Energy-bandwidth Tradeoff in Wireless Networks

by

Changhun Bae

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Electrical Engineering: Systems)
in The University of Michigan
2010

Doctoral Committee:
Professor Wayne E. Stark, Chair
Associate Professor Achilleas Anastasopoulos
Associate Professor Hendrikus G. Derksen
Associate Professor Sandeep P. Sadanandarao

© Changhun Bae
All Rights Reserved 2010

To my wife Sunjoo Boo

and my daughter Jessica Eunsae Bae.

ACKNOWLEDGEMENTS

My graduate study at the University of Michigan is one of the most memorable periods of my life. I felt very fortunate to have learned from and worked with so many incredible people. Numerous people have supported me during the development of this dissertation, and my graduate experience more generally. A few words' mention here cannot adequately capture all my appreciation.

First of all, I would like to thank my advisor Professor Wayne E. Stark for his great support and guidance throughout the years. His boundless energy, continual encouragement, and devotion to his students have been truly inspirational. Professor Stark not only advises me on how to do great research, but also teaches me how to be a rightful person. He has been and will be the role model for me in both my academic and my personal lives.

I would also like to thank the other members of my dissertation committee, Professor Achilleas Anastasopoulos, Professor Sandeep S. Pradhan, and Professor Hendrikus Derksen for many useful interactions and for contributing their broad perspective in refining the ideas in this dissertation.

The financial support of the Air Force Office of Scientific Research, the Office of Naval Research, and the department of Electrical Engineering and Computer Science of University of Michigan are also gratefully acknowledged.

I would also like to thank Ms. Becky Turanski who work made my life at the department of electrical engineering and computer science much easier while I'm not in Ann Arbor.

I'm also deeply indebted to my lab members, who are always great sources for discussion on everything and made my life at Ann Arbor a happy and joyful time. At the risk of leaving out many names, I would like to thank Shihyu Chang, Kar-Peo Yar, Chih-Wei Wang and Jinho Kim for their help at the Wireless Communications Lab. In addition, I would like to thank those researchers in other labs at the University of Michigan, including but not limited to Dongkwan Lee, Junho Choi, Ilju Na, Suhan Choi, Dongsook Kim, and Hyunseok Lee for numerous motivating discussions. Of course, I have to thank all my friends in South Korea for their continual encouragement, and especially Dongkyu Seo, who passed away one year ago without seeing my graduation.

As always, I thank my parent and my aunt for inspiring me to pursue an academic career and letting me do what I needed to do. Their encouragement, support, and understanding through these many years have meant more I can ever express.

Last, my most tender and sincere thanks go to my loving wife, Sunjoo. Aside from perhaps myself, no one has been more impacted by this dissertation and the latter years of my graduate program. This dissertation is not good enough to dedicate to her, but the one we are writing together will be.

TABLE OF CONTENTS

DEDICATION ii

ACKNOWLEDGEMENTS iii

LIST OF TABLES viii

LIST OF FIGURES ix

LIST OF APPENDICES xiii

CHAPTERS

1 Introduction 1
- 1.1 Overview of Wireless Network 1
- 1.2 Challenges and Motivations 2
- 1.3 Overview of Contributions 5
- 1.4 Outline of the Dissertation 5

2 End-to-End Energy-Bandwidth Tradeoff in Multi-hop Wireless Networks 7
- 2.1 Introduction 8
- 2.2 System Model 10
- 2.3 Problem Formulation and Performance Measures 13
  - 2.3.1 End-to-end Rate 13
  - 2.3.2 End-to-end Energy Consumption 14
- 2.4 Common Power, Common Rate Transmission Strategies 20
  - 2.4.1 Performance of the Common Rate Strategy 20
  - 2.4.2 Performance of the Common Power Strategy 24
- 2.5 Performance of Equidistant Multi-hop Routing 27
  - 2.5.1 Problem IIIa: Optimization of the number of hops . . . 29
  - 2.5.2 Problem IIIb: Optimization of the end-to-end rate . . . 30
  - 2.5.3 Problem IVa: Optimization of the end-to-end rate . . . 32
  - 2.5.4 Problem IVb: Optimization of the number of hops . . . 33
- 2.6 Numerical Results 35
- 2.7 Conclusion 39

3 Minimum Energy-Per-Bit Wireless Multi-hop Networks with Spatial Reuse . . . 52
3.1 Introduction . . . 53
3.2 System Model . . . 55
3.3 Performance Measures . . . 57
3.3.1 End-to-end rate . . . 57
3.3.2 End-to-end energy consumption-per-bit . . . 58
3.3.3 Minimum energy consumption-per-bit with arbitrary transmission energy . . . 59
3.4 Minimum energy consumption-per-bit . . . 61
3.4.1 Minimum energy-per-bit for a given end-to-end rate . . . 61
3.4.2 Minimum energy-per-bit for an optimal end-to-end rate 64
3.5 Performance of equi-distant multi-hop routing . . . 66
3.5.1 Optimization of the number of hops . . . 68
3.5.2 Optimization of the end-to-end rate . . . 69
3.6 Numerical Results . . . 71
3.7 Conclusion . . . 75

4 Energy-Bandwidth Tradeoff in Multi-hop Wireless Networks with Reed-Solomon Codes . . . 87
4.1 Introduction . . . 88
4.2 System Model . . . 91
4.3 Performance Measures and Problem Formulation . . . 94
4.3.1 End-to-end Rate . . . 94
4.3.2 End-to-end Energy Consumption . . . 94
4.3.3 Minimum energy consumption per bit . . . 95
4.4 Performance Evaluation . . . 97
4.4.1 BPSK . . . 98
4.4.2 Orthogonal Modulation . . . 100
4.4.3 Parametric Solution to the Minimum Energy-Bandwidth Tradeoff . . . 102
4.5 Performance of Equidistant Multi-hop Routing . . . 102
4.6 Numerical Results . . . 106
4.7 Conclusion . . . 108

5 On the Energy-Bandwidth Tradeoff for AWGN Relay Channels . . . 115
5.1 Introduction . . . 116
5.2 System Model and Problem Formulation . . . 117
5.3 Decode and Forward Relaying . . . 120
5.3.1 Without Cooperation . . . 120
5.3.2 Upper Bound with Cooperation . . . 121
5.3.3 Lower Bound with Cooperation . . . 123
5.3.4 Minimum energy per bit . . . 124
5.4 Amplify and Forward Relaying . . . 126
5.4.1 With Cooperation . . . 126

5.4.2 Without Cooperation 127
5.4.3 Minimum energy per bit 128
5.5 Numerical Results 130
5.6 Conclusion 132

6 Conclusions 138
6.1 Summary of Contributions 138
6.2 Future Research 141

APPENDICES 143

BIBLIOGRAPHY 155

LIST OF TABLES

Table

2.1 Optimal energy consumption constants for various AWGN channels . . 40
3.1 Optimal energy consumption constants for various AWGN channels . . 78

LIST OF FIGURES

Figure

1.1 A typical multi-hop routing network. 3

2.1 Illustration of 2-dimensional network model where distance between any two nodes denotes the normalized relay distance by the end-to-end distance. 41

2.2 Normalized energy-bandwidth characteristic when $\gamma_c = 4.69$dB for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m. 41

2.3 Normalized energy-bandwidth characteristic when $\gamma_c = -5.31$dB for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m. 42

2.4 Normalized energy-bandwidth characteristic when $\gamma_c = 0$ for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m. . 42

2.5 Normalized energy-bandwidth characteristic for various AWGN channel models when $\gamma_c = -5.31$dB for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m. 43

2.6 Energy-bandwidth characteristic when $\gamma_c \neq 0$ with single hop ($k = 1$) and multi-hop ($k = 2$) for selected location of relays, $\alpha_1 : \alpha_2 = 5 : 5, 7 : 3$, and $99 : 1$ at d_e=3000m. 44

2.7 Normalized energy-bandwidth characteristic when $\gamma_c = 0$ with $k = 1$ single hop and $k = 2$ multi-hops for selected location of relays, $\alpha_1 : \alpha_2 = 5 : 5, 7 : 3$, and $99 : 1$ at d_e=3000m. 45

2.8 Normalized energy-bandwidth characteristic of 3 relays over AWGN channels when $\gamma_c = 4.69$dB at d_e=3000m. 45

2.9 Normalized energy-bandwidth characteristic of 3 relays over AWGN channels when $\gamma_c = -17.36$dB at d_e=3000m. 46

2.10 Normalized energy-bandwidth characteristic when $\gamma_c = 0$ for equi-spaced relays with $k = 1, ..., 20$, and the analytical result in (2.51) at d_e=3000m. 46

2.11 Normalized energy-bandwidth characteristic when $\gamma_c = 4.69$dB for equi-spaced relays with $k = 1, ..., 20$, and the analytical result in (2.51) at d_e=3000m. 47

2.12 Energy-bandwidth characteristic comparison between the numerical optimization and equi-spaced relays case when $\gamma_c = 4.69$dB. 47

2.13 Normalized energy-bandwidth characteristic of the analytical result in (2.51) with the single hop case when $\gamma_c = 0$ for various AWGN channels at d_e=3000m. 48
2.14 Normalized energy-bandwidth characteristic of the analytical result in (2.51) with the single hop case when $\gamma_c = 4.69$dB for various AWGN channels at d_e=3000m. 49
2.15 Normalized energy-bandwidth characteristic of different end-to-end distance and number of hops for equi-spaced relays when $E_p = 0.95\mu$J/symbol. 50
2.16 Feasible region of the relay location where a two hop network is more energy efficient than a single hop network for selected end-to-end rates. 50
2.17 Feasible region of the relay location where a two hop network is better than a single hop network in terms of the minimum total energy consumption at the optimal end-to-end rate for selected receiver circuit processing energy. 51
3.1 Illustration of network model with spatial reuse, $\rho = 3$. 76
3.2 Illustrative example of multi-hop transmission scenario. 76
3.3 Example of two possible routes in 2-dimensional network model where distance between any two nodes denotes the normalized relay distance by the end-to-end distance with spatial reuse, $\rho = 3$. 77
3.4 Comparison of normalized energy-bandwidth tradeoff when $\gamma_c = -5.31$dB with $k = 4$ hops for selected location of relays at $d_e = 3000$m and $\rho = 3$. 77
3.5 Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot when $\gamma_c = -17.36$dB with $k = 4$ and $k = 5$ hops for selected location of relays at $d_e = 3000$m and $\rho = 3$. . . 78
3.6 Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot when $\gamma_c = 0$ with $k = 4$ and $k = 5$ hops for selected location of relays at $d_e = 3000$m and $\rho = 3$. 79
3.7 Comparison of lower bounds of normalized energy-bandwidth tradeoff for two different routes: $k = 4$ hops for selected location of relays when $\gamma_c = -5.31$dB at $d_e = 3000$m. 80
3.8 Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot: $k = 6$ hops for selected minimum separation of simultaneous transmitters when $\gamma_c = -5.31$dB for equi-distant relays at $d_e = 3000$m. 81
3.9 Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot: $k = 6$ hops for selected minimum separation of simultaneous transmitters when $\gamma_c = -17.36$dB for equi-distant relays at $d_e = 3000$m. 82
3.10 Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot: $k = 6$ hops for selected minimum separation of simultaneous transmitters when $\gamma_c = 0$ for equi-distant relays $d_e = 3000$m. 83

