Fujita and Ramey (2007).³⁵ Figure 2.9 shows the dynamic adjustment of the V/U ratio in response to such shock, while Figure 2.10 shows the evolution of productivity. On impact Θ increases by around 3%. Thereafter, propagation of the shock occurs: Θ continues to rise for 3 months. In addition, out of the total increase in Θ , around 16% owes to the period over which it slowly increases. Given the evidence in Fujita and Ramey (2007), in the MA-OS model the length of time it takes adjustment in Θ to peak is around 25% of its empirical counterpart. Moreover, the relative fraction of the total rise in Θ owing to propagation in the MA-OS model is approximately 27% of that which occurs empirically.

I now turn to the dynamic adjustment of aggregate vacancies and unemployment themselves. Because $\Theta = v/u$, there are three ways through which propagation in Θ can emerge. Consider an increase in p . Of course, u will decline. Therefore, as long as $d \log v_t > d \log u_t$, Θ will be increasing. This can occur either because after the shock vacancies are increasing, vacancies remain fixed, or vacancies decrease at a slower rate than unemployment does. As shown in Figure 2.11, it is the last of these possibilities that generates propagation in Θ in the MA-OS model. The post-shock slow-moving decline in π_2^1 maintains incentives for vacancy posting higher than otherwise. Therefore, after an initial jump vacancies decrease, but at a slower rate relative to unemployment than they would in the absence of a post-shock decline in π_2^1 . In the standard and standard-OS models vacancies do not exhibit propagation either; however, after a productivity shock vacancies return to their mean value too

³⁵Fujita and Ramey (2007) show the empirical effects of a temporary shock in productivity of 0.7 percent, with persistence parameter at quarterly frequency approximately equal to 0.95.

fast. This lies at the center of these models' lack of propagation in Θ .

If vacancies did exhibit propagation in the MA-OS model, then the model-generated propagation in the V/U ratio would be enhanced. The model's two worker types, two production sectors structure serves to intuitively uncover the latent propagation channel implied by the firm-side probability of skill-mismatched employment. However, a clear path for future research involves exploring extensions of the model in order to understand how slow-moving changes in π_2^1 *could* induce propagation in vacancies. Intuitively, two reasons for which the effects of changes in π_2^1 could be magnified are greater heterogeneity in the labor force, and job-training costs that are decreasing in the actual quality of the employment match. The former is sensible given that heterogeneity is the key to the model's propagation channel. The latter would imply that slow-moving increases in the firm-side probability of skill-matched employment would induce declines in expected training costs; this could increase expected vacancy-posting incentives enough so that vacancies would in fact exhibit propagation.

2.4.4 Match Quality

Although a main focus of this essay has been understanding propagation in the V/U ratio, the heterogeneous worker framework of the MA-OS model allows for broader applications. One interesting issue that I consider is related to match quality itself, which in the model is captured by the fraction of skill-mismatched employment χ/e .

There is no readily applicable index of match quality. However, in a broad sense of match quality Barlevy (2002) notes a dichotomy: while some research suggests that recessions are times in which resources are exclusively reallocated to better uses, other research suggests the opposite. For example, using US manufacturing plant-level data, Bailey, Bartelsman, and Haltiwanger (2005) use the Longitudinal Research Database and find that the labor shares of less productive plants fall in recessions, suggesting that under such circumstances there is a relative reallocation of resources to better uses. Alternatively, Bowlus (1995) assumes that matches of better quality last a longer time. Using job-tenure National Longitudinal Survey of Youth data,

Bowlus finds that during recessions workers increase their acceptance of jobs that on average dissolve faster, suggesting a decline in match quality. In the spirit of reconciling this evidence, Barlevy (2002) argues that given a negative productivity shock, matches of the worst quality may be destroyed, thus increasing average match quality. At the same time, a worsening of overall economic conditions can slow down the reallocation of workers across jobs enough so that individuals have a relatively more difficult time moving into jobs for which they are best suited for; this tends to decrease average match quality.³⁶

Figure 2.12 shows the dynamic response of the fraction of skill-mismatched employment χ/e to a 1% permanent increase in productivity in the MA-OS model. Of course, the response of χ/e closely mimics that of χ shown earlier in Figure 2.8. Note that χ/e first increases (implying a decrease in average match quality) and thereafter declines (which implies an improvement in average match quality). The intuition for this result is as noted earlier for the dynamic response of χ . Employed skill-mismatched job seekers have a more attractive outside option than unemployed job seekers. Therefore, the effective search of unemployed individuals is more procyclical than that of OTJ searchers. This means that, on impact of a positive productivity shock, entries into skill-mismatched employment exceed outflows. Overall, results imply that changes in productivity can yield different stages of adjustment over which increases and decreases in match quality are observed.³⁷

2.5 Related Literature

The model developed in the present essay builds on the search-and-matching framework as developed through the work of, for example, Diamond (1982), Mortensen (1982), Pissarides (1985), Mortensen and Pissarides (1994), and Pissarides (2000). Shimer (2005) revealed several quantitative limitations of the standard, homogeneous-

³⁶The model in Barlevy (2002) is based on non-directed search, and does not account for endogenous effective search, the main focus of the paper being the cyclical behavior of match quality itself, and not the impact of worker-side heterogeneity on the coexistence of aggregate vacancies and unemployment.

³⁷However, recall from analysis of the MA model that this is not necessary for propagation in the V/U ratio to occur in response to changes in productivity.

agent model of equilibrium unemployment theory in terms of explaining the empirical behavior of aggregate vacancies and unemployment. Since then, much research within the homogenous-agent framework has focused on addressing these limitations. Among others, this includes Hall (2005), Fujita and Ramey (2007), Mortensen and Nagypal (2007), and Hagedorn and Manovskii (2008).

Turning towards heterogeneity, Albrecht and Vroman (2002), Gautier (2002), Chassamboulli (2009), and Dolado, Jansen, and Jimeno (2009) focus on explaining differences in unemployment and wages between high- and low-skill workers when high-skilled individuals can work in low-skill jobs, but not the other way around, and effective search is exogenous. Bils, Chang, and Kim (2009) study worker-side heterogeneity in a context in which “comparative advantage” refers to individuals who have high market productivity relative to their home productivity. However, labor markets are segmented: although the labor force is heterogeneous, conditional on idiosyncratic characteristics individuals seek employment in only one production sector. Bils, Chang, and Kim focus on understanding differences in unemployment and work hours across labor force participants.

Krause and Lubik (2006) explore the role of endogenous OTJ search by developing a single-worker, two-sector model in which one sector is “good” and the other is “bad.” The value of output in the good sector is always higher than the value of output in the bad sector. Since there is only one type of worker, there is no firm-side uncertainty with regards to expected match quality. Moreover, the search behavior of unemployed individuals is not endogenized. Although all workers are identical, in equilibrium a fraction of unemployed individuals only search for jobs in the good sector, and the remaining fraction only searches for jobs in the bad sector. Krause and Lubik show that in an expansion, endogenous OTJ search along with a decrease in the fraction of unemployed individuals searching for good jobs can induce amplification in the V/U ratio in excess of the standard model, as well as propagation.

Pries (2007) reverses the argument in Krause and Lubik (2006) by modeling an economy in which there is a unique production sector (hence, no OTJ search is possible), and two types of workers: “high” and “low.” High types always produce more

than low types. Pries shows that because the employment surplus associated with low types is smaller and thus more sensitive to productivity shocks, the greater the number of low types, the greater the amplification of shocks in the V/U ratio. Pries also shows that when the job-destruction probability associated with low-type individuals is (exogenously) countercyclical, in a recession expected employment surpluses are reduced through an increase in the probability of a match being formed with a low type. This enhances the amplification channel generated by the presence of low-type workers. Pries does not address issues of propagation. In addition, in his model individuals' search behaviors are not endogenous.

2.6 Conclusions

This essay develops an understanding of the effects of worker-side heterogeneity and optimal job-seeking behavior on aggregate labor-market fluctuations. I show that these worker-side factors can aid in accounting for the amplification of productivity shocks in the aggregate vacancy-unemployment (V/U) ratio. Moreover, slow-moving changes in expected vacancy-posting gains that stem from the presence of worker-side heterogeneity can help explain why, empirically, the V/U ratio exhibits a stage of sluggish adjustment (propagation) in response to changes in productivity. Sluggish adjustment in the V/U ratio is a key feature of the data that the standard, homogenous-agent model of equilibrium unemployment theory cannot account for.³⁸ Understanding the slow-moving behavior of the V/U ratio is important, since it captures the extent to which the labor market responds persistently, for example, in the wake of a recession.

The model's channel for slow-moving changes in the V/U ratio lies in changes in the composition of sectoral searchers. In an expansion, as the pool of unemployed workers declines, optimal job-seeking behavior (in particular, directed search) induces the quality of the pool of sector-specific searchers to improve. Therefore, at the sectoral level, the firm-side probability of skill-matched employment increases. Slow-moving increases in this probability lead to slow-moving increases in the expected

³⁸See, for example, Fujita and Ramey (2007).

gains from posting vacancies. This keeps vacancy-posting incentives higher than otherwise. Coupled with declining unemployment, this results in sluggish adjustment in the V/U ratio.

Although slow-moving changes in the probability of skill-matched employment generate propagation in the V/U ratio in response to changes in productivity, in the present context they do not induce propagation in aggregate vacancies. As shown in Fujita and Ramey (2007), propagation in vacancies indeed occurs empirically. If vacancies did exhibit propagation in the model developed in the present essay, then the model-generated propagation in the V/U ratio would be enhanced.

The model's two worker types, two production sectors structure serves to intuitively uncover and analyze the latent V/U -ratio propagation channel implied by the firm-side probability of skill-matched employment. However, a clear path for future research involves exploring extensions of the model in order to understand how slow-moving changes in the firm-side probability of skill-matched employment could in fact induce propagation in vacancies. Intuitively, two reasons for which the effects of changes in the firm-side probability of skill-matched employment could be magnified are greater heterogeneity in the labor force, and job-training costs that are decreasing in the quality of an employment match. The former is sensible, given that heterogeneity is the key to the model's propagation channel. The latter would imply that slow-moving increases in the firm-side probability of skill-matched employment would induce declines in expected training costs; this could increase expected vacancy-posting incentives enough so that vacancies would in fact exhibit propagation.

2.7 Figures

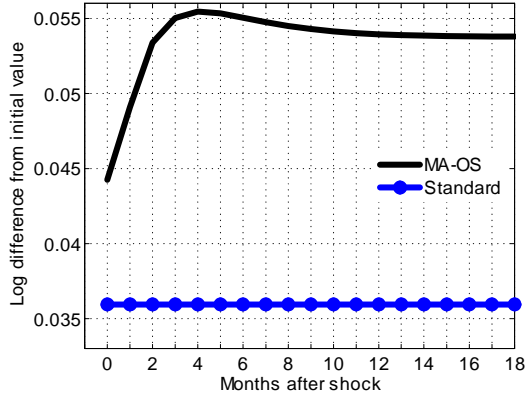


Figure 2.1

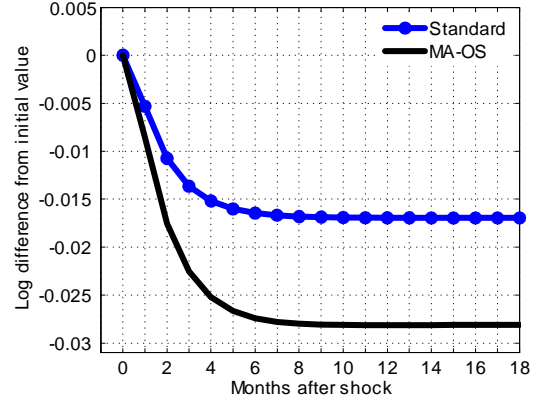


Figure 2.2

Figure 2.1: response of the V/U ratio to a 1% permanent increase in economy-wide productivity p . Figure 2.2: response of aggregate unemployment to a 1% permanent increase in p .

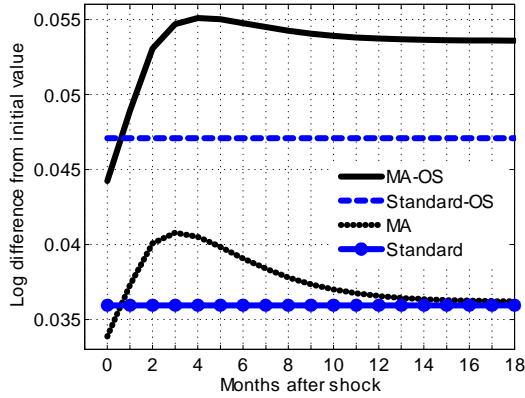


Figure 2.3

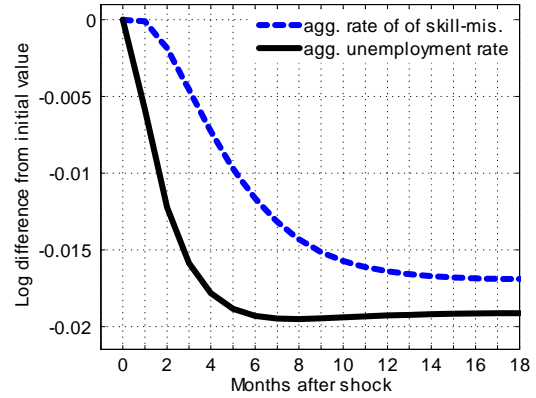


Figure 2.4

Figure 2.3: response of the V/U ratio to a 1% permanent increase in economy-wide productivity p . Figure 2.4: response of χ and u to a 1% permanent increase in economy-wide productivity p in the MA model.

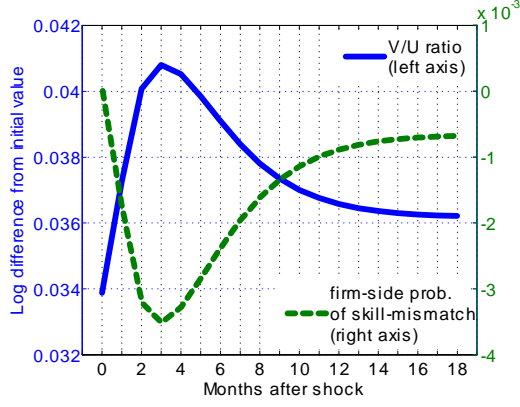


Figure 2.5

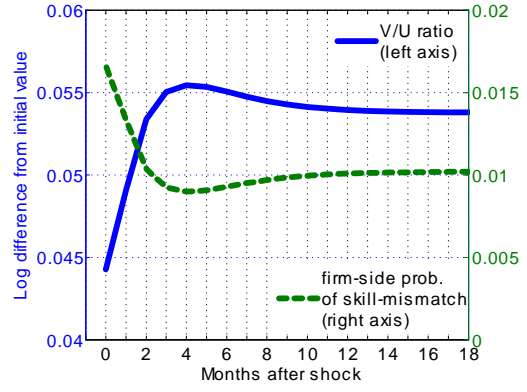


Figure 2.6

Figure 2.5: response of π_2^1 and Θ to a 1% permanent increase in p in the MA model. Figure 2.6: response of π_2^1 and Θ to a 1% permanent increase in p in the MA-OS model.

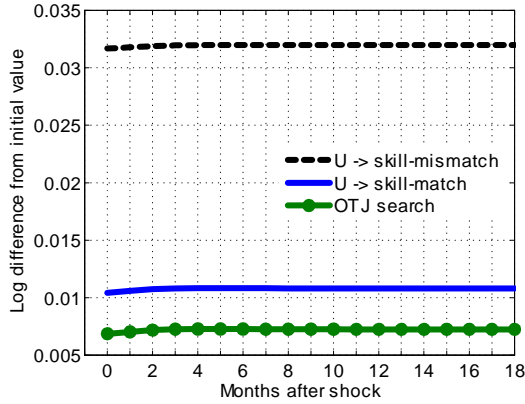


Figure 2.7

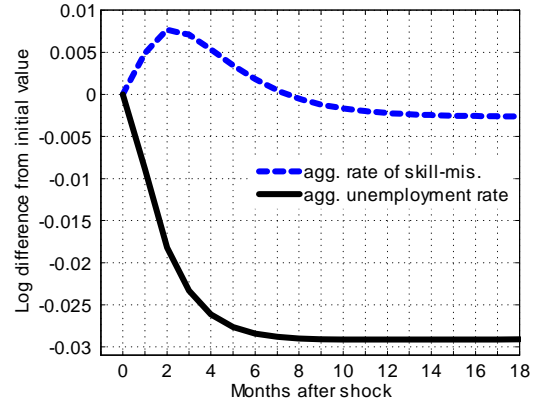


Figure 2.8

Figure 2.7: response of effective search to a 1% permanent increase in economy-wide productivity p in the MA-OS model. Figure 2.8: response of χ and u to a 1% permanent increase in economy-wide productivity p in the MA-OS model.

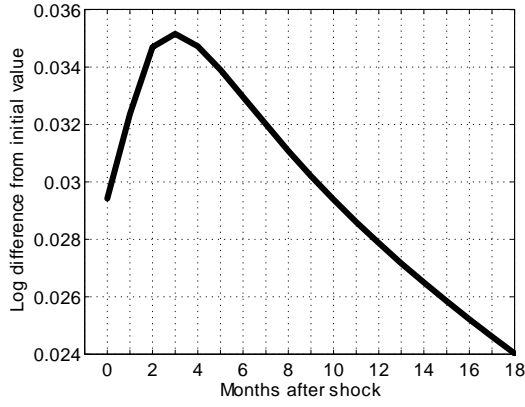


Figure 2.9

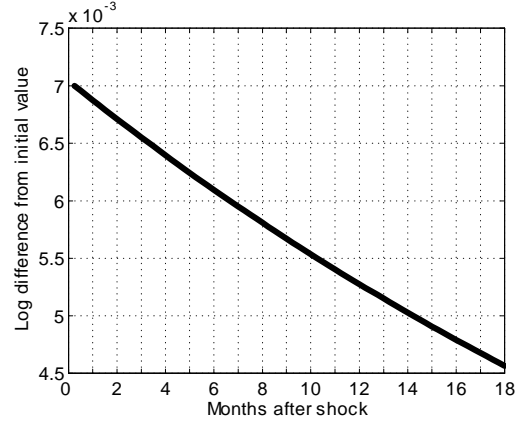


Figure 2.10

Figure 2.9: response of the V/U ratio to a temporary increase in economy-wide productivity p in the MA-OS model (see text for details). Figure 2.10: response of economy-wide productivity p given a shock of size 0.007 (see text for details).

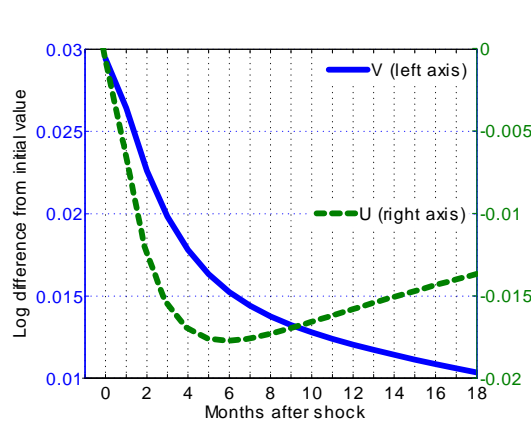


Figure 2.11

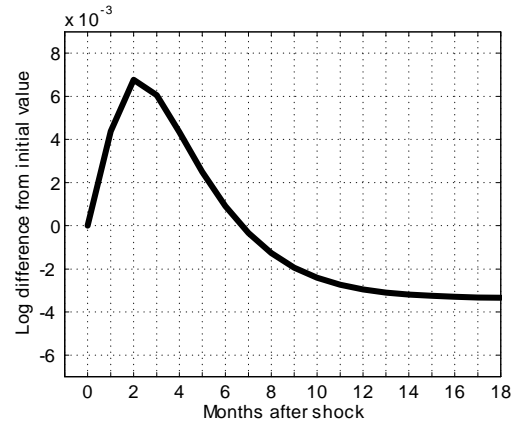


Figure 2.12

Figure 2.11: response of aggregate vacancies and unemployment to a temporary increase in economy-wide productivity p in the MA-OS model (see text for details). Figure 2.12: response of the fraction of skill-mismatched employment χ/e to a 1% permanent increase in p .

2.8 Tables

Table 2.1: MA-OS model calibration (weekly frequency)

Parameter	Value	Reason
b	0.772	$z = b - C = 0.71$, Hall and Milgrom (2008)
α	0.371	Petrongolo and Pissarides (2001)
A	0.128	data: target $u = 0.057$
Γ	0.0526	Lise, Seitz, and Smith (2003)
δ	0.0082	data: implied by $u = 0.057$
η	0.529	$c/q^i=14\%$, Silva and Toledo (2007)
c	0.438	$\Theta = 0.72$, data: Pissarides (2009)
ϕ	0.151	$\omega = 0.84$, McLaughlin and Bils (2001)
ε	0.338	Beveridge curve slope
β	0.999	Quarterly interest rate 0.012
ρ	0.99	US OPW, quarterly autocorrelation 0.88
σ_ϵ^2	0.0034	US OPW, quarterly std=0.02

Table 2.2: Elasticities with respect to output per worker (OPW)

Variable	Description	US Data	Model	
			MA-OS	Standard
Θ	V/U ratio	7.261	5.201	3.462
u	Agg. unemployment rate	-3.580	-2.611	-1.537

Notes: All elasticities are calculated using the data's log deviations from trend by applying a Hodrick-Prescott filter with smoothing parameter 10^5 (as in Shimer (2005)). The standard error on the US aggregate vacancy-unemployment ratio Θ is 1.175, while that on the US unemployment rate u is 0.581. Following the literature, I use the Conference Board's Help-Wanted Advertising Index for vacancies. Data on Θ and u spans 1951:Q1 through 2006:Q2 (220 quarters), and is subject to availability of the Conference Board's Index. Data on US unemployment is available from the Bureau of Labor Statistics.

Table 2.3: Standard model calibration

b	α	A	η	c	ε	ϕ
0.762	0.5	0.159	0.515	0.336	0.000	0.000

Notes: For the standard model, I assume that $z = b - C(1)$ given that effective search is fixed at unity. Moreover, for consistency with the MA-OS model the matching function becomes $m = v^\alpha u^{1-\alpha}$. All applicable parameters not explicitly noted are as in Table 2.1, and all applicable calibration targets are as described for the MA-OS model. The calibration is at weekly frequency.

Table 2.4: Standard-OS and MA model calibrations

Parameter	b	α	A	η	c	ϕ	ε
Standard-OS	0.775	0.330	0.148	0.514	0.337	0.000	0.338
Multi-Agent	0.815	0.572	0.151	0.554	0.479	0.165	0.000

Notes: For the MA model I assume $z = b - C(1, 1)$ given that effective search is fixed at unity. This means that OTJ searchers incur the search cost $C(1)$. In the standard-OS model $z = b - \Gamma \ell^{(1+\varepsilon)/\varepsilon}$, and consistent with the MA-OS model the matching function becomes $m = v^\alpha (\ell u)^{1-\alpha}$. All applicable parameters not explicitly noted are as in Table 2.1, and all calibration targets are as described for the MA-OS model except for my use of ε in the standard-OS model. To gauge the relative contribution of the elasticity of effective search to results stemming from the MA-OS model, in the standard-OS model I keep this parameter as in Table 2.1. In spite of this, the calibration target otherwise associated with ε remains almost unchanged relative to its value in the MA-OS model. The calibration is at weekly frequency.

2.9 Proofs

As noted in Shimer (2005) and Mortensen and Nagypal (2007), the cyclical properties of models of the sort developed in this essay are well assessed by considering differences between steady states. Therefore, proofs in this section are based on steady-state to steady-state changes.

Proof of Proposition 1. Given symmetry, for $i \neq j$ $u_i = u_j = 0.5u$, and $\chi_i = \chi_j = 0.5\chi$. Then, using equation (2.3) and rearranging implies that $\pi_j^i = (2 + \chi/u)^{-1}$, where $i \neq j$, and I have used the assumption that all effective search is fixed at unity. Therefore, if χ/u is procyclical, then π_j^i is countercyclical. For $i \neq j$ symmetry implies that $f^i = f^j = f$. Then,

$$\chi_i = \frac{f}{\delta + f} u_i \implies \frac{d \log (\chi/u)}{d \log p} = \frac{\delta}{f + \delta} \frac{d \log f}{d \log p} > 0, \quad (2.22)$$

since f is procyclical. \square

Proof of Proposition 2. Using symmetry ($\chi_i = 0.5\chi$, $u_i = 0.5u$, $\psi_i = 0.5$) implies that

$$\chi = \frac{f}{\delta + f} \cdot \frac{\delta}{\delta + 2f} \implies \frac{d \log \chi}{d \log p} = \frac{\delta^2 - 2f^2}{(\delta + f)(\delta + 2f)} \frac{d \log f}{d \log p}. \quad (2.23)$$

For reasonable parameter values and job-finding probability f , $\delta^2 < 2f^2$.³⁹ Given that the probability of finding a job is procyclical, it follows that $d \log \chi / d \log p < 0$.
□

Proof of Proposition 3. In text.

Proof of Proposition 4.⁴⁰ Consistent with the functional forms assumed for the MA-OS model, for the standard-OS model let $C = \Gamma \ell^{(1+\varepsilon)/\varepsilon}$ and $m = v^\alpha s^{1-\alpha}$, where $s = \ell u$. In this case, $q = A\theta^{\alpha-1}$, where $\theta = v/\ell u$. Moreover, denote the job-finding probability of unemployed individuals by $F = \ell f$, where $f = A\theta^\alpha$. The relevant first-order condition for optimal search implies that

$$\Gamma \frac{(1+\varepsilon)}{\varepsilon} \ell^{1/\varepsilon} = \beta \eta f S, \quad (2.24)$$

where S is the employment surplus (recall that individuals choose effective search taking market conditions as given). Moreover, the job-creation condition is now given by

$$\beta (1 - \eta) S = c/q. \quad (2.25)$$

Substituting this into equation (2.24) implies, after total differentiation, that

$$\frac{d \log \ell}{d \log p} = \varepsilon \frac{d \log \theta}{d \log p}. \quad (2.26)$$

Since θ is procyclical, the equation above implies that so is effective search ℓ . Showing formally that θ is procyclical, to which I now proceed, offers additional insight. Using the definition of surplus, $S = W - U + J$, and substituting in for the relevant value functions implies that

$$S = p - z + \beta(1 - \delta - F\eta)S \implies S = \frac{p - z}{1 - \beta(1 - \delta - F\eta)}. \quad (2.27)$$

³⁹See the calibration section.

⁴⁰This proof assumes specific functional forms for ease of exposition.

Combining with equation (2.25) yields

$$\frac{p-z}{1-\beta(1-\delta-F\eta)} = \frac{c}{\beta(1-\eta)q}. \quad (2.28)$$

Therefore,

$$\left(\frac{p}{p-z}\right) d\log p - \left(\frac{z}{p-z}\right) d\log z = \frac{\beta\eta dF}{1-\beta(1-\delta-F\eta)} - d\log q. \quad (2.29)$$

Using the fact that $F = \ell f$ and $z = b - \Gamma\ell^{(1+\varepsilon)/\varepsilon}$, this can be stated as

$$\left(\frac{p}{p-z}\right) d\log p = \left(1 - \alpha + (\varepsilon + \alpha) \frac{\beta\eta F}{1-\beta(1-\delta-F\eta)} - \frac{\beta\eta FS}{p-z}\varepsilon\right) d\log \theta. \quad (2.30)$$

Of course, $1 - \alpha > 0$ and $p - z > 0$, the latter by assumption: otherwise, it would not be optimal for individuals to seek employment. Moreover, $1 > \beta(1 - \delta - F\eta)$. Then, given equation (2.30), to show that $d\log \theta / d\log p > 0$ it is enough to show that the second term in the coefficient on $d\log \theta$ is greater than the third term. Note that

$$\begin{aligned} (\varepsilon + \alpha) \frac{\beta\eta F}{1-\beta(1-\delta-F\eta)} &> \frac{\beta\eta FS}{p-z}\varepsilon \\ \iff (\varepsilon + \alpha) \frac{p-z}{1-\beta(1-\delta-F\eta)} &> \varepsilon S \\ \iff (\varepsilon + \alpha) S &> \varepsilon S, \end{aligned} \quad (2.31)$$

which will always hold (in the second line above, I have made use of equation (2.27)).

Therefore, $d\log \theta / d\log p > 0$, $d\log z / d\log p < 0$, and $d\log \ell / d\log p > 0$. \square

Proof of Proposition 5.⁴¹ Consistent with the functional forms assumed for the MA-OS model, for the standard model let $m = v^\alpha u^{1-\alpha}$. In this case, $q = A\Theta^{\alpha-1}$, where $\Theta = v/u$. Moreover, denote the job-finding probability of unemployed individuals by F , where $F = A\Theta^\alpha$. Using the definition of surplus, $S = W - U + J$, and substituting in for the relevant value functions implies that

$$S = p - z + \beta(1 - \delta - F\eta)S \implies S = \frac{p-z}{1-\beta(1-\delta-F\eta)}. \quad (2.32)$$

⁴¹This proof assumes specific functional forms for ease of exposition.

Combining with the job-creation condition $S = c/\beta (1 - \eta) q$ implies that

$$\frac{p - z}{1 - \beta(1 - \delta - F\eta)} = \frac{c}{\beta(1 - \eta)q}. \quad (2.33)$$

Hence,

$$\left(\frac{p}{p - z}\right) d \log p = \frac{\beta \eta dF}{1 - \beta(1 - \delta - F\eta)} - d \log q, \quad (2.34)$$

and

$$\frac{d \log \Theta^{EXO}}{d \log p} = \left(\frac{p}{p - z}\right) (1 - \alpha + \alpha X)^{-1}, \quad (2.35)$$

Above, $X = \beta \eta F / (1 - \beta(1 - \delta - F\eta))$, and EXO denotes that effective search is exogenous (and fixed at unity: $\ell = 1$). Moreover, for reasonable values for β , η , δ , and F , $X \in (0, 1)$.