3.11 Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot when $\rho = 3$ and $\gamma_c = -45.31$dB for equi-distant relays $d_e = 30$km. 84
3.12 Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot for selected number of hops, $k = 6, 7, 8$ when $\gamma_c = -5.31$dB for equi-distant relays $d_e = 3$km and $\rho = 3$. 85
3.13 Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot for selected number of hops, $k = 10, 20, 30$ when $\gamma_c = -45.31$dB for equi-distant relays $d_e = 3$km and $\rho = 3$. 86
3.14 Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot for various AWGN channel models when $\gamma_c = -5.31$dB and $k = 6$ hops for equi-distant relays $d_e = 3$km and $\rho = 3$. 86
4.1 Illustration of 2-dimensional network model where distance between any two nodes denotes the normalized relay distance by the end-to-end distance. 109
4.2 Comparison of energy-bandwidth tradeoff for practical modulation with $L = 5$ when $\gamma_c = 0$ for a multihop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 6 : 4$ at $d_e = 3000m$. 110
4.3 Comparison of energy-bandwidth tradeoff for practical modulation with $L = 5$ when $\gamma_c = -15.31$dB for a multihop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 6 : 4$ at $d_e = 3000m$. 110
4.4 Comparison of energy-bandwidth tradeoff for practical modulation with $L = 5$ when $\gamma_c = -5.31$dB for a multihop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 6 : 4$ at $d_e = 3000m$. 111
4.5 Comparison of energy-bandwidth tradeoff between the case without retransmission and with retransmission for BPSK modulation with $L = 5$ when $\gamma_c = -5.31$dB for a multihop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 6 : 4$ at $d_e = 3000m$. 112
4.6 Comparison of normalized energy-bandwidth characteristic of 2 relays over AWGN channels when $\gamma_c = -5.31$dB at d_e=3000m. 113
4.7 Normalized energy-bandwidth tradeoff with $L = 5$ when $\gamma_c = -5.31$dB for equi-spaced relays with $k = 1, 2, 3, 4$ and the analytical result in (4.49) at $d_e = 3000m$. 114
5.1 Illustration of relay channel. 132
5.2 Comparison of the decode-and-forward scheme when $\gamma_c = -5.31$dB with a selected location of relay, $\alpha_1 : \alpha_2 = 7 : 3$ for a common power scheme. 133
5.3 Comparison of the amplify-and-forward scheme when $\gamma_c = -5.31$dB with a selected location of relay, $\alpha_1 : \alpha_2 = 7 : 3$ for a common power scheme. 134
5.4 Energy-bandwidth tradeoff when $\gamma_c = -5.31$dB with single hop and a selected location of relay, $\alpha_1 : \alpha_2 = 7 : 3$ for a common power scheme. . 134
5.5 Energy-bandwidth tradeoff when $\gamma_c = -5.31$dB with single hop and a selected location of relay, $\alpha_1 : \alpha_2 = 3 : 7$ for a common power scheme. . 135

5.6 Energy-bandwidth tradeoff when $\gamma_c = 0$ with single hop and a selected location of relay, $\alpha_1 : \alpha_2 = 7 : 3$ for a common power scheme. 135
5.7 Energy-bandwidth tradeoff when $\gamma_c = 0$ with single hop and a selected location of relay, $\alpha_1 : \alpha_2 = 3 : 7$ for a common power scheme. 136
5.8 Comparison of end-to-end rates with a selected location of relay, $\alpha_1 : \alpha_2 = 3 : 7$ for a common power scheme. 136
5.9 Comparison of energy-bandwidth tradeoffs for amplify-and-forward scheme when $\gamma_c = 0$ with a selected location of relay, $\alpha_1 : \alpha_2 = 3 : 7$ for a common power scheme. 137

LIST OF APPENDICES

APPENDIX

A Convexity and Solutions of Chapter 2 144
A.1 Convexity and Solution of Problem Ia 144
A.2 Convexity and solution of Problem Ib 146
A.3 Proof of Property 2.1 147
A.4 Conditions for convexity of Problem III 148
B Convexity of Chapter 3 150
B.1 Condition for convexity 150
C Convexity of Chapter 4 153
C.1 Convexity of Problem III 153

CHAPTER 1

Introduction

Exploding demand for various wireless services has fueled significant development of wireless networks, especially several generations of cellular networks, and more recently, ad-hoc data networks with cellular-Wi-Fi convergence. Intense development of radio hardware and wireless applications has driven system designers and service providers to design more efficient and cost effective systems that require better understanding of the channel characteristics and multi-user communications. In particular, technological advances in integrated circuits and radio-frequency devices increasingly allow for more sophisticated signal processing and channel coding algorithms that enable us to approach the fundamental performance limits. However, compared to point-to-point single hop communications, we are only beginning to understand the fundamental performance limits of wireless multi-hop networks and practical ways to approach them.

1.1 Overview of Wireless Network

On a typical single link communication, Shannon published his seminal work on channel coding theory for point-to-point communication [1] in an additive white Gaussian noise (AWGN) channel, where he characterized the point-to-point communication channel by a parameter, which he named the *capacity* of the channel. He showed that communication with arbitrarily small error probability is possible only when the com-

munication rate is less than the channel capacity. For more than fifty years, researchers have tried to design coded systems of reasonable complexity to achieve low error probability with signal-to-noise ratio close to the Shannon limit. The advent of turbo codes [2] and low-density parity check codes [3]-[5] allows for point-to-point communication in AWGN channels close to the Shannon limit with the help of technological advances. Other than the point-to-point communication, there have been extensions of Shannon's theory to the broadcast and multiple access communication channels. The most successful extensions have been for the broadcast channels [6] and the multiple access channels [7].

With the rapid hardware advances that allow more signal processing functionality to be integrated into a single chip, there are more expectations on future wireless network: reliable communications among devices are desired regardless of physical limitations. Wireless cellular networks allow people on the move to communicate with anyone or anything using multimedia services. Wireless sensor networks allow devices to cooperate in monitoring of stress and strain in buildings and bridges, detection of hazardous events, tracking of enemy targets, and the support of unmanned robotic vehicles. Wireless broadband access networks allow people and devices to have high-speed connections to the backbone network from any place at any time.

1.2 Challenges and Motivations

Of all potential wireless applications, wireless ad-hoc networks are special due to their self-configurable characteristic to form a network without the aid of any established infrastructure and their emphasis on communication between devices. Multi-hop routing, whereby intermediate nodes relay packets towards their final destination, can improve the throughput and energy efficiency of the network as shown in Fig. 1.1. These networks have hard energy constraints since each node is powered by a small

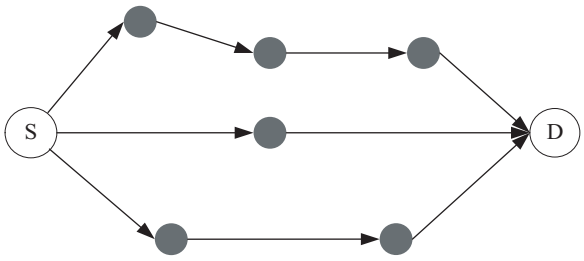


Figure 1.1: A typical multi-hop routing network.

battery that may not be rechargeable or renewable [40]. Hence, the nodes can only transmit a finite number of bits before they run out of energy and reducing the energy consumption per bit for end-to-end data transmission is the most important design consideration for such networks. In addition, efficient use of the available bandwidth for end-to-end transmission must be exploited to optimize the system performance. These two goals of minimizing energy consumption and maximizing bandwidth utilization cannot be achieved simultaneously, and this motivates analyzing the energy-bandwidth tradeoff in wireless multi-hop networks.

To date, there have been tremendous research on the network throughput and capacity maximization (bits per second) or transport capacity (bits-meter per second) in which the performance measure related to data rate has been regarded as a primary metric. In data-rate centric applications such as video communication where there is a continuous data stream, throughput or transport capacity are important with a tolerable delay constraint. However most of energy-constrained ad-hoc wireless networks have typically intermittent small traffic requirements with limited energy supplies. Hence, the energy and the bandwidth should be considered jointly. To measure the performance of transmitting a finite number of data through the network, the bits-per-Joule capacity of energy-limited wireless networks are considered in [11, 12]. The energy-bandwidth tradeoff under various assumption on interference with a linear network model are described in [21][24]. In addition, in many wireless multi-hop networks which deliver the data by multi-hop routing, the receiver energy consumption is not negligible since there

is a significant energy dissipation to process the received signals. Usually the circuit energy consumption is negligibly small relative to the transmission energy in long-range communications. However, in short-range multi-hop transmissions such as wireless sensor networks, the receiver energy consumption at each relay can affect the energy-bandwidth tradeoff especially in low rate communications [17, 18, 19, 28, 29, 30, 31]. Thus the overall spectrum utilization across the network needs to be jointly considered with the minimum energy consumption [21, 24, 28, 41]. In particular, the end-to-end bandwidth efficiency to characterize the energy-bandwidth tradeoff is analyzed through the optimization of the bandwidth allocation in [24]. The optimal routing to maximize the bandwidth efficiency is described in [26].

Even though much work has been done on cross-layer design issues in general wireless networks, we still lack a characteristic between the total energy consumption and the end-to-end bandwidth efficiency in wireless multi-hop networks, especially for short-range networks where circuit processing energy must be jointly considered. In this thesis, we focus on the energy-bandwidth tradeoff in order to optimize the network performance in an energy-constrained network, where we include receiver energy consumption. We characterize the tradeoff between energy efficiency $\frac{E_b}{N_0}$ and bandwidth efficiency R. This is similar in nature to the tradeoff in a single hop transmission over AWGN channel determined by Shannon as

$$\frac{E_b}{N_0} > \frac{2^{2R}-1}{2R}.$$

We model the optimization problem in a systematic way such that efficient closed-form solutions can be applied to find the optimal or near-optimal solutions. The main contributions are listed in the next section.

1.3 Overview of Contributions

We address the following specific questions in this thesis:

1. For a multi-hop wireless network with arbitrary distributed relays in a 2-dimensional plane, what is the fundamental tradeoff between the total energy consumption and the end-to-end rate, and how can we achieve the optimal tradeoff to minimize the total energy consumption?

2. Given the energy-bandwidth tradeoff for a multi-hop route, how can we find the optimal routing path which provides the minimum total energy consumption for a given target end-to-end rate, and how does the relays location affect the overall performance?

3. How is the energy-bandwidth tradeoff affected without spatial reuse and with spatial reuse?

4. How is the energy-bandwidth tradeoff different when practical channel codes with modulation schemes are employed to reflect the successful packet reception instead of the channel capacity?

5. Is node collaboration beneficial in terms of increasing the end-to-end bandwidth efficiency? How much can we gain in terms of the energy-bandwidth tradeoff using different cooperative techniques at relays?

1.4 Outline of the Dissertation

This dissertation continues as follows. Chapter 2 analyzes the energy-bandwidth tradeoff without spatial reuse in wireless multi-hop networks. Chapter 3 treats energy-bandwidth tradeoff in wireless multi-hop routing networks, when spatial reuse is employed to increase the bandwidth efficiency. In this analysis the minimum separation

between simultaneous transmitters is considered to determine which relays are selected to operate in the same transmission slot. Chapter 4 considers energy-bandwidth trade-off when Reed-Solomon codes are employed with BPSK and orthogonal modulation to incorporating the effect of unsuccessful packet reception. Chapter 5 treats cooperative relaying in a typical relay network, in particular when decode-and-forward and amplify-and-forward schemes can be exploited by the forwarding strategy. Finally, Chapter 6 summarizes our results and points to areas for future research.

CHAPTER 2

End-to-End Energy-Bandwidth Tradeoff in Multi-hop Wireless Networks

In this chapter energy-constrained wireless multi-hop networks with a single source-destination pair are considered. A network model that incorporates both the energy radiated by the transmitter and the energy consumed by the circuits that process the received signals is proposed. The rate of communication is the number of information bits transmitted (end-to-end) per coded symbol transmitted by any node in the network that is forwarding the data. The tradeoff between the total energy consumption and the end-to-end rate of communication is analyzed. The performance (either energy or rate) depends on the transmission strategy of each node, the location of the relay nodes, and the data rate used by each node. Communication strategies include the rate of transmission on each link, the scheduling of links, and the power used for each link. Strategies that minimize the total energy consumption for a given rate are found. Two communication strategies that capture the inherent constraints of some practical networks are also considered and compared with the optimum strategies. In the case of equi-spaced relays, analytical results for the tradeoff between the energy and the end-to-end data rate are provided. The minimum energy over all possible data rates is also obtained. Low rates incur a significant penalty because the receiver is on for a long time period while high rates require high transmission energy. At high rates routes with

fewer hops minimize the energy consumption while at lower rates more hops minimize the energy consumption.

2.1 Introduction

Wireless ad-hoc networks may be widely deployed in the near future. Wireless nodes in such a network are typically equipped with a small battery. This makes minimizing the energy consumption a critical design issue for such wireless ad-hoc networks. In addition, efficient use of the available spectrum to transmit information bits across the network is also an important design consideration. These two goals of minimum energy consumption and maximizing bandwidth utilization cannot be achieved simultaneously and motivate analyzing the energy-bandwidth tradeoff in wireless multi-hop networks.

Previous research in energy-constrained networks has mainly focused on transmission schemes to minimize the radiated energy. The capacity per cost with general capacity cost functions on a single link is considered in [11], and the bits-per-Joule capacity to measure the efficiency of finite data transmission is described in [12]. Some optimal signaling methods in the wideband regime that minimize the energy per bit while considering the fundamental energy-bandwidth tradeoff is described [13]. Optimal scheduling problems to minimize the transmission energy by varying packet transmission times are considered in [14]. Emphasis on the transmission energy is quite reasonable for typical long range communications where transmission energy dominates the total energy consumption. However, for short range communication, energy dissipation in the circuit is not negligible and results in a more complicated tradeoff in the power-limited regime [29][30]. In [17]-[19] the circuit energy consumption is incorporated to minimize the total energy consumption but the bandwidth efficiency is not considered. Besides the energy consumption, the end-to-end throughput of wireless networks or equivalently the bandwidth efficiency also needs to be considered at the same time. Important guidelines

are provided for bandwidth efficient network design in [20] where the cooperative relaying is exploited, and in [21]-[26] where every relay is located equidistantly on a straight line. However, the energy-bandwidth tradeoff and the optimal routing to maximize the bandwidth efficiency are analyzed without considering the circuit energy consumption [23]-[26].

In this chapter, we consider the tradeoff between the total energy consumption and the end-to-end rate in wireless multi-hop networks where each node can be placed arbitrarily between a single source node and a destination node. The problem and solution we consider is limited to multi-hop transmissions based on point-to-point communication and interference-free scheduling. This requires perfect synchronization among all nodes and knowledge of the overall network characteristics at all nodes. We formulate the overall costs in terms of the end-to-end bandwidth utilization (channel uses) and energy consumption to deliver one information bit across the network in order to fairly evaluate different multi-hop routing strategies. The end-to-end rate measures how efficiently the network utilizes the bandwidth to forward one information bit. The total energy consumption includes the transmission energy and the circuit processing energy to forward data. For an arbitrary placement of nodes we find the optimal rate of transmission for each link, the scheduling of links, and the power level for each link that minimizes the energy required for a given end-to-end rate. We also find the end-to-end rate that minimizes the total energy consumption. If the circuit processing energy is negligible, then the rate that minimizes the energy consumption is zero in many single-hop communication systems. We find the dependence of the minimum energy on the location of the nodes in the network. We also propose two practical communication strategies namely (1) a common rate strategy and (2) a common power strategy. For both strategies, we characterize the tradeoff between energy efficiency, measured in terms of the required E_b/N_0 for reliable communications and bandwidth efficiency, measured in terms of the end-to-end rate R. This is similar in nature to the tradeoff in

a single hop transmission for the AWGN channel determined by Shannon. Typically, the energy-bandwidth tradeoff has been studied with linear equi-distant placed nodes, and only the transmission energy has been included in the model [11]-[14], [21]-[26]. Our work differs from previous works in two fundamental ways. First, to account for the general topologies, each relay node can be placed arbitrarily between source and destination. Second, our energy consumption model incorporates both transmission energy and circuit processing energy.

In summary, the main contribution of this chapter is twofold. First, we propose a framework to analyze the tradeoff between total energy consumption and end-to-end rate. In particular, we quantify the impact of the relay's configuration on the tradeoff between energy and rate. Secondly, we show that there is a minimum total energy consumption per information bit that is obtained by the optimal selection of the end-to-end rate. We find the minimum energy for the optimal transmission scheme and for more practical transmission schemes.

The rest of this chapter is organized as follows. We describe the system model in Section 2.2 and the performance measures of wireless networks (energy and rate) in Section 2.3. In Section 2.4, we investigate the performance in terms of the end-to-end energy-bandwidth tradeoff for two proposed strategies called common power transmission and common rate transmission. In Section 2.5 the special case of equi-spaced relays is considered. We provide numerical results in Section 2.6 and summarize the results in Section 2.7.

2.2 System Model

Consider a wireless network where a source node communicates with a destination node, separated by a distance d_e, through a multi-hop route with the information traversing through $k-1$ relay nodes that are arbitrarily located between the source

and the destination as shown in Fig. 2.1. We denote the relay distance of hop i as d_i and let $\alpha_i = d_i/d_e$ where $0 < \alpha_i < 1$ and $\overline{\alpha} = (\alpha_1, ..., \alpha_k)$. We assume each relay is in the far-field region of the corresponding transmitter [36]. For RF carrier frequencies from 400 MHz to 10 GHz, the far-field assumption is satisfied by relays separated at least 1m [37]. The received power $P_{r,i}$ in link i in the far-field is modeled as

$$P_{r,i} = \frac{\beta}{d_i^{\eta}} P_{out}, \quad d_i > 1$$

where P_{out} is the transmitter output power and η is a path-loss exponent (typically between 2 and 4), and β is a constant from the antenna characteristics. For simplicity we assume $\beta = 1$.

A simple decode-and-forward protocol is considered in which relay i decodes the message sent from relay $i-1$, re-encodes it, and then forwards the message to relay $i+1$. We consider link connectivity only between intermediate neighboring relays, and thus any cooperative relaying scheme to exploit cooperative diversity is not allowed. Each transmission is assumed to employ capacity-achieving codes with the same time duration for each coded symbol for each link. We impose no peak power constraint for each transmitter output or delay constraints. The rate of transmission R in information bits per channel use must be less than the capacity C for reliable communication. That is

$$R < C(\gamma) \Longleftrightarrow \gamma > g(R)$$

where γ is the received energy per channel use-to-noise power spectral density ratio and $g(R) = C^{-1}(R)$ is the inverse of channel capacity $C(\gamma)$. We now make the following assumptions on the channel capacity function $C(\gamma)$.

1. $C(\gamma)$ is continuous in $\gamma \geq 0$ and twice differentiable, non-decreasing and concave in γ.

2. $C(\gamma)$ has an inverse function. That is, $C^{-1}(x) = g(x)$ which is also continuous, twice differentiable in γ.