Note from the proof of Proposition 4 that equation (2.29) can be written as

$$\begin{aligned} (1 - \alpha + \alpha X) d \log \Theta &= \left(\frac{p}{p - z}\right) d \log p - \left(\frac{z}{p - z}\right) d \log z + (1 - \alpha)(1 - X) d \log \ell \\ \Rightarrow \frac{d \log \Theta^{ENDO}}{d \log p} &= \left(\frac{p}{p - z}\right) (1 - \alpha + \alpha X)^{-1} - \frac{(z/(p - z)) d \log z}{(1 - \alpha + \alpha X) d \log p} \\ &\quad + \frac{(1 - \alpha)(1 - X) d \log \ell}{(1 - \alpha + \alpha X) d \log p}, \end{aligned} \quad (2.36)$$

where $ENDO$ denotes that effective search is endogenous. From earlier, $X \in (0, 1)$; therefore, $1 - \alpha + \alpha X > 0$. Moreover, from the proof of Proposition 4 $d \log z / d \log p < 0$ and $d \log \ell / d \log p > 0$. Of course, $p > z$. Hence, using equation (2.35), equation (2.36) can be stated as

$$\begin{aligned} \frac{d \log \Theta^{ENDO}}{d \log p} &= \frac{d \log \Theta^{EXO}}{d \log p} - \frac{(z/(p - z)) d \log z}{(1 - \alpha + \alpha X) d \log p} + \frac{(1 - \alpha)(1 - X) d \log \ell}{(1 - \alpha + \alpha X) d \log p} \\ &\Rightarrow \frac{d \log \Theta^{ENDO}}{d \log p} > \frac{d \log \Theta^{EXO}}{d \log p}. \quad \square \end{aligned} \quad (2.37)$$

2.10 References

- Albrecht, James, and Susan Vroman.** 2002. "A Matching Model with Endogenous Skill Requirements." *International Economic Review*, 43(1): 283-305.
- Bailey, Martin, Eric Bartelsman, and John Haltiwanger.** 2001. "Labor Productivity: Structural Change and Cyclical Dynamics." *The Review of Economics and Statistics*, 83(3): 420-433.
- Barlevy, Gadi.** 2002. "The Sullyng Effects of Recessions." *Review of Economic Studies*, 69(10): 65-96.
- Bils, Mark, Yongsung Chang, and Sun-Bin Kim.** 2009. "Comparative Advantage and Unemployment." NBER Working Paper No. w15030.
- Bowlus, Audra.** 1995. "Matching Workers and Firms: Cyclical Fluctuations in Match Quality." *Journal of Labor Economics*, 13(2): 335-350.
- Chassamboulli, Andri.** 2009. "Cyclical Upgrading of Labor and Unemployment Differences Across Skill Groups." University of Cyprus. Unpublished manuscript.
- Christensen, Bent, Rasmus Lentz, Dale Mortensen, George Neumann, and Axel Werwatz.** 2005. "On-the-Job Search and the Wage Distribution." *Journal of Labor Economics*, 23(1): 31-58.
- Davis, Steven, Jason Faberman, and John Haltiwanger.** 2009. "The Establishment Level Behavior of Vacancies and Hiring." Federal Reserve Bank of Philadelphia Working Paper No. 09-14.
- Diamond, Peter.** 1982. "Aggregate Demand Management in Search Equilibrium." *The Journal of Political Economy*, 90(5): 881-894.
- Dolado, Juan, Marcel Jansen, and Juan Jimeno.** 2008. "On-the-Job Search in a Matching Model with Heterogeneous Jobs and Workers." *The Economic Journal*, 119(534): 200-228.
- Elsby, Michael, Ryan Michaels, and Gary Solon.** 2009. "The Ins and Outs of Cyclical Unemployment." *American Economic Journal: Macroeconomics*. 1(1): 84-110.
- Fujita, Shigeru, and Garey Ramey.** 2007. "Job Matching and Propagation." *Journal of Economic Dynamics and Control*, 31(11): 3671-3698.
- Gautier, Pieter.** 2002. "Unemployment and Search Externalities in a Model with Heterogeneous Jobs and Workers." *Economica*, 69(273), 21-40.

- Hagedorn, Marcus, and Iourii Manovskii.** 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *The American Economic Review*, 98(2): 1692-1706.
- Hall, Robert.** 2005. "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review*, 95(1), 50-65.
- Hall, Robert., and Paul Milgrom.** 2008. "The Limited Influence of Unemployment on the Wage Bargain." *The American Economic Review*, 98(4): 1653-1674.
- Krause, Michael, and Thomas Lubik.** 2006. "The Cyclical Upgrading of Labor and On-the-Job Search." *Labour Economics*, 13(4): 459-488.
- Krueger, Alan, and Andreas Mueller.** 2008. "The Lot of the Unemployed." IZA discussion paper No. 3490.
- Lise, Jeremy, Shannon Seitz, and Jeffrey Smith.** 2003. "Equilibrium Policy Experiments and the Evaluation of Social Programs." IZA Discussion Paper No. 758.
- McLaughlin, Kenneth, and Mark Bils.** 2001. "Interindustry Mobility and the Cyclical Upgrading of Labor." *Journal of Labor Economics*, 19(1): 94-135.
- Mortensen, Dale.** 1982. "Property Rights and Efficiency in Mating, Racing, and Related Games." *The American Economic Review*, 72(5): 968-979.
- Mortensen, Dale, and Eva Nagypal.** 2007. "More on Unemployment and Vacancy Fluctuations." *Review of Economic Dynamics*, 10(3): 327-347.
- Mortensen, Dale, and Christopher Pissarides.** 1994. "Job Creation and Job Destruction in the Theory of Unemployment." *The Review of Economic Studies*, 61(3): 397-415.
- Nagypal, Eva.** 2006. "Amplification of Productivity Shocks: Why Don't Vacancies Like to Hire the Unemployed?" *Structural Models of Wage and Employment Dynamics*, vol. 275 of *Contributions to Economic Analysis*, ed. H. Bunzel, B. J. Christensen, G. R. Neumann, and J.-M. Robin, pp. 481-506. Amsterdam: Elsevier.
- Nagypal, Eva, and Dale Mortensen.** 2007. "More on Unemployment and Vacancy Fluctuations." *Review of Economic Dynamics*, 10(3): 327-347.
- Petrongolo, Barbara, and Christopher Pissarides.** 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature*, 39(2): 390-431.
- Pissarides, Christopher.** 1985. "Short-Run Equilibrium Dynamics of Vacancies, Unemployment, and Real Wages." *The American Economic Review*, 75(4): 676-690.
- Pissarides, Christopher.** 2000. *Equilibrium Unemployment Theory*. Cambridge, MA: MIT Press.

Pissarides, Christopher. 2009. “The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?” *Econometrica*, 77(5): 1339-1369.

Shimer, Robert. 2005. “The Cyclical Behavior of Equilibrium Unemployment and Vacancies.” *The American Economic Review*, 95(1): 25-49.

Silva, Jose, and Manuel Toledo. 2006. “Labor Turnover Costs and the Behavior of Vacancies and Unemployment.” Universidad Autonoma de Barcelona and Universidad Carlos III de Madrid. Unpublished manuscript.

CHAPTER III

THE PARADOX OF HARD WORK

3.1 Introduction

Over the last centuries there has been a dramatic world-wide increase in real output, consumption, and wages. Keynes predicted a large increase in leisure in his 1930 essay “Economic Possibilities for Our Grandchildren.” Indeed, income effects on labor supply are substantial.¹ However, the leisure boom predicted by Keynes has not taken place; instead, work hours have remained approximately constant. This is highlighted in Figures 3.0.A through 3.0.D, which show the natural logarithm of consumption per population and work hours per population over the period 1960-2004 for the United States, Japan, the remainder of the G-7 countries, and a large set of European countries, relative to their 1960 values.² With the exception of relatively small declines in hours per population in European countries and relatively small increases in the US, the extent to which hours per population have remained mostly constant across countries, given ongoing and substantial increases in consumption per population, is striking. The objective of this essay is to understand why people are still working so hard, and what the implications of this *paradox of hard work* are for the economy as a whole.³

There are in principle four alternative, although not mutually exclusive, explanations through which the paradox of hard work can be rationalized. The first is

¹See, for example, Shapiro and Kimball (2008).

²Data is at yearly frequency, and taken from the Penn World Tables and the Total Economy Database from The Groningen Growth and Development Centre. The “European Aggregate” consists of Austria, Belgium, Canada, Finland, Greece, Ireland, the Netherlands, Norway, Portugal, Spain, Sweden, and Switzerland.

³This essay is co-written with Miles S. Kimball, Professor of Economics at the University of Michigan, Ann Arbor.

assuming that the elasticity of intertemporal substitution is large. However, empirical evidence suggests exactly the contrary. Hall (1988) finds this elasticity to be approximately zero, Basu and Kimball (2002) find that plausible values are less than 0.7, and Kimball, Sahm, and Shapiro (2011) find a value of approximately 0.08. The second is an increasing marginal-wage to consumption ratio. This can be the result of, for instance, a reduction in the progressivity of the tax system, an intensification of competition for promotions within firms, and increasing educational debts. The third relates to anything that keeps the marginal utility of consumption high. This can occur, for example, because of habit formation, both internal and external (keeping up with the Joneses), as well as the development of new goods. The fourth explanation relates to anything that serves to keep the marginal disutility of work low. This can be, for instance, the result of technological progress in household production, non-separability between consumption and leisure (King, Plosser, and Rebelo (1988), Basu and Kimball (2002)), and jobs getting nicer. Of the set of possible explanations, in this essay we focus particular attention on the impact of improvements in on-the-job utility both within the context of separable and non-separable preferences. Indeed, economists have long understood that cross-sectional differences in on-the-job utility at a particular time give rise to compensating differentials. This essay develops a theory that focuses on a less-studied topic: understanding the long-run macroeconomic consequences of trends in on-the-job utility.⁴

Given our focus, a natural point of departure is the theory of compensating differences, which originates in the first ten chapters of Book I of “The Wealth of Nations” (Smith (1776)). A standard modern reference on this topic is Rosen (1986). Figures 3.1 and 3.2, where W denotes the real wage, J on-the-job utility, and Y output, show two well-known results from the theory compensating differences. In particular, the solid line in Figure 3.1 is a wage/job-utility frontier showing that jobs that offer lower on-the-job utility should be expected to be associated with higher real wages as a means of compensating individuals for such lower on-the-job utility: individuals face a trade-off between these two variables. As the solid line in Figure 3.2 shows, a

⁴See Coulibaly (2006) for complementary research.

similar trade-off is faced by firms in terms of a job-utility/output frontier: offering higher on-the-job utility is costly in terms of output. This is because, as noted in Rosen (1986), firms can divert part of their productive resources towards making the quality of their jobs better. Given workers' individual preferences and firms' idiosyncratic costs of on-the-job utility in terms of output, each economic actor respectively optimizes by choosing a feasible point on the (J, W) plane and the (Y, J) plane.

Of course, the secular increases in real wages and consumption referred to earlier are the result of increases in output. In Figures 3.1 and 3.2, all else equal, increases in output and the real wage are consistent with movements along the plotted curves as indicated by the accompanying arrows, moving the economy from points a to b and c to d . Such movements are associated with decreases in on-the-job utility. Therefore, if on-the-job utility is positively related to the amount of hours individuals desire to spend at work, trendless labor hours in the face of strong income effects consistent with increases in output and wages require shifts in the wage/job-utility and job-utility/output frontiers to the dashed lines shown in Figures 3.1 and 3.2. Therefore, the economy's choice set should be expanding and optimal choices should be moving in the northeast direction as exemplified by points a' and d' .

Our principal analysis focuses on the macroeconomic implications of factors that shift the economy's output/on-the-job-utility possibility frontier across time. In Section 3.2 we develop a benchmark model that allows us to study the interaction of *work hours* (which stands in for all aspects of the job that interfere with leisure and home production), *effort* (which stands in for all aspects of a job whose cost is in terms of proportionate changes in effective productive input from labor), *amenities* (which we define to be job characteristics whose cost is in terms of goods), and *drudgery* (which is a variable capturing everything else that matters for job utility). A novel result with respect to the Frisch elasticity of labor supply emerges, which is that this elasticity is decreasing in job utility. Therefore, the higher job utility is, the lower the volatility of work hours attributable to labor supply given temporary changes in the real wage.

Section 3.3 examines the determination of equilibrium, which is partly captured by

way of two theoretical objects that are the result of explicitly accounting for on-the-job utility: *labor-earnings supply and labor-earnings demand*. Using our analytical framework, we show that there exists complementarity between optimal firm-level effort requirements and ordinary changes in technology, providing a means through which the effect on productivity of short-run technological fluctuations can be amplified. Moreover, we show that ongoing declines in drudgery will, all else constant, eventually induce unambiguous increases in work hours. This stands in contrast to the long-run impact of ordinary technological progress, the effects of which can eventually result in income effects outweighing substitution effects. Overall, the analysis suggests that drudgery should be thought of as an extended concept of technology.

Section 3.4 studies the implications of heterogeneity in production when considering differences in drudgery and technology in final goods producers, as well as across industries, and also in a setting of monopolistic competition. We show that firm- and industry-level job utility offerings play a critical role in determining the ability of firms and industries to endure across time given changes in economic conditions. In particular, increases in aggregate productivity and decreases in the marginal value of real wealth will tend to endogenously drive out firms and industries characterized by lower job utility offerings. This result is particularly interesting in light of the substantial degree of outsourcing that developed economies have engaged in over the last several decades. Moreover, we argue that there are strong firm-level incentives for developing innovations that increase job utility. In particular, this owes to the fact that firms offering higher job utility gain a competitive advantage by enduring a lower real wage per unit of worker productivity.

In Section 3.5 we examine the role of amenities. We show that the temporal evolution of amenities is inversely related to the temporal evolution of the marginal value of real wealth. As economies become richer, firms find it endogenously optimal to increase job utility via increases in amenities in order to (partially) mute the reduction in work hours that higher wealth would otherwise tend to induce.

Then, in Section 3.6, we explicitly relate our theory to the empirical trend(less) behavior of work hours. We show that within our framework, given large increases

in wealth, the extent to which work hours can remain high, and for that matter, higher than expected, is necessarily a reflection of ongoing increases in on-the-job utility. Moreover, the model can in principle explain a positive asymptote for work hours if people enjoy work as much as the marginal leisure activity. We also address welfare effects given changes in job utility in the alternative contexts of separable and non-separable preferences, and argue that the welfare effects associated with the paradox of hard work can be substantial under either case.

Finally, Section 3.7 concludes. Our research contributes to the labor economics literature by developing a theoretical framework through which an intertemporal understanding of the primitives that determine the economy's available trade-offs between output, wages, and job utility can be attained. In addition, we contribute to the macroeconomics literature by showing that the paradox of hard work is not necessarily evidence of a low intertemporal elasticity of substitution or non-separability of preferences in consumption and leisure. Secular improvements in on-the-job utility are such that it is possible for work hours to remain approximately constant over time even if the income effect of higher wages on labor supply exceeds the substitution effect of higher wages. In turn, secular improvements in on-the-job utility can themselves be a substantial component of the welfare gains from technological progress. These two implications are connected by an identity: improvements in on-the-job utility that have a significant effect on labor supply tend to have large welfare effects.

3.2 The General Framework

The model is cast in continuous time. Throughout the essay we omit time indexes in order to avoid notational clutter. Since our focus is on the labor market, we assume the context of a small open economy in which agents can freely borrow and lend at the exogenously determined real interest rate r (equal to ρ , the rate at which all economic agents discount the future, in steady state).

3.2.1 Households

For simplicity, we begin our analysis by focusing on effort and drudgery. The treatment of amenities is deferred until further in the essay.⁵ First, consider effort: several dimensions impact this variable. For instance, the intensity of a worker's concentration on a task while at his or her work station, the amount of time spent at the water cooler or in other forms of on-the-job leisure, time spent cleaning and beautifying the work place, time spent in office parties during work hours, morale building exercises, amount of time spent pursuing worker interests that have some productivity to the firm but would not be the boss's first priority, etc. Let \mathcal{E} be a vector describing all such dimensions of what the average hour of work is like, including the fraction of time spent in each different activity at work. We assume \mathcal{E} is determined optimally by firms, and for simplicity focus on perfect monitoring so that moral hazard problems are not an issue. Let D denote the drudgery level associated with work and $\mathcal{J} = \mathcal{J}(\mathcal{E}, D)$ be the function that maps \mathcal{E} and D into hourly utility associated with being at work.

The maximized value of \mathcal{J} can in principle take on any sign. In particular, let

$$J(E, D) = \max_{\mathcal{E}} \{ \mathcal{J}(\mathcal{E}, D) \} \text{ such that } \Theta(\mathcal{E}) = E.$$

Above, Θ is a function mapping the vector \mathcal{E} into the number E , and E gives effective productive input from an hour of labor before multiplication by labor-augmenting technology. We henceforth refer to E as effort per worker and $J(E, D)$ as the job utility function. We assume that $J \gtrless 0$ is such that $J_D < 0$, and allow for the possibility of job utility being increasing in effort at relatively small effort levels, and decreasing in effort at relatively high levels of effort.⁶ Of course, $J_D < 0$ implies that in (E, J) space a decrease in drudgery causes an upward shift in the job utility function. That is, lower drudgery results in higher job utility at any given effort level.

⁵Understanding the role of amenities is straightforward once the implications of drudgery are clear.

⁶We consider this to be the more intuitive case, although our results are unaltered by assuming that job utility is always decreasing in effort.

As an example of how to interpret J , consider two production techniques: 1 and 2. Suppose that production technique 1, \mathcal{J}_1 , results in relatively higher job-utility levels at lower effort levels, and production technique 2, \mathcal{J}_2 , results in relatively higher job-utility levels at higher effort levels. Then, as shown in Figure 3.3, in (E, J) space the job-utility function J is the upper envelope (bold) of these two techniques. We return to this issue when we analyze cross-industry implications.

Let a representative household's utility be a function of consumption of the final good C , work hours H , effort E , and drudgery D . We assume that households are infinitely lived, consist of a representative worker, and seek to maximize

$$\int e^{-\rho t} \mathcal{U} dt = \int e^{-\rho t} (U(C) + \Phi(T - H) + H \cdot J(E, D)) dt. \quad (3.1)$$

Above, ρ is the rate at which all economic agents discount the future, t denotes time, T is an individual's total per-period time endowment, U represents consumption utility and is such that $U' > 0$ and $U'' < 0$, and Φ denotes utility from off-the-job leisure, satisfying $\Phi' > 0$ and $\Phi'' < 0$. In this additively separable case of \mathcal{U} , we normalize J and Φ so that $\Phi'(T) = 0$. Given this normalization, $J > 0$ means that if the worker has no other job options, then the worker would be willing to spend some time on the job even if unpaid. On the other hand, $J < 0$ means that the worker would never do such job unless paid.⁷ The functional form in equation (3.1) is additively separable in consumption and leisure in line with the overall theme of our research: a rationalization of the paradox of hard work that does not require non-separability. Nonetheless, we contrast this framework with the non-separable case when we address welfare issues.

Consider a worker employed in a job that demands effort E and is characterized by drudgery D . The individual's utility maximization problem is, taking the hourly real wage rate per worker W paid by the firm as given, to choose a path for consumption, assets M , and work-hours to maximize equation (3.1) subject to $\dot{M} = rM + \Pi + WH - C$, where the price of consumption has been normalized to 1, Π represents non-

⁷See Section 3.9.1 for further details on this normalization.

labor, non-interest income, and for any variable X , \dot{X} refers to its change over time. The current-value Hamiltonian associated with the household's problem is given by

$$\mathcal{H} = U(C) + (\Phi(T - H) + H \cdot J(E, D)) + \lambda(rM + \Pi + WH - C), \quad (3.2)$$

where λ is the costate variable giving the marginal value of real wealth in the household's dynamic control problem. The first-order condition for consumption implies that $U'(C) = \lambda$. Substituting the underlying expression for optimized consumption into the Hamiltonian we can state the Hamiltonian maximized over C as

$$\bar{\mathcal{H}} = [U(U'^{-1}(\lambda)) - \lambda(rM + \Pi - C)] + [\Phi(T - H) + H \cdot (\lambda W + J(E, D))]. \quad (3.3)$$

Maximizing \mathcal{H} over both C and H is the same as maximizing $\bar{\mathcal{H}}$ over H . Note that only the second term on the right-hand-side of equation (3.3) depends on H . Therefore, to study the household's labor-supply decision we can focus on the optimization subproblem

$$\max_H \Phi(T - H) + H \cdot B, \quad (3.4)$$

where

$$B = \lambda W + J(E, D) \quad (3.5)$$

represents *hourly (marginal) net job benefits*.⁸ Note that B captures the utils per hour that an individual derives from on-the-job activities.

The individuals's optimization subproblem implies that for any $H > 0$ the first-order necessary condition for optimal per worker labor hours satisfies

$$\Phi'(T - H) = B. \quad (3.6)$$

Therefore, at the optimal level of hours per worker the marginal utility from off-the-job leisure is set equal to hourly net job benefits. It follows that, as shown in Figure 3.4, $\Phi'(T - H)$ is the labor-hours supply function. Moreover, the market clearing

⁸This solution method is similar to the one used in Shapiro and Kimball (2008).

device for work hours is in fact marginal net job benefits B .

Proposition 1. *The Frisch elasticity of labor supply is decreasing in job utility.*

Proof. Consider once more the solution to the worker's optimization subproblem. Since work hours are a direct function of marginal net job benefits, we can write $d \log H = \bar{\eta} d \log B$. Given $B = \lambda W + J$, holding everything constant except wages $d \log B = \lambda dW / (\lambda W + J)$. Rearranging, it follows that $d \log B$ is equal to $d \log W / (1 - \zeta)$, where $\zeta = -J/\lambda W$. Therefore,

$$d \log H = \bar{\eta} d \log B \implies d \log H / d \log W = \bar{\eta} / (1 - \zeta) \quad (3.7)$$

is the Frisch elasticity of labor supply. \square

Proposition 1 implies that the higher job utility is, the lower the volatility of work hours attributable to labor supply given temporary changes in the real wage. Moreover, note that $B = \lambda W + J$ implies that $B = \lambda(W(1 - \zeta))$. Therefore, ζ can be interpreted as the fraction of the wage that is a compensating differential.

3.2.2 Firms

Consider a representative firm whose jobs are characterized by drudgery D . Firms produce output Y by means of a function that takes as inputs capital K , workers N , hours per worker H , effort per worker E , and is subject to an exogenous labor-augmenting technology parameter Z . In particular, let a firm's production function be given by $Y = K^\alpha (Z E H N)^{1-\alpha}$, where $\alpha \in (0, 1)$.

Let R denote the rental rate of capital.⁹ It follows that for any output level \bar{Y} a firm's cost minimization problem involves choosing capital K and total work hours HN to minimize $RK + W(HN)$ such that $K^\alpha (Z E H N)^{1-\alpha} = \bar{Y}$. Solving this problem, it is straightforward to show that the firm's cost function is given by

$$\mathcal{C}(\omega, R, Y) = R^\alpha / ((\alpha^\alpha (1 - \alpha)^{1-\alpha}) \omega^{1-\alpha} Y), \quad (3.8)$$

where $\omega = W/(ZE)$ is the *effective wage*.

⁹We assume no adjustment costs, so that $R = r + \delta$, where δ is the capital depreciation rate.

Since R is exogenous to the firm, given equation (3.8) the remaining issue in solving the firm's optimization subproblem is minimizing the effective wage. In solving this optimization subproblem we assume that firms take the marginal value of real wealth λ as given, as they do the rental rate of capital R and equilibrium hourly net job benefits B . Of course, λ and B may differ for different types of workers. For now, we assume the existence of a representative household, but return to this issue later in the essay. It follows that the firm's optimization subproblem amounts to

$$\min_{W, E} \omega = W / (ZE)$$

such that

$$\lambda W + J(E, D) = B. \quad (3.9)$$

Combining these implies that $W / ZE = (B - J(E, D)) / (\lambda ZE)$, which after rearranging yields

$$J = B - \lambda Z \omega E. \quad (3.10)$$

In (E, J) space equation (3.10) is representative of a firm's isocost lines in (E, J) space: it traces out all the effort and job utility combinations that are consistent with any given effective wage. As the firm's optimization subproblem involves minimizing the effective wage, in (E, J) space the solution to its subproblem involves being on the isocost line that has the algebraically greatest feasible slope. Such feasibility is determined by the firm's job utility function, since this function represents all the job utility and effort combinations that a firm is able to offer.

The left panel of Figure 3.5 shows the solution to the firm's optimization subproblem. Given equilibrium marginal net job benefits B , lower effective wages are consistent with counter clockwise rotations in the firm's isocost lines. As seen in the left panel of Figure 3.5 $\omega'' > \omega > \omega'$ and ω is the firm's optimal effective wage: it can do better than ω'' , and although ω' is preferred to ω , the former is not feasible conditional on the firm's job utility function. In turn, the right panel of Figure 3.5 shows the determination of the number of hours per worker given marginal net job

benefits B and hours supply H^S . The relevance of job utility for the firm's optimization subproblem now becomes clear: job utility is the firm's effective constraint in its optimization subproblem. Clearly, the solution to the firm's optimization subproblem occurs at a point of tangency between the firm's job utility function and one of its isocost lines.¹⁰ Optimality is defined by

$$J_E = -\lambda Z\omega \implies J_E E = -\lambda W. \quad (3.11)$$

Since $\lambda, E > 0$, for positive wages it is an endogenous result from equation (3.11) that at the optimal choice of effort $J_E < 0$.¹¹

The possibility of real wages of different signs implies that the theory developed so far is consistent with the theory of compensating differences. The left panel of Figure 3.5 is similar in spirit to the discussion in Rosen (1986) that points to a trade-off between “job attributes” and productivity. In particular, Rosen explains that firms may shift resources from production to improving job attributes, in which case there is a loss in output, but at the same time for any given level of output the associated real wage is lower. This is because higher job attributes constitute the means for compensating differences. In that sense, Rosen's discussion can be read as one in which firms have a choice between investing in capital that can be used in producing output, or capital that is used as an input in the production of job attributes (and useless in the production of output). In terms of the left panel of Figure 3.5, the previous is consistent a north-west movement along the job utility function, and relates back to our discussion in the introductory section regarding Figures 3.1 and 3.2. Part of our research therefore amounts to an extension of the theory of compensating differentials by focusing on the primitives that determine the position of the job utility function itself, one of which is drudgery. Moreover, in our discussion of

¹⁰Note that the tangency optimization method we use is robust to situations as shown in Figure 1. More generally, we have not needed to assume concavity of the firm's job utility function in order to solve its optimization subproblem.

¹¹The solution methodology employed in solving a firm's optimization subproblem is the same regardless of the sign of the wage. For ease of exposition we henceforth restrict attention to cases under which the real wages associated with any given job are positive. Of course, the canonical example of a real wage equal to zero is volunteer work. Moreover, note that *Dude Ranches* are an interesting example of negative real wages.

amenities we show that aspects of job utility other than effort that are in the firm's control and lead to *shifts* in the job utility function as opposed to movements along. Ultimately, our framework allows us to make inferences regarding the evolution of job utility across time, and also predictions of its evolution conditional on changes in the economy's labor-augmenting technology and marginal value of real wealth. This results in a novel time-series understanding of the impact of changes in job utility that is complementary to the long-standing static framework of compensating-differences analysis.

3.3 Equilibrium

Continue to assume a representative-agent framework with regards to both firms and workers. Although firms set the real wage they pay their employees, we assume that firms are price-takers in the product market. Moreover, let all firms be a producers of the final consumption good itself. In this context profit maximization for any firm entails a marginal cost of production equal to the price of output, which is normalized to one. Using equation (3.8) implies that under perfect competition firms with positive output must have

$$1 = (R^\alpha / (\alpha^\alpha (1 - \alpha)^{1-\alpha})) \omega^{1-\alpha}. \quad (3.12)$$

Rearranging,

$$W / (ZE) = (\alpha^\alpha (1 - \alpha)^{1-\alpha} / R^\alpha)^{1/(1-\alpha)}. \quad (3.13)$$

Thus, under perfect competition the effective wage is an exogenously determined constant from the firm's point of view.¹²

The economy's general equilibrium can be determined by way of two graphical tools. The first of these is shown in Figure 3.6 and extends the intuition developed via Figure 3.5. In this case, the slope of an isocost line $-\lambda Z\omega$ is exogenously determined. Nonetheless, equilibrium requires that cost minimization be satisfied. Therefore, optimality continues to be summarized by a point of tangency between

¹²Recall that we refer to W as the real wage and to ω as the effective wage.

the job utility function and an isocost line. The left panel of Figure 3.6 shows optimal effort requirements E and job utility J , which implicitly define the optimal real wage $W = \omega ZE$ and hourly net job benefits B . The right panel of Figure 3.6 shows the determination of work hours H . As shown earlier, hours supply per worker H^S is given by $\Phi'(T - H) = B$.

What remains to be determined is the economy's marginal value of real wealth λ . Given this, equilibrium consumption - and therefore production - are implicitly pinned down. In general equilibrium, our open-economy framework has $r = \rho$. The household's budget constraint establishes a direct relationship between consumption C and labor earnings WH , which defines an implicit relationship between labor earnings and λ . In equilibrium $C = rM + \Pi + WH$. Given the household's first-order condition for consumption, this implies that

$$\lambda = U'(rM + \Pi + WH). \quad (3.14)$$

Since $U'(\cdot)$ is decreasing in C , equation (3.14) yields a negative relationship between λ and labor earnings WH , which we call the *demand for labor-earnings* (LE^D) *function*.

Now, consider the determinants of the configuration shown in Figure 3.6, which was shown for a given λ . Suppose that the marginal value of real wealth increases from λ to λ' . Then, as the left panel of Figure 3.7 shows, since ω is fixed and Z does not change, the firm's isocost lines become steeper. The tangency condition summarizing optimality is such that B and E increase, while J decreases. The right panel of Figure 3.7 shows that the increase in B induces an increase in H . In addition, given that ω cannot change but E increases, W must increase so that ω remains fixed. This implies a positive relationship between λ and labor earnings WH , which we call the *supply of labor-earnings* (LE^S) *function*:

$$WH = Z\omega E(\lambda Z, D) \cdot H(B(\lambda Z, D)). \quad (3.15)$$

Note that it is in fact the case that both E and B are increasing in the *product* λZ .

As shown in Figure 3.8, demand and supply for labor earnings, equations (3.14) and (3.15), jointly determine the economy's level of λ and WH .

3.3.1 Analysis: Changes in Technology and Drudgery

Consider a change from Z to $Z' > Z$. Figure 3.9 shows the adjustments that occur holding the marginal value of real wealth constant. As shown in the figure's left panel, for given λ and ω firms' isocost lines become steeper. This implies optimally higher hourly net job benefits, lower job utility, and higher effort requirements. Additionally, the real wage increases. This is because $W = \omega ZE$, ω is fixed, and ZE increases. In turn, as shown in the right panel of Figure 3.9, given no change in hours supply higher hourly net job benefits result in an increase in hours per worker. Because an increase in Z induces an increase in E , the λ held constant response in effective labor productivity ZE is greater than proportional to the increase in Z . Therefore, the model suggests a channel through which shocks to labor-augmenting technology can be amplified in terms of their effect on productivity for short-run fluctuations in Z .

To understand the full implications of changes in Z given the resulting adjustment in λ , consider once more Figure 3.8. Since for given λ both W and H increase due to a change in the exogenous parameter Z , labor-earnings supply shifts out. The outward shift in LE^S implies a decrease in equilibrium λ and an increase in equilibrium WH . Returning to Figure 3.9, note that a decrease in λ means that after all adjustments in λ take place firms' isocost lines will be less steep than before adjustment in λ . Less steep isocost lines mean lower B , H , and E , the last of these translating into lower W and higher J . The extent to which isocost lines become less steep than for λ held fixed ultimately depends on the magnitude of the change in λ . Thus, the final level of B , H , J , W , and J relative to their values prior to the change in Z is ambiguous. What is unambiguous is a resulting decrease in λ and an increase in labor earnings WH . It could well be the case that in the new equilibrium W is higher than before the change in Z , but H is lower. This would be a situation in which the income effect dominates the substitution effect.

Now, suppose instead that drudgery changes from D to $D' < D$. Three possibili-

ties emerge conditional on $J_{ED} = 0$, $J_{ED} > 0$, or $J_{ED} < 0$. The first of these means that changes in drudgery do not affect how taxing extra effort is, the second that less drudgery makes extra effort more taxing, and the last that lower drudgery makes increases in effort less taxing. We focus on $J_{ED} < 0$, as it is the most intuitively appealing possibility.

Suppose $J_{ED} < 0$. When drudgery decreases from D to D' the job-utility function shifts up and for given λ becomes less steep at every effort level. As shown in the left panel of Figure 3.10, for given λ the result of this change is an increase in effort and an increase in hourly net job benefits. In turn, under these circumstances an increase in effort along with no change in technology or the equilibrium effective wage means that W increases. As the right panel of Figure 3.10 shows, the increase in hourly net job benefits induces an increase in hours per worker. Moreover, there is ambiguity in terms of job utility. As shown in Figure 3.10 the change in drudgery results in an increase in job utility. However, this need not always be the case. This is because for a sufficiently small upward shift in the job-utility function, it could be that the level of job utility remains constant or actually decreases. In particular, note that if job utility decreases, then for given λ the qualitative effects of a decrease in drudgery and an increase in labor-augmenting technology are identical. Moreover, regardless of the change in job utility, a decrease in drudgery when $J_{ED} < 0$ induces an increase in optimal effort requirements which results in an increase in hourly effective labor productivity.