For the AWGN channel the capacity $C(\gamma)$ and its inverse function $g(R)$ are given by

$$C(\gamma) = \frac{1}{2}\log_2(1+2\gamma) \Longleftrightarrow g(R) = \frac{2^{2R}-1}{2}.$$

We assume that each node has the same circuitry which requires circuit energy E_p Joules to process a single received coded symbol. In practice, the circuit processing energy depends on a transceiver design and architecture, and coding and modulation techniques. In some cases the energy consumption of the receiver is dominated by the RF front-end. In this case the energy per symbol is just the product of the receiver power consumption and the time duration for each channel use (modulation symbol). We assume E_p is a constant throughout this chapter. We note that the total energy consumed per information bit is the energy consumed per coded bit, E_p, divided by the rate of the code. As such, very low rates require a large amount of energy per information bit to process the received signal. This leads to an optimal rate that minimizes the total energy. High rates require high transmitted energy per information bit. Low rates require high receiver processing energy per information bit.

We assume that each node operates in half-duplex mode, hence it cannot transmit and receive simultaneously. Thus each transmission needs to be scheduled to avoid conflicts. Inter-link interference is not considered here since in this chapter we assume that the network operates without spatial reuse. Perfect synchronization is assumed to be achieved among the nodes, hence each relay node transmits in its own slot (time or frequency) without any interference.

2.3 Problem Formulation and Performance Measures

In this section, we describe the performance measures considered, namely the total energy consumption per information bit by the network and the end-to-end rate. The total energy consumption includes the transmission energy and the circuit processing energy. The end-to-end rate is the number of information bits transmitted (end-to-end) per coded symbol transmitted by any node in the network that is forwarding the data.

2.3.1 End-to-end Rate

The end-to-end rate is defined as the number of information bits transmitted end-to-end per channel use across the network. Consider N total uses of the channel by the k links. Link i connecting node $i-1$ and node i is allocated N_i exclusive channel uses. Node $i-1$ transmits at rate R_i on link i. The maximum end-to-end rate that can be achieved is [23][24]

$$R_e = \max_{\sum_i N_i = N} \min \left\{ \frac{N_i}{N} R_i \right\} = \max_{\sum_i q_i = 1} \min\{q_i R_i\}$$

where $q_i = N_i/N$. The maximum end-to-end rate is achieved by optimally assigning a fraction of total channel uses to each transmission. This optimal assignment of channel uses to each transmission is obtained by using an equalizer strategy. That is, the optimal solution must have $q_i R_i$ being a constant for all i. Together with the constraint that $\sum_i q_i = 1$ this yields the solution $q_i^* = R_i^{-1} / \sum_j R_j^{-1}$. Given the optimum channel use allocation, q_i^*, among the links, the maximum *end-to-end rate* for this route is

$$R_e = \frac{1}{\sum_{i=1}^{k} R_i^{-1}} \text{ (bits/channel use).} \tag{2.1}$$

2.3.2 End-to-end Energy Consumption

Consider the i-th link which is communicating at rate R_i. The energy consumption per information bit of link i, E_i, consists of transmission energy consumption and receiver energy consumption to process received symbols. Thus, we have

$$E_i = \frac{E_{i,tx} + E_p}{R_i}$$

where $E_{i,tx}$ is the energy transmitted per coded symbol and E_p is the energy consumed by the circuitry at hop i for each coded symbol. Typically, the minimum received energy (rather than the transmitted energy) is found such that reliable communication is possible. This avoids the need to know the distance between the transmitter and receiver. However, the distance and propagation loss must be taken into account to investigate the actual energy consumption. In order to compare with standard results on energy required for reliable communications (for instance, a single-hop case) we take the total energy consumption (transmitted and processing energy of the receiver) and normalize by the end-to-end propagation loss. In the absence of receiver processing energy then this is exactly what is typically used for energy (or power) consumption. Thus if the total energy is normalized by the propagation loss then we should normalize both the transmitted energy and the receiver processing energy by this same quantity. From this normalized quantity we can find the actual energy consumption easily. So we normalize the circuit processing energy by the propagation loss, which depends on distance. The normalized energy $E_{i,tx}$ is defined as

$$\frac{E_{i,tx}}{N_0} d_e^{-\eta} = \frac{E_{i,tx}}{N_0} \alpha_i^{\eta} d_i^{-\eta} = \alpha_i^{\eta} \gamma_i$$

where $\gamma_i = \frac{E_{i,tx}}{N_0} d_i^{-\eta}$ denotes the received energy per coded symbol-to-noise power spectral density ratio at hop i. Similarly a normalized circuit processing energy γ_c is defined

as

$$\gamma_c \triangleq \frac{E_p}{N_0} d_e^{-\eta}.$$

Our major interest in this chapter is the total energy consumption that includes the transmission energy and the circuit processing energy required to communicate a bit across the network. In order to fairly represent the total energy consumption for different hops, we define the *normalized total energy consumption per information bit-to-noise power spectral density ratio* as

$$\frac{E_{tot}}{N_0} \triangleq \sum_{i=1}^{k} \frac{E_i}{N_0} d_e^{-\eta} = \sum_{i=1}^{k} \frac{\alpha_i^{\eta}\gamma_i + \gamma_c}{R_i}. \tag{2.2}$$

Note that for low rates, the processing energy for the circuitry will dominate the total energy consumption whereas for high rates the transmitted energy will dominate.

For a single link the received energy is the usual performance measure as opposed to the transmitted energy. In order to be consistent with this, we define the *end-to-end energy received per information bit-to-noise power spectral density ratio* as

$$\frac{E_b}{N_0} \triangleq \sum_{i=1}^{k} \frac{E_{i,tx}}{R_i} \frac{d_e^{-\eta}}{N_0} = \sum_{i=1}^{k} \frac{\alpha_i^{\eta}\gamma_i}{R_i} \equiv \left(\frac{E_{tot}}{N_0}\right)_{\gamma_c=0} \tag{2.3}$$

which denotes the sum of transmission energy per information bit in each link that is involved in the end-to-end transmission, and a special case of the total energy consumption when $\gamma_c = 0$.

Our fundamental goal of this chapter is, for a given multi-hop routing path $\overline{\alpha} = (\alpha_1, ..., \alpha_k)$, to find the optimal set of transmission energies $(E_{1,tx}, ..., E_{k,tx})$ or equivalently $(\gamma_1, ..., \gamma_k)$ that provide the smallest total energy consumption for a given end-to-end data rate R_e. This problem corresponds to scarce wireless multi-hop networks where only a few relays are available and thus location of relays are not design variables.

This general problem of minimizing total energy consumption for a given end-to-end rate can be formulated as the following optimization problem.

Problem Ia:

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \min_{(\gamma_1,..,\gamma_k)} \quad & \sum_{i=1}^{k} \frac{\alpha_i^{\eta}\gamma_i + \gamma_c}{R_i} \\ \text{s.t.} \quad & R_i = C(\gamma_i),\ i = 1, ..., k \\ & \textstyle\sum_{i=1}^{k} R_i^{-1} = R_e^{-1} \end{aligned} \tag{2.4}$$

where the optimization variables are γ_i's and $C(\gamma_i)$ denotes the channel capacity which is increasing and concave. Problem Ia can be converted into an equivalent convex optimization problem as shown in Appendix A. The solution for the above problem can be obtained in a parametric form as follows.

Parametric Solution to Problem Ia: Let $g(x) = C^{-1}(x)$ be the inverse function of the channel capacity function. By solving the following set of equations for x_i

$$\lambda = \alpha_i^{\eta}\left[g(x_i^{-1}) - \frac{1}{x_i}g'(x_i^{-1})\right] + \gamma_c,\ i = 1, ..., k \tag{2.5}$$

where $x_i > 0$ and $\gamma_c \geq 0$, the solution to Problem Ia is as follows

$$\gamma_i = C^{-1}(x_i^{-1}), \quad R_i = \frac{1}{x_i} \tag{2.6}$$

$$R_e^{-1} = \sum_{i=1}^{k} x_i \tag{2.7}$$

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \sum_{i=1}^{k} x_i\left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c\right]. \tag{2.8}$$

Proof: See Appendix A.1. ■

We can obtain the solution to Problem Ia from the following procedure using numerical methods:

Step 1. Fix λ.

Step 2. Solve (2.5) for x_i for $i = 1, ..., k$.

Step 3. Determine γ_i, R_i and R_e from (2.6) and (2.7).

Step 4. Determine E_{tot}/N_0 from (2.2) or (2.8).

By varying λ we can determine the tradeoff between E_{tot}/N_0 and R_e. For the case of the AWGN channel, the channel capacity is given by

$$\begin{aligned} \frac{1}{C(\gamma_i)} &= \frac{2}{\log_2(1+2\gamma_i)} = x_i \Rightarrow \gamma_i = C^{-1}(x^{-1}) \\ g(x^{-1}) &= C^{-1}(x^{-1}) = \frac{1}{2}(2^{2/x} - 1). \end{aligned}$$

Then the parametric solution to Problem Ia for the AWGN channel is obtained by solving the following equation for x_i.

Solution to Problem Ia for the AWGN channel: Let $\mathcal{W}(x)$ be the principal branch of the Lambert W-function, for which $\mathcal{W}(x) \geq -1$ [34]. Let $u_i = 1 + \mathcal{W}\left(\frac{2\alpha_i^{-\eta}(\gamma_c - \lambda) - 1}{e}\right)$. Then from (2.5) we obtain

$$\begin{aligned} 2\lambda &= \alpha_i^{\eta}\left[e^{2\ln 2/x_i} - 1 - \frac{2\ln 2 e^{2\ln 2/x_i}}{x_i}\right] + 2\gamma_c \\ \frac{1}{x_i} &= \frac{1}{2\ln 2}u_i = C(\gamma_i) \Rightarrow \gamma_i = \frac{1}{2}(e^{u_i} - 1). \end{aligned}$$

Therefore the solution to Problem Ia for the AWGN channel is as follows:

$$\begin{aligned} R_e^{-1} &= 2\ln 2 \sum_{i=1}^{k} \frac{1}{u_i} \\ \frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} &= \ln 2 \sum_{i=1}^{k} \frac{\alpha_i^{\eta}(e^{u_i} - 1) + 2\gamma_c}{u_i} \\ &= \ln 2 \sum_{i=1}^{k} \left[\frac{2\lambda}{u_i} + \alpha_i^{\eta} e^{u_i}\right] \end{aligned}$$

where $x_i \geq 0$ for all i. The next problem is to also optimize E_{tot}/N_0 over R_e.

Problem Ib:

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \min_{(\gamma_1,..,\gamma_k)} \sum_{i=1}^{k} \frac{\alpha_i^\eta \gamma_i + \gamma_c}{C(\gamma_i)} \tag{2.9}$$

where the optimization variables are γ_i's. Problem Ib can be converted into an equivalent convex problem as shown in Appendix A.2. The solution for Problem Ib can be obtained by solving the following equation

Solution to Problem Ib: From the solution x_i^* to the following equation

$$\frac{g'(1/x_i)}{x_i} - g(1/x_i) = \alpha_i^{-\eta}\gamma_c \tag{2.10}$$

we can determine γ_i^*, R_e^*, and $E_{tot}(\overline{\alpha}, R_e^*)/N_0$ from (2.6)-(2.8).

Proof: See Appendix A.2. ■

Note that $\gamma_c = 0$ results in $x_i = \infty$ or equivalently $\gamma_i^* = 0$ since $h(x) \triangleq xg'(x) - g(x)$ is increasing in $x \geq 0$, which leads to $C(\gamma_i^*) = 0$ and $R_e^* = 0$ respectively. Hence using the following property, the minimum end-to-end received energy per information bit without circuit energy consumption can be obtained when $\gamma_i \to 0$ or equivalently $R_e \to 0$. [1]

Property 2.1. $\left(\frac{E_{tot}}{N_0}\right)_{\gamma_c=0} = \frac{E_b}{N_0}$ is strictly increasing in $R_e > 0$.

Proof: See Appendix A.3. ■

Therefore the minimum end-to-end received energy per information bit for a given multi-hop routing path without any processing energy is as follows.

Solution to Problem Ib with $\gamma_c = 0$:

$$\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \lim_{\gamma_i \to 0} \sum_{i=1}^{k} \frac{\alpha_i^\eta \gamma_i}{C(\gamma_i)} = \frac{1}{C'(0)} \sum_{i=1}^{k} \alpha_i^\eta \tag{2.11}$$

[1] In (4) for each set of values of γ_i there will be a value for R_i and hence a value for R_e from (2.1). After optimizing over γ_i with the constraint on the end-to-end rate R_e, there is γ_i^*, a function of R_e. When R_e is small the optimal value for γ_i is also small.

where $C'(0) = \lim_{\gamma \to 0} \frac{\gamma}{C(\gamma)}$.

For the case of the AWGN channel, the solution to Problem Ib for the minimum E_{tot}/N_0 over R_e is as follows.

Solution to Problem Ib for the AWGN channel: Let $u_i^* = 1 + \mathcal{W}\left(\frac{2\alpha_i^{-\eta}\gamma_c - 1}{e}\right) = 1 + \mathcal{W}\left(\frac{2d_i^{-\eta}E_p - N_0}{eN_0}\right)$. Then the optimal solution to Problem Ib is given as follows:

$$\gamma_i^* = \frac{1}{2}(e^{u_i^*} - 1) \tag{2.12}$$

$$R_e^*(\overline{\alpha})^{-1} = 2\ln 2 \sum_{i=1}^{k} \frac{1}{u_i^*} \tag{2.13}$$

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} &= \ln 2 \sum_{i=1}^{k} \frac{\alpha_i^{\eta}(e^{u_i^*} - 1) + 2\gamma_c}{u_i^*} \\ &= \ln 2 \sum_{i=1}^{k} \left(\frac{d_i}{d_e}\right)^{\eta} e^{u_i^*}. \end{aligned} \tag{2.14}$$

Proof: See Appendix A.2. ∎

We note from (2.14) that if the end-to-end distance is doubled by adding nodes in such a way that each link distance is duplicated then the total end-to-end energy consumption *without* the end-to-end distance normalization (at the optimal rate R_e^*) also doubles. That is, the minimum total energy consumption grows linearly with distance. Also note that $\gamma_c = 0$ results in $u_i = 0$ and $\gamma_i^* = 0$ since $\mathcal{W}(-e^{-1}) = -1$, which leads to $R_i^* = 0$ and $R_e^* = 0$ respectively. Hence using the property 2.1, the minimum end-to-end received energy per information bit can be obtained when $\gamma_i \to 0$ or equivalently $R_e \to 0$.

Solution to Problem Ib with $\gamma_c = 0$ for the AWGN channel:

$$\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \lim_{\gamma_i \to 0} \sum_{i=1}^{k} \frac{\alpha_i^{\eta}\gamma_i}{0.5\log_2(1 + 2\gamma_i)} = \ln 2 \sum_{i=1}^{k} \alpha_i^{\eta}. \tag{2.15}$$

Note that the above result can be verified directly from (2.13) by setting $u_i = 0$ for $i = 1, ..., k$.

Besides determining the best achievable performance, we also want to find the relation between $\frac{E_{tot}}{N_0}$ and R_e for practical transmission schemes. In the following section we consider two practical schemes for rate selection and transmission energy selection of physical layer at each hop and investigate the optimization problem in (2.9).

2.4 Common Power, Common Rate Transmission Strategies

In this section we consider two practical strategies for selecting the rate and energy in each relay link. For the first scheme, we fix the transmission rate of each link and vary the transmission energy of each link. We call this the "Common Rate" strategy. For the second scheme, we fix the transmission energy of each symbol on each link and vary the rate of each link. We call this "Common Power" strategy. Common power strategy may be a more practical strategy than common rate strategy. Common rate strategy requires nodes to adjust their transmit power levels according to the distances to the next relay which may, for example, require changing the drive level of an amplifier. Changing the rate of communication, however, is something that can be done easily, for example, by puncturing.

2.4.1 Performance of the Common Rate Strategy

We first consider multi-hop transmission where each node communicates at a common fixed rate, $R_i = R$ for all i. For reliable communication at each hop, the common rate R should be achievable in any link which is involved in forwarding data. Under multi-hop transmission where each link communicates using channel-capacity achieving codes over the channel, the relation between the rate and the signal-to-noise power

spectral density ratio of each link is

$$\gamma_i = \gamma = g(R),\ i = 1, ..., k. \tag{2.16}$$

Hence the transmission energy of link i is obtained as

$$E_{i,tx} = \gamma d_i^{\eta} N_0. \tag{2.17}$$

Notice that to communicate at a common rate, each link needs to achieve the same received energy per coded symbol-to-noise ratio. Therefore the transmission energy in each link is adjusted so as to achieve the same amount of received code symbol energy-to-noise ratio

$$E_{i,tx} = \left(\frac{d_i}{d_{max}}\right)^{\eta} E_{tx} \leq E_{tx} \tag{2.18}$$

where $E_{i,tx}$ denotes the transmission energy per coded symbol in link i, and E_{tx} denotes the transmission energy per coded symbol in the link with the largest relay distance $d_{max} = \max_i d_i$. Therefore each link uses a rate R while it adjusts its transmission energy to $\left(\frac{d_i}{d_{max}}\right)^{\eta} E_{tx}$.

1) End-to-end rate: With a fixed rate at each hop, the end-to-end rate in (2.1) is simplified to

$$R_e = \frac{1}{\sum_{i=1}^{k} R^{-1}} = \frac{R}{k} = \frac{C(\gamma)}{k} \tag{2.19}$$

where the factor k^{-1} follows from bandwidth sharing among relays.

2) End-to-end energy-bandwidth tradeoff: With the common rate strategy, the

energy consumption for a given end-to-end rate is

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \sum_{i=1}^{k} \frac{\alpha_i^{\eta}\gamma + \gamma_c}{R}. \tag{2.20}$$

From (2.16) and (2.19), solving for γ in terms of R_e and substituting into (2.20) yields the relation between total energy consumption and end-to-end rate as

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \frac{\gamma \sum_i \alpha_i^{\eta} + k\gamma_c}{C(\gamma)} = \frac{g(kR_e)\sum_i \alpha_i^{\eta} + \gamma_c}{R_e}. \tag{2.21}$$

Using (2.21) we can find the relation between the total energy and the end-to-end rate for the common rate transmission strategy.

3) Problem IIb: Consider the optimal end-to-end rate that minimizes the total energy consumption and the corresponding minimum total energy consumption. Problem IIb for the common rate scheme is the following optimization problem.

Problem IIb for the common rate case:

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \min_{\gamma>0} \quad \frac{\gamma \sum_{i=1}^{k} \alpha_i^{\eta} + k\gamma_c}{C(\gamma)} \tag{2.22}$$

where the optimization variable is γ. From (A.3) the above optimization problem can be converted into an equivalent convex problem with the transformation $x = 1/C(\gamma)$

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \min_{x>0} \quad \left[\left(\sum_{i=1}^{k} \alpha_i^{\eta} \cdot x g(x^{-1})\right) + k\gamma_c x\right]. \tag{2.23}$$

By taking the derivative with respect to x and setting it to 0, we have

$$\frac{g'(x^{-1})}{x} - g(x^{-1}) = \frac{k\gamma_c}{\sum_{i=1}^{k} \alpha_i^{\eta}}. \tag{2.24}$$

Let x^* be the solution to the above equation. Then the optimal end-to-end rate and

the minimum total energy consumption are as follows

$$R_e^* = \frac{1}{kx^*} \tag{2.25}$$

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = g'(1/x^*) \cdot \sum_{i=1}^{k} \alpha_i^{\eta}. \tag{2.26}$$

For the case of the AWGN channel, the optimal solution to (2.24) is given by

$$x^* = \frac{2 \ln 2}{t} \tag{2.27}$$

where

$$t = \mathcal{W}\left(\frac{2k\gamma_c - \sum_{i=1}^{k} \alpha_i^{\eta}}{e \sum_{i=1}^{k} \alpha_i^{\eta}} \right) + 1.$$

Therefore the optimal end-to-end rate and the minimum total energy consumption for the AWGN channel are as follows

$$R_e^* = \frac{1}{kx^*} = \frac{t}{2k \ln 2} \tag{2.28}$$

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \ln 2 \cdot 2^{2/x^*} = \left[\ln 2 \cdot \sum_{i=1}^{k} \alpha_i^{\eta} \right] e^t. \tag{2.29}$$

4) Problem IIb with $\gamma_c = 0$: Now consider minimizing the total energy consumption when the circuit processing energy $\gamma_c = 0$. Problem IIb with $\gamma_c = 0$ is as follows.