Since the decrease in D under consideration induces an increase in both H and W it follows that the product WH increases for any given λ . In terms of labor-earnings supply and demand, this means that the labor-earnings supply function shifts out, delivering a new long-run equilibrium value of λ that is lower and WH that is higher than before the decline in drudgery. Returning to Figure 3.10, lower λ makes isocost lines less steep. Therefore, the final tangency condition capturing optimality will imply lower B , H , E , and W , and higher J than before the adjustment of λ . This makes the final levels of these variables relative to their original ones in principle ambiguous. However, ambiguity only exists if the peak of the J curve after the

decrease in drudgery is less than or equal to the original level of B . If after the change in D the new peak of the J curve lies above the original level of B , then any new tangency condition consistent with positive wages will necessarily deliver a new equilibrium value of B , and therefore H , that lies above the original one. This highlights that given positive wages, ongoing decreases in drudgery, regardless of the sign of J_{ED} , will eventually lead to increases in work hours. This is the result of decreases in D inducing upward shifts in the J curve, and stands in contrast to changes in labor-augmenting technology in which the ultimate change in work hours is always, in principle, ambiguous.¹³

3.3.2 Heterogeneity in the Labor Force

At this point, a natural question of interest is what the effects of worker heterogeneity are. In particular, given the relevance of marginal net job benefits in determining work hours and the importance of labor-augmenting technology in the firm's optimization subproblem, it is of particular interest to understand the impact of a labor force that is heterogeneous in wealth and productive capacity. To address this, let there be a continuum of agents inhabiting the economy and indexed by m . A type m individual's optimization problem is as earlier in the essay, but we allow for differences in individual marginal values of real wealth λ_m and idiosyncratic productivity θ_m . Appropriately re-indexed the solution to an individual's utility maximization problem and optimization subproblem remain as before.

We consider the case in which workers are perfect substitutes in production. The natural extension of a firm's production function becomes

$$Y = K^\alpha \left(Z \int \theta_m E_m H_m N_m dm \right)^{1-\alpha}.$$

The firm's cost minimization problem and solution involves appropriately re-indexing

¹³Given the analytical methodology developed above, it is straightforward to show that when $J_{ED} = 0$ a decrease in drudgery leads to a decrease in the marginal value of real wealth, effort, and the real wage, along with an increase in equilibrium work hours that ultimately induces an increase in the product WH . Moreover, in the less intuitive case $J_{ED} > 0$ the effects of a decrease in drudgery on all of the model's endogenous variables is entirely ambiguous.

the ones obtained earlier. Then, for a given worker of type- m the firm's optimization subproblem is such that it chooses the real wage it pays this worker W_m and the corresponding effort requirement E_m to minimize $\omega = W_m/(\theta_m Z E_m)$ such that $\lambda_m W_m + J(E_m, D) = B_m$. The firm takes as given the marginal value of real wealth of type- m workers λ_m , as well as their equilibrium marginal net job benefits B_m .

As shown in Figure 3.11 the intuition and solution methodology developed under a representative worker carries over to the present context of worker heterogeneity. Interestingly, notice that from the firm's point of view what is relevant about worker types is the product $\lambda_m \theta_m$. Let this product denote a worker's *hungriness*. Then, we can class individuals into supra types M , which are any arbitrary worker types for which the product $\lambda \theta$ is equal to some value Γ_M . Given perfect competition in the product market it is straightforward to show that the equilibrium effective wage is once more determined exogenously by equation (3.13). Under these circumstances the firm is always indifferent in terms of employing any given worker type. However, across individuals there are differences in the associated isocost-line slopes, which are given by $-\Gamma_M Z \omega$. Hence, the isocost lines associated with individuals characterized by higher values of Γ are steeper relative to those with lower values of Γ .

Consider the left panel of Figure 3.12. As shown there, for workers of type- M and - N such that $\Gamma_M > \Gamma_N$, individuals characterized by less hungriness are predicted to exert lower effort, enjoy higher job utility, and receive lower hourly net job benefits than their counterparts with greater hungriness. Moreover, as shown in the right-hand panel of Figure 3.12, workers with greater hungriness are predicted to work more hours than those with lower ones. Note, however, that relative real wages are in principle ambiguous and depend on the idiosyncratic productivity of the individuals under consideration. Overall, it follows that relatively wealthy workers (lower λ) who are highly productive (high θ) can actually have relatively high hungriness and therefore be found to work relatively high hours at high effort levels.

Finally, note that given constant returns to scale in production and mobile capital, different worker types can be thought of as being on their own individual islands. Therefore the comparative steady state analysis for a small open economy with one

type of worker is exactly identical to the analysis of an economy with the differences in workers considered above. Thus, for ease of exposition, in what follows we revert to a representative worker framework.

3.4 Heterogeneity in Production

3.4.1 Differences in Final-Good Producers

Let there be a continuum of firms indexed by i and suppose that each firm is a producer of the final consumption good. In principle, we allow firms to differ in their labor-augmenting technology, drudgery levels, and job utility functions. Regardless of the object over which firms are different it is still the case that marginal net job benefits are the market clearing device for labor hours. In addition, the solution to firms' cost minimization problem remains as earlier. With generality in mind, let a firm's labor-augmenting technology now be equal to the product of an economy-wide productivity parameter P and a firm-specific one Z_i . All results from our earlier development carry over except that the slope of firm i 's isocost lines are given by $-\lambda P Z_i \omega$.

To make our points concise, consider two firms: 1 and 2. First, assume that these firms have the same functional form for J , but firm 1's jobs are characterized by lower drudgery than firm 2's, that is, $D_1 < D_2$. As before, there is an economy-wide λ and exogenously set ω . Suppose that $Z_1 = Z_2$. Then, as shown in Figure 3.13, firm 1's tangency condition suggests higher marginal net job benefits than firm 2's. Under these circumstances, firm 1 implicitly sets the economy's level of marginal net job benefits. Since workers take the jobs with the highest B , firm 2 would be unable to operate. Note that given $D_1 < D_2$ this result can even emerge when $Z_2 > Z_1$, which highlights the fundamental importance of job utility for the (ongoing) existence of firms relative to the traditional concept of technology.

Consider instead a case with differences in technology countervailing differences in drudgery. Figure 3.14 shows that if firm 2 was endowed with technology $Z'_2 > Z_2$ then both firms would offer the same marginal net job benefits and the individuals

would be indifferent between working at either firm. Moreover, if firm 2's technology were given by $Z_2'' > Z_2'$, then it would actually be able to offer higher marginal net job benefits than firm 1. In this case individuals would strictly prefer being employed at firm 2, and firm 1 would not be able to operate. Therefore, an illuminating way to view technology and drudgery jointly is to see drudgery as a component of an expanded concept of technology.

Now, focus on the impact of different marginal values of real wealth λ or economy-wide productivity P as captured by the product λP that enters the slope of firms' isocost lines. In the cases considered above, the results were robust to different values of λP . However, a different situation emerges when firms also differ in the actual functional form of their J curves. Consider, for example, Figure 3.15, where firms differ in their J curves, potentially in ways that cannot be described by differences in their drudgery levels. Without loss of generality, assume $Z_1 = Z_2 = Z$. Given λP , as pictured in Figure 3.15, firm 1 is able to offer the highest marginal net job benefits and therefore firm 2 would not be able to attract any employees. As Figure 3.16 shows, the previous need not always be the case. Indeed, given $(\lambda P)' > \lambda P$ then both firms are able to offer the same marginal net job benefits in which case workers are indifferent between them. Moreover, for $(\lambda P)'' > (\lambda P)'$ the initial situation is now reversed meaning that firm 1 is unable to attract employees.

We defer the treatment of applicable versions of labor-earnings supply and demand until further below, where we consider the implications of industry-level differences. At this point, however, note that because in equilibrium B is the same across all employment opportunities, then in equilibrium individuals are willing to supply work hours to all firms. However, individuals need not be willing to spend the same amount of time working at each type of job.

Proposition 2. *Consider firms 1 and 2. Suppose that firm i offers the job-utility/real wage bundle (J_i, W_i) and $J_2 > J_1$. Let ξ be the fraction of total work hours an individual devotes to working in firm 1. Then, $\lambda = -(J_1 - J_2) / (W_1 - W_2)$, and*

$$\xi = \frac{1}{W_1 - W_2} \left(\frac{U'^{-1}(\lambda) - rM - \Pi}{T - \Phi'^{-1}(B)} - W_2 \right).$$

Proof. $J_2 > J_1$ implies that $W_1 > W_2$. In equilibrium both firms offer the same B . Hence, $B = \lambda W_1 + J_1$ and $B = \lambda W_2 + J_2$. Combining yields

$$\lambda = -(J_1 - J_2) / (W_1 - W_2).$$

Using the household's budget constraint

$$C = H (\xi W_1 + (1 - \xi) W_2) + rM + \Pi.$$

Since the household's choice of total work-hours supply satisfies $\Phi'(T - H) = B$, then $H = T - \Phi'^{-1}(B)$. Combining implies that

$$C = (T - \Phi'^{-1}(B)) (\xi W_1 + (1 - \xi) W_2) + rM + \Pi.$$

Given the household's condition for optimal consumption, $U'^{-1}(\lambda) = C$. Combining these two final equations and rearranging yields ξ . \square

Note that $J_2 > J_1$ can be interpreted as job 2 being a “dream job” and job 1 being a “day job.” Therefore, Proposition 2 is quite intuitive: the more time an individual has to spend on overall work activities, or the more exogenous wealth he or she is endowed with, the more time said individual will devote to the dream job. Interestingly, note that the fraction of time devoted to the day job ξ is also decreasing in the difference $W_1 - W_2$. To the extent that this difference implicitly captures how much lower job utility the day job offers relative to the dream job, this highlights that higher real wages offered as compensation for relatively lower job utility are not a sufficient factor to induce individuals to devote more hours to such job, all else equal.

3.4.2 Industry-Level Differences

Suppose now that there is a continuum of industries indexed by i . Each industry produces a different type of good, but firms within industries are perfectly competitive. For ease of exposition, let there be a representative firm per industry. In

analogous fashion to earlier analysis, let P be an economy-wide productivity parameter and Z_i the labor-augmenting technology characterizing industry i . Let p_i be the relative price of the good produced by industry i . Worker-side optimization is just as before, and appropriately re-indexed the same is true of a firm's cost minimization problem. Moreover, the industry-level optimization subproblem is now to choose W_i and E_i to minimize the industry-level effective wage ω_i , which is equal to W_i/PZ_iE_i . The relevant constraint is $\lambda W_i + J_i(E_i, D_i) = B_i$, where B_i are equilibrium marginal net job benefits in industry i . As before this implies that $W_i/PZ_iE_i = (B_i - J_i(E_i, D_i))/(\lambda PZ_iE_i)$, which after rearranging yields $J_i = B_i - \lambda PZ_i\omega_iE_i$. In (E_i, J_i) space the solution to an industry's optimization subproblem again involves being on the isocost line that has the algebraically greatest feasible slope. However, in this case profit maximization implies that for industries with positive output

$$p_i = (R^\alpha/(\alpha^\alpha(1-\alpha)^{1-\alpha}))\omega_i^{1-\alpha}.$$

Rearranging,

$$(1-\alpha)\alpha^{\alpha/(1-\alpha)}/R^{\alpha/(1-\alpha)} = W_i \cdot p_i^{-1/(1-\alpha)}/(PZ_iE_i) = \bar{\omega},$$

where we have used the definition of ω_i . Therefore, in this case, from an industry's point of view $\bar{\omega}$ is an exogenously determined constant.

To fully appreciate the solution to a firm's optimization subproblem, note that an isocost line can be stated as

$$J_i = B_i - \lambda\bar{\omega} \left(PZ_iE_i p_i^{1/(1-\alpha)} \right),$$

and furthermore

$$J_i(E_i, D_i) = J_i \left(x_i/PZp_i^{1/(1-\alpha)}, D_i \right),$$

where $x_i = PZ_i p_i^{1/(1-\alpha)} E_i$. Figure 3.17 shows the solution to a industry-level representative firm's optimization subproblem in $(x_i/(PZp_i^{1/(1-\alpha)}), J_i)$ space. Since the slope of isocost lines, $-\lambda\bar{\omega}$, is the same across industries, then industry-level optimal

operations and marginal net job benefits are determined by the point of tangency between a representative firm's isocost line and job utility function. Given Figure 3.17, note that in any industry the vertical distance between B_i and J_i is equal to $-\lambda W_i$. Moreover, the horizontal distance between $PZ_i E_i p_i^{1/(1-\alpha)}$ and the origin is equal to $W_i/\bar{\omega}$.

Now, consider two industries, $i = 1, 2$ with job utility functions given by $J_1 = \mathcal{J}_1$ and $J_2 = \mathcal{J}_2$ as depicted in Figure 3.2. As explained earlier, what is now relevant is the upper envelope of these job utility functions. To see this, consider Figure 3.18. Note that for a low marginal value of real wealth such as λ' industry 1 is able to offer the highest marginal net job benefits in which case industry 2 does not operate. For a higher marginal value of real wealth such as $\lambda'' > \lambda'$ both industry 1 and 2 are able to offer the same marginal net job benefits in which case the worker chooses hours allocation across industries according to Proposition 2. Finally for even higher marginal values of real wealth such as $\lambda''' > \lambda''$ industry 2 is able to offer the highest marginal net job benefits, in which case industry 1 is unable to operate.

We now turn to the determination of labor-earnings supply and demand. LE^D is a simple extension of that derived earlier. In particular, this function now satisfies

$$\lambda = U'(rM + \Pi + H(\xi W_1 + (1 - \xi) W_2)),$$

where ξ is the fraction of total work hours devoted to industry 1. The appropriate version of LE^S is slightly different than that considered earlier. Note that given Figure 3.18, for low values of λ only industry 1 operates, and the associated real wages, marginal net job benefits, and work hours are relatively low. Therefore, in terms of labor-earnings supply, low values of λ are associated with low labor earnings. Continuing with the analysis of Figure 3.18, at λ'' both industries are operational, and wages, marginal net job benefits, and hours are higher than under λ' , meaning that so are labor earnings. Note, however, that for given hours, at λ'' any level of labor earnings in the range $[W_1'' H'', W_2'' H'']$ is an equilibrium, implying a perfectly elastic portion in labor-earnings supply. Finally, for high values of λ only industry

2 is operational with associated higher wages, marginal net job benefits, and hours, meaning that in terms of labor-earnings supply high values of λ are associated with high values of labor earnings. Figure 3.19 shows these considerations by depicting an equilibrium in which both industries are operational.

In this multiple industry context two comparative statics are particularly interesting: changes in relative prices, and changes in exogenous wealth. We consider these cases in relation to labor-earnings demand and supply in a dual industry framework.

Suppose that p_1 increases and p_2 decreases, meaning that the relative price of good 1 increases. This results in a horizontal expansion of industry 1's job utility function and a horizontal contraction in industry 2's job utility function. It immediately follows that, as shown in Figure 3.20, the lower portion of LE^S shifts out, the upward portion of LE^S shifts back, and therefore its perfectly elastic portion - that is, the range over which the worker is indifferent between industries - shrinks.

Now, suppose that non-interest, non-labor income Π increases. Then, LE^S is not affected. However, as shown in Figure 3.21, LE^D shifts back. As hinted in Proposition 2, it is interesting to note that every single bit of the increase in Π is devoted towards shifting hours of work to the industry with a lower wage and higher job utility.

3.4.3 Firm-Level Incentives for Drudgery Declines

The development so far highlights the importance on several dimensions of changes in drudgery. Thus, a natural question is whether there are incentives for innovations that lead to decreases in drudgery. To explore this issue, continue to assume the existence of a representative household. However, suppose that there is a continuum of firms indexed by i with production function $Y_i = K_i^\alpha (P Z_i E_i H_i N_i)^{1-\alpha}$. As earlier in the essay, P is an economy-wide productivity parameter. Let the firms under consideration be monopolistic competitors producing intermediate inputs that are used in the production of a final good Y . In particular, assume $Y = \left(\int Y_i^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} di$, where $\varepsilon > 1$ and there are no other factors used in the production of final output. It is straightforward to show that the optimal demand for input i satisfies $Y_i = Y/p_i^\varepsilon$,

where p_i is the input's price.

Given monopolistic competition and the effective cost function in equation (3.8), profit maximization at the intermediate inputs stage solves, for any firm i ,

$$\max_{Y_i} p_i(Y_i) \cdot Y_i - \left(R^\alpha / (\alpha^\alpha (1 - \alpha)^{1-\alpha}) \right) \omega_i^{1-\alpha} Y_i.$$

Using $Y_i = Y/p_i^\varepsilon$ this problem's first-order condition implies that for each firm

$$p_i = (\varepsilon / (\varepsilon - 1)) \left(R^\alpha / (\alpha^\alpha (1 - \alpha)^{1-\alpha}) \right) \omega_i^{1-\alpha}. \quad (3.16)$$

Therefore, once the firm's effective wage is established, so is the price of its output and hence its level of production. Clearly, firms with the lowest effective wage will have the lowest price for their output, and hence a higher demand for their production.

As before, assume firms take as given the equilibrium level of hourly net job benefits B and the economy's marginal value of real wealth λ . A firm's optimization subproblem is to choose W_i and E_i to minimize $\omega_i = W_i / (P Z_i E_i)$ such that $\lambda W_i + J_i(E_i, D_i) = B$. Firm-level optimality is captured by $J_{iE} = -\lambda P Z_i \omega_i$. Given equation (3.16), under imperfect competition effective wages can indeed vary across firms. Since there is an economy-wide B , it follows that workers will decide how to allocate their time across firms with a decision rule analogous to that shown earlier.

Under imperfect competition changes in labor-augmenting technology and drudgery that affect one firm need not affect all or any other firms. This may be the result of these variables being protected by individual firms, for example, quite simply by secrecy or through patent laws. In what follows, we thus focus on firm-specific comparative statics.

Figure 3.22 shows the solution to a firm's problem both before and after idiosyncratic labor-augmenting technology decreases temporarily from Z_i to Z'_i . When Z_i decreases, given no change in equilibrium hourly net job benefits, the firm's isocost lines rotate clockwise. To restore optimality, the firm lowers the effective wage until it is on an isocost line that is once more tangent to its job utility function. This amounts to a counter-clockwise rotation until the isocost line with slope $-\lambda Z'_i \omega'_i$ is

reached. Since drudgery has not changed $-\lambda Z'_i \omega'_i = -\lambda Z_i \omega_i$ and therefore the same level of job utility, effort, and hours per worker ensue as before the change in Z_i . In addition, $\omega'_i < \omega_i$ holds by way of a decrease in the real wage. The overall result is that the firm expands: the decrease that occurs in the effective wage induces a decrease in the firm's marginal cost and therefore a decrease in the price of its output. In fact, as Proposition 3 shows, firms with lower drudgery (or higher job utility per unit of effort) have a competitive advantage. To the extent that decreases in drudgery further this competitive advantage, it is even plausible that firms might set above-optimal effort requirements in order to induce workers themselves to think of ways to decrease drudgery. This amounts to a costless form of research and development.

Proposition 3. *For any sign of J_{iED} , under imperfect competition the marginal value of real wealth held fixed effect of a decrease in drudgery is to decrease the effective wage.*

Proof. The firm's choice set expands. \square

As shown in Section 3.9.2, conditional on the sign of J_{iED} several different results can emerge given a change in drudgery. We limit to noting the interesting case shown in Figure 3.23, which depicts the λ held constant effects of a decrease in drudgery when $J_{iED} < 0$ and $J_{iE}dE_i/dD_i > -J_{iD}$. The latter condition amounts by total differentiation of the job utility function to $dJ_i/dD_i > 0$ and, as Section 3.9.2 shows in detail, to $dW_i/dD_i < 0$. Note that the case under consideration is such that decreases in drudgery make marginal effort less taxing on job utility. Under these circumstances a decrease in drudgery results in an increase in effort requirements, a decrease in job utility and the effective wage, and an increase in the real wage.

This result is interesting if one were to consider two firms, say 1 and 2, for which $D_1 > D_2$. As seen above, firm 2 would optimally demand more effort than firm 1. This ultimately results in firm 2 offering a higher real wage than firm 1 but also lower job utility. In spite of this, firm 2's jobs offer higher job utility at any *given* effort level. Moreover, this situation yields a circumstance under which real wages and drudgery move in *opposite* directions. This highlights an issue related to workers comparing jobs in terms of pleasantness. If individuals think of more pleasantness as

lower drudgery, then as shown above they may report that a more pleasant jobs also offer higher wages. This is also true if workers think of pleasantness as job utility per effort.

3.5 The Role of Amenities

For ease of exposition we revert to the context of a representative final-good producer and household, where the firm is perfectly competitive in the product market. Recall that we have defined amenities to be job characteristics whose costs are in terms of goods. Thus, let p_A denote the price of amenities relative to the final consumption good. In addition, A denotes the level of amenities per hour of work that the firm offers to each employee. Let $J(E, D, A)$ denote the job-utility function extended to account for amenities and assume $J_A > 0$, $J_{AA} < 0$, and the same properties over E and D as $J(E, D)$. Following steps entirely analogous to those in Section 2, the solution to the worker's labor-hours supply optimization subproblem is just as before. In turn, the firm's cost minimization problem is given by

$$\min_{K, HN} RK + \mathcal{W}HN$$

such that $K^\alpha (ZEHN)^{1-\alpha} = \bar{Y}$, where $\mathcal{W} = W + p_A A$ is the *inclusive wage*. Following similar steps as earlier in the essay, the relevant cost function becomes

$$\mathcal{C}(\omega, R, \bar{Y}) = (R^\alpha / (\alpha^\alpha (1 - \alpha)^{1-\alpha})) \omega^{1-\alpha} \bar{Y},$$

which is similar to the one derived in Section 2 except that now $\omega = \mathcal{W}/(ZE)$ is the effective wage. The firm's new optimization subproblem is to choose a real wage W , effort per worker E , and amenities per worker A to minimize $\omega = \mathcal{W}/(ZE)$ subject to $\lambda W + J(E, A, D) = B$. Let ψ be the multiplier associated with the firm's optimization subproblem. Then, the first-order conditions are

$$W : 1/ZE - \psi\lambda = 0, \quad A : p_A/ZE - \psi J_A = 0, \quad \text{and} \quad E : -(W + p_A A)/ZE^2 - \psi J_E = 0.$$

Combine the first and last first-order conditions derived earlier to yield $EJ_E = -\lambda(W + p_A A)$. Dividing this by E , and multiplying and dividing the right side by Z yields, in (E, J) space, the exact same optimality condition as earlier in the essay: $J_E = -\lambda Z \omega$. In addition, as shown earlier, combining the first and second of the first-order conditions implies that $p_A \lambda = J_A$. Together, these last two equations implicitly define the firm's optimal choice of effort, amenities, and real wage given the exogenous parameters λ , Z , p_A , and D . Alternatively, the optimality conditions $J_E = -\lambda Z \omega$ and $p_A \lambda = J_A$ can be combined to eliminate λ , which after rearranging implies that $-\varepsilon_{JE} = \varepsilon_{JA} (1 + W/p_A A)$. That is, at the firm's optimal choices are such that the absolute value of the elasticity of job utility with respect to effort ε_{JE} equals that with respect to amenities ε_{JA} weighted by 1 plus the ratio of the hourly per worker real wage to the hourly cost of amenities per worker. Since for any variables x and y $\varepsilon_{xy} = d \log x / d \log y$, then noted optimality condition can be restated as

$$-\varepsilon_{AE} = 1 + W/p_A A.$$

Therefore, when amenities are a choice variable the firm's optimal operations occur where a 1% increase in effort per worker induces a $(1 + W/p_A A)$ % increase in amenities per worker. It is straightforward to show that the optimal level of amenities is the same if instead households are assumed to choose A .

The abstract functional form $J(E, D, A)$ is not as amenable for graphical analysis as was the case without amenities. Consider, however, a special case that proves illuminating. Suppose that $J(E, D, A) = G(E, D) + F(A)$. The first-order condition for amenities and the real wage together imply that $p_A \lambda = J_A$, which in this case amounts to $p_A \lambda = F'(A)$. Moreover, note that

$$\lambda W = \lambda (\mathcal{W} - p_A A) = \lambda (\omega Z E - p_A A).$$

Therefore,

$$\lambda \omega Z E + G(E, D) + F(A) - \lambda p_A A = B,$$

and hence the firm's optimal choice of amenities is alternatively the result of an optimization problem in which A is chosen to maximize $F(A) - \lambda p_A A$. Let

$$S(\lambda p_A) = \max_A \{F(A) - \lambda p_A A\} = F(F'^{-1}(\lambda p_A)) - \lambda p_A F'^{-1}(\lambda p_A)$$

be the surplus the individual receives from the optimal choice of amenities, and note that $S_{p_A}, S_\lambda < 0$. In $(E, G + S)$ space the firm's isocost lines now satisfy $G + S = B - \lambda Z \omega E$. As before the less steep this line, the lower the associated effective wage. Moreover, the optimality condition for effort $J_E = -\lambda Z \omega$ is such that in $(E, G + S)$ space the slope of the job utility function $J_E = G_E$ is equal to the slope of an isocost line $-\lambda Z \omega$. In other words, once the level of amenities is determined, the solution the firm's problem in the present context is entirely analogous to that which we presented earlier. This is shown in Figure 3.24.

It is straightforward to re-derive all comparative statics as developed earlier in the essay when amenities are accounted for. However, given the direct relationship between amenities and λ , we now consider differences between an economy with marginal value of real wealth λ and one with $\lambda' < \lambda$. As derived above, a lower marginal value of real wealth is consistent with higher amenities, meaning that under λ' the firm's job utility function in $(E, G + S)$ space shifts up and its isocost lines become less steep. This is shown in Figure 3.25, where lower effort and higher job utility under λ' are also implied. Since E is lower and both Z and ω remain constant, then under λ' the real wage is lower. Although it may seem ambiguous, as shown in Section 3.9.3 marginal net job benefits are actually lower under λ' , meaning that so are equilibrium work hours. Note however, that given the endogenously optimal higher A consistent with a lower λ , the difference in equilibrium work hours between steady states is less than in the absence of amenities. In that sense, amenities can be seen as partially muting changes in work hours given changes in the marginal value of real wealth.¹⁴

¹⁴For simplicity, we have not considered the production-side of amenities. However, note that if these are interpreted as fractions of the consumption good transformed into amenities, then $p_A = 1$. Otherwise, for example, the sectoral analysis developed earlier can be applied in straightforward fashion. Whichever the case, the main points of this section are not altered.

3.6 Work Hours and Welfare

3.6.1 Equilibrium Work Hours

Recall that marginal net job benefits B implicitly determine work hours. Using the definition of B and assuming a continuum of firms indexed by i , it follows that

$$dB = (\lambda dW_i + dJ_i) + W_i d\lambda. \quad (3.17)$$

Above, the first term shows that increases in real wages or on-the-job utility would induce individuals to work more hours. However, the second term shows that increases in consumption - and therefore decreases in the marginal value of real wealth λ - do the opposite. Therefore, in this framework, to the extent to which work hours remain high, and for that matter, higher than expected, in the face of enormous increases in consumption is a reflection of increases in $\lambda W_i + J_i$. This can be operationalized given that dB captures change in hours per worker, $d\lambda$ changes in consumption, and $dB - W_i d\lambda = (\lambda dW_i + dJ_i)$. Labor hours will be trendless if and only if $dB = 0$, which amounts to $dJ_i = -W_i d\lambda - \lambda dW_i$. Of course, if income effects dominate substitution effects, then $W_i d\lambda < -\lambda dW_i$ holds. Under such circumstances $dB = 0$ if and only if $dJ_i > 0$. In addition, note that even if $\lambda W_i \rightarrow 0$ because the income effect overwhelms the substitution effect, work hours H_i will tend to some constant $\bar{H}_i > 0$ as long as job utility J_i tends to some constant $\bar{J}_i > \Phi'(0)$. That is, the model can in principle explain a positive asymptote for work hours if people enjoy work as much as the marginal leisure activity.

The data suggests relatively trendless labor hours in the face of increases in both productivity and the real wage. If the income effect dominates the substitution effect, then, as noted above, labor hours will be relatively trendless if and only if on-the-job utility J_i is increasing over time via northeast movements of the job utility function in (E, J) space. We have shown that such outward shifts can be triggered by decreases in drudgery or increases in amenities. Moreover, there are strong firm-level microeconomic incentives to focus on innovations that decrease drudgery, and amenities are inversely related to the economy's marginal value of real wealth. This means that as

economies get richer, amenities are expected to increase, the direct effect of which is to partially mute income effects that would otherwise lead to large decreases in work hours. Since both decreases in drudgery and increases in amenities shift the job-utility function outwards, our analysis suggests intuitive channels through which observed patterns in the data can be explained.

3.6.2 Welfare Under Additive Separability

In order to address the theory's welfare implications we continue to allow for a variety of job options so that $H = \sum_i H_i$. Parameter-induced changes in welfare are well assessed via steady-state to steady-state considerations. In steady state, given $r = \rho$, an individual's problem is equivalent to the static optimization problem

$$\max_{C, H, H_i \geq 0} U + \Phi + \sum_i H_i J_i$$

such that $C = rM + \Pi + \sum_i W_i H_i$ and total hours $H = \sum_i H_i$. Given the multipliers λ and b , let

$$\mathcal{L}^* = \max_{C, H, H_i \geq 0} \{U + \Phi + \sum_i H_i J_i + b(H - \sum_i H_i) + \lambda(rM + \Pi + \sum_i W_i H_i - C)\}.$$

Note that the optimal choice of H_i renders two cases: $H_i = 0$ and $J_i + \lambda W_i < b$, or $H_i > 0$ and $J_i + \lambda W_i = b$. Therefore, $b = B$, where, as before, B denotes the economy's level of equilibrium marginal net job benefits.

Using the envelope theorem,

$$d\mathcal{L}^*/\lambda = \sum_i H_i dJ_i/\lambda + \sum_i H_i dW_i + d(\Pi + rM) \quad (3.18)$$

Above, each of the three terms on the right-hand-side highlights a distinct way in which the economy's opportunity set becomes larger. Changes in welfare owing to changes in on-the-job utility are captured by the first term, modifications due to increases in consumption are reflected in the second term, and modifications due to changes in exogenous wealth appear in the last term. In particular, $(\sum_i H_i) dJ_i/\lambda$

can be interpreted as the portion of the change in the maximized value of utility that answers the question: “how much would the household have to be paid in order to go back to working in yesterday’s conditions?”

To better understand the implications of the envelope theorem, note that

$$d(\sum_i H_i W_i) = \sum_i H_i dW_i + \sum_i W_i H_i \cdot dH/H + \sum_i W_i (dH_i - H_i \cdot dH/H)$$

That is, the change in labor earnings is equal to the sum of a term reflecting the change in wages for narrowly defined job categories, a term reflecting the change in total hours, and a term reflecting the change in the composition of jobs between relatively high paid jobs with low job utility and low paid jobs with high job utility. The change in wages for narrowly defined jobs is a key component of welfare from the envelope theorem perspective. Note that

$$\sum_i H_i dW_i = d(\sum_i H_i W_i) - [\sum_i W_i H_i \cdot dH/H] - [\sum_i W_i (dH_i - H_i \cdot dH/H)].$$

Therefore, to gauge this component of welfare, we need to adjust the change in overall labor earnings by subtracting not only extra earnings from people working longer hours overall, but also extra earnings coming from people switching towards jobs that are more highly paid and have lower job utility. If λW is moving down, then the overall trend should involve compositional shifts towards jobs with higher job utility and relatively lower pay than other available jobs. This means that the increase in labor earnings will tend to understate the true increase in welfare (leaving aside changes in overall hours). Empirically, it should be possible to obtain a direct measure of the change in wages for narrowly defined jobs $\sum_i H_i dW_i$.