Problem IIb for the common rate strategy with $\gamma_c = 0$:

$$\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \min_{R_e > 0} \frac{g(kR_e)}{kR_e} \sum_{i=1}^{k} \alpha_i^{\eta}. \tag{2.30}$$

Note that the right hand side of (2.30) is increasing in $R_e > 0$ and thus the minimum occurs when $R_e \to 0$. Using L'Hôpital's rule, the minimum end-to-end received energy

per information bit is

$$\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \lim_{R_e \to 0} \frac{E_b}{N_0} = g'(0) \sum_{i=1}^{k} \alpha_i^{\eta} \tag{2.31}$$

where notice that for the AWGN channel, $g'(0) = \ln 2$. Note that $\sum_i \alpha_i^{\eta} < 1$ represents the gain over multi-hop transmission. We see that increasing the number of hops, without violating the far-field constraint, is more energy-efficient in terms of the received energy per information bit in the power-limited regime. Comparing the result of (2.31) with the result of (2.26) or (2.29), we identify the penalty in the normalized signal-to-noise ratio resulting from the circuit processing energy consumption.

2.4.2 Performance of the Common Power Strategy

Now consider multi-hop transmission where each node transmits with same energy per coded symbol, *i.e.* $E_{i,tx} = E_{tx}$ for all i while each rate of communication is determined by the received signal-to-noise ratio on each link. The received signal-to-noise ratio is determined by the location of relays. Therefore the common power strategy can be interpreted as an adaptive rate communication strategy. Normalizing by the end-to-end distance d_e and relay distance d_i, and noise power spectral density N_0, we have $\gamma_i \alpha_i^{\eta} = \gamma_j \alpha_j^{\eta}$. Using the minimum SNR from a link with the largest relaying distance, we have

$$\gamma_i = \pi_i \gamma_{min} \geq \gamma_{min} \tag{2.32}$$

$$\pi_i \triangleq \left(\frac{\alpha_{max}}{\alpha_i} \right)^{\eta} \tag{2.33}$$

where γ_{min} is the minimum link SNR and $\alpha_{max} = \max_i \alpha_i$, and π_i denotes the ratio of the maximum relay distance to the i-th relay distance.

1) End-to-end rate: With a common transmission energy at each hop, the rate on

link i is

$$R_i = C(\gamma_i),\ i = 1, ..., k \tag{2.34}$$

where $\gamma_i = E_{tx} d_i^{-\eta}/N_0$. The end-to-end rate in (2.1) is given by

$$R_e = \frac{1}{\sum_i R_i^{-1}} = \left[\sum_{i=1}^{k} \frac{1}{C(\pi_i \gamma_{min})}\right]^{-1} \tag{2.35}$$

where $\gamma_{min} = \frac{E_{tx}}{N_0} d_{max}^{-\eta}$.

2) End-to-end energy-bandwidth tradeoff: The end-to-end energy consumption can be determined from the minimum signal-to-noise ratio.

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \sum_{i=1}^{k} \frac{E_{tx} + E_p}{R_i} \frac{d_e^{-\eta}}{N_0}. \tag{2.36}$$

Substituting the relation of (2.32) into (2.36) yields the relation between total energy consumption and end-to-end rate as

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \sum_{i=1}^{k} \frac{\alpha_{max}^{\eta} \gamma_{min} + \gamma_c}{C(\pi_i \gamma_{min})} = \frac{\alpha_{max}^{\eta} \gamma_{min} + \gamma_c}{R_e}. \tag{2.37}$$

Thus we can parametrically determine the end-to-end rate and the corresponding normalized total energy consumption. As we vary either E_{tx} or γ_{min} the end-to-end rate changes as does the total energy consumption.

3) Problem IIb: Consider the optimal end-to-end rate minimizing the total energy consumption and the corresponding minimum total energy consumption. Problem IIb of the common rate case can be simplified to the following single variable optimization problem.

Problem IIb for the common power case:

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \min_{\gamma_{min}>0} \quad \sum_{i=1}^{k} \frac{\alpha_{max}^{\eta}\gamma_{min} + \gamma_c}{C(\pi_i \gamma_{min})}. \tag{2.38}$$

From (A.3) (2.38) can be converted into an equivalent problem with k variables where a transformation of variables $x_i = C(\gamma_i)$ turns the objective function into a convex function as follows

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \min_{(x_1,..,x_k)} \quad & \sum_{i=1}^{k} \frac{\alpha_i^{\eta} g(x_i) + \gamma_c}{x_i} \\ \text{s.t.} \quad & \alpha_{max}^{\eta} g(x_{min}) = \alpha_i^{\eta} g(x_i). \end{aligned} \tag{2.39}$$

Since the inverse function of channel capacity function is generally not an affine function, the equality constraint is not an affine function which renders (2.39) not a convex problem. However the optimal end-to-end rate and the corresponding minimum total energy consumption can be obtained by numerical techniques. Also note that without the equality constraint, (2.39) becomes Problem Ib, an unconstrained optimization problem. Therefore the minimum total energy consumption of common power scheme is bounded by the result of Problem Ib.

4) Problem IIb with $\gamma_c = 0$: Now consider minimizing the total energy consumption when the circuit processing energy $\gamma_c = 0$. From (2.39), we have

Problem IIb for the common power case with $\gamma_c = 0$:

$$\begin{aligned} \frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \min_{(\gamma_1,..,\gamma_k)} \quad & \sum_{i=1}^{k} \frac{\alpha_i^{\eta} g(x_i)}{x_i} \\ \text{s.t.} \quad & \alpha_{max}^{\eta} g(x_{min}) = \alpha_i^{\eta} g(x_i). \end{aligned} \tag{2.40}$$

It can be easily shown that for any x_i, from the objective function of (2.40), $f(x_i) = g(x_i)/x_i$ is increasing in $x_i \geq 0$ and thus the minimum occurs at $x_i = 0$. To see this,

we need to show that $f'(x_i) > 0$. We can express $f'(x_i)$ in terms of $g(x_i)$ as

$$f'(x_i) = \frac{x_i g'(x_i) - g(x_i)}{x_i^2}. \tag{2.41}$$

To show $f'(x_i) > 0$, it is sufficient to show that $h(x_i) = x_i g'(x_i) - g(x_i)$ from the numerator of $f'(x_i)$ is increasing and $h(0) = 0$. Since $h'(x_i) = x_i g''(x_i) > 0$ and $h(0) = 0$, $f(x_i)$ is increasing in $x_i \geq 0$. Hence the minimum end-to-end received energy per information bit is achieved when $x_i \to 0$ for all i from the equal energy constraint. By taking the limit as $x_i \to 0$ and using L'Hôpital's rule, the minimum end-to-end received energy per information bit is

$$\lim_{R_e \to 0} \frac{E_b}{N_0} = \lim_{x_i \to 0} \sum_{i=1}^{k} \alpha_i^{\eta} \frac{g(x_i)}{x_i} = g'(0) \sum_{i=1}^{k} \alpha_i^{\eta} \tag{2.42}$$

which is same as the result of the common rate in (2.31). Within the far-field region, using multiple hops for low rates is more energy efficient compared to single hop when the circuit processing energy is not taken into account.

2.5 Performance of Equidistant Multi-hop Routing

In this section we derive the optimal end-to-end rate and the optimal number of hops to minimize the overall energy consumption of equi-spaced relays. The optimal solutions are obtained by decoupling the joint optimization problem in rate and number of hops into two sub-problems. We first obtain the optimal number of hops as a function of R_e. Then the optimal value of R_e is derived [18][38]. Finally we derive the solution to Problem IIb for the equi-spaced relays.

Consider the case of equi-spaced relays so that $\alpha_i = 1/k$. In this case the common rate strategy and the common power strategy are identical. The end-to-end rate R_e is

then

$$R_e = \frac{R}{k} = \frac{C(\gamma)}{k} \tag{2.43}$$

where γ denotes the link SNR. From (2.2) and (2.43), the total energy consumption per information bit is

$$\frac{E_{tot}(k, R_e)}{N_0} = \frac{k^{-\eta} g(kR_e) + \gamma_c}{R_e}. \tag{2.44}$$

With a transformation of variables $e^{-x} = k$ and $R_e^{-1} = e^y$, the minimum total energy consumption can be converted into an equivalent optimization problem

$$\frac{E_{tot}(x^*, y^*)}{N_0} = \min_{y \in \mathbf{R}, x \in \mathbf{R}} e^y \left[e^{\eta x} g(e^{-x-y}) + \gamma_c \right]. \tag{2.45}$$

Proposition 2.1. The objective function $E_{tot}(x, R_e)/N_0$ is convex in x, R_e for $R_e > 0$ and $x \in \mathbf{R}$ if $g(u)$ satisfies the following conditions

1. $a' = u^2 g''(u) - u g'(u) + g(u) > 0$.
2. $a'c' - (b')^2 > 0$ where $b' = \eta u g'(u) - u^2 g''(u) - n g(u)$ and $c' = u^2 g''(u) + \eta^2 g(u) + (1 - 2\eta) u g'(u)$.

Proof: See Appendix A.4. ■

Therefore, if the sufficient conditions are satisfied, the optimization problem is a convex problem and thus the locally optimal solution is also globally optimal. Let $\hat{x}^*$ and $\hat{y}^*$ be the locally optimal solution to (2.45). The locally optimal solutions can provide an upper bound to the global optimum

$$\frac{E_{tot}(x^*, y^*)}{N_0} \leq \frac{E_{tot}(\hat{x}^*, \hat{y}^*)}{N_0} \tag{2.46}$$

where the equality holds if the convexity is guaranteed. For the case of the AWGN

channel, it can be easily verified that the objective function in (2.45) is convex (see appendix A.4). Other channels of interest are the binary input AWGN channel and the binary input hard decision AWGN channel whose channel capacity functions are given by respectively [39]

$$C_{BISO}(\gamma) = 1 - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-2\sqrt{2\gamma})^2}{2}} \log_2(1 + e^{-2x\sqrt{2\gamma}}) dx$$

and

$$C_{BIBO}(\gamma) = 1 - H_2(Q(\sqrt{2\gamma}))$$

where $H_2(x) = -x\log_2(x) - (1-x)\log_2(1-x)$ is the binary entropy function. Numerical methods for the above two channels can verify that the sufficient conditions for problem to be convex are valid.