In terms of the remaining welfare components, consider once more equation (3.17). Using this, rearranging, and substituting in equation (3.18) implies that

$$\begin{aligned} d\mathcal{L}^*/\lambda &= (\sum_i H_i) dB/\lambda - (\sum_i H_i W_i) d\lambda/\lambda + d(\Pi + rM) \\ \implies \frac{d\mathcal{L}^*}{\lambda \sum_i H_i W_i} &= \frac{H}{\sum_i H_i W_i} \frac{dB}{\lambda} - \frac{d\lambda}{\lambda} + \frac{d(\Pi + rM)}{\sum_i H_i W_i}. \end{aligned} \quad (3.19)$$

The last term on the right-hand side of equation (3.19) is well understood. As noted above, $\sum_i H_i dW_i$ can in principle be computed. Hence, we would like a measure for the first two terms on the right-hand side.

Define $\gamma = -CU_{CC}/U_C$. Then, $1/\gamma$ is the elasticity of intertemporal substitution, and $d\lambda/\lambda = -\gamma dC/C$. Moreover, for any job i the Frisch elasticity of labor supply satisfies $\eta_i(1 - \zeta_i)$. Then,

$$dB/B = (1/\bar{\eta}) dH/H \implies dB = ((1 - \zeta_i) \lambda W_i) dH/H \implies dB/\lambda = (W_i/\eta_i) dH/H.$$

Substituting these derivations into equation (3.19) and simplifying yields

$$\frac{d\mathcal{L}^*}{\lambda \sum_i H_i W_i} = \frac{(W_i/\eta_i) dH}{\sum_i H_i W_i} + \frac{\gamma dC}{C} + \frac{d(\Pi + rM)}{\sum_i H_i W_i}. \quad (3.20)$$

Evidence about γ can be found from workers' job choices. Consider an individual working two jobs satisfying $J_2 > J_1$. Then, $\lambda W_1 + J_1 = \lambda W_2 + J_2$, meaning that

$$\lambda = \frac{J_2 - J_1}{W_2 - W_1} \implies \frac{d\lambda}{\lambda} = \frac{dJ_1 - dJ_2}{J_1 - J_2} - \frac{dW_1 - dW_2}{W_1 - W_2}.$$

For any individual for whom $dJ_1 - dJ_2 = 0$, for example, $dJ_1, dJ_2 = 0$, then

$$d\lambda/\lambda = -(dW_1 - dW_2) / (W_1 - W_2).$$

and using $d\lambda/\lambda = -\gamma dC/C$ it follows that

$$\gamma = ((dW_1 - dW_2) / (W_1 - W_2)) / (dC/C).$$

More generally, the short-run elasticity of intertemporal substitution has been suggested by Hall (1988) to be approximately zero, and by Kimball, Sahm, and Shapiro (2011) to be 0.08. However, there are reasons suggesting that the long-run elasticity of intertemporal substitution should be higher than its short-run counterpart. This includes taking account of full adjustment, new goods, habit formation, and “keeping up with the Joneses.” In the context of our analysis, it is precisely the

long-run elasticity of intertemporal substitution which should be applied. Say the long-run elasticity of intertemporal substitution is 0.5, in which case $\gamma = 2$. Using this value for γ along with equation (3.20) implies that for $d\Pi, dM = 0$ and $dH = 0$, a 1% increase in consumption would be associated with a welfare increase of at least 2%.

A natural question that follows is what fraction of welfare gains are attributable to higher job utility. To see this, note that dividing equation (3.18) by $\sum_i H_i W_i$ and combining with equation (3.20) yields

$$\begin{aligned} \frac{\sum_i H_i dJ_i}{\lambda \sum_i H_i W_i} + \frac{\sum_i H_i dW_i}{\sum_i H_i W_i} &= \frac{(W_i/\eta_i) dH}{\sum_i H_i W_i} + \frac{\gamma dC}{C} \\ \Rightarrow \frac{\sum_i H_i dJ_i}{\lambda \sum_i H_i W_i} + \left(\frac{d \sum_i H_i W_i}{\sum_i H_i W_i} - \frac{\sum_i W_i dH_i}{\sum_i H_i W_i} \right) &= \frac{(W_i/\eta_i) dH}{\sum_i H_i W_i} + \frac{\gamma dC}{C}. \end{aligned}$$

Then, given $dH = dH_i = 0$ and a 1% increase in consumption resulting from a 1% increase in labor earnings (that is, with *all* of the increase in labor earnings being put towards consumption), continuing to assume $\gamma = 2$ it follows that

$$\begin{aligned} \frac{\sum_i H_i dJ_i}{\lambda \sum_i H_i W_i} &= \frac{\gamma dC}{C} - \frac{d \sum_i H_i W_i}{\sum_i H_i W_i} \\ &= 2\% - 1\% = 1\%. \end{aligned}$$

Hence, given constant labor hours, up to half of the welfare gains associated with a 1% increase in consumption can result from increases in on-the-job utility.

3.6.3 Welfare Under Non-separability

Given the development so far, it is of interest to understand the welfare implications of job utility when consumption and leisure are non-separable. Hence, suppose now that

$$\mathcal{U} = \mathbb{U}(C, H) + \sum_i H_i J_i,$$

where $\mathbb{U}_C > 0$, $\mathbb{U}_H < 0$, and $\mathbb{U}_{CH} > 0$. Then, an individual's problem involves choosing C , H , and $H_i \geq 0$ to maximize \mathcal{U} such that $C = rM + \Pi + \sum_i W_i H_i$ and

total hours $H = \sum_i H_i$.

Let

$$\mathcal{L}^* = \max_{C, H, H_i} \mathbb{U}(C, H) + \sum_i H_i J_i + \lambda (rM + \Pi + \sum_i W_i H_i - C) + B (H - \sum_i H_i).$$

Then,

$$d\mathcal{L}^* = \sum_i H_i (dJ_i + \lambda dW_i) + \lambda (d\Pi + rdM).$$

Using equation (3.17), summing over hours, and dividing by λC the previous can be stated as

$$d\mathcal{L}^*/\lambda C = (H/C) \cdot dB/\lambda - (\sum_i H_i W_i/C) \cdot d\lambda/\lambda + (d\Pi + rdM)/C. \quad (3.21)$$

Other than dB/λ and $d\lambda/\lambda$, it is straightforward to obtain empirical counterparts to all variables on the right-hand side of the equation (3.21). Thus, it is of interest to find expressions for dB/λ and $d\lambda/\lambda$ that can be operationalized. To this end, define

$$V(\lambda, H) = \max_C \mathbb{U}(C, H) - \lambda C \quad (3.22)$$

and

$$\max_H V(\lambda, H) + \lambda (H \sum_i \xi_i W_i + \Pi) + H \sum_i \xi_i J_i, \quad (3.23)$$

where ξ_i is the fraction of total hours that the individual spends on job i . Note that

$$H (\lambda \sum_i \xi_i W_i + \sum_i \xi_i J_i) = H \sum_i \xi_i (\lambda W_i + J_i) = HB$$

since in equilibrium $B = \lambda W_i + J_i$, and also $\sum_i \xi_i = 1$. Therefore, the statement in equation (3.23) becomes

$$\max_H V(\lambda, H) + H (\lambda \sum_i \xi_i W_i + \sum_i \xi_i J_i) + \lambda \Pi.$$

The first-order condition is $-V_H(\lambda, H) = B$. Therefore, $dB = -V_{HH}dH - V_{H\lambda}d\lambda$. If

$d\lambda = 0$, then

$$dB/B = -(V_{HH}H/B) \cdot dH/H = (V_{HH}H/V_H) \cdot dH/H,$$

where the second equality follows from the earlier FOC. It follows that,

$$(dB/B) / (dH/H) = (V_{HH}H/V_H) = 1/\bar{\eta},$$

where $\bar{\eta}$ is defined as the λ -held-constant elasticity of H with respect to B . Given

$$dB = -V_{HH}dH - V_{H\lambda}d\lambda,$$

as shown in the appendix

$$dB/\lambda = (1 - \zeta_i) W_i/\bar{\eta} \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda,$$

where $\zeta_i = -J_i/W_i\lambda$.

From Proposition 1, it follows that the Frisch elasticity of labor supply for any job i is given by $\eta_i = \bar{\eta}/(1 - \zeta_i)$. Therefore the previous can be stated as,

$$dB/\lambda = (W_i/\eta_i) \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda. \quad (3.24)$$

The first term on the right-hand side above has straightforward empirical counterparts. However, we still require an expression for $d\lambda/\lambda$, and are now also in need of one for $V_{H\lambda}$. Note from the expression in (3.22) that $V_\lambda = -C(\lambda, H)$ and $V_{\lambda H} = -C_H(\lambda, H)$. Furthermore, as shown in Section 3.9.4

$$dC/C = (V_{\lambda\lambda}\lambda/V_\lambda) \cdot d\lambda/\lambda + (V_{\lambda H}H/V_\lambda) \cdot dH/H.$$

Define $-1/\gamma = V_{\lambda\lambda}\lambda/V_\lambda$ and $\Theta = V_{\lambda H}H/V_\lambda$. That is, $\Theta = d \ln C / dH$ for constant λ . Then,

$$d\lambda/\lambda = \gamma (\Theta \cdot dH/H - dC/C). \quad (3.25)$$

A value for Θ can be estimated by noting that

$$\Delta \ln C + \alpha + \beta r + \Theta \Delta \ln H + \varepsilon.$$

Basu and Kimball (2002) suggest that a higher-end estimate for Θ is 0.3. Moreover,

$$\Theta = V_{\lambda H} H / V_{\lambda} = -V_{\lambda H} H / C \implies -V_{\lambda H} = \Theta C / H$$

Substituting into equation (3.24),

$$\begin{aligned} dB/\lambda &= (W_i/\eta_i) \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda \\ \implies dB/\lambda &= (W_i/\eta_i) \cdot dH/H + (\Theta C/H) \cdot d\lambda/\lambda. \end{aligned} \quad (3.26)$$

We set out to search empirical counterparts for $d\lambda/\lambda$ and dB/λ for use in equation (3.21), which we now have in equations (3.25) and (3.26). Combining these three equations, as shown in Section 3.9.4 it now follows that

$$\frac{d\mathcal{L}^*}{\lambda C} = \frac{W_i}{\eta_i} \frac{dH}{C} + \left(\Theta - \frac{\sum_i H_i W_i}{C} \right) \gamma \left(\Theta \cdot \frac{dH}{H} - \frac{dC}{C} \right) + \frac{(d\Pi + rdM)}{C}. \quad (3.27)$$

Consider an example. Suppose $dC/C = 1\%$, $dH = 0$, $\Theta = 0.3$, $\gamma = 2$. Moreover, suppose $\sum_i H_i W_i = C$ so that there is no non-labor income, and $d\Pi = dM = 0$. Then, using equation (3.27)

$$d\mathcal{L}^*/\lambda C = (.3 - 1) \cdot 2 \cdot (-1\%) = 1.4\%.$$

Hence, in this case .4% beyond the welfare increase from the increase in consumption owes to changes in on-the-job utility. Note that in terms of welfare there is no fundamental difference between increases in J from compositional effects and increases in J in any given job - it is only a matter of how detailed the definitions of jobs are.

3.7 Conclusions

The *paradox of hard work* refers to the fact that, given enormous world-wide increases in consumption, work hours have remained relatively trendless across countries. Given a low elasticity of intertemporal substitution¹⁵ and income effects on labor supply being substantial,¹⁶ work hours should have in fact shown a significant decline. In principle, this can be rationalized by an increasing marginal-wage to consumption ratio, something that keeps the marginal utility of consumption high, or something that keeps the marginal disutility of work low. We focus attention on the last of these explanations. Economists have long understood that cross-sectional differences in on-the-job utility at a particular time give rise to compensating differentials. In this essay, we develop a theory that focuses on a less-studied topic: understanding the long-run macroeconomic consequences of trends in on-the-job utility. Two main implications emerge. First, secular improvements in on-the-job utility are such that it is possible for work hours to remain approximately constant over time even if the income effect of higher wages on labor supply exceeds the substitution effect of higher wages. Secondly, secular improvements in on-the-job utility can themselves be a substantial component of the welfare gains from technological progress. These two implications are connected by an identity: improvements in on-the-job utility that have a significant effect on labor supply tend to have large welfare effects.

Of course, if ongoing increases in job utility help rationalize the paradox of hard work, a natural question is whether the proper incentives are in place for such changes in job utility to occur. We argue that the answer to this is an overwhelming yes. These incentives exist on several dimensions, and are compatible with helping explain broader empirical phenomena in addition to trendless labor hours. In particular, we show that as economies become richer, endogenous channels will lead to firms and industries whose jobs are characterized by relatively low job utility to be driven out of the market. This offers a novel explanation for outsourcing, which has been a marked characteristic of industrialized economies over the last several decades. In

¹⁵See, for example, Hall (1988), Barsky et al. (1997), and Basu and Kimball (2002).

¹⁶See, for example, Shapiro and Kimball (2008).

addition, firms whose jobs offer relatively higher job utility endure a lower real wage per unit of worker productivity. Thus, firms with relatively higher job utility have a competitive advantage. Finally, we show that firms will find it endogenously optimal to increase job utility via increases in amenities as a method of countervailing income effects so that individuals will not substantially decrease their work hours. These last two findings help shed light on recent trends in non-pecuniary improvements in working conditions, as exemplified by, for example, Google's emphasis on its general work environment.

This essay's research contributes to the labor economics literature by developing a theoretical framework through which an intertemporal understanding of the primitives that determine the economy's available trade-offs between output, wages, and job utility can be attained. Moreover, we contribute to the macroeconomics literature by offering a novel explanation for the paradox of hard work, thus showing that this paradox is not necessarily evidence of a low intertemporal elasticity of substitution or non-separable preferences in consumption and leisure.

3.8 Figures

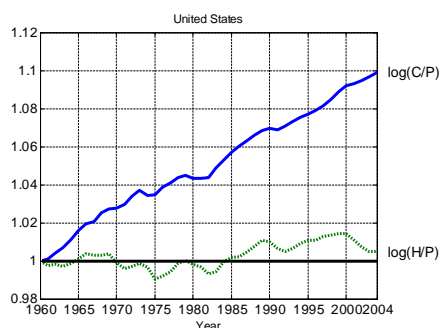


Figure 3.0.A

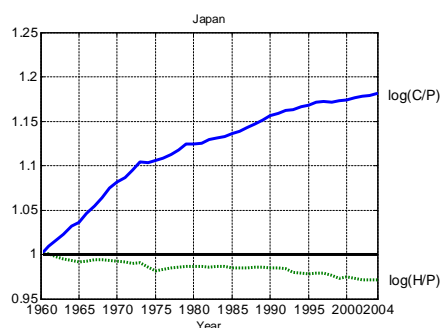


Figure 3.0.B

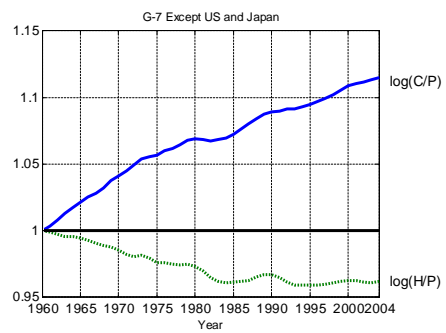


Figure 3.0.C

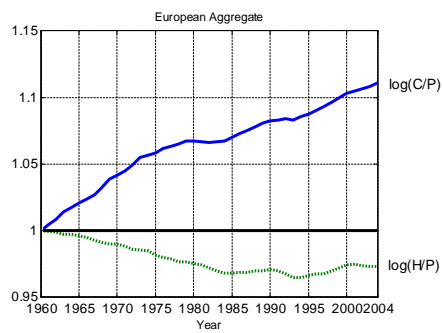


Figure 3.0.D

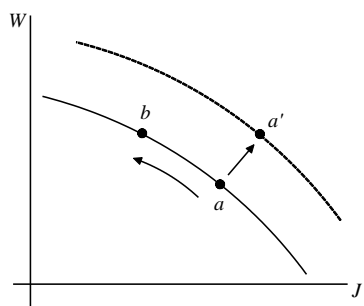


Figure 3.1

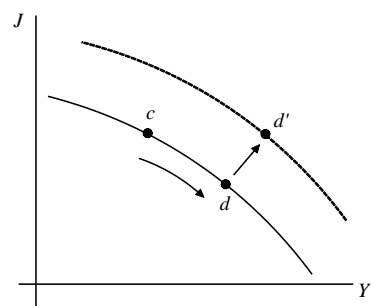


Figure 3.2

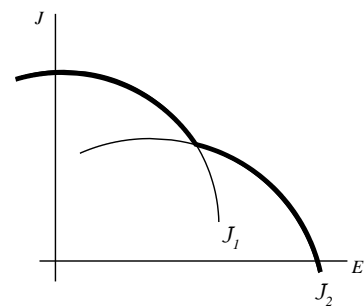


Figure 3.3

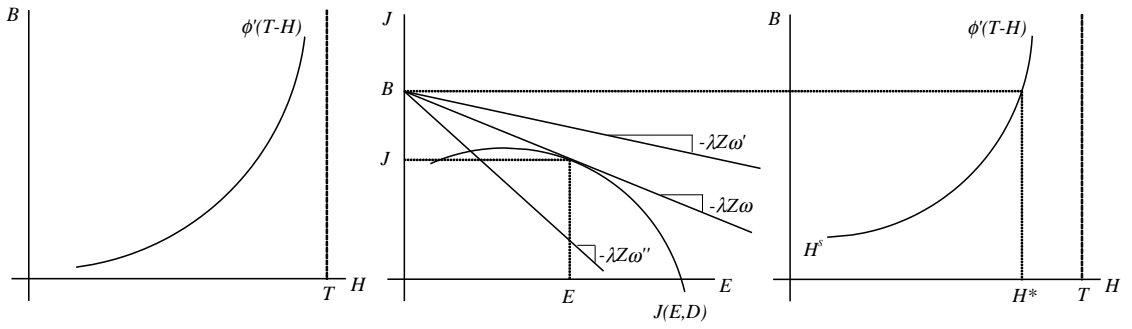


Figure 3.4

Figure 3.5

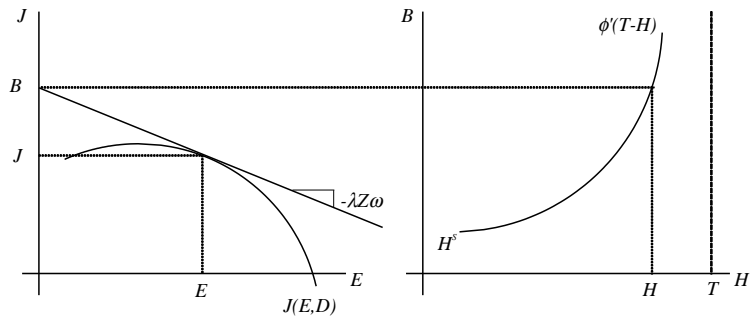


Figure 3.6

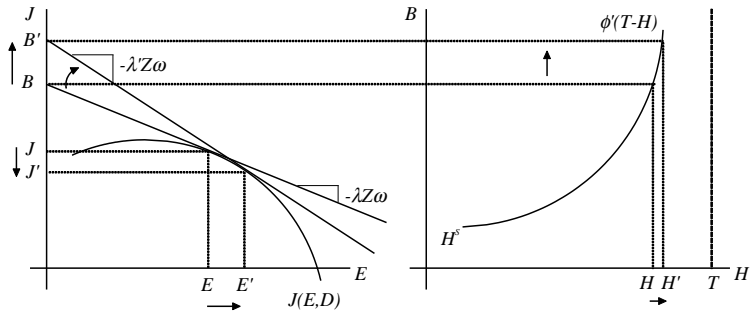


Figure 3.7

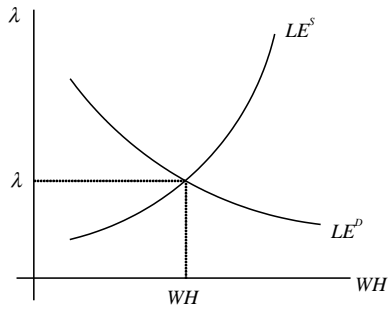


Figure 3.8

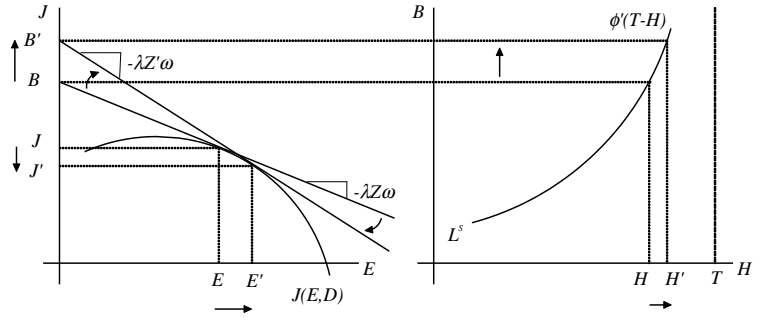


Figure 3.9

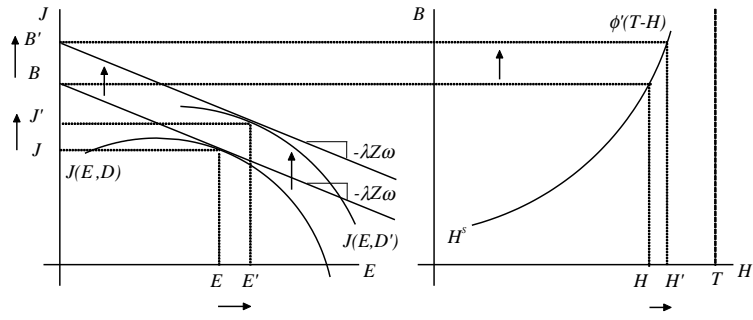


Figure 3.10

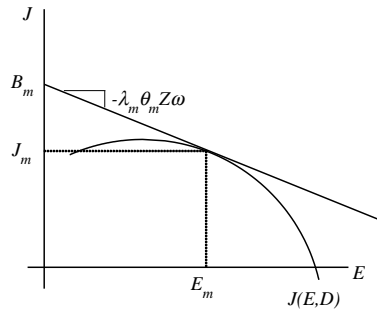


Figure 3.11

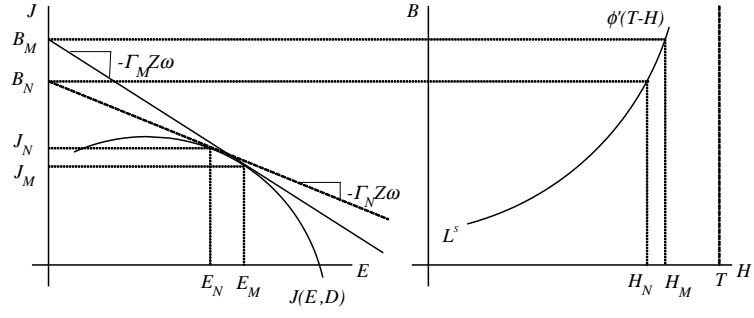


Figure 3.12

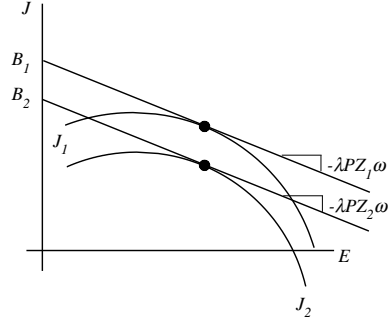


Figure 3.13

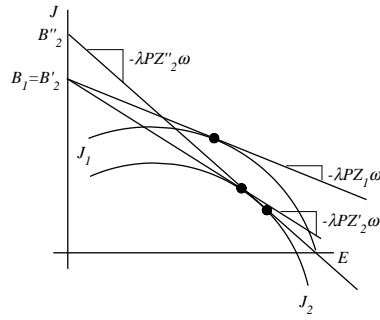


Figure 3.14

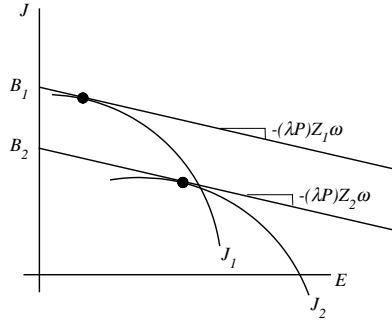


Figure 3.15

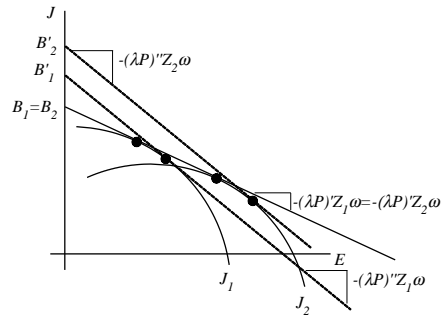


Figure 3.16

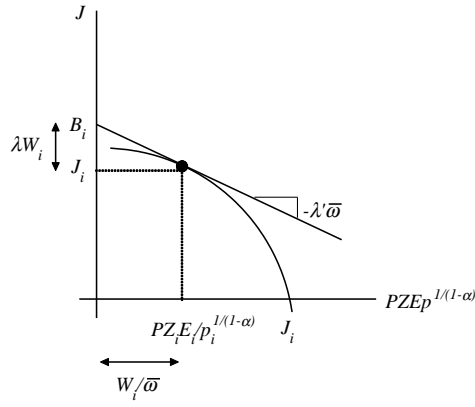


Figure 3.17

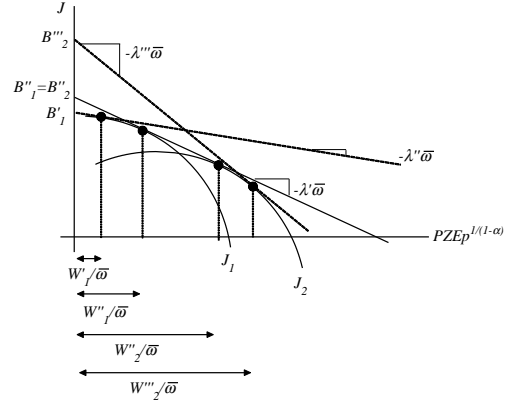


Figure 3.18

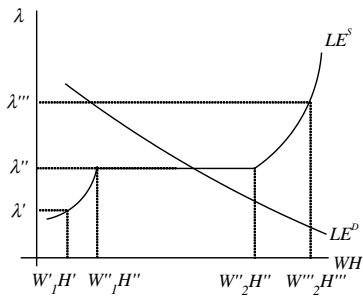


Figure 3.19

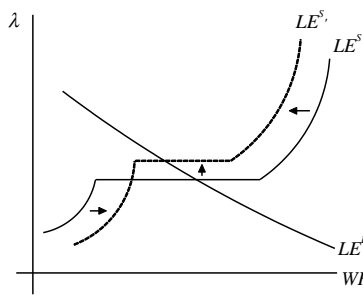


Figure 3.20

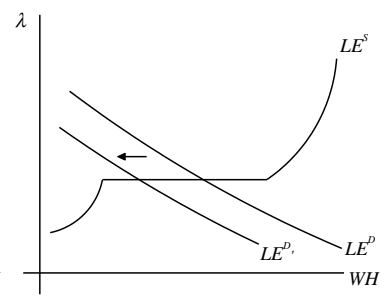


Figure 3.21

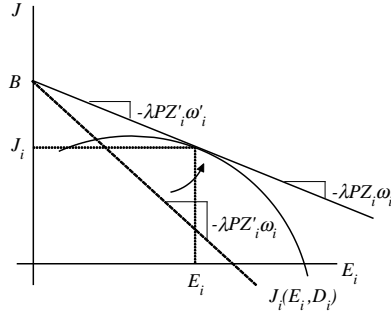


Figure 3.22

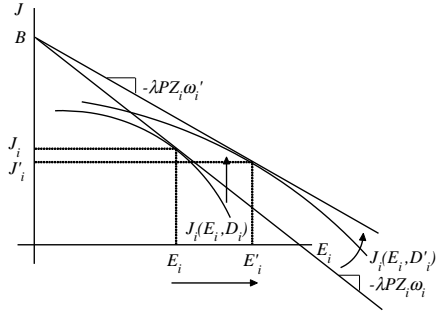


Figure 3.23

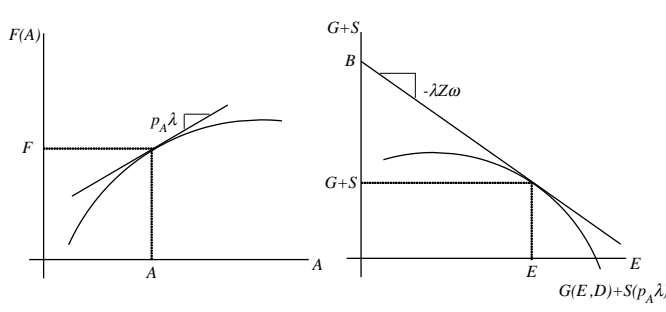


Figure 3.24

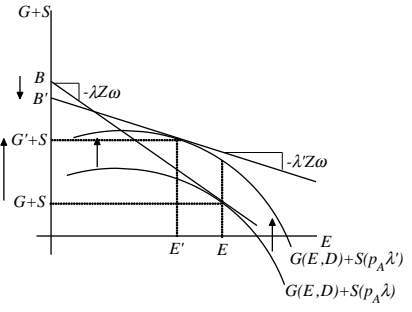


Figure 3.25

3.9 Derivations

3.9.1 Normalization

To show this, consider $U + \tilde{\Phi} + H\tilde{J}$ with $\tilde{\Phi}'(T) = \kappa$, where κ is a constant. Define $\Phi(X) = \tilde{\Phi}(X) - \kappa X$ and $J = \tilde{J} - \kappa H$. Then, $\Phi'(T) = 0$, and

$$\mathcal{U} = U + \tilde{\Phi}(T - H) + H\tilde{J} - \kappa(T - H) - \kappa H \implies \mathcal{U} = U + \Phi(T - H) + H\tilde{J} - \kappa T,$$

which is equivalent to $U + \Phi + H\tilde{J}$.

3.9.2 Details on the Incentives for Drudgery Declines

Given a change in drudgery - and keeping all else constant, in particular equilibrium net job benefits -, total differentiation of the firm's constraint, $\lambda W_i + J_i = B$, yields

$$J_{iE}dE_i/dD_i + \lambda dW_i/dD_i = -J_{iD}. \quad (3.28)$$

Similarly, total differentiation of the optimality condition $-J_{iE}E_i = \lambda W_i$ implies that

$$J_{iE}dE_i/dD_i + \lambda dW_i/dD_i = -J_{iEE}E_idE_i/dD_i - J_{iED}E_i. \quad (3.29)$$

Combining the previous two equations results in

$$dE_i/dD_i = (J_{iD} - J_{iED}E_i) / J_{iEE}E_i. \quad (3.30)$$

By assumption $J_{iD}, J_{iEE} < 0$. Whereas $J_{iED} \geq 0$ ensures that dE_i/dD_i is strictly positive, $J_{iED} < 0$ allows for $dE_i/dD_i \leq 0$. Moreover, note that rearranging equation (3.28) implies that

$$dW_i/dD_i = -J_{iD}/\lambda - (J_{iE}/\lambda) dE_i/dD_i. \quad (3.31)$$

Before proceeding, totally differentiate the job function. This yields

$$dJ_i/dD_i = J_{iE}dE_i/dD_i + J_{iD}, \quad (3.32)$$

where we refer to $J_{iE}dE_i/dD_i$ as the *effort substitution effect* and J_{iD} as the (*pure*) *drudgery effect*. Note that whereas the drudgery effect is always negative, the sign of the effort substitution effect is ambiguous and depends directly on that of dE_i/dD_i .¹⁷

The effects of a change in drudgery depend on the sign of J_{iED} . Consider the case in which $J_{iED} < 0$. This initially gives way to three additional possibilities conditional on the sign of the numerator in equation (3.30). Assume $-J_{iD} < -J_{iED}E_i$. In this

¹⁷Recall that $J_{iE} < 0$ is an endogenous result of the firm's optimization subproblem given positive, as we have assumed throughout the essay.

case, equation (3.30) implies that $dE_i/dD_i < 0$. However, by equation (3.28)

$$dW_i/dD_i = -J_{iD}/\lambda - (J_{iE}/\lambda)(dE_i/dD_i) \leq 0.$$

Using equation (3.32) the fact that $dE_i/dD_i < 0$ implies that in this case the effort substitution and drudgery effects work in opposite directions. If the effort substitution effect dominates the drudgery effect, then $dJ_i/dD_i > 0$. Hence,

$$J_{iE}dE_i/dD_i > -J_{iD} \implies 0 > -(J_{iE}/\lambda)(dE_i/dD_i) - J_{iD}/\lambda$$

which using (3.31) implies that $dW_i/dD_i < 0$.

3.9.3 Amenities

To see that under λ' marginal net job benefits are lower than under λ , consider the firm's constraint $\lambda Z\omega E + G = B + F$. Since both ω and Z remain constant, it follows that

$$Z\omega E d\lambda + \lambda Z\omega dE + G_E dE = d(B - F + F'A).$$

The firm's optimality condition $G_E = -\lambda Z\omega$ implies that $G_E dE = -\lambda Z\omega d\lambda$. Using this in the differentiated version of the firm's constraint results, after rearranging, in $Z\omega E = dB/d\lambda + (F''A) dA/d\lambda$. Then, use of the optimality condition $p_A\lambda = F'$ implies that $dA/d\lambda = cp_A/F''$. Substituting this into the firm's differentiated constraint implies that the second term on its right side reduces to p_AA . Moreover, note that the left side of this equation is actually $(W + p_AA)ZE/ZE$. Given the previous, rearranging and simplifying the firm's differentiated constraint results in $dB/d\lambda = W > 0$ assuming, as we have throughout the essay, a positive real wage.

3.9.4 Welfare Under Non-Separability

Given

$$\frac{dB}{\lambda} = (-V_H/\lambda\bar{\eta}) \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda,$$

it follows that

$$\begin{aligned}
dB/\lambda &= (B/\lambda\bar{\eta}) \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda \\
\implies dB/\lambda &= (\lambda W_i + J_i) / \lambda\bar{\eta} \cdot dH/H - V_{H\lambda} d\lambda/\lambda \\
\implies dB/\lambda &= (W_i + J_i/\lambda) / \bar{\eta} \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda \\
\implies dB/\lambda &= (1 - \zeta_i) W_i / \bar{\eta} \cdot dH/H - V_{H\lambda} \cdot d\lambda/\lambda.
\end{aligned}$$

Now, consider $C(\lambda, H) = -V_\lambda(\lambda, H)$. This implies that

$$\begin{aligned}
dC &= C_\lambda d\lambda + C_H dH \\
\implies dC &= -V_{\lambda\lambda} d\lambda - V_{\lambda H} dH \implies dC/C = (V_{\lambda\lambda}\lambda/V_\lambda) \cdot d\lambda/\lambda + (V_{\lambda H}H/V_\lambda) \cdot dH/H.
\end{aligned}$$

Finally, note that

$$\begin{aligned}
\frac{d\mathcal{L}^*}{\lambda C} &= \frac{H}{C} \left(\frac{W_i}{\eta_i} \frac{dH}{H} + \left(\Theta \frac{C}{H} \right) \frac{d\lambda}{\lambda} \right) - \frac{\sum_i H_i W_i}{C} \frac{d\lambda}{\lambda} + \frac{(d\Pi + rdM)}{C} \\
\implies \frac{d\mathcal{L}^*}{\lambda C} &= \frac{W_i}{\eta_i} \frac{dH}{C} + \Theta \frac{d\lambda}{\lambda} - \frac{\sum_i H_i W_i}{C} \frac{d\lambda}{\lambda} + \frac{(d\Pi + rdM)}{C} \\
\implies \frac{d\mathcal{L}^*}{\lambda C} &= \frac{W_i}{\eta_i} \frac{dH}{C} + \left(\Theta - \frac{\sum_i H_i W_i}{C} \right) \gamma \left(\Theta \cdot \frac{dH}{H} - \frac{dC}{C} \right) + \frac{(d\Pi + rdM)}{C}.
\end{aligned}$$

3.10 References

Barsky, Robert, Thomas Juster, Miles Kimball, and Matthew Shapiro. 1997. “Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study.” *Quarterly Journal of Economics*, 112: 537-579.

Basu, Susanto, and Miles Kimball. 2002. Long-Run Labor Supply and the Elasticity of Intertemporal Substitution for Consumption. Unpublished manuscript.

Coulibaly, Brahma. 2006. “Changes in Job Quality and Trends in Labor Hours.” *International Finance Discussion Papers* 882. Washington: Board of Governors of the Federal Reserve System.

Hall, Robert E. “Intertemporal Substitution in Consumption.” 1988. *The Journal of Political Economy*, 96: 339-357.

Keynes, John Maynard. 1930. “Economic Possibilities for Our Grandchildren.” Printed in Vol. IX of *The Collected Writings of JM Keynes*, 1973. London: Macmillan for The Royal Economic Society.

Kimball, Miles, Claudia Sahm and Matthew Shapiro. 2011. “Measuring Time Preference and the Elasticity of Intertemporal Substitution Using Web Surveys.” Unpublished manuscript.

Kimball, Miles, and Matthew Shapiro. 2008. “Labor Supply: Are the Income Effects Both Large or Both Small?” NBER Working Paper No. 14208.

Rosen, Sherwin. “The Theory of Equalizing Differences.” 1986. *Handbook of Labor Economics*, 1: 641-692.

Smith, Adam. 1776. “An Inquiry Into the Nature and Causes of The Wealth of Nations.” Chicago: University of Chicago Press, 1976. Edwin Cannan, Ed.

CHAPTER IV

BEYOND TAXES: UNDERSTANDING THE LABOR WEDGE

4.1 Introduction

Recent literature has focused on examining the determinants of the long-run behavior of aggregate labor hours per working-age population (H/P) within and across countries.¹ This literature's analytical framework is based on a standard neoclassical macroeconomic model. The theory behind this model implies that equilibrium H/P is implicitly defined through a static optimality condition that equates the marginal rate of substitution of consumption for leisure with the marginal product of labor. The extent to which this condition fails to hold has been coined the *labor wedge*. More concisely, the labor wedge is a residual that captures the percent difference between model-predicted H/P and its empirical counterpart. The labor wedge has been found to be substantial across a large sample of OECD countries. Recent studies focus on understanding what factors can account for the labor wedge.² These studies argue that within any given country a considerable fraction of the labor wedge can be explained by accounting for the presence of taxes, which are typically left unaccounted for in related macroeconomic analysis. This conclusion is based on the finding that when the standard neoclassical model is enhanced to incorporate taxes, the model's predictions regarding the long-run behavior of H/P improve considerably. However, this improvement is limited to European countries. In particular, the model's predictions regarding the trend behavior of US and Canadian hours per population are for

¹See, for example, Prescott (2004) and Ohanian et al. (2008).

²See, for example, Shimer (2009) in addition to the earlier cited research.

all purposes contrary to the data. While over the last several decades these two countries have exhibited an upward trend in hours per population, the standard model enhanced with taxes predicts that the opposite should have occurred. This suggests that the labor wedge amounts to more than just taxes.

The aim of this essay is to understand what factors, in addition to taxes, can account for the labor wedge.³ This is particularly important towards understanding the effects of tax policy. Indeed, the extent to which the standard model enhanced with taxes fails to account for the trend behavior of H/P in the US and Canada is a reflection of the extent to which taxes have not had the expected effect on work hours in these countries relative to European countries. In the present essay, the analysis we develop implies a surprising result, which is that the limitations of the standard model in accounting for the long-run behavior of H/P in Canada and the US are actually evidence of the model's *overall* inability to explain the behavior of H/P . This is the result of the model lacking appropriate theoretical foundations for capturing the optimal behavior of the extensive margin of labor supply: the employment-to-population ratio.

The theory behind the standard neoclassical macroeconomic model results in an equation that defines total equilibrium labor hours. When testing the model's predictions, the consensus is to normalize all variables by the working-age population.⁴ Hence, the model is explicitly assumed to yield predictions regarding H/P . In Section 2.1 we review the standard model's failures in accounting for the empirical trend behavior of H/P , and in Section 2.2 we review the relative improvements that result from extending the model to account for taxes.⁵

Of course, H/P is equal to the product of hours per worker (H/E) and the employment-to-population ratio (E/P). In Section 3, we present evidence implying that the standard model has limited, if any, long-run explanatory power regarding E/P . We show that once taxes are accounted for, the model's implied equilibrium

³This essay is co-written with Shanthi P. Ramnath, Economist of the US Treasury. The views and opinions expressed in this essay are not necessarily those of the US Treasury.

⁴From now on, for simplicity we refer to population and working-age population interchangeably.

⁵These sections serve as a general recap of the analysis in relevant past literature, which includes Prescott (2004), Ohanian et al. (2008), and Shimer (2009), among others.

equation for H/P is in fact a relatively good predictor of H/E . Therefore, whenever E/P does not change much relative to H/E , which has been the case for most European countries, the empirical behavior of H/E and H/P are virtually indistinguishable and the standard model gives the impression of correctly predicting H/P . On the other hand, when E/P *does* change considerably relative to H/E , as has been the case in Canada and the US over the last several decades, the standard model implicitly reveals its limitations in predicting E/P , which in turn inhibits its ability to accurately predict H/P . Because the labor wedge is defined as a residual, that is, the portion of the data that model-predicted hours per population cannot explain, the E/P ratio automatically becomes part of the labor wedge. This finding represents an important contribution in terms of the interpretation of the labor wedge. The research in Prescott (2004) and Ohanian et al. (2008) successfully identifies that part of the existence of the labor wedge results from ignoring the role of taxes. We complement this research by showing that another part of the labor wedge actually holds by construction. We argue that a portion of the labor wedge exists because of an inherent inability on the standard model's part to predict extensive-margin changes in labor supply.

In Section 4, we develop a model that allows for heterogeneity in terms of employment status. In our framework, a household planner maximizes the joint utility of all household members by optimally choosing the fraction of the population that is employed, the hours that each employed individual works, and the distribution of household consumption across individuals conditional on employment status. Our model accounts for non-employment disutility and a time-varying fixed cost associated with employment. We show that *net employment disutility*, which is a weighted difference between time-varying employment fixed costs and non-employment disutility, has a significant influence in the household planner's optimal choice regarding the distribution of employed and non-employed individuals.⁶ Moreover, the theory we develop yields an equation for equilibrium work hours that is explicitly in per worker terms, and almost identical to the standard model's implicit equation for equilibrium

⁶In the neoclassical spirit of market clearing, we do not focus on involuntary aspects of unemployment.

H/P . Hence, our model rationalizes why the standard model's equation for equilibrium hours per population is instead a substantially more accurate predictor of hours per worker.

In Section 5.1 we show our theory's empirical applications relative to the trend behavior of hours per worker over a large set of OECD countries for the period 1960-2006. Then, in Section 5.2, we derive the implied trend behavior of net employment disutility consistent with empirical long-run cross-country trends in employment. Our analysis suggests that within our sample period, trends in employment across countries require a relatively small average yearly decline in net employment disutility in order to be rationalized.

In Sections 5.3 and 5.4, we focus on the model's theoretical implications regarding tax policy. In particular, we analyze how changes to the net-of-tax rate impact hours per worker and employment. Intuitively, we find that both are increasing in the net-of-tax rate, and therefore have an unambiguous impact on hours per population, which is that hours per population are increasing in the net-of-tax rate. However, we show that the relative elasticity of hours per worker and employment with respect to the net-of-tax rate is in principle ambiguous. We then assess the differential impact that changes in average tax rates versus marginal tax rates can have on hours per population. We find that a decrease in the average tax rate will lead to a decrease in hours per worker accompanied by an increase in employment. Thus, a decrease in the average tax rate could potentially have an ambiguous impact on hours per population. However, we show that in general the increase in employment will outweigh the decline in hours per worker implying that overall, hours per population will tend to increase given a decline in the average tax rate. The noted ambiguity in the impact of tax policy arises from the structure of the model, since the presence of net employment disutility implies differences in marginal disutility from changes in hours per worker and employment. This makes the household's decision over which margin to adjust hours per population, given changes in economic conditions, directly dependent on optimal current levels of employment and hours per worker. Finally, Section 6 reviews related literature, and Section 7 concludes.

This essay makes three main contributions to the literature. First, it provides substantial empirical- and theory-based evidence that implies that the standard neoclassical macroeconomic model lacks explanatory power regarding the extensive margin of labor supply. This limitation of the model leads to inaccurate predictions of H/P when there are large changes in E/P relative to those in H/E . Second, based on the previous finding, we rationalize the so-far puzzling fact that the standard neoclassical macroeconomic model extended to account for taxes is unable to match the trend behavior of hours per population in the US and Canada, even though it has been relatively successful in doing so for most European countries. In contrast to the US and Canada, in most European countries E/P has not changed much relative to H/E . Hence, in these countries the long-run behavior of H/P and H/E is virtually indistinguishable. Third, this essay provides an understanding of the labor wedge that complements previous studies. Earlier research successfully identifies that part of the existence of the labor wedge results from ignoring the role of taxes. We complement this research by showing that another part of the labor wedge actually holds by construction, and stems from an inherent inability of the standard model to predict extensive-margin changes in labor supply.

4.2 The Labor Wedge

In a standard neoclassical macroeconomic model, setting labor supply equal to labor demand yields a straightforward equation that defines equilibrium work hours. Work hours are a function of output, consumption, and the structural parameters of the model. Hence, it is possible to generate a predicted series of hours using aggregate data on consumption and output, along with assumed parameter values. The extent to which model-predicted hours differ from the actual data on hours is captured by the labor wedge, which is defined as the ratio of model-predicted hours to actual data on hours. If the model were able to perfectly predict hours, then this ratio would be one. Realistically, many of the assumptions of the standard neoclassical model are likely to not hold. However, the exercise of comparing model-generated hours with the data is useful for measuring the explanatory power of the model given its simplifying

assumptions. In what follows, we derive the equation for labor hours that stems from the standard neoclassical model. For ease of exposition, throughout the essay our main analysis focuses on Canada, France, Germany, and the US as benchmarks of the accuracy of the model's predictions. These countries are representative of the differences that earlier research has found between European and North-American countries in terms of hours per working-age population. Whereas in Europe, on average, H/P has decreased over the last several decades, in North America H/P has increased. We derive the labor wedge associated with the standard model given data for the US, Canada, France and Germany over the span of roughly 40 years. We then relax the assumption that tax distortions are non-existent in the standard model and re-derive the model assuming a broad set of taxes on labor, capital, investment, and consumption. The effective tax rate that had been previously excluded from the standard model factors explicitly into the prediction for hours worked. The analysis in this section is broadly analogous to that in Prescott (2004), Ohanian et al. (2008), and Shimer (2009).

4.2.1 The Standard Model

In the standard neoclassical macroeconomic model a representative household maximizes its present discounted value of utility subject to an intertemporal budget constraint. This infinitely lived household derives utility from household consumption C and disutility from household labor hours H . Thus, the household seeks to maximize

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U(C_t, H_t),$$

subject to

$$W_t H_t + R_t K_t \geq C_t + I_t.$$

Above, β is the discount factor, W is the real wage, I is investment, and the price of consumption is normalized to 1. The household is assumed to own the economy's capital, K , and a representative firm rents the capital from the household at rate R .

The capital accumulation equation is given by

$$K_{t+1} = I_t + (1 - \delta)K_t,$$

where δ is the capital depreciation rate.

Following Shimer (2009), we assume that the household's instantaneous utility function is given by

$$U(C_t, H_t) = \ln C_t - \gamma \frac{\varepsilon}{1 + \varepsilon} H_t^{\frac{1+\varepsilon}{\varepsilon}}, \quad (4.1)$$

where ε is the Frisch (marginal value of real wealth held constant) elasticity of labor supply, and γ is a positive constant.⁷

Output Y is determined by a representative firm with Cobb-Douglas production function

$$Y_t = Z_t K_t^\alpha H_t^{1-\alpha},$$

where $\alpha \in (0, 1)$ and Z is technology. The representative firm chooses capital and work hours to maximize profits, given by

$$\Pi_t = Z_t K_t^\alpha H_t^{1-\alpha} - W_t H_t - R_t K_t.$$

Assuming all markets are perfectly competitive, in equilibrium labor supply equals labor demand. Therefore, combining the household's first-order conditions for labor supply and consumption with the firm's first-order condition for labor demand yields an equation for the equilibrium level of hours worked

$$H_t^{NM} := \left(\frac{(1 - \alpha)Y_t}{\gamma C_t} \right)^{\frac{\varepsilon}{1+\varepsilon}}, \quad (4.2)$$

where NM stands for *neoclassical model*.⁸ This is a static condition that must hold

⁷The assumed functional form for instantaneous utility differs from that used in Prescott (2004) and Ohanian et al. (2008). However, as shown in Shimer (2009), the choice of utility function has little impact on both the qualitative and quantitative results. Chetty (2009) offers a detailed comparison between the Frisch elasticity, which is commonly used in macroeconomic literature, and the compensated elasticity, which is frequently used in the public finance and labor literature.

⁸The notation $:=$ means that the object on the left-hand side of this symbol is defined by the object on its right-hand side.

within any time period.

There are diverse ways to test the validity of models such as structural estimation and numerical simulation. However, given that the condition in equation (4.2) is static, we can easily test its accuracy by using aggregate data on output and consumption to generate the model's prediction for hours worked, subject to choices for the model's parameters. This approach has been used in past literature including Parkin (1988), Rotemberg and Woodford (1999), and Mulligan (2002). The extent to which the model's predicted hours H^{NM} differ from actual hours H^{ACTUAL} is captured by the labor wedge, $(1 - \Delta_t)$, which satisfies

$$(1 - \Delta_t) := \frac{H_t^{ACTUAL}}{H_t^{NM}}.$$

The difference between the labor wedge and unity, $-\Delta_t$, measures the percent deviation between actual hours and model hours. We follow Prescott (2004), Ohanian et al. (2008), and Shimer (2009) and focus on the long-run behavior of labor hours by using yearly data. In order to gauge the performance of the standard model we compare model-predicted hours to actual hours.

A country's model-predicted hours H^{NM} are generated using equation (4.2) as follows. As is standard in the literature, we normalize all within-country variables by the working-age population; thus, the standard model is assumed to predict hours per working-age population. We use annual data from 1960 through 2006, which is detailed in Section 4.8. Using data on real output and consumption, and assuming a value for ε , we generate the series $(Y_t/C_t)^{\frac{\varepsilon}{1+\varepsilon}}$ for each country. The final version of model hours requires scaling this series by $(1 - \alpha)/\gamma$. Let $\kappa = (1 - \alpha)/\gamma$. In the model, κ is constant over time. Therefore, we can use κ as a free parameter to calibrate the model to achieve a predetermined target. Following Shimer (2009), we define

$$\kappa^{NM} := \frac{\text{mean} \left((H_t/P_t)^{ACTUAL} \right)}{\text{mean} \left((Y_t/C_t)^{\frac{\varepsilon}{1+\varepsilon}} \right)},$$

which implicitly allows for cross-country heterogeneity in κ . Hence, for each country we choose the scaling parameter κ such that mean model-generated hours H^{NM} are

equal to that country's mean actual hours H^{ACTUAL} .

Although micro estimates of the Frisch elasticity of labor supply usually imply values of ε less than unity, macro estimates are on average slightly higher than 1. Some studies develop explanations by which these difference can be reconciled (Chetty (2009), Rogerson and Wallenius (2009)). However, a line of research, especially that regarding real business cycle analysis, tends to impute ε by choosing a value for this parameter that makes the model-predicted cyclical fluctuations in labor hours most closely match the cyclical fluctuations in the data. This approach leads to much higher choices of ε than those mentioned earlier. A similar approach, focusing on the trend behavior of labor hours rather than their cyclical fluctuations, leads Prescott (2004) to impute $\varepsilon = 3$, and Shimer (2009) to impute $\varepsilon = 4$.

We compare actual hours per working-age population with model H/P generated alternatively with $\varepsilon = 1$ and $\varepsilon = 4$. Figure 4.1 shows the model generated hours per working age population with ε set to 1 and alternatively to 4.⁹ As noted above, the appropriate parameter value for the Frisch labor supply elasticity is debatable. However, the results in Figure 4.1 show that the values for ε under consideration make little difference in terms of their effect on the trend behavior of model-generated hours. Henceforth, we follow Shimer (2009) and set ε equal to 4. Figure 4.2 shows actual hours per working age population along with their model-generated counterparts using $\varepsilon = 4$. The figure illustrates that although the model performs relatively well when predicting the trend behavior in Canada, this is not the case in the other countries. The residual of the model's predictions relative to the data is captured by the resulting labor wedge, which is graphed in Figure 4.3 for each country under consideration and relative to its corresponding value in 1960. The normalization owes to the fact that since our study focuses on the trend behavior of H/P , the level is irrelevant.

⁹Sources and summary statistics of all data used in this essay can be found in Section 4.8. Section 4.9 details mathematical derivations associated with Sections 4.5.3 and 4.5.4. Finally, all figures noted throughout the text can be found in Section 4.10, and tables relevant to our analysis can be found in Section 4.11.

4.2.2 The Model with Taxes

That trends in hours per working age population are vastly different across countries stimulated interest into potential causes for reconciling this stylized fact. One explanation is that taxes contribute to these differences both across countries and within countries over time. Thus a growing body of literature (for instance, Prescott (2004), Ohanian et al. (2008), and Shimer (2009)) incorporates taxation into the standard neoclassical model in order to address this issue. This literature argues that a fraction of the labor wedge is accounted for by taxes. Below, we re-derive the equation for labor hours allowing for a broad set of taxes.

Following related literature we assume that the statutory incidence of all taxes is on consumers, making the household's budget constraint

$$(1 + \tau_t^c)C_t + (1 + \tau_t^i)I_t \leq (1 - \tau_t^h)W_tH_t + (1 - \tau_t^k)R_tK_t + T_t.$$

Above, τ^c , τ^i , τ^h , and τ^k , are, respectively, consumption, investment, labor, and capital taxes, and T_t are lump-sum government transfers. The counterpart of equation (4.2) is now

$$H_t^{NMT} := \left((1 - \tau_t) \frac{(1 - \alpha)Y_t}{\gamma C_t} \right)^{\frac{\varepsilon}{1+\varepsilon}}, \quad (4.3)$$

where $\tau = (\tau^h + \tau^c) / (1 + \tau^c)$ is the effective tax rate and NMT stands for *neoclassical model with taxes*. The tax-inclusive model reveals that $(1 - \tau)^{\frac{\varepsilon}{1+\varepsilon}}$ is part of the labor wedge of the standard model when taxes are ignored. Hence when taxes are included, the labor wedge satisfies,

$$(1 - \Delta_t) := \frac{H_t^{ACTUAL}}{H_t^{NMT}}. \quad (4.4)$$

We generate model hours for the period 1960-2006 using equation (4.3) normalized by the working-age population, along with data on C and Y . Following Ohanian et al. (2008), we use the effective marginal tax series created by McDaniel (2007), which includes calculated taxes on both income and consumption. McDaniel's methods are similar to those of Mendoza et al. (1994), although her data is mainly derived from

national accounts publications. Income and expenditure data and tax revenue are all categorized into labor or capital income, and consumption or private investment. Tax rates are then calculated by dividing the tax revenue by either the income or expenditure for that category. This method for calculating tax rates has the appealing feature that taxes can be derived independently of tax return data using only aggregate data. However, a trade-off exists by which strong assumptions are required for classifying the data into categories, which necessarily impacts the results. An additional drawback from this method is that the calculated tax rates are average tax rates rather than marginal tax rates. However, McDaniel (2007) provides a comparison of the average tax rates and average marginal tax rates series calculated from past studies for the US and finds a similar trend behavior in each of the two series. The McDaniel (2007) tax data is summarized in Section 4.8.¹⁰ The normalized model is once again assumed to predict hours per population. To generate model hours, we continue to set ε to 4. For each country in our OECD sample we generate the series $((1 - \tau)Y_t/C_t)^{\frac{\varepsilon}{1+\varepsilon}}$. In this case note that for any given country the scaling parameter $\kappa = (1 - \alpha)/\gamma$ now satisfies

$$\kappa^{NMT} := \frac{\text{mean}((H_t/P_t)^{ACTUAL})}{\text{mean}\left((1 - \tau_t)(Y_t/C_t)^{\frac{\varepsilon}{1+\varepsilon}}\right)}.$$

Figure 4.4 presents the resulting model-generated H/P and actual H/P . For the purposes of comparison, Figure 4.5 shows the labor wedges generated by the standard model and the labor wedges generated by standard model augmented with taxes; as before, all of these are shown relative to their 1960 value. The wedge generated by the model with taxes is closer to one for France and Germany. This highlights the improvement made in terms of predicting the long-run behavior of H/P relative to the standard model for these countries. However, as noted by both Prescott (2004) and Ohanian et al. (2008), when taxes are included, the model's predictions for the US and Canada fail to account for the data.¹¹ Returning to Figure 4.5, note that this

¹⁰See McDaniel (2007) for a more detailed explanation for how tax rates are calculated.

¹¹We calculate a sum of the squared differences, where the difference is between the actual data on hours and the model predicted hours with and without taxes. The sum of squared differences

failure is nontrivial: while US and Canadian hours per population have exhibited an upward trend over the last several decades, the neoclassical model with taxes actually predicts a downward trend in H/P for these countries, thus contradicting the data.

4.3 The Role of the E/P Ratio

Given that $H/P = (H/E) \cdot (E/P)$, understanding why the standard model with taxes fares poorly for some countries may be illuminated by understanding on which margin it is failing: H/E , E/P , or both. Thus, it is useful to disentangle the relative influence of H/E and E/P in shaping the observed patterns in H/P . Figure 4.6, shows actual hours per population for Canada, France, Germany, and the US. In addition, the graph illustrates the behavior of H/P , had E/P remained fixed at its 1960 value and only H/E changed, and also the behavior of H/P , had H/E remained fixed at its 1960 value and only E/P changed. In Canada and the US, hours per population have been increasing while hours per worker have experienced a relatively small decrease. Thus, in Canada and the US, the long-run trend in H/P has predominately been driven by changes in E/P . This stands in contrast to France and Germany, where both hours per population and hours per worker have been decreasing while E/P has remained relatively constant.

Recall that the standard model with taxes was shown to provide relatively accurate predictions of H/P for France and Germany, but not for Canada and the US. Combined with the patterns in Figure 4.6, this suggests that the standard model may be inherently incapable of predicting changes in employment and instead is a better predictor of hours per worker.

The theoretical predictions stemming from NM and NMT regarding equilibrium hours of work do not specify whether they are in per worker or per population terms. However, it is standard in the literature to normalize all variables in the model by population, P . As noted above, the model is therefore assumed to predict H/P . Normalizing by the population implies that all household members share consumption

is lower when taxes are included in the model for France and Germany, but higher when taxes are included in the model for Canada and the U.S.

utility as well as work-hours disutility equally. In both NM and NMT, assuming that disutility from work-hours is shared across the population is the same as assuming both that everyone works the same amount of hours and that the entire population is employed.

Alternatively, suppose that equation (4.3) actually satisfied

$$\frac{H_t}{E_t} := \left((1 - \tau_t) \frac{(1 - \alpha)Y_t}{\gamma C_t} \right)^{\frac{\varepsilon}{1+\varepsilon}}. \quad (4.5)$$

That is, suppose that the model's prediction for hours were explicitly in per-worker terms. In this case, model hours per worker can be generated for each country by creating the series $((1 - \alpha)Y_t/\gamma C_t)^{\frac{\varepsilon}{1+\varepsilon}}$ and then scaling it by the parameter

$$\kappa := \frac{\text{mean} \left((H_t/E_t)^{ACTUAL} \right)}{\text{mean} \left((1 - \tau_t) (Y_t/C_t)^{\frac{\varepsilon}{1+\varepsilon}} \right)}.$$

Figure 4.7 shows the actual hours per worker for Canada, France, Germany, and the US along with hours per worker generated using equation (4.5). When the model is assumed to predict H/E , as in equation (4.5), the model-generated data is very close to the actual data.

If the standard model with taxes provides good predictions of hours per worker, then it should also be the case that multiplying these hours by a correct prediction of E/P would yield correct predictions of H/P for all countries, including the US and Canada. Unfortunately, the standard model only provides an equation for labor hours. However, if equation (4.5) is a good approximation to the actual behavior of H/E , then multiplying the implied model hours by each country's actual E/P ratio should yield a largely correct approximation of each country's actual H/P ratio. Let

$$\left(\frac{H_t}{P_t} \right)^{HYBRID} := \left(\frac{E_t}{P_t} \right)^{ACTUAL} \cdot \left(\frac{H_t}{E_t} \right),$$

where H/E are model-generated hours per worker as implied by equation (4.5). Hybrid hours per working-age population as well as actual H/P are shown in Figure

4.8. Hybrid hours per working-age population perform extremely well in approximating actual H/P for each country. In particular, for Canada and the US, the trend behavior of H/P is correct.

The analysis thus far suggests that the standard neoclassical model extended to account for taxes is better suited for predicting H/E rather than H/P . The degree to which this is true can be gauged by one final test. If the standard neoclassical model is incapable of predicting E/P , then E/P will in practice fall into the labor wedge defined in equation (4.4). For each country under consideration Figure 4.9 shows the labor wedge implied by equation (4.4), as before normalized by its 1960 value, along with the (empirical) E/P ratio. Except for a scaling constant, the long-run behavior of these two series track one another surprisingly well. This indicates that a significant part of the labor wedge in equation (4.4) corresponds to the E/P ratio.

Our conclusions have mixed implications for the success of the standard model. Our analysis shows that the current theory produces a relatively accurate prediction for H/E , which implies a strong theoretical underpinning for the determinants of this variable, but not of E/P . Hence, the standard model with taxes can provide mostly accurate predictions of the long-run behavior of H/P only when the E/P ratio does not change much relative to H/E . In other words, the model is successful when the behaviors of H/E and H/P are similar. However, when E/P does change substantially relative to H/E , as has been the case over the last several decades in Canada and the US, the model's predictions necessarily fail.

Prescott (2004), Ohanian (2008), and Shimer (2009) identify taxes as part of the labor wedge. Our research complements these findings by showing that the residual generated by the standard model with taxes is additionally comprised to a large extent by the E/P ratio itself. Hence, our findings imply that the standard model lacks predictive power for changes in employment, which is a fundamental component for understanding, in particular, the total impact that tax policy has on the behavior of H/P .