2.5.1 Problem IIIa: Optimization of the number of hops

For a given end-to-end rate, treating the number of hops as a continuous variable, we optimize the total energy consumption over x

$$\frac{E_{tot}(\hat{x}^*, y)}{N_0} = \min_{x \in \mathbf{R}} e^y \left[e^{\eta x} g(e^{-x-y}) + \gamma_c \right]. \tag{2.47}$$

Setting the derivative with respect to x equal to zero yields $\eta g(e^{-x-y}) = e^{-x-y} g'(e^{-x-y})$. Let $\hat{u}^*$ be the solution to

$$\eta g(u) = u g'(u). \tag{2.48}$$

The optimal number of hops for a given end-to-end rate is

$$\hat{k}^* = e^{-x^*} = e^y \hat{u}^* = \frac{\hat{u}^*}{R_e}. \tag{2.49}$$

The minimum of the total energy consumption at the optimal number of hops is given by

$$\frac{E_{tot}(\hat{x}^*, y)}{N_0} = g(\hat{u}^*)(\hat{u}^*)^{-\eta} e^{(1-\eta)y} + \gamma_c e^y \tag{2.50}$$

$$= g(\hat{u}^*)(\hat{u}^*)^{-\eta} R_e^{\eta-1} + \gamma_c R_e^{-1}. \tag{2.51}$$

2.5.2 Problem IIIb: Optimization of the end-to-end rate

The problem of determining the optimal end-to-end rate is formulated by substituting (2.49) into (2.45). This results in

$$\frac{E_{tot}(\hat{x}^*, \hat{y}^*)}{N_0} = \min_y \min_x e^y \left[e^{\eta x} g(e^{-x-y}) + \gamma_c \right] \tag{2.52}$$

$$= \min_y \left[g(\hat{u}^*)(\hat{u}^*)^{-\eta} e^{(1-\eta)y} + \gamma_c e^y \right].$$

Setting the derivative with respect to y equal to zero yields the optimal end-to-end rate $\hat{R}_e^* = e^{-\hat{y}^*}$ as follows

$$e^{-\hat{y}^*} = \hat{R}_e^* = \frac{\gamma_c^{1/\eta} \hat{u}^*}{\left[(\eta - 1) g(\hat{u}^*)\right]^{1/\eta}}. \tag{2.53}$$

The optimal number of hops $\hat{k}^*(\hat{R}_e^*)$ is obtained by substituting (2.53) into (2.49),

$$\hat{k}^*(\hat{R}_e^*) = g(\hat{u}^*)^{1/\eta} \left[\frac{\gamma_c}{\eta - 1} \right]^{-1/\eta}$$

$$= g(\hat{u}^*)^{1/\eta} \left[\frac{E_p}{N_0} \frac{1}{\eta - 1} \right]^{-1/\eta} d_e. \tag{2.54}$$

The resulting minimum total energy consumption is then

$$\begin{aligned}\frac{E_{tot}(\hat{x}^*, \hat{R}_e^*)}{N_0} &= \frac{g(\hat{u}^*)^{1/\eta}}{\hat{u}^*}\eta\left[\frac{\gamma_c}{\eta-1}\right]^{1-1/\eta} \\ &= \frac{g(\hat{u}^*)^{1/\eta}}{\hat{u}^*}\left[\frac{\eta}{(\eta-1)^{1-1/\eta}}\right]\left[\frac{E_p}{N_0}\right]^{1-1/\eta} d_e^{1-\eta} \\ &= f_c(\eta)\left[\frac{E_p}{N_0}\right]^{1-1/\eta} d_e^{1-\eta}\end{aligned} \tag{2.55}$$

where $f_c(\eta)$ is a function of the channel and the propagation loss exponent. From the above results we observe that the optimal rate decreases as d_e^{-1} and the optimal number of hops grows linearly with d_e. In addition, there are two key observations for the minimum normalized total energy consumption: it (i) grows as the $\frac{\eta-1}{\eta}$ power of the processing energy and (ii) decreases as $d_e^{1-\eta}$. Therefore we can conclude that the actual unnormalized minimum total energy consumption increases linearly with d_e from the relation between the normalized total energy consumption and the end-to-end distance in (2.2).

For the case of the AWGN channel, it can be easily verified that the objective function in (2.45) is convex (see Appendix A.4). Therefore the locally optimal solution is indeed the globally optimal solution. The optimal solution to (2.48) is $u^* = a/2\ln 2$ where $a = \mathcal{W}(-\eta e^{-\eta}) + \eta$ which is determined from the path-loss exponent. Therefore the optimal number of hops $k^*(R_e^*)$ is

$$k^*(R_e^*) = \left[\frac{ae^a}{2\eta}\right]^{1/\eta}\left[\frac{\gamma_c}{\eta-1}\right]^{-1/\eta} \tag{2.56}$$

and the optimal end-to-end rate R_e^* is given by

$$R_e^* = \frac{a}{2\ln 2}\left[\frac{2\eta}{ae^a}\right]^{1/\eta}\left[\frac{\gamma_c}{\eta-1}\right]^{1/\eta}. \tag{2.57}$$

The minimum total energy consumption found by optimizing over the number of hops

and the end-to-end rate is then

$$\begin{aligned}\frac{E^*_{tot}(k^*, R^*_e)}{N_0} &= \frac{2\ln 2}{a}\left[\frac{ae^a}{2\eta}\right]^{1/\eta}\eta\left[\frac{\gamma_c}{\eta-1}\right]^{1-1/\eta} \\ &= \frac{2\ln 2}{a}\left[\frac{ae^a}{2\eta}\right]^{1/\eta}\frac{\eta}{(\eta-1)^{1-1/\eta}}\left[\frac{E_p}{N_0}\right]^{1-1/\eta} d_e^{1-\eta}.\end{aligned} \tag{2.58}$$

Notice that the optimal energy consumption constant

$$f_c(\eta) = \frac{g(\hat{u}^*)^{1/\eta}}{\hat{u}^*}\frac{\eta}{(\eta-1)^{(1-1/\eta)}} \tag{2.59}$$

from (2.55) depends on the channel capacity function. For different types of AWGN channels the resulting energy consumption constants are given in Table 2.1 along with the loss incurred relative to the AWGN channel.

2.5.3 Problem IVa: Optimization of the end-to-end rate

Now consider optimizing over the rate first and then the number of hops using the previous results for common rate and common power strategies because they are equivalent for equi-spaced relays. The result gives the minimum total energy consumption per information bit as

$$\begin{aligned}\frac{E_{tot}(k, R^*_e)}{N_0} &= \min_{v>0}\left[k^{1-\eta}vg(v^{-1}) + k\gamma_c v\right] \\ &= k^{1-\eta}g'(1/\hat{v}^*).\end{aligned} \tag{2.60}$$

where $\hat{v}^*$ is the solution to $v^{-1}g'(v^{-1}) - g(v^{-1}) = k^\eta\gamma_c$ and $v = 1/C(\gamma)$. The corresponding optimal rate for k equi-spaced relays is given by

$$\hat{R_e}^* = \frac{1}{k\hat{v}^*}. \tag{2.61}$$

For the case of the AWGN channel where the locally optimum is the global optimum,

the optimal rate is given by

$$R_e^* = \frac{1}{2k \ln 2} \left[\mathcal{W}\left(\frac{2\gamma_c k^\eta - 1}{e} \right) + 1 \right].$$

and the resulting minimum energy consumption is

$$\frac{E_{tot}(k, R_e^*)}{N_0} = k^{1-\eta} \ln 2 \exp\left(\mathcal{W}\left(\frac{2\gamma_c k^\eta - 1}{e} \right) + 1 \right). \tag{2.62}$$

When the circuit processing energy is ignored, $\gamma_c = 0$, from (2.43) and (2.44), we have

$$\left(\frac{E_{tot}}{N_0} \right)_{\gamma_c = 0} = \frac{E_b}{N_0} = \frac{g(kR_e)}{kR_e} k^{1-\eta} \tag{2.63}$$

which is an increasing function of R_e. Therefore the minimum end-to-end energy received per information bit for a given number of hops is given by

$$\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} = \lim_{R_e \to 0} \frac{E_b}{N_0} = k^{1-\eta} g'(0) \tag{2.64}$$

which can be also verified in (2.60) for $x \to \infty$. Note that from (2.64), multiple hop routing can provide more energy-efficiency for low rates than single hop routing. However (2.63) is increasing in k for high rate that implies a single hop transmission is the most energy-efficient.

2.5.4 Problem IVb: Optimization of the number of hops

Now we further optimize the result of problem IVa over the number of hops similar to problem III. Since $v^{-1} g'(v^{-1}) - g(v^{-1}) = k^\eta \gamma_c$, by setting the derivative of (2.60)

with respect to k equal to zero, we obtain

$$\hat{k}^* = \left[\frac{(\eta-1)\hat{v}^* g(1/\hat{v}^*)}{\gamma_c}\right]^{1/\eta}. \tag{2.65}$$

Substituting the above result into (2.60) yields

$$\frac{E_{tot}(k^*, R_e^*)}{N_0} = \left[\frac{\gamma_c}{\eta-1}\right]^{1-1/\eta} [\hat{v}^* g(1/\hat{v}^*)]^{1/\eta}\, g'(1/\hat{v}^*). \tag{2.66}$$

For the AWGN channel since $\frac{\partial \mathcal{W}(x)}{\partial x} = \frac{\mathcal{W}(x)}{x(\mathcal{W}(x)+1)}$, by setting the derivative of (2.62) with respect to k equal to zero, we obtain the optimal number of hops as

$$k^* = \left(\frac{b}{2\gamma_c}\right)^{1/\eta} \tag{2.67}$$

where b is the solution to

$$\frac{\eta-1}{\eta} = \frac{b\mathcal{W}(\frac{b-1}{e})}{(b-1)(\mathcal{W}(\frac{b-1}{e})+1)}. \tag{2.68}$$

Note that, as expected, the optimal number of hops varies inversely with the processing energy at each hop. Then substituting (2.67) into (2.62) yields

$$\frac{E_{tot}^*(k^*, R_e^*)}{N_0} = \ln 2 \left(\frac{2}{b}\right)^{\frac{\eta-1}{\eta}} e^{1+\mathcal{W}(\frac{b-1}{e})} (\gamma_c)^{\frac{\eta-1}{\eta}}. \tag{2.69}$$

By using numerical methods we can identify that the above results for the optimal number of hops and the minimum energy consumption are same as the results of (2.56) and (2.58).