4.4 Heterogeneity

We have argued that the equation for hours implied by the standard theory is relatively better suited for predicting hours per worker than hours per working-age population. Of course, when the model is normalized by employment, consumption utility will also be on a per worker basis. On the other hand, if the model is normalized by population, then a within-household distribution is implicitly established by which all household members share utility from consumption and disutility from work-hours equally. In other words, normalizing by population establishes that all household members consume the same amount and work the same number of hours. This implicitly dampens the representative household's labor disutility since aggregate hours are normalized by a group that includes non-workers. The differences stemming from choice in normalization highlight an aspect of the model that is lacking, which is its ability to distinguish between the intensive and extensive margins of labor supply.

A perfectly competitive model assumes market clearing with full employment, which makes using population to normalize hours seem natural. However, empirically, we observe the existence of non-employed individuals. As an alternative, we develop a model that explicitly incorporates the possibility that some individuals are employed and others (voluntarily) are not.¹² That is, we allow for heterogeneity in employment status. In our model, hours per worker and employment are disentangled as choice variables to be optimized by a household planner. The result is an equation for equilibrium labor hours in terms of hours per worker. Interestingly, this equation is almost identical to the one stemming from the employment-normalized standard model. In fact, we show that the hours equation stemming from both the population-normalized and the employment-normalized models are special cases of the hours equation from the model developed below. This model provides a theoretical rationalization for our earlier finding that the standard model is relatively better at predicting hours per worker.

Our model allows us to derive conditions that show the impact that taxes have

¹²In the spirit of market clearing, we focus on non-employment rather than unemployment.

on both hours per worker and employment. Tax policy can have differential impacts on the extensive and intensive margins of labor supply decisions. In particular, the average tax rate is associated with changes on the extensive margin while the marginal tax rate impacts the intensive margin. Thus, to understand the role that taxes play in explaining aggregate trends in labor hours, we must account for their impact on both individuals' choices of whether to work or not, and, conditional on working, their choices regarding how many hours to work. We further our discussion by relaxing the assumption of a flat tax on wage income to allow for a more realistic graduated wage income tax. By allowing for a graduated wage tax, we can also isolate the differential impact that average versus marginal taxes have on labor supply. In particular, we derive conditions for how both average tax rates and marginal tax rates each impact hours per worker and employment.

In standard representative agent macroeconomic models, the population implicitly consists of a continuum of infinitely divisible individuals that is normalized to 1. As individuals are assumed to be identical, a household's instantaneous utility is equivalent to that of a single representative agent multiplied by the number of individuals (i.e., 1). The model we develop maintains the assumption of an infinitely divisible population (that we will normalize to unity), but extends the household planner's problem to include the possibility of non-employment.

Suppose that the within-period utility of an employed individual is given by

$$U^E = \ln(c_t) - \gamma \left(\phi_t - \frac{\varepsilon}{1 + \varepsilon} (h_t)^{(1+\varepsilon)/\varepsilon} \right),$$

and that of a non-employed individual, for whom $h = 0$, is

$$U^N = \ln(c_t) - D_t.$$

Above, c is an individual's consumption, $\phi > 0$ is a fixed cost associated with employment, and h is an individual's market work hours. As before, γ is a labor disutility parameter and ε is the Frisch elasticity of labor supply. In addition, D is a catch-all

variable that reflects disutility endured by non-employed individuals.¹³

Let P denote the population and assume that all individuals in the population are grouped within a single household in which resources are pooled. We assume individuals are altruistic and their joint objective is to maximize the household's utility. Thus, suppose a household planner maximizes the joint utility \mathcal{U} of the household's P members, taking prices and government policy as given. In particular, the household planner's objective is to maximize

$$\mathcal{U} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \psi_t^E E_t \left(\ln \left(\frac{C_t^E}{E_t} \right) - \gamma \phi_t - \gamma \frac{\varepsilon}{1 + \varepsilon} \left(\frac{H_t}{E_t} \right)^{(1+\varepsilon)/\varepsilon} \right) + \psi_t^N (P_t - E_t) \left(\ln \left(\frac{C_t^N}{P_t - E_t} \right) - D_t \right) \right\}$$

subject to

$$(1 + \tau_t^c) (C_t^E + C_t^N) + (1 + \tau_t^i) I_t \leq (1 - \tau_t^l) W_t H_t + (1 - \tau_t^k) R_t K_t + T_t.$$

Above, E is the number of employed individuals, C^E is the fraction of total household consumption C that employed individuals receive, H is total work hours, and C^N is the fraction of total household consumption that non-employed individuals receive. In addition, $\psi^E \in (0, 1)$ and $\psi^N = 1 - \psi^E$ are, respectively, the weights that the household planner places on the utility of employed and non-employed individuals. As we will show below, these weights guarantee that at any given point in time the solution to the planner's problem satisfies a no-utility arbitrage condition, meaning that no individual can obtain greater utility by switching between employment states. All other variables, as well as the capital accumulation equation, are as described in Section 4.2.

The choice of employment versus non-employment matters on two important dimensions. First, note that since non-employed individuals do not work, they contribute no labor disutility to \mathcal{U} . Also, what matters for total labor income is total

¹³With the exception of allowing for the employment fixed cost to be time varying and for the possibility of non-employed individuals to receive (dis)utility from sources other than consumption, the instantaneous utility functions we use are the same as that used in Kimball and Shapiro (2008).

labor hours H . Once there is a choice between employment E and hours per worker H/E , where explicit disutility from the former is linear and from the latter is convex, making the decision over which margin to adjust total hours given changes in economic conditions becomes explicitly relevant. Indeed, note that the relative disutilities of hours-per-worker and employment change at different rates.

Let λ_t be the Lagrange multiplier associated with the household planner's problem. Our focus throughout this essay has been on the labor market and as such, we focus on the first-order conditions for consumption, total work hours, and employment, which after rearranging imply that

$$\psi_t^E E_t / C_t^E = \lambda_t (1 + \tau_t^c), \quad (4.6)$$

$$\psi_t^N (P_t - E_t) / C_t^N = \lambda_t (1 + \tau_t^c), \quad (4.7)$$

$$\psi_t^E \gamma \left(\frac{H}{E_t} \right)^{1/\varepsilon} = \lambda_t (1 - \tau_t^l) W_t, \quad (4.8)$$

and

$$\begin{aligned} \psi_t^E \left(\ln \left(\frac{C_t^E}{E_t} \right) - \gamma \phi_t \right) - \psi_t^N \ln \left(\left(\frac{C_t^N}{(P_t - E_t)} \right) - D_t \right) \\ - \psi_t^E + \psi_t^N + \psi_t^E \frac{\gamma}{1 + \varepsilon} \left(\frac{H_t}{E_t} \right)^{(1+\varepsilon)/\varepsilon} = 0. \end{aligned} \quad (4.9)$$

Combining the consumption first-order conditions, it follows that within any period

$$C_t^N = \frac{\psi_t^N (P_t - E_t)}{\psi_t^E E_t} C_t^E.$$

Using this, and the fact that the sum of C_t^E and C_t^N must equal total household consumption, C_t , implies that

$$C_t^E = \zeta_t C_t, \quad (4.10)$$

where

$$\zeta_t = \psi_t^E E_t / (\psi_t^E E_t + \psi_t^N (P_t - E_t))$$

is the fraction of total household consumption that the planner assigns to employed individuals.

In order to close the model, we must once more consider the firm's problem. The firm chooses total work hours H and capital K to maximize

$$\Pi_t = Z_t K_t^\alpha (H_t)^{1-\alpha} - W_t H_t - R_t K_t. \quad (4.11)$$

The first-order conditions for hours per worker and employment both result in

$$\begin{aligned} (1 - \alpha) Z_t K_t^\alpha (H_t)^{-\alpha} &= W_t \\ \implies (1 - \alpha) Y_t &= W_t H_t. \end{aligned} \quad (4.12)$$

Combining (4.12) with (4.8) yields

$$\psi_t^E \gamma \left(\frac{H_t}{E_t} \right)^{(1+\varepsilon)/\varepsilon} = \lambda_t (1 - \tau_t^l) (1 - \alpha) \frac{Y_t}{E_t}.$$

After substituting for λ_t using (4.6), it follows that

$$\gamma \left(\frac{H_t}{E_t} \right)^{(1+\varepsilon)/\varepsilon} = (1 - \alpha) (1 - \tau_t) \frac{Y_t}{C_t^E},$$

where τ is the effective tax rate, defined as in Section 4.2.2. Substituting in for C_t^E , using (4.10), and rearranging yields

$$h_t = \left(\left(\frac{\psi_t^E E_t + \psi_t^N (P_t - E_t)}{\psi_t^E E_t} \right) \cdot \left((1 - \tau_t) (1 - \alpha) \frac{Y_t}{\gamma C_t^E} \right) \right)^{\varepsilon/(1+\varepsilon)}, \quad (4.13)$$

where $h_t = H_t/E_t$ is hours per worker.

The right-hand side of equation (4.13) defines the hours per worker that are theoretically consistent with the household's optimal choice of employment, taxes, and the output to consumption ratio. Note that as $\psi_t^E \rightarrow 1$,

$$(\psi_t^E E_t + \psi_t^N (P_t - E_t)) / \psi_t^E E_t \rightarrow 1.$$

Therefore, as $\psi_t^E \rightarrow 1$, equation (4.13) converges to the prediction of hours that the standard model enhanced with taxes yields when all variables are normalized by the level of employment. On the other hand, when $\psi_t^E \rightarrow \psi_t^N \rightarrow 0.5$, slight rearrangement of equation (4.13) implies that the prediction for hours converges to that of the standard model with all variables normalized by population.

We now derive an implicit expression for employment. Consider once more equation (4.9). Substituting in for C_t^E , C_t^N , and rearranging yields

$$\begin{aligned} \psi_t^N \left(\ln \left(\frac{\psi_t^N}{\psi_t^E E_t + \psi_t^N (P_t - E_t)} C_t \right) - D_t \right) \\ - \psi_t^E \left(\ln \left(\frac{\psi_t^E}{\psi_t^E E_t + \psi_t^N (P_t - E_t)} C_t \right) - \gamma \phi_t \right) \\ = \psi_t^E \frac{\gamma}{1 + \varepsilon} h_t^{(1+\varepsilon)/\varepsilon} + \psi_t^N - \psi_t^E. \end{aligned}$$

Further rearrangement implies that

$$F(E_t) = \psi_t^E \frac{\gamma}{1 + \varepsilon} h_t^{(1+\varepsilon)/\varepsilon} - (\psi_t^E - \psi_t^N) (1 - \ln C_t) - (\psi_t^E \gamma \phi_t - \psi_t^N D_t), \quad (4.14)$$

where

$$F(E_t) = \psi_t^N \ln (\psi_t^N / (\psi_t^E E_t + \psi_t^N (P_t - E_t))) - \psi_t^E \ln (\psi_t^E / (\psi_t^E E_t + \psi_t^N (P_t - E_t))).$$

The right-hand of equation (4.14) implicitly defines the level of employment that is theoretically consistent with the household's optimal choices regarding aggregate consumption C and hours per worker h , given the time- t employment fixed cost ϕ and non-employment disutility D . Note that as $\psi_t^E \rightarrow 1$, equation (4.14) converges to

$$\ln(E_t) = \frac{\gamma}{1 + \varepsilon} h_t^{(1+\varepsilon)/\varepsilon} - (1 - \ln(C_t)) - \gamma \phi_t,$$

which implies complementarity between employment and hours per worker and, as expected, that employment is decreasing in ϕ .

Returning to the household's planning weights, no-utility arbitrage implies that

within any period

$$\ln \left(\frac{C_t^E}{E_t} \right) - \gamma \phi_t - \gamma \frac{\varepsilon}{1 + \varepsilon} \left(\frac{H_t}{E_t} \right)^{(1+\varepsilon)/\varepsilon} = \ln \left(\frac{C_t^N}{P_t - E_t} \right) - D_t. \quad (4.15)$$

Subtract $\psi_t^E \gamma \varepsilon / (1 + \varepsilon) (H/E)^{(1+\varepsilon)/\varepsilon}$ from both sides of equation (4.9), use the fact from equation (4.15) that no-utility arbitrage implies that $U^E = U^N = U$, and rearrange to obtain

$$\psi_t^E = \left(2 - \frac{\gamma (\varepsilon - 1)}{(U - 1) (1 + \varepsilon)} h^{(1+\varepsilon)/\varepsilon} \right)^{-1}. \quad (4.16)$$

Note that given equation (4.10) and the fact that $\psi^N = 1 - \psi^E$, it follows that

$$U = U(\psi_t^E, \phi_t, D_t, \gamma, \cdot).$$

Hence, equation (4.16) implicitly determines the planning weights consistent with no-utility arbitrage. The extent to which these weights vary over time depends on the variation of U relative to h . Finally, recall from earlier in the analysis that as $\psi_t^E \rightarrow 0.5$, equation (4.13) implies that our model's prediction of hours converges to that of the standard neoclassical model's with all variables normalized by population. Given equation (4.16), it follows that the standard neoclassical model's predicted hours per population are special cases of our model, which occur either as $\gamma \rightarrow 0$, as $U \rightarrow \infty$, or both.

4.5 Applications of the Model

The planner's optimal weights cannot be operationalized given that they depend on the unknown parameter, γ , and the unknown employment and non-employment disutility variables, ϕ and D . However, the extent to which variation in ψ^E occurs can be gauged implicitly by use of equation (4.13). Indeed, if the planning weight ψ^E is approximately constant (and the model is an accurate representation of reality), then there should exist a constant value of ψ^E for which the model's predictions

of hours per worker will accurately replicate the empirical behavior of hours per worker. In this section, we explore the model's implications under the assumption of approximately constant optimal planning weights, and show that such an assumption is indeed consistent with the model yielding broadly accurate predictions regarding the trend behavior of hours per worker across countries.

4.5.1 Hours Per Worker

As a first approximation in gauging whether constant values of ψ^E are consistent with providing a relatively close approximation of the household planner's utility-weighting process, we generate model-predicted hours per worker using empirical measures for the variables on the right-hand side of equation (4.13). We apply data-generating methods analogous to those detailed in Section 4.2. Figures 4.10-4.13 show the actual and model-predicted hours per worker for Canada, France, Germany, and the US, the latter being generated using $\psi^E \in \{0.1, 0.5, 0.9\}$. Inspection of the relevant figures shows that for each country model-predicted hours per worker are a closer match to their empirical counterparts as ψ^E increases. In particular, the analysis suggests that the main importance of the planning weight ψ^E lies in conditioning the model's ability to match the trend behavior of hours per worker, which can indeed be closely approximated for given, and constant, ψ^E . Thus, the various model-generated hours per worker suggest that the variation in ψ^E necessary to reconcile the model with the data is relatively small, and therefore that assuming constant planning weights is a good approximation to the actual behavior of these weights.

We extend our analysis to 15 OECD countries for which the McDaniel (2007) tax series is available. For each country we impute an appropriate value for ψ^E by selecting the constant household planning weight consistent with minimizing the sum of squared percent deviations of model-generated hours per worker relative to empirical hours per worker. The second column of Table 4.1 shows the implied constant weight for each country. Except for Finland, the Netherlands, and Switzerland, in all cases $\psi^E > 0.5$, with the mean of ψ^E across all countries equal to 0.78, as shown in the last row of Table 4.1. The third column of Table 4.1 shows the correlation of model-

predicted hours per worker with their empirical counterparts. Model-predicted hours are now obtained for each country by using the relevant planning weights shown in the second column of Table 4.1 and the methodology described earlier in this section. In all cases the correlation is above 0.75 and averages 0.90 across countries, as shown in the last row of Table 4.1. This implies not only that constant planning weights are a good approximation to the actual behavior of these weights, but also that the model performs extremely well in generating hours-per-worker predictions that match the *trend* behavior of their empirical counterparts. In that sense, it is noteworthy that although the planning weights shown in Table 4.1 are consistent with maximizing the model's ability to match the relevant empirical data, it is not inherently the choice of weights, but rather, the theoretical soundness of the model which allows for the correlations to be so high. If the model were theoretically unsound, then even if the planning weights were chosen to maximize the model's predictive capabilities, relative to the actual data these predictions could still be quite poor, for instance, yielding very low correlations of model-generated hours per worker with their empirical counterparts. The model's predictive power is further confirmed graphically in Figure 14, which shows empirical and model-predicted hours per worker for the four countries we have centered on throughout this study, with model-predicted hours per worker generated using the relevant planning weights from Table 4.1.

4.5.2 Net Employment Disutility

Given equation (4.14), as was the case with the planning weights, the determination of employment depend directly on the unknown employment and non-employment disutility variables ϕ and D . Hence, the model's employment predictions cannot be directly backed out. However, the behavior of ϕ relative to D can, in principle, be imputed. To see this note that after rearranging equation (4.14) it follows that

$$\gamma\phi_t - \frac{\psi_t^N}{\psi_t^E}D_t = \frac{\gamma}{1+\varepsilon}h_t^{(1+\varepsilon)/\varepsilon} - \frac{F(E_t)}{\psi_t^E} - \frac{(\psi_t^E - \psi_t^N)(1 - \ln(C_t))}{\psi_t^E}. \quad (4.17)$$

Define $\gamma\phi_t - (\psi_t^N/\psi_t^E) D_t$ to be the household's *net employment disutility* - that is, net of weighted non-employment disutility. The evolution of net employment disutility can be imputed given knowledge of the right-hand side of equation (4.17). The parameter γ is unknown, making it impossible to impute the level value of net employment disutility. However, as long as the second and third terms on the right-hand side of this equation are always negative, then the actual value of γ is irrelevant in gauging the trend behavior and percent change of net employment disutility relative to some benchmark value. Indeed, under these circumstances the entirety of the right-hand side of equation (4.17) will always be positive, making the direction of change of net employment disutility obvious. However, if the second or third terms on the right-hand side of equation are positive, then the value of γ does become particularly important. In this case, it is still possible to gauge relative changes in the absolute value of net employment disutility, but it is not possible to infer whether these changes are occurring because net employment disutility is becoming more or less negative or positive.

For reasonable empirical values of total household consumption C , $1 - \ln(C)$ will be negative. If $\psi^E > 0.5$, then the last third term in equation (4.17) will be unambiguously positive. Moreover, it is straightforward to show that, given the definition of $F(E)$ from earlier in the essay, $F(E) = 0$ if $\psi^E = 0.5$, $F(E) < 0$ if $\psi^E > 0.5$, and $F(E) > 0$ if $\psi^E < 0.5$. Hence, if $\psi^E > 0.5$ both the second and third terms in equation (4.17) are positive, and if $\psi^E < 0.5$ they are both negative.¹⁴ Recall from the results shown in Table 4.1 that except for Finland, the Netherlands, and Switzerland, in all cases the imputed planning weight ψ^E was found to be greater than 0.5. Hence, for any given country and arbitrary γ , we generate a series for

¹⁴Note that $F(E) > 0$ if and only if

$$\begin{aligned} & \psi_t^N \ln \left(\frac{\psi_t^N}{\psi_t^E E_t + \psi_t^N (P_t - E_t)} \right) < \psi_t^E \ln \left(\frac{\psi_t^E}{\psi_t^E E_t + \psi_t^N (P_t - E_t)} \right) \\ \iff & \frac{\psi_t^E}{\psi_t^N} \left(\ln \psi_t^E - \ln (\psi_t^E E_t + \psi_t^N (P_t - E_t)) \right) > \left(\ln \psi_t^N - \ln (\psi_t^E E_t + \psi_t^N (P_t - E_t)) \right) \end{aligned}$$

As $\psi^E \rightarrow 1$ the right-hand side of the first inequality tends to $-\ln(E_t)$ and its left-hand side to $-\infty - \ln E_t$, so the inequality holds. However, as $\psi^E \rightarrow 0$ the right-hand side of this inequality tends to 0 and its left-hand side to $-\ln(P_t - E_t)$, so the inequality is reversed.

net employment disutility using equation (4.17) and the relevant constant planning weight from Table 4.1, and scale each resulting series by its value in 1960. Given the discussion of the relevance of the value of ψ^E for the sign of net employment disutility, it follows that we can make inferences regarding the direction of change of net employment disutility for all countries except Finland, the Netherlands, and Switzerland.

Since employment and hours per worker are in fact jointly determined by equations (4.13) and (4.14), although it is true that all else equal employment will be decreasing in net employment disutility, this need not occur when all else is not equal. Therefore, it should not be expected that within any country imputed changes in net employment disutility will necessarily be negatively correlated with changes in employment. Rather, imputed changes in net employment disutility should be interpreted as the changes in net employment disutility consistent with reconciling the trend behavior of employment within any given country, given changes in that country's consumption and work hours.

The second column of Table 4.2 shows average yearly percent changes in net employment disutility from 1960 through 2006. For each country, net employment disutility is generated as noted above, and scaled by its 1960 value. In all cases, except for Finland, the Netherlands, and Switzerland, the implied changes imply average yearly declines in net employment disutility across countries. With regards to Finland, the Netherlands, and Switzerland, recall from Table 4.1 that because the imputed weight ψ^E is less than 0.5 for these countries, it follows that only the absolute value of mean average yearly changes in $\gamma\phi_t - (\psi_t^N/\psi_t^E) D_t$ has meaning. Hence the relevant notation in Table 4.2 with regards to these countries, the interpretation of the average yearly changes being that in these countries the difference $\gamma\phi_t - (\psi_t^N/\psi_t^E) D_t$ is getting smaller either because net employment disutility is actually negative and increasing, or net unemployment disutility is positive and decreasing. In all other countries, of course, the appropriate interpretation is that net unemployment disutility is positive and decreasing.

The third column of Table 4.2 shows, for each country, average yearly changes

in the (empirical) employment-to-population ratio, also scaled by its 1960 value. As shown in the last row of Table 4.2, on average, across countries, an average yearly decrease in net employment disutility of 0.68% has been consistent with a mean year-to-year increase in E/P of 0.04%. Given the earlier discussion, this can be interpreted as follows: if net employment disutility had *not* decreased by as much on a percent-wise year-to-year basis, then, all else equal, on average E/P might have actually exhibited a substantial average yearly percent *decline*. In other words, the results suggest that, on average, consistent declines in net employment disutility are required to reconcile observed changes in employment given the trend behavior of other aggregate variables.

4.5.3 Comparative Statics

In the standard model, the impact of the effective tax rate is captured through only one margin of adjustment: aggregate hours worked. If the effective tax rate were increased, then aggregate hours would respond by decreasing. In our model, there are two margins that can explicitly adjust given changes in the effective tax rate: hours per worker and employment. To examine how changes in the effective tax rate and net employment disutility impact hours per worker and the number of workers, we totally differentiate the relevant equations presented above. We can then isolate four objects of interest, which are the elasticities of both hours per worker and employment with respect to the net-of-tax rate and net employment disutility, holding all other variables fixed. Given our earlier finding that constant planning weights are a good approximation to the actual behavior of these weights, the analysis in this and the following subsection proceeds under the assumption that the planning weights are indeed constant.

As shown in Section 4.9,

$$\begin{aligned}
& \left(1 + \frac{\psi^N \psi^E}{(\psi^E - \psi^N)^2 (E_t/P_t)} \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) d \ln (h_t) \\
&= \left(\frac{\psi^N}{(\psi^E - \psi^N)^2 (E_t/P_t)} \right) \Gamma_t d \log (\Gamma_t) \\
&+ \frac{\varepsilon}{1+\varepsilon} d \log (1 - \tau_t) + \frac{\varepsilon}{1+\varepsilon} d \ln (Y_t/C_t) \\
&- \left(\frac{\psi^N}{(\psi^E - \psi^N)^2 (E_t/P_t)} \right) d \ln (C_t/P_t), \quad (4.18)
\end{aligned}$$

where Γ_t denotes net employment disutility $\gamma \phi_t - (\psi_t^N / \psi_t^E) D_t$, and

$$\begin{aligned}
& \left(1 + \frac{\psi^N \psi^E}{(\psi^E - \psi^N)^2 (E_t/P_t)} \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) d \ln (E_t/P_t) = \\
& - \left(\frac{\psi^E E_t/P_t + \psi^N (1 - (E_t/P_t))}{(\psi^E - \psi^N)^2 (E_t/P_t)} \right) \Gamma_t d \log (\Gamma_t) \\
& + \left(\frac{\psi^E (E_t/P_t) + \psi^N (1 - (E_t/P_t))}{(\psi^E - \psi^N)^2 (E_t/P_t)} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) \frac{\varepsilon}{1+\varepsilon} (d \ln (1 - \tau_t) + d \ln (Y_t/C_t)) \\
& + \left(\frac{\psi^E E_t/P_t + \psi^N (1 - (E_t/P_t))}{(\psi^E - \psi^N)^2 (E_t/P_t)} \right) d \ln (C_t/P_t). \quad (4.19)
\end{aligned}$$

This means that hours per worker are increasing in net employment disutility, $\gamma \phi_t - (\psi_t^N / \psi_t^E) D_t$, while employment is decreasing in net employment disutility. This result intuitively illustrates the substitution between hours per worker and employment. Moreover, note that both hours per worker and employment are increasing in the net-of-tax rate, $(1 - \tau)$. Thus, an increase in the effective tax rate τ causes the household to decrease both hours per worker and employment, *ceteris paribus*.

From equations (4.18) and (4.19), we can obtain elasticities of hours per worker and of employment with respect to Γ holding all other variables fixed. For hours per worker,

$$\frac{d \ln h_t}{d \ln \Gamma_t} = \frac{\left(\frac{\psi^N}{(\psi^E - \psi^N)^2 (E_t/P_t)} \right) \Gamma_t}{1 + \frac{\psi^N \psi^E}{(\psi^E - \psi^N)^2 (E_t/P_t)} \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon}},$$

while for employment,

$$\frac{d \ln(E_t/P_t)}{d \ln \Gamma_t} = \frac{\left(\frac{\psi^E(E_t/P_t) + \psi^N(1-(E_t/P_t))}{(\psi^E - \psi^N)^2(E_t/P_t)} \right) \Gamma_t}{\left(1 + \left(\frac{\psi^N}{(\psi^E - \psi^N)^2(E_t/P_t)} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) \right)}.$$

Given a 1 percent increase in net employment disutility Γ , the percentage decrease in employment is greater than the percentage increase in hours per worker if and only if

$$\frac{\left(\frac{\psi^E(E_t/P_t) + \psi^N(1-(E_t/P_t))}{(\psi^E - \psi^N)^2(E_t/P_t)} \right) \Gamma_t}{\left(1 + \frac{\psi^N}{(\psi^E - \psi^N)^2(E_t/P_t)} \psi^E \frac{\gamma}{\varepsilon} \left(h_t^{(1+\varepsilon)/\varepsilon} \right) \right)} > \frac{\left(\frac{\psi^N}{(\psi^E - \psi^N)^2(E_t/P_t)} \right) \Gamma_t}{1 + \frac{\psi^N \psi^E}{(\psi^E - \psi^N)^2(E_t/P_t)} \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon}}$$

$$\iff \psi^E > \psi^N, \Gamma_t > 0 \text{ or } \psi^E < \psi^N, \Gamma_t < 0.$$

Earlier, we imputed for all countries except Finland, the Netherlands, and Switzerland, $\psi^E > \psi^N$, in which case we showed that it was unambiguously the case that $\Gamma_t > 0 \forall t$. Hence, in general, our findings suggest that a 1 percent increase in net employment disutility will be consistent with a greater percent decrease in employment than the associated percent increase in hours per worker, thus leading to an overall decline in hours per population.

Next, we consider the elasticities of hours per worker and of employment with respect to the net-of-tax rate $(1 - \tau)$, again holding all other variables fixed. The elasticity of hours per worker with respect to $(1 - \tau)$ is

$$\frac{d \ln h_t}{d \ln(1 - \tau_t)} = \frac{\frac{\varepsilon}{1+\varepsilon}}{1 + \frac{\psi^N \psi^E}{(\psi^E - \psi^N)^2(E_t/P_t)} \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon}},$$

and the elasticity of employment with respect to $(1 - \tau_t)$ is

$$\frac{d \ln(E_t/P_t)}{d \ln(1 - \tau_t)} = \frac{\left(\frac{\psi^E(E_t/P_t) + \psi^N(1-(E_t/P_t))}{(\psi^E - \psi^N)^2(E_t/P_t)} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) \frac{\varepsilon}{1+\varepsilon}}{\left(1 + \left(\frac{\psi^N}{(\psi^E - \psi^N)^2(E_t/P_t)} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) \right)}.$$

This means a 1 percent increase in $(1 - \tau)$ causes a greater percentage increase in employment than in hours per worker if and only if

$$\begin{aligned} \frac{\left(\frac{\psi^E(E_t/P_t) + \psi^N(1 - (E_t/P_t))}{(\psi^E - \psi^N)^2(E_t/P_t)} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right)^{\frac{\varepsilon}{1+\varepsilon}}}{\left(1 + \left(\frac{\psi^N}{(\psi^E - \psi^N)^2(E_t/P_t)} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) \right)} &> \frac{\frac{\varepsilon}{1+\varepsilon}}{1 + \frac{\psi^N \psi^E}{(\psi^E - \psi^N)^2(E_t/P_t)} \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon}} \\ \iff \left(\frac{\psi^E(E_t/P_t) + \psi^N(1 - (E_t/P_t))}{(\psi^E - \psi^N)^2(E_t/P_t)} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) &> 1. \end{aligned}$$

Thus, the relative percent-wise response of hours per worker and employment to a tax change is ultimately ambiguous. For example, consider a country with $\varepsilon = 4$, $E_t/P_t = 0.7$ and suppose that $h_t = 2,080$. Then,

$$\left(\frac{(\psi^E - \psi^N) 0.7 + \psi^N}{(\psi^E - \psi^N)^2 0.7} \right) \left(\psi^E \frac{\gamma}{4} 2,080^{1.25} \right).$$

As $\psi^E \rightarrow 1$

$$\left(\frac{(\psi^E - \psi^N) 0.7 + \psi^N}{(\psi^E - \psi^N)^2 0.7} \right) \left(\psi^E \frac{\gamma}{4} 2,080^{1.25} \right) \rightarrow \left(\frac{2,080^{1.25}}{4} \right) \gamma$$

As $\psi^E \rightarrow \psi^N$

$$\left(\frac{(\psi^E - \psi^N) 0.7 + \psi^N}{(\psi^E - \psi^N)^2 0.7} \right) \left(\psi^E \frac{\gamma}{4} 2,080^{1.25} \right) \rightarrow \infty$$

As $\psi^N \rightarrow 1$

$$\left(\frac{(\psi^E - \psi^N) 0.7 + \psi^N}{(\psi^E - \psi^N)^2 0.7} \right) \left(\psi^E \frac{\gamma}{4} 2,080^{1.25} \right) \rightarrow 0$$

Hence, this particular example suggests that for $\gamma > 1/(2,080^{1.25}/4)$, an increase in $(1 - \tau_t)$ will cause a greater percentage increase in participation than hours per worker as long as ψ^E is not particularly small. Of course, $1/(2,080^{1.25}/4) \simeq 0$. Hence, the previous is virtually valid for all $\gamma > 0$, which is indeed the assumption on this parameter. Moreover, from earlier, imputed values for ψ^E across countries are almost entirely greater than 0.5. Therefore, the results suggest that in general it should be

expected that any given change in the net of tax rate will cause greater percent changes in participation than hours per worker, everything else held constant.

4.5.4 Average vs. Marginal Tax Rates

So far, we have assumed that wage income is taxed at a flat rate, making the marginal tax rate on wage income equivalent to its respective average tax rate. However, the true tax system is often a graduated tax schedule on personal income, where the marginal tax rate increases with income. For simplicity, consider a graduated schedule for wage income with two marginal tax rates, τ^A and τ^B , where $\tau_B > \tau_A$. The lower rate τ^A , applies to the first WH_1 dollars of wage income earned and WH_1 is the income cut off where the marginal rate changes, while τ^B applies to the next $W(H - H_1)$ dollars. Also, to isolate responses stemming from differences in the average and marginal tax rates on wage income, we focus on the case where $\tau^c = \tau^i = \tau^k = 0$. The representative household's budget constraint is now written as,

$$(C_t^E + C_t^N) + I_t \leq W_t H_t - W_t H_1 \tau^A + R_t K_t + T_t$$

for $H \leq H_1$, and for $H > H_1$

$$(C_t^E + C_t^N) + I_t \leq W_t H_t - W_t H_1 \tau^A - W_t (H_t - H_1) \tau^B + R_t K_t + T_t.$$

Thus, a graduated tax on wage income induces a kink in the household's budget constraint. This means the household's utility maximization problem is non-differentiable at WH_1 . Nonetheless, the first-order conditions will remain valid for each segment of the household's problem. We focus on the maximization that occurs on the segment where $H > H_1$, as it highlights the household's problem when the marginal tax rate is not equivalent to the average tax rate.

Given that the average and marginal tax rates are no longer equivalent, we can analyze the impact of the average tax rate on h and E/P holding the marginal tax

rate fixed. The average tax rate $\bar{\tau}$ is defined as

$$\bar{\tau}_t = \frac{W_t H_1 \tau_t^A + W_t (H_t - H_1) \tau_t^B}{W_t H_t},$$

where τ^B is the marginal tax rate for $H > H_1$. If there is an increase in τ^A with no subsequent change in τ^B , then for $H > H_1$, this represents an increase in the average tax rate with no change to the marginal tax rate. The household budget constraint can be rewritten as

$$C_t^E + C_t^N + I_t \leq W_t H_t (1 - \tau^B) + W_t H_1 (\tau^B - \tau^A) + R_t K_t + T_t.$$

Given that $W H_1$ is constant, $W H_1 (\tau^B - \tau^A)$ is also constant. This means that changes to the average tax rate that do not impact the marginal tax rate will have only a pure income effect. For instance, a decrease to τ^A will allow the household to increase total consumption for a given amount of investment. Thus, the impact of the average tax rate on h , and E/P is realized through changes in consumption. Equations (4.18) and (4.19) show that changes to consumption impact both changes to h and E/P where an increase in consumption leads to a decrease in h and an increase E/P . Given earlier derivations, a 1% increase in consumption will be consistent with a greater percentage-wise increase participation than percent decrease in hours per worker if and only if

$$\frac{\left(\frac{\psi^E E_t + \psi^N (1 - E_t)}{(\psi^E - \psi^N) E_t} \right)}{1 + \left(\frac{\psi^N}{(\psi^E - \psi^N)^2 E_t} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right)} > \frac{\left(\frac{\psi^N}{(\psi^E - \psi^N) E_t} \right)}{\left(1 + \frac{\psi^N \psi^E}{(\psi^E - \psi^N)^2 E_t} \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right)}$$

$$\iff \psi^E > \psi^N.$$

Thus, a decrease in τ^A , which in turn decreased $\bar{\tau}$, will lead to an overall increase in H/P even though h falls, if the household planner places greater weight on the utility of employed individuals than on that of non-employed individuals.

To analyze the impact of the marginal tax rate, suppose τ^B decreased, while

τ^A increased, keeping the average tax rate constant. This assumption allows us to abstract from the additional income effect that would occur if the average tax rate had also changed. Conditional on $H > H_1$, the planner seeks to maximize the household's utility subject to the budget constraint with varying marginal rates. The new first order condition for hours is,

$$H_t : \psi^E \gamma \left(\frac{H}{E_t} \right)^{1/\varepsilon} = \lambda_t (1 - \tau_t^B) W_t.$$

After taking first-order conditions for consumption and setting labor supply equivalent to labor demand, we obtain a new hours per worker equation in terms of only the marginal tax rate,

$$h_t = \left(\frac{\psi^E E_t + \psi^N (P_t - E_t)}{\psi^E E_t} \right) \left((1 - \tau^B) \frac{(1 - \alpha) Y_t}{\gamma C_t} \right).$$

The equations used in the comparative statics above can be used to study the impact of the marginal tax rate on h and E/P , though replacing $(1 - \tau)$ with $(1 - \tau^B)$, since we have assumed only wage taxes. The comparative statics illustrated that a decrease in τ^B (and thus an increase in $(1 - \tau^B)$) will increase both h and E/P .

Overall we find that a decrease in the average tax rate, holding the marginal tax rate constant, increases the overall H/P ratio by increasing E/P more than the fall in h when $\psi^E > \psi^N$. A decrease in the marginal tax rate, holding the average tax rate fixed, will increase both h and E/P . Although the relative magnitudes of their responses is ambiguous, the overall effect will be a subsequent increase to H/P . The sensitivity of h and E/P with respect to the average and marginal tax rates depends on ψ^E and ψ^N because the household planning weights influence how much additional disutility the household endures by increasing employment relative to increasing hours per worker.

4.6 Related Literature

Conceptually, the labor wedge discussed in this essay stems from the analysis in Prescott (2004). Prescott seeks to explain differences in hours per population between the US and a set of European countries. Using aggregate data, Prescott derives effective tax rates for a group of OECD countries between 1970 and 1974, and also between 1993 and 1996. He then uses the first-order conditions derived from a standard neoclassical model with taxes to generate data on hours per population. Prescott argues that over his reference periods, cross-country differences in effective tax rates can account for a considerable fraction of the level differences in hours per population. However, he notes that the model predicts that in the US, H/P should have gone down, when in reality this ratio has gone up. He suggests that this failure of the model owes to the fact that the US experienced an increase in married women's labor force participation along with a flattening of the tax schedule during the 1980's. Thus, the marginal tax rate for large changes in income when moving to a two-earner household was significantly higher in the earlier period compared to the later period even though the calculated marginal tax rate used for predicting hours remained the same.

Alesina et al. (2005) argue that the assumptions underpinning the model in Prescott (2004) actually drive the results. In particular, they argue that the choice of log utility function as well as an implied labor supply elasticity with respect to the tax rate roughly equal to 3 are what allow for taxes to explain most of the differences in labor hours across countries. Moreover, the authors suggest that an omitted variable in Prescott's analysis is cross-country differences in the degree of unionization. Alesina et al. posit that in the absence of changes in market regulations imposed by unions, changes in effective tax rates would not have affected hours worked to the extent implied by Prescott (2004).

More recently, Ohanian et al. (2008) extend the analysis in Prescott (2004) by studying a larger set of countries over a longer time frame and using a slightly different functional form for utility. The analysis uses annual effective tax rates derived in McDaniel (2007). This allows for a broad and detailed documentation of the long-

run behavior of H/P , as well as extensive testing of the standard macroeconomic model's explanatory power. Ohanian et al. agree with Prescott in that augmenting the neoclassical model with taxes can broadly account for changes within country changes in H/P over time. This conclusion is robust to controlling for institutional differences. However, Ohanian et al. note that for Canada and the US, model-generated hours per population fail to match their empirical counterparts.

Shimer (2009) reviews literature on the labor wedge. A central point of Shimer's paper involves shifting attention from the trend to the cyclical behavior of the labor wedge. In particular, Shimer notes the strong procyclicality of the US labor wedge, and discusses possible explanations for this. As argued in Prescott (2004) and Ohanian et al. (2008), part of the labor wedge can be accounted for by taxes. Thus, Shimer notes that one potential explanation for the cyclical behavior of the labor wedge could be cyclical fluctuations in taxes. However, Shimer argues that this explanation is unreasonable, since it is inconsistent with actual tax policy, which on average amounts to taxes being countercyclical. Instead, he suggests that a more plausible explanation for these cyclical fluctuations may involve noncompetitive aspects of the labor market.

4.7 Conclusions

This essay re-examines the ability of the standard neoclassical macroeconomic model augmented with taxes to match the trend behavior of hours per population across countries. Past work has argued that by including taxes in the standard neoclassical model, much of the long-run changes in hours per population both within and across countries can be accounted for. However, two countries stand out as exceptions: Canada and the US. We delve deeper into this puzzle and highlight that unlike most other countries, over the last several decades Canada and the US both experienced large changes in their employment-to-population ratios relative to changes in hours per worker. We find compelling evidence that the failure of the standard neoclassical model with taxes to predict hours per population in Canada and the US stems from an inherent inability of the standard model to account for changes in the employment-

to-population ratio.

After identifying the shortcomings of the standard model, we develop a model that explicitly incorporates employment as a choice variable. In our model, a household planner maximizes household utility, which is a weighted sum of all employed and non-employed individuals' utility. Our model incorporates a time-varying fixed cost associated with employment, as well as a general non-employment disutility variable.

We define net employment disutility to be a weighted difference between the fixed cost of employment and non-employment disutility. Net employment disutility enters linearly into the household's utility maximization problem. The household's optimal decision with regards to aggregate hours per population then involves a trade-off between (linear) net employment disutility costs and (convex) costs associated with work hours. This leads hours and employment to be substitutes with regards to net employment disutility. In particular, an increase in net employment disutility induces a decrease in employment and an increase in hours per worker.

We find that as the weight that the household places on the utility of employed individuals increases, the relative magnitude of changes in employment given a change in net employment disutility also increases. Thus, when the household planner weights employed individuals' utility greater than that of non-employed individuals, given changes in net employment disutility the impact on hours per population will be dominated by the response in employment. In other words, for a large enough household planning weight on employed individuals, an increase in net employment disutility will lead to a decrease in hours per population driven by a decrease in employment. However, this decrease in hours per population is mitigated by the associated increase in hours per worker. Net employment disutility can include any number of costs that are relevant for an individual's decision to work. Past studies have found that the manner in which the government uses its revenue can have an important impact on the incentive to work (Ragan (2006) and Rogerson (2007)). For example, if the government were to subsidize child care, as is the case in certain Scandinavian countries, then there would be an increased incentive to substitute home production with market work (Shimer (2009)). This lower cost of entering the

work force can be interpreted as a decrease in net employment disutility, as it makes working relatively more attractive.

We also analyze the effects of changes in the net-of-tax rate and find that the relative magnitude between the hours per worker response and the employment response is ambiguous. However, both respond in the same direction and therefore have an unambiguous impact on hours per population, which is that hours per population move in the same direction as the net-of-tax rate. Additionally, we highlight the model's relevance as a tool for analyzing policy by exploring the differential impact that changes in average tax rates versus marginal taxes can have on hours per population. We find that a decrease in the average tax rate is consistent with a decrease in hours per worker and an increase in the employment-to-population ratio. Similarly to a change in net employment disutility, the dominating response will depend on how the household weights the utility of employed individuals relative to that of non-employed individuals. Thus, a decrease in the average tax rate could potentially be associated with either an increase or decrease in hours per population. This finding is particularly interesting in the case of the US, where between 1960 and 2006 - which is our sample period - there were large decreases in the average tax rate at the same time that there were increases in hours per population. When viewing each component of hours per population separately, as in Figure 4.6, hours per worker and employment per population in the US behave as the model predicts, with hours per worker falling while employment per population rises.

Finally, using our model and available data, we impute the weights that the household places on the utility of employed and non-employed individuals and find that the model yields accurate predictions for the trend behavior of hours per worker across countries when applying approximately constant planning weights. We find that, on average, the model best matches the data on hours per worker when the household's weight on the utility of employed individuals is between 0.5 and 1. In terms of net employment disutility, our analysis suggests that over the time period under consideration, trends in employment across countries require an average yearly decline in this variable of approximately 0.7% in order to be rationalized given the

trend behavior of other macroeconomic variables.

4.8 Data Sources and Summary Statistics

All data is at yearly frequency. Data on hours per worker H/E and employment E are from The Groningen Growth and Development Centre. This data is used to back out total hours H . Data on working-age population P is taken from the Source OECD Database, and data on consumption C and output Y are from the Penn World tables. The McDaniel (2007) tax data is available at her website, and is derived using similar methods as in Mendoza et al. (1994). All of the data we use is summarized in Tables (4.0A) through (4.0G) over the period 1960-2006 for the 15 OECD for which the McDaniel (2007) tax series is available.

Table 4.0.A: H/P summary statistics 1960-2006

Country	Mean	St. Dev.	Min	Max
Australia	1226	36.57	1128	1283
Austria	1253	181.91	1019	1605
Belgium	1070	149.01	916	1389
Canada	1206	46.176	1125	1305
Finland	1339	178.01	1051	1675
France	1165	161.80	974	1416
Germany	1131	187.35	910	1489
Italy	1034	133.82	908	1420
Japan	1474	82.08	1327	1648
Netherlands	1077	108.94	944	1291
Spain	1076	169.83	841	1296
Sweden	1250	57.10	1177	1395
Switzerland	1409	105.51	1287	1616
UK	1263	134.42	1111	1517
US	1266	57.38	1159	1375

Table 4.0.B: H/E summary statistics 1960-2006

Country	Mean	St. Dev.	Min	Max
Australia	1818	44.773	1751	1945
Austria	1762	178.25	1495	2073
Belgium	1802	194.15	1603	2289
Canada	1858	82.562	1760	2040
Finland	1868	115.31	1719	2074
France	1857	227.58	1532	2227
Germany	1736	224.57	1438	2163
Italy	1783	186.17	1590	2234
Japan	2054	142.50	1786	2224
Netherlands	1680	228.67	1399	2135
Spain	1954	134.85	1737	2137
Sweden	1648	116.76	1508	1900
Switzerland	1719	127.74	1551	1936
UK	1810	162.42	1614	2134
US	1854	62.76	1782	1981

Table 4.0.C: E/P summary statistics 1960-2006

Country	Mean	St. Dev.	Min	Max
Australia	0.67	0.02	0.63	0.71
Austria	0.70	0.03	0.66	0.77
Belgium	0.59	0.02	0.54	0.62
Canada	0.65	0.04	0.57	0.72
Finland	0.71	0.05	0.59	0.81
France	0.62	0.02	0.59	0.65
Germany	0.64	0.03	0.60	0.69
Italy	0.57	0.02	0.55	0.63
Japan	0.71	0.02	0.68	0.75
Netherlands	0.64	0.05	0.59	0.76
Spain	0.54	0.06	0.45	0.61
Sweden	0.76	0.04	0.71	0.83
Switzerland	0.81	0.03	0.76	0.87
UK	0.69	0.02	0.64	0.72
US	0.68	0.04	0.62	0.74

Table 4.0.D: Y/P summary statistics 1960-2006

Country	Mean	St. Dev.	Min	Max
Australia	32269	8148	19429	49697
Austria	32351	9816	14925	48353
Belgium	30970	9202	14802	47030
Canada	33540	7665	20811	49010
Finland	26785	8110	13961	42984
France	31117	8342	15438	44959
Germany	33398	8391	17778	47212
Italy	27679	8694	12497	41902
Japan	28541	10905	8613	43757
Netherlands	32723	8029	19486	47891
Spain	24291	7710	9235	38675
Sweden	31442	7177	18253	45455
Switzerland	42195	6839	27555	51676
UK	28964	8086	17305	45697
US	41187	10705	24672	61156

Table 4.0.E: C/P summary statistics 1960–2006

Country	Mean	St. Dev.	Min	Max
Australia	17881	3952	11428	26628
Austria	18684	5565	8541	26976
Belgium	16869	4804	8737	24531
Canada	18906	3587	12848	26088
Finland	13129	3897	6790	20615
France	17589	4733	8651	25545
Germany	19695	5580	9595	28379
Italy	15476	5167	6156	23536
Japan	14585	5549	4904	22950
Netherlands	17378	3956	9832	24429
Spain	13798	4054	5672	21159
Sweden	16990	3216	10740	22841
Switzerland	23828	3764	15734	29051
UK	17851	5639	10592	29735
US	27739	7585	16325	42632

Table 4.0.F: C/Y summary statistics 1960–2006

Country	Mean	St. Dev.	Min	Max
Australia	1.79	0.06	1.68	1.91
Austria	1.73	0.03	1.63	1.79
Belgium	1.83	0.05	1.69	1.91
Canada	1.76	0.07	1.58	1.91
Finland	2.03	0.07	1.87	2.17
France	1.77	0.02	1.71	1.82
Germany	1.71	0.07	1.63	1.85
Italy	1.80	0.08	1.70	2.04
Japan	1.94	0.07	1.75	2.07
Netherlands	1.87	0.07	1.72	1.98
Spain	1.74	0.05	1.62	1.84
Sweden	1.83	0.08	1.69	1.99
Switzerland	1.77	0.04	1.65	1.85
UK	1.63	0.06	1.52	1.75
US	1.49	0.03	1.42	1.56

Table 4.0.G: $(1 - \tau)$ summary statistics 1960–2006

Country	Mean	St. Dev.	Min	Max
Australia	0.77	0.04	0.73	0.84
Austria	0.54	0.06	0.47	0.67
Belgium	0.53	0.08	0.44	0.69
Canada	0.70	0.05	0.63	0.80
Finland	0.55	0.09	0.41	0.73
France	0.52	0.06	0.44	0.62
Germany	0.54	0.05	0.48	0.64
Italy	0.58	0.07	0.49	0.69
Japan	0.75	0.06	0.67	0.82
Netherlands	0.51	0.06	0.43	0.68
Spain	0.66	0.08	0.51	0.76
Sweden	0.48	0.08	0.37	0.68
Switzerland	0.79	0.04	0.73	0.86
UK	0.63	0.03	0.59	0.73
US	0.74	0.02	0.71	0.79

4.9 Derivations

The following derivations are based on the assumption of constant planning weights ψ^E and ψ^N , hence the lack of time subscript on these weights. Normalize all variables by P_t , and let $e_t = E_t/P_t$, and $c_t = C_t/P_t$. Let Γ_t denote net unemployment disutility $\psi_t^E \gamma \phi_t - \psi_t^N D_t$.

First, consider

$$\begin{aligned}
F(e_t) &= \psi^E \frac{\gamma}{1+\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} - (\psi^E - \psi^N) (1 - \ln(c_t)) - \Gamma_t \\
\implies dF(e_t) &= \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{1/\varepsilon} \right) dh_t + (\psi^E - \psi^N) d \ln(c_t) - d\Gamma_t \\
&= \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{1/\varepsilon} \right) h_t \frac{dh_t}{h_t} + (\psi^E - \psi^N) d \ln(c_t) - d\Gamma_t, \\
&= \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) d \log h_t + (\psi^E - \psi^N) d \ln(c_t) - \Gamma_t d \log(\Gamma_t),
\end{aligned}$$

Now, consider

$$\begin{aligned}
F(e_t) &= \psi^N \log \left(\frac{\psi^N}{\psi^E e_t + \psi^N (1 - e_t)} \right) - \psi^E \log \left(\frac{\psi^E}{\psi^E e_t + \psi^N (1 - e_t)} \right) \\
&= \psi^N \log(\psi^N) - \psi^N \log(\psi^E e_t + \psi^N (1 - e_t)) \\
&\quad - \psi^E \log(\psi^E) + \psi^E \log(\psi^E e_t + \psi^N (1 - e_t)) \\
\implies dF(e_t) &= \frac{-\psi^N (\psi^E - \psi^N) de_t}{\psi^E e_t + \psi^N (1 - e_t)} + \frac{\psi^E (\psi^E - \psi^N) de_t}{\psi^E e_t + \psi^N (1 - e_t)} \\
&= \left(\frac{(\psi^E - \psi^N)^2}{\psi^E e_t + \psi^N (1 - e_t)} \right) de_t, \\
&= \left(\frac{(\psi^E - \psi^N)^2 e_t}{\psi^E e_t + \psi^N (1 - e_t)} \right) d \log(e_t).
\end{aligned}$$

Combining the previous:

$$\begin{aligned}
&\left(\frac{(\psi^E - \psi^N)^2 e_t}{\psi^E e_t + \psi^N (1 - e_t)} \right) d \log(e_t) = \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) d \log h_t \\
&\quad + (\psi^E - \psi^N) d \ln(c_t) - \Gamma_t d \log(\Gamma_t) \\
\implies d \log(e_t) &= \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) d \log h_t \\
&\quad + \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) (\psi^E - \psi^N) d \ln(c_t) \\
&\quad - \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) \Gamma_t d \log(\Gamma_t),
\end{aligned}$$

where the implication follows from multiplying both sides by $\left(\frac{(\psi^E - \psi^N)^2 e_t}{\psi^E e_t + \psi^N (1 - e_t)}\right)$.

Now, consider

$$\begin{aligned}
h_t &= \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{\psi^E e_t}\right) \left(\frac{(1 - \tau_t)(1 - \alpha) Y_t}{\gamma C_t}\right)^{\varepsilon/(1+\varepsilon)} \\
\Rightarrow \log(h_t) &= \log(\psi^E e_t + \psi^N (1 - e_t)) - \log(\psi^E) - \log(e_t) \\
&\quad + \frac{\varepsilon}{1 + \varepsilon} \log(1 - \tau_t) + \frac{\varepsilon}{1 + \varepsilon} \log(1 - \alpha) \\
&\quad - \frac{\varepsilon}{1 + \varepsilon} \log(\gamma) + \frac{\varepsilon}{1 + \varepsilon} \log(Y_t/C_t) \\
\Rightarrow d \log(h_t) &= \left(\frac{\psi^E - \psi^N}{\psi^E e_t + \psi^N (1 - e_t)}\right) de_t \\
&\quad - d \log(e_t) + \frac{\varepsilon}{1 + \varepsilon} d \log(1 - \tau_t) + \frac{\varepsilon}{1 + \varepsilon} d \log(Y_t/C_t) \\
&= \left(\frac{(\psi^E - \psi^N) e_t}{\psi^E e_t + \psi^N (1 - e_t)}\right) d \log(e_t) \\
&\quad - d \log(e_t) + \frac{\varepsilon}{1 + \varepsilon} d \log(1 - \tau_t) + \frac{\varepsilon}{1 + \varepsilon} d \log(Y_t/C_t) \\
&= \left(\frac{(\psi^E - \psi^N) e_t}{\psi^E e_t + \psi^N (1 - e_t)} - 1\right) d \log(e_t) \\
&\quad - d \log(e_t) + \frac{\varepsilon}{1 + \varepsilon} d \log(1 - \tau_t) + \frac{\varepsilon}{1 + \varepsilon} d \log(Y_t/C_t) \\
&= \left(\frac{-\psi^N}{\psi^E e_t + \psi^N (1 - e_t)}\right) d \log(e_t) \\
&\quad + \frac{\varepsilon}{1 + \varepsilon} d \log(1 - \tau_t) + \frac{\varepsilon}{1 + \varepsilon} d \log(Y_t/C_t)
\end{aligned}$$

Therefore, the relevant two equations in the two "unknowns" $d \log(e_t)$ and $d \log(h_t)$

are

$$\begin{aligned}
d \log (e_t) &= \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) d \log h_t \\
&+ \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) (\psi^E - \psi^N) d \ln (c_t) \\
&- \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) \Gamma_t d \log (\Gamma_t)
\end{aligned} \tag{4.20}$$

and

$$\begin{aligned}
d \log (h_t) &= \left(\frac{-\psi^N}{\psi^E e_t + \psi^N (1 - e_t)} \right) d \log (e_t) \\
&+ \frac{\varepsilon}{1 + \varepsilon} d \log (1 - \tau_t) + \frac{\varepsilon}{1 + \varepsilon} d \log (Y_t / C_t).
\end{aligned} \tag{4.21}$$

Insert (4.21) in (4.20):

$$\begin{aligned}
d \log (e_t) &= \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) (\psi^E - \psi^N) d \ln (c_t) \\
&- \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) \Gamma_t d \log (\Gamma_t) \\
&- \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) \left(\frac{\psi^N}{\psi^E e_t + \psi^N (1 - e_t)} \right) d \log (e_t) \\
&+ \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) \frac{\varepsilon}{1 + \varepsilon} (d \log (1 - \tau_t) + d \log (Y_t / C_t)).
\end{aligned}$$

After rearranging and simplifying this yields

$$\begin{aligned}
\left(1 + \left(\frac{\psi^N}{(\psi^E - \psi^N)^2 e_t} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) \right) d \log (e_t) &= \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N) e_t} \right) d \ln (c_t) \\
&- \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) \Gamma_t d \log (\Gamma_t) \\
&+ \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) \frac{\varepsilon}{1 + \varepsilon} (d \log (1 - \tau_t) + d \log (Y_t / C_t)).
\end{aligned}$$

Now, insert (4.20) in (4.21):

$$\begin{aligned}
d \log (h_t) = & - \left(\frac{\psi^N}{\psi^E e_t + \psi^N (1 - e_t)} \right) \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) \left(\psi^E \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) d \log h_t \\
& - \left(\frac{\psi^N}{\psi^E e_t + \psi^N (1 - e_t)} \right) \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) (\psi^E - \psi^N) d \ln (c_t) \\
& + \left(\frac{\psi^N}{\psi^E e_t + \psi^N (1 - e_t)} \right) \left(\frac{\psi^E e_t + \psi^N (1 - e_t)}{(\psi^E - \psi^N)^2 e_t} \right) \Gamma_t d \log (\Gamma_t) \\
& + \frac{\varepsilon}{1 + \varepsilon} d \log (1 - \tau_t) + \frac{\varepsilon}{1 + \varepsilon} d \log (Y_t / C_t) .
\end{aligned}$$

After simplifying and rearranging, this yields

$$\begin{aligned}
\left(1 + \frac{\psi^N \psi^E}{(\psi^E - \psi^N)^2 e_t} \frac{\gamma}{\varepsilon} h_t^{(1+\varepsilon)/\varepsilon} \right) d \log (h_t) = & - \left(\frac{\psi^N}{(\psi^E - \psi^N) e_t} \right) d \ln (c_t) \\
& + \left(\frac{\psi^N}{(\psi^E - \psi^N)^2 e_t} \right) \Gamma_t d \log (\Gamma_t) \\
& + \frac{\varepsilon}{1 + \varepsilon} d \log (1 - \tau_t) + \frac{\varepsilon}{1 + \varepsilon} d \log (Y_t / C_t) .
\end{aligned}$$

4.10 Figures

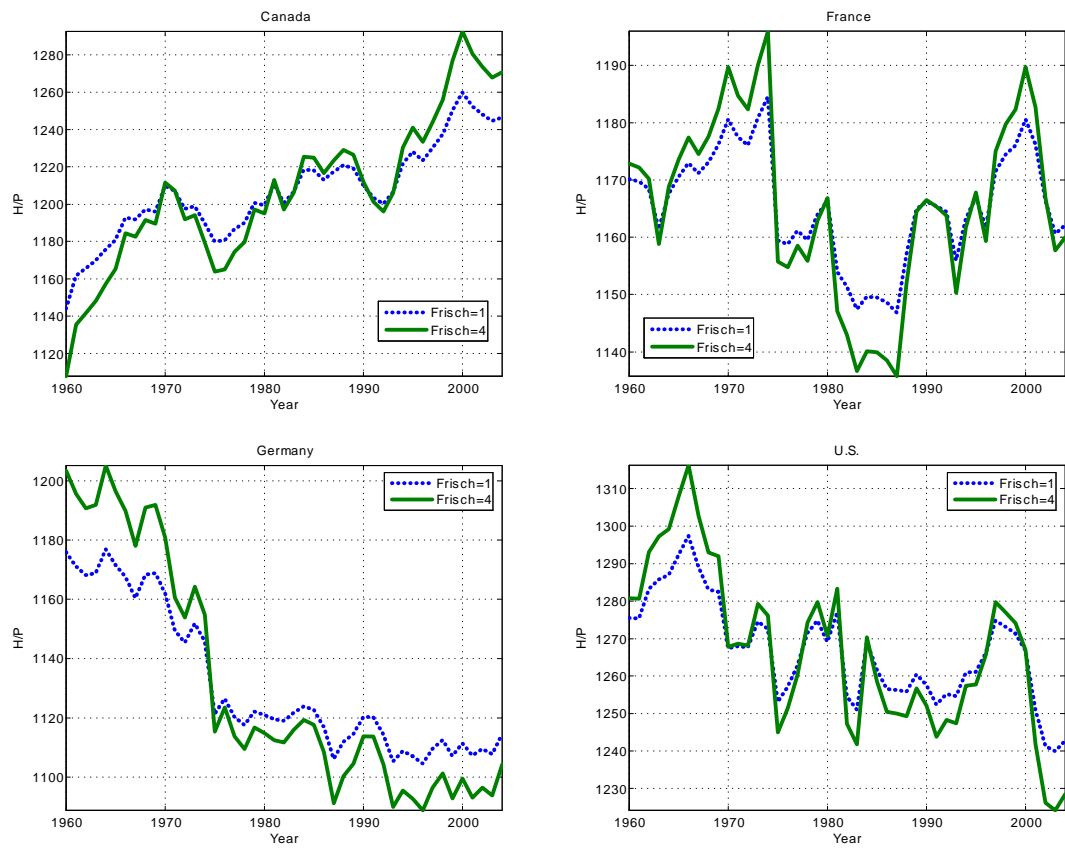


Figure 4.1

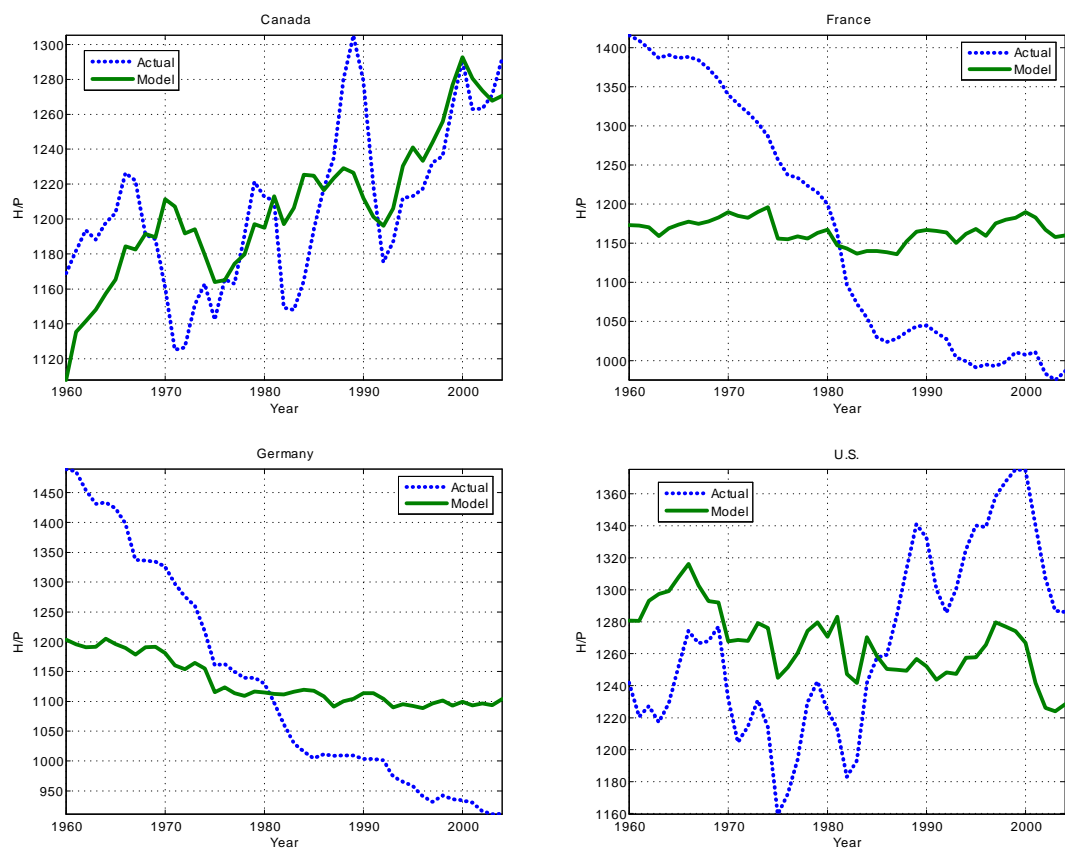


Figure 4.2

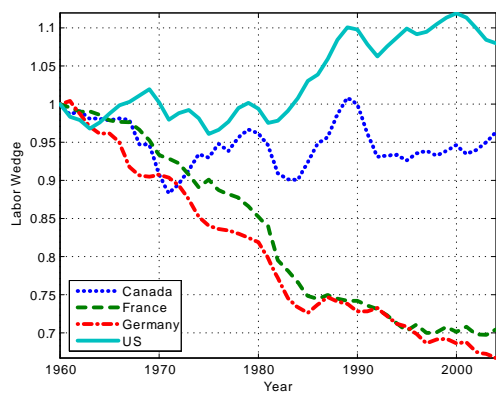


Figure 4.3

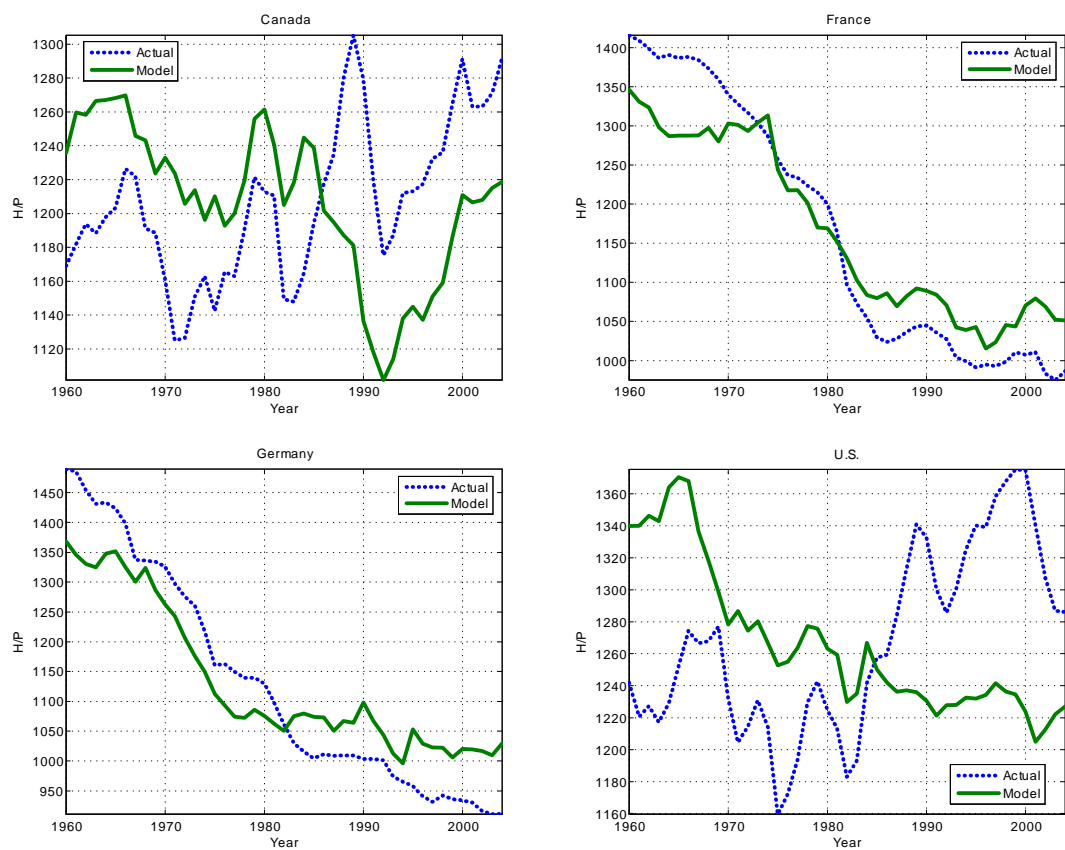


Figure 4.4

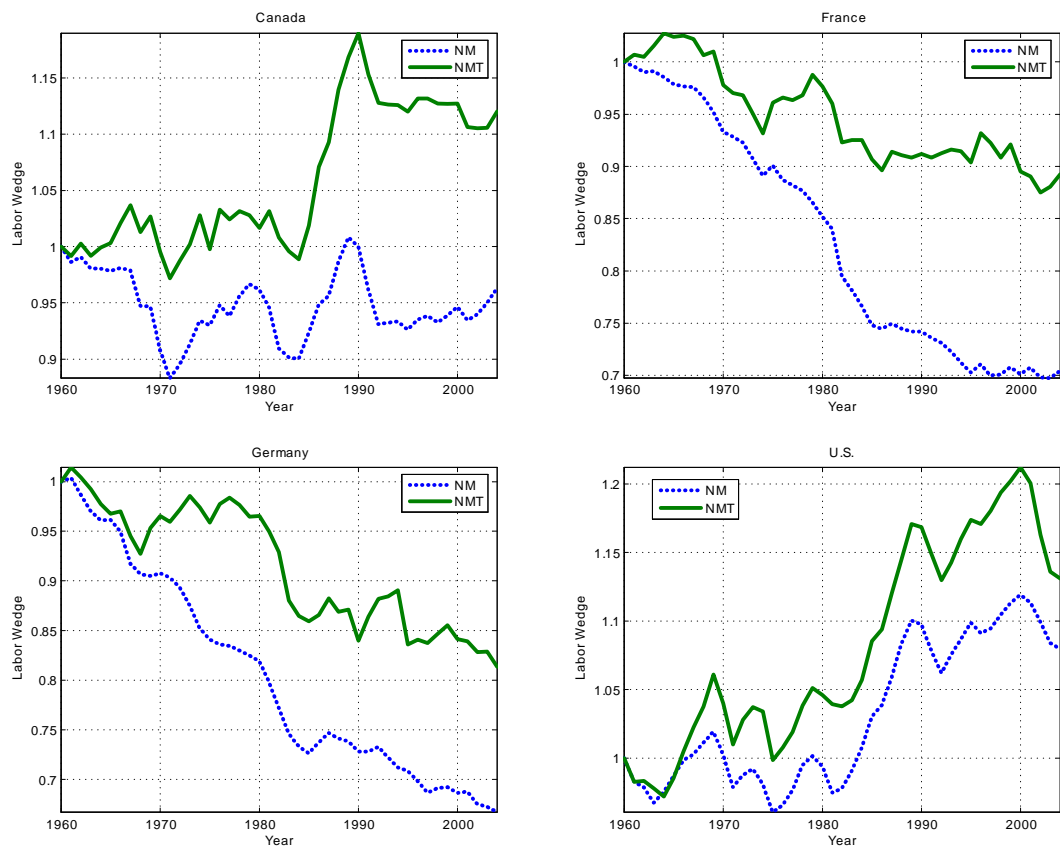


Figure 4.5

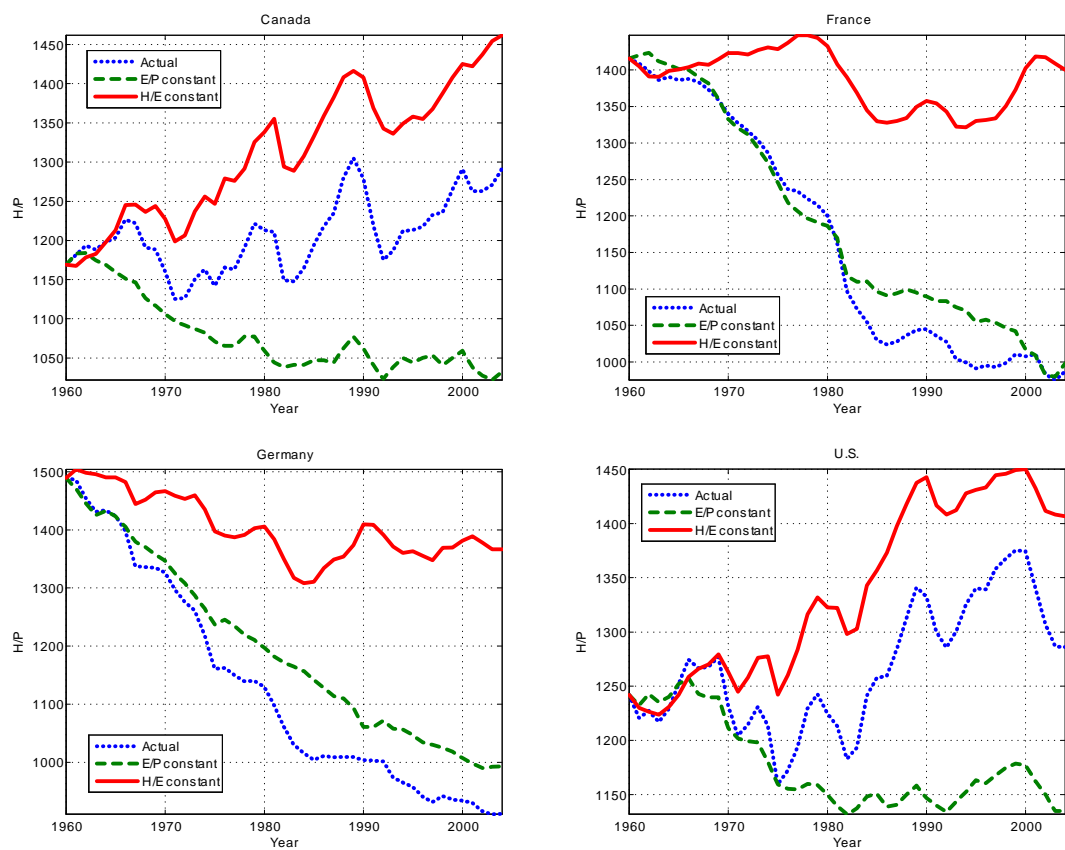


Figure 4.6

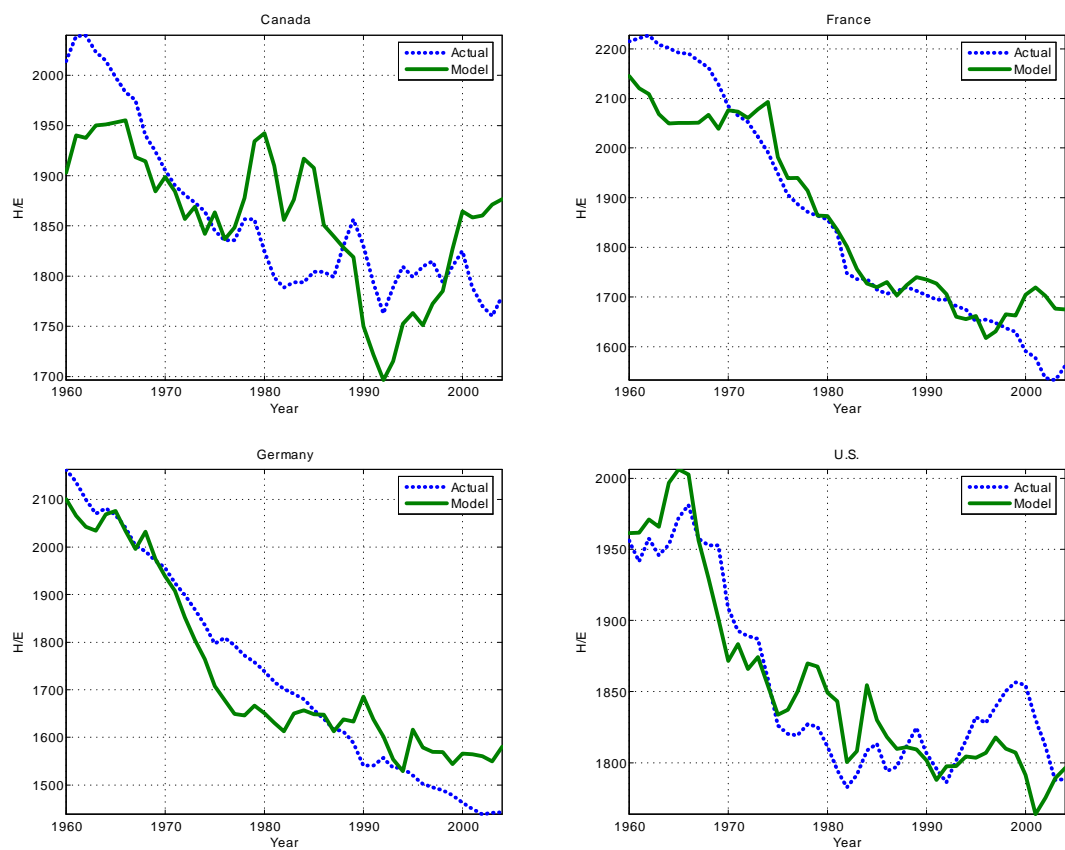


Figure 4.7

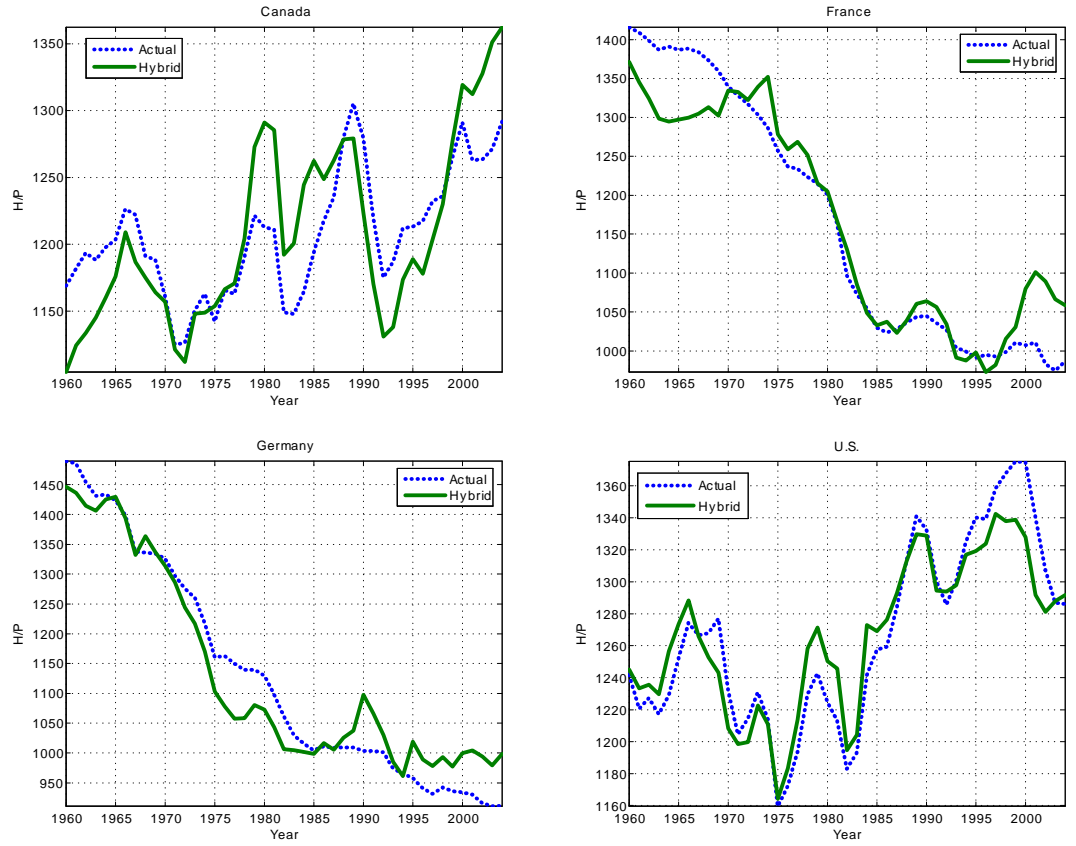


Figure 4.8

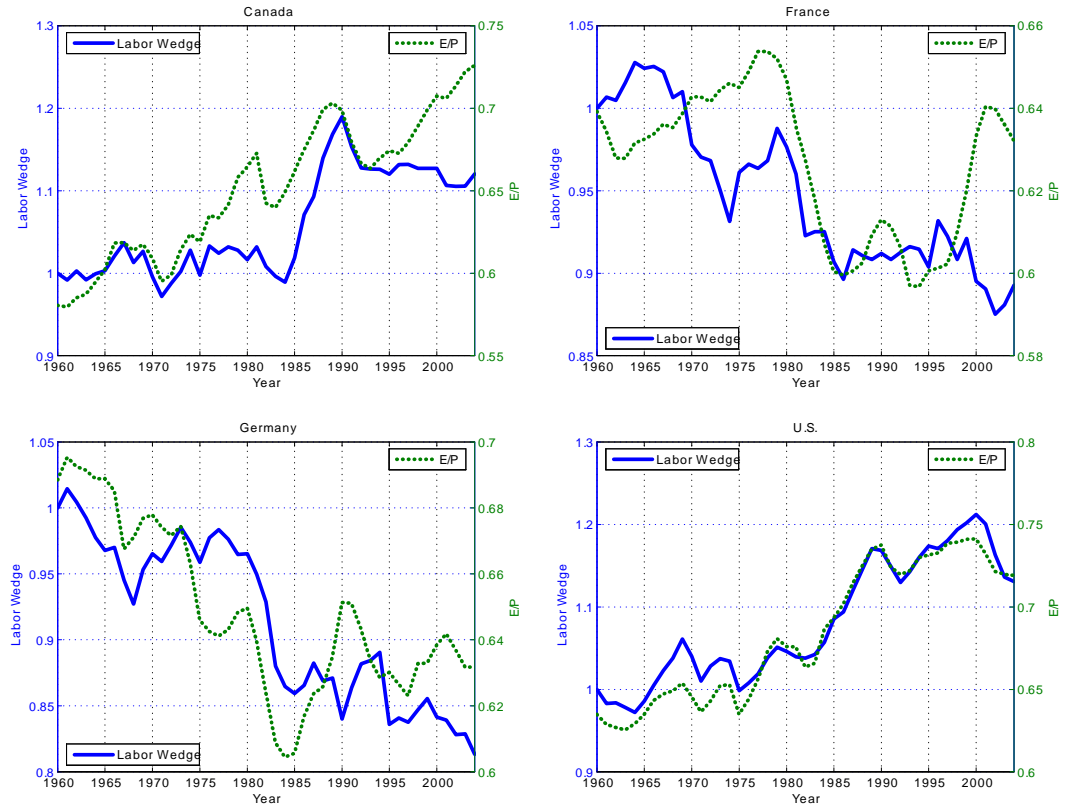


Figure 4.9

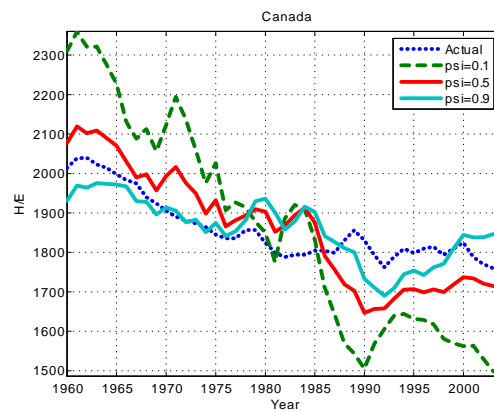


Figure 4.10

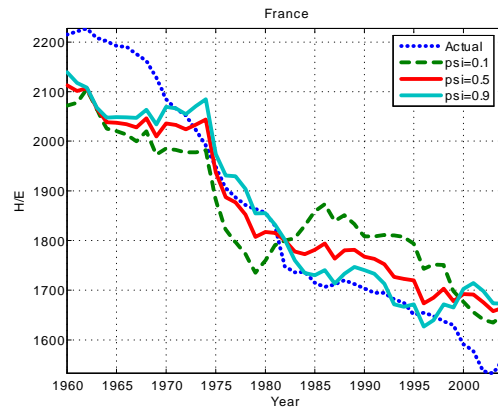


Figure 4.11

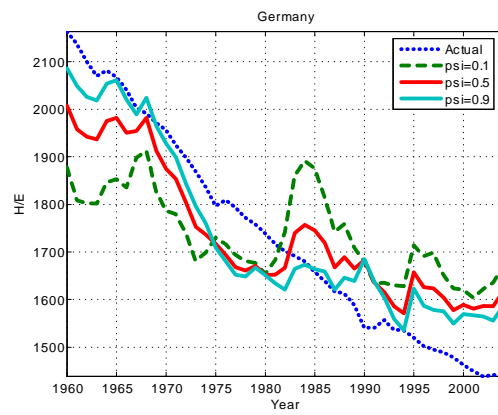


Figure 4.12

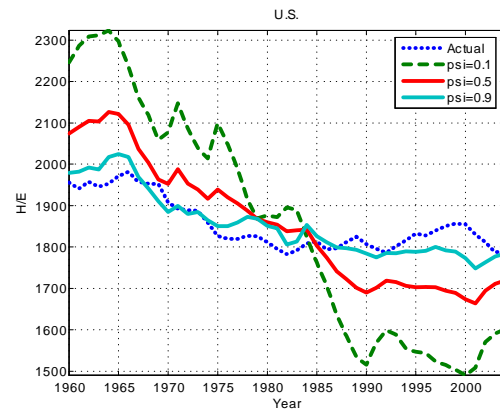


Figure 4.13

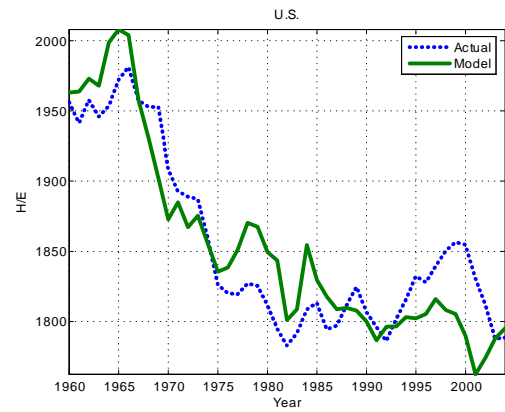
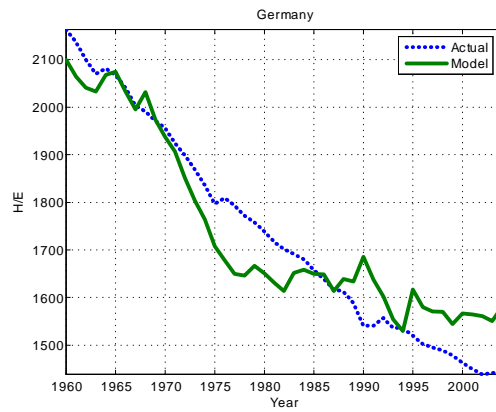
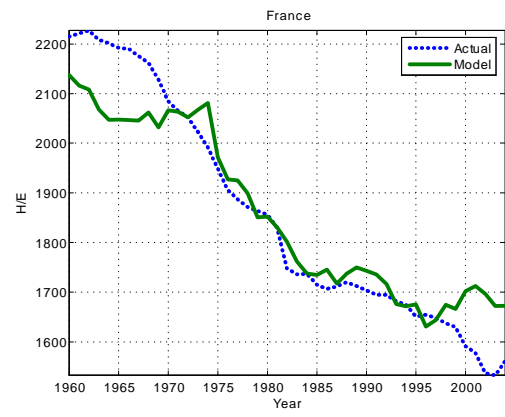
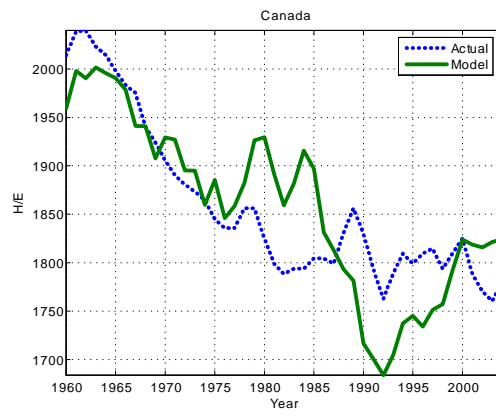


Figure 4.14

4.11 Tables

Table 4.1

Country	ψ^E	Corr.
Austria	0.99	0.89
Australia	0.58	0.76
Belgium	0.60	0.95
Canada	0.81	0.78
Finland	0.47	0.87
France	0.86	0.98
Germany	0.99	0.96
Italy	0.99	0.96
Japan	0.82	0.86
Netherlands	0.27	0.93
Spain	0.88	0.87
Sweden	0.99	0.92
Switzerland	0.43	0.94
United Kingdom	0.99	0.95
United States	0.99	0.89
Mean	0.78	0.90

Table 4.2

Country	$\gamma\phi_t - (\psi_t^N/\psi_t^E) D_t$	E/P
Austria	-0.89	-0.31
Australia	-0.28	0.19
Belgium	-1.01	0.01
Canada	-0.35	0.51
Finland	-0.51	-0.41
France	-0.99	-0.03
Germany	-1.15	-0.20
Italy	-0.96	-0.03
Japan	-0.62	0.01
Netherlands	-1.21	0.48
Spain	-0.44	0.00
Sweden	-0.47	0.02
Switzerland	-0.56	0.06
United Kingdom	-0.79	0.00
United States	-0.25	0.28
Mean ¹⁵	-0.68	0.04

¹⁵Means do not include data for Finland, the Netherlands, and Switzerland. See text for details.

4.12 References

- Alesina, Alberto, Edward Glaeser, and Bruce Sacerdote.** 2005. "Work and Leisure in the United States and Europe: Why So Different?" *NBER Macroeconomics Annual*, 20: 1-64.
- Chetty, Raj.** 2009. "Bounds on Elasticities with Optimization Frictions: An Application to Taxation and Labor Supply." NBER Working Paper 15616.
- Kimball, Miles and Matthew Shapiro.** 2008. "Labor Supply: Are the Income Effects Both Large or Both Small?" NBER Working Paper No. 14208.
- McDaniel, Cara.** 2007. "Hours worked in the OECD 1960-2003: Driving forces and propagation mechanisms." Unpublished manuscript.
- Mendoza, Enrique, Assaf Razin, and Linda Tesar.** 1994. "Effective tax rates in macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption." *Journal of Monetary Economics*, 34 (3): 297-323.
- Mulligan, Casey.** 2002. "A Century of Labor-Leisure Distortions." NBER Working Paper 8774.
- Ohanian, Lee, Andrea Raffo, and Richard Rogerson.** 2008. "Long-term changes in labor supply and taxes: Evidence from OECD countries, 1956-2004." *Journal of Monetary Economics*, 55 (8): 1353-1362.
- Parkin, Michael.** 1988. "A Method for Determining Whether Parameters in Aggregative Models Are Structural." *Carnegie-Rochester Conference Series on Public Policy*, 29: 215-252.
- Prescott, Edward.** 2004. "Why Do Americans Work So Much More Than Europeans?" *Federal Reserve Bank of Minneapolis Quarterly Review*, 2004, 28 (1): 2-13.
- Ragan, Kelly.** 2006. "Taxes, Transfers, and Time Use: Fiscal Policy in a Household Production Model." Ph.D. dissertation, University of Chicago.
- Rogerson, Richard.** 2007. "Taxation and Market Work: Is Scandinavia an Outlier?" *Economic Theory*, 32 (1): 59-85.
- Rogerson, Richard, and Johanna Wallenius.** 2009. "Micro and Macro Elasticities in a Life Cycle Model with Taxes," *Journal of Economic Theory*, 144 (6): 2277-2292.
- Rotemberg, Julio and Michael Woodford.** 1999. "The Cyclical Behavior of Prices and Costs" in John B. Taylor and Michael Woodford, Eds., *Handbook of Macroeconomics*, Vol. 1B, Cambridge, MA: MIT Press: 1051-1135.
- Shimer, Robert.** 2009. "Convergence in Macroeconomics: the Labor Wedge." *American Economic Journal: Macroeconomics*, 1 (1): 280-297.

CHAPTER V

CONCLUSIONS

In this dissertation, I have examined several dimensions of macroeconomics linked to labor economics. In particular, the first essay develops an understanding of the effects that worker-side heterogeneity regarding comparative advantage in production, along with optimal job-seeking behavior, have on aggregate labor-market fluctuations. I show that these worker-side factors can aid in accounting for the amplification of productivity shocks in the aggregate vacancy-unemployment (V/U) ratio. Moreover, slow-moving changes in expected vacancy-posting gains that stem from the presence of worker-side heterogeneity can help explain why, empirically, the V/U ratio exhibits a stage of sluggish adjustment in response to changes in productivity. In particular, in an expansion, endogenous changes in the composition of the pool of individuals searching for any given type of job lead to a slow-moving improvement in the firm-side probability of comparative advantage employment, which translates into a slow-moving improvement in the expected gains from posting vacancies. Sluggish adjustment in the V/U ratio is a key feature of the data that the standard, homogenous-agent model of equilibrium unemployment theory cannot account for.

The second essay - joint with Miles S. Kimball¹ - focuses on the *paradox of hard work*, which refers to the fact that, given enormous world-wide increases in consumption, work hours have remained relatively trendless across countries. Given a low elasticity of intertemporal substitution and income effects on labor supply being substantial, work hours should have in fact shown a significant decline. In principle, this can be rationalized by an increasing marginal-wage to consumption ratio, something that keeps the marginal utility of consumption high, or something that keeps the

¹Miles is a Professor of Economics at the University of Michigan, Ann Arbor.

marginal disutility of work low. We focus attention on the last of these explanations. Economists have long understood that cross-sectional differences in on-the-job utility at a particular time give rise to compensating differentials. The second essay develops a theory that focuses on a less-studied topic: understanding the long-run macroeconomic consequences of trends in on-the-job utility. Two main implications emerge. First, secular improvements in on-the-job utility are such that it is possible for work hours to remain approximately constant over time even if the income effect of higher wages on labor supply exceeds the substitution effect of higher wages. Secondly, secular improvements in on-the-job utility can themselves be a substantial component of the welfare gains from technological progress. These two implications are connected by an identity: improvements in on-the-job utility that have a significant effect on labor supply tend to have large welfare effects.

Finally, the third essay - joint with Shanthi P. Ramnath² - re-examines the ability of the standard neoclassical macroeconomic model augmented with taxes to match the trend behavior of hours per population across countries. Past work has argued that by including taxes in the standard neoclassical model, much of the long-run changes in hours per population both within and across countries can be accounted for. However, two countries stand out as exceptions: Canada and the US. We delve deeper into this puzzle and highlight that unlike most other countries, over the last several decades Canada and the US both experienced large changes in their employment-to-population ratios (E/P) relative to changes in hours per worker (H/E). We find compelling evidence that the failure of the standard neoclassical model with taxes to predict hours per population in Canada and the US stems from an inherent inability of the standard model to account for changes in E/P . Therefore, whenever E/P does not change much relative to H/E , which has been the case for most European countries, the empirical behavior of H/E and hours per population (H/P) are virtually indistinguishable and the standard model gives the impression of correctly predicting H/P . On the other hand, when E/P *does* change considerably relative to H/E , as has been the case in Canada and the US over the last

²Shanthi is an Economist at the US Treasury. The views and opinions expressed in this essay do not necessarily represent those of the US Treasury.

several decades, the standard model implicitly reveals its limitations in predicting E/P , which in turn inhibits its ability to accurately predict H/P . It follows that E/P automatically becomes part of the labor wedge, which is the name given to the residual that captures the extent to which the standard model's predicted H/P do not match up with their empirical counterparts.

Overall, this dissertation has contributed to developing a better understanding of when heterogeneity matters for macroeconomic outcomes, of what factors can play an important role in the determination of the relative trend behavior of work hours, and also of what factors, in addition to taxes, can account for the labor wedge. Each of these topics is representative of recent and ongoing research in the joint area of macro and labor economics. Hence, the theoretical and empirical analyses carried out in this dissertation jointly provide a natural contribution to the continued development of these areas of research.