2.6 Numerical Results

In the results we assume the path-loss exponent $\eta = 4$, end-to-end distance $d_e = 3000$m, noise power spectral density $N_0 = -174$dBm/Hz and circuit processing energy $E_p = 0.95\mu$J/symbol ($\gamma_c = 4.69$dB) and 0.095μJ/symbol ($\gamma_c = -5.31$dB) unless otherwise specified.

In Fig. 2.2 we plot the normalized energy-bandwidth tradeoff for a two hop network with the ratio of link distances equal to 99. This is done for the optimal scheme obtained from the solution of Problem Ia as well as for the common rate transmission scheme from (22) and common power transmission scheme from (34) and (36). At low rates the energy consumption due to the transmitter alone decreases because of increasing error protection with lower rate codes, while the overall energy consumption increases due to the energy needed for processing. At high rates the energy consumption increases due to the increase in required transmitted energy for high rate communication. Because of this there is an optimal nonzero rate that minimizes the total energy consumption. These curves also show that the common power transmission strategy is nearly as good as the optimal transmission scheme whereas the common rate scheme requires significantly higher energy. One explanation of the reason common power is significantly better than common rate is that with a common power scheme each node can adjust the rate of communications as well as the amount of time each link is scheduled for communication. With common rate, however, each node uses the same rate of communication and as a result has the same amount of time allocated for transmission. So common power with adaptive rate has more flexibility than common rate with adaptive power.

In Fig. 2.3 we plot the normalized energy-bandwidth tradeoff for $E_p = 0.095\mu$J/symbol. As the circuit processing energy becomes small, the optimal rate for minimum total energy consumption becomes small. The energy-bandwidth tradeoff shows similar performance without circuit processing energy because the transmission energy is more dominant at low processing energy. In Fig. 2.4 the normalized energy-bandwidth char-

acteristic without processing energy is shown. The same minimum energy consumption is achieved when $R_e \to 0$ from the solution of Problem Ib in (2.15), the common rate scheme in (2.31) and the common power scheme in (2.42).

Fig. 2.5 depicts the normalized energy-bandwidth tradeoff for various AWGN channel models for a two hop network where the ratio of relay distances is equal to 99. This is done by applying the procedures to find the solution of Problem Ia to the binary input with hard decisions AWGN channel (BIBO AWGN), the binary input AWGN channel (BISO AWGN), and the unquantized input/ouput AWGN channel (SISO AWGN). As expected it is observed that AWGN channels with input constraint show limited bandwidth utilization. The end-to-end rate of input constrained AWGN channels is bounded at 0.5 (bits/channel use) in a two hop network. In addition, the total energy consumption of input constrained AWGN channels shows a degraded performance compared to the unquantized AWGN channel because of their limited bandwidth utilization. For this case the loss by using binary input is about 0.83dB in energy use and a 40% reduction in the optimal data rate. For binary input and hard decisions there is an additional loss in energy of about 0.91dB in energy but only a very slight additional loss in optimal data rate.

Fig. 2.6 plots the normalized energy-bandwidth tradeoff with various locations of relays. The common power case outperforms the common rate case by allowing adaptive rate in each link. In particular the common power case with an irregular distribution of relays may outperform the equi-spaced relays case at high rates. This is because the gain from the irregular distribution of relays at high SNR dominates the performance. It implies that the equi-spaced relay case is not always an optimal route for a given number of hops, which further suggests to reduce the bandwidth sharing at high SNR. Therefore at high rates a single hop transmission shows the best performance while the minimum energy consumption is achieved at the equi-spaced relays. In Fig. 2.7 the normalized energy-bandwidth characteristic for various locations of relays without

processing energy is plotted. As expected multi-hop transmissions within the far-field region is more energy-efficient at low rates without considering processing energy.

To observe how relays can affect the energy-bandwidth tradeoff, a wireless network with 3 relays is considered as an example where the locations of the nodes are given in Fig. 2.1. Using the solution of Problem Ia, we plot the performance of all possible multihop routes for two circuit processing energy values in Fig. 2.8 and Fig. 2.9 where the numbers in parenthesis denote the selected node(s) for multihop relays. It is observed from these figures that when $\gamma_c = 4.69$dB, optimal routing is accomplished with two hops using node 3 as a relay while when $\gamma_c = -5.31$dB, optimal routing is done with four hops using all available nodes as relays. We calculate the energy-rate tradeoff for all sensible routes (that is, we do not consider routes that provide negative progress in reaching the destination such as S $\rightarrow$ 4$\rightarrow$1$\rightarrow$ D). At high rates we find that fewer hops require less energy than more hops while at low rates the reverse is true. Currently we do not know of an algorithm that can find the optimal route for a given rate in polynomial time with regard to the number of nodes. However, the insight into which routing strategy works best at which rates is useful. We can determine optimal routes provided that the network has linear, equi-spaced nodes as discussed next.

In Fig. 2.10 and Fig. 2.11 the normalized energy-bandwidth characteristic for the equi-spaced relays from the result of (2.51) is shown. The analytical result treating k as a continuous variable gives a very good approximation to the minimum energy as a function of rate. These figures also illustrate the result that at high rates fewer hops should be used whereas at low rates a larger number of hops is better. In Fig. 2.12 we plot the result of (2.51) and the minimum energy consumption for a single and a two hop network for comparison. From this figure we can see that there is a region of rates around $R_e = 2$ whereby the use of time sharing between single hop and multi-hop transmissions can improve the performance slightly (about 0.2dB). Time sharing does not help significantly at other rates as can be seen from the previous figures.

Fig. 2.13 and Fig. 2.14 compare the normalized energy-bandwidth tradeoff for various AWGN channel models from the result of (2.51). Since we approximated k by a continuous variable, the optimal number of hops could be less than 1 at high rates which is impossible practically. The curves for single hop case, $k = 1$ are also plotted respectively for BIBO and BISO AWGN channels to show where the continuous k approximation breaksdown. The threshold end-to-end rate where a single hop is the most energy-efficient is 0.907, 0.894 and 2.856 for BIBO, BISO and SISO AWGN channel respectively. This threshold does not depend on γ_c. The energy loss incurred by using binary input for the case of a linear network with the optimal number of hops is about 2.2dB with a reduction of the rate by about a 40%. This is roughly the same for different values of γ_c. For binary input hard decisions there is an additional loss of about 0.3 dB and a slight additional reduction in the optimal rate (less than 7%). Fig. 2.15 depicts the energy-bandwidth characteristic for the equi-spaced relays with selected end-to-end distances and number of hops. At low end-to-end rates the transmission energy consumption is $k^{1-\eta} \ln 2$ which is negligible compared to the receiver processing energy consumption. Hence from (2.44) the energy consumption is dominated by the receiver energy consumption. For low end-to-end rates the receiver energy depends on the end-to-end distance but not the number of hops. However, at high rates the receiver energy consumption is negligible compared to the transmission energy consumption. Therefore, the energy consumption at high rates depends on the number of hops but not the end-to-end distance.

Finally consider a source, destination and a single relay. We have shown that when the three nodes are located on a straight line at high rates the single hop routing strategy is better than a two hop strategy while at low rates the two hop strategy is more energy efficient. However, when the nodes are not collinear then the additional distance for a two hop strategy may require more energy than a single hop strategy. In Fig. 2.16 we plot the feasible region of relay locations for selected end-to-end rates

where the minimum energy consumption of a two hop network for a given end-to-end rate is less than that of a single hop network using the result of (2.5) and (2.6). As the end-to-end rate increases, the feasible region of relays shrinks. At high rates a two hop network needs significantly more *received* energy because each link must operate at twice the rate of a single hop route to have the same end-to-end data rate. This is partially compensated for by the decrease in distance between nodes when relaying. However the decrease in transmitted power with distance is an inverse power relation whereas the increase in received power is an exponential relation at sufficiently high rates. Hence the increase in required received power will dominate the decrease in power due to a smaller distance when relaying.

Observe that in the region where a two hop network is more energy efficient than a single hop network, the optimal distribution of relays is that of equi-distant relays. Note that the processing energy and the end-to-end distance, *i.e.* γ_c do not affect the region for a given end-to-end rate. However in terms of the minimum total energy consumption over the end-to-end rate, the feasible region depends on the processing energy and distance through γ_c. This is because the gain from multi-hop routing decreases as the circuit energy consumption increases. In Fig. 2.17 we plot the feasible region of relay locations for selected receiver processing energies where two hop routing is better than single hop in terms of the minimum total energy consumption at the optimal end-to-end rate. As explained the feasible region of two hop network becomes smaller as γ_c increases.

2.7 Conclusion

In this chapter, we considered interference-free communication through multi-hop routing where each relay is not necessarily placed on a line between a single source-destination pair. We formulated the normalized total energy consumption and the end-

to-end bandwidth utilization to transmit information bits for the optimal transmission energy and rate on each hop and two practical communication schemes: (1) common rate scheme and (2) common power scheme. The results showed that the common power case yields better energy-bandwidth tradeoff than the common rate case due to the rate adaptation in each link, while both cases have the same asymptotic performance in the power-limited regime when the circuit processing energy is not taken into account. In addition, we showed that an irregular distribution of relays with common power scheme can outperform equi-spaced relays at high end-to-end rates, which implies reducing the number of hops is more energy-efficient. We also showed that the total energy consumption can be minimized by optimally choosing the rate, determined from the location of relays and the end-to-end distance. Therefore, by comparing the total energy consumption for different location of relays, a routing path which achieves the best energy-bandwidth tradeoff can be obtained. In addition, we showed that by optimizing jointly over the end-to-end rate and number of hops for equi-spaced relays, the optimal number of hops and the optimal end-to-end rate can be found. In the future, more practical considerations such as spatial reuse and cooperative strategies or multiple source-destination pairs should be taken into account.

Table 2.1: Optimal energy consumption constants for various AWGN channels

Type	Energy constant			Loss (dB)		
	$\eta = 2$	$\eta = 3$	$\eta = 4$	$\eta = 2$	$\eta = 3$	$\eta = 4$
SISO	2.43	1.85	1.38	-	-	-
BISO	2.76	2.55	2.28	-0.55	-1.39	-2.18
BIBO	3.26	2.82	2.44	-1.27	-1.83	-2.47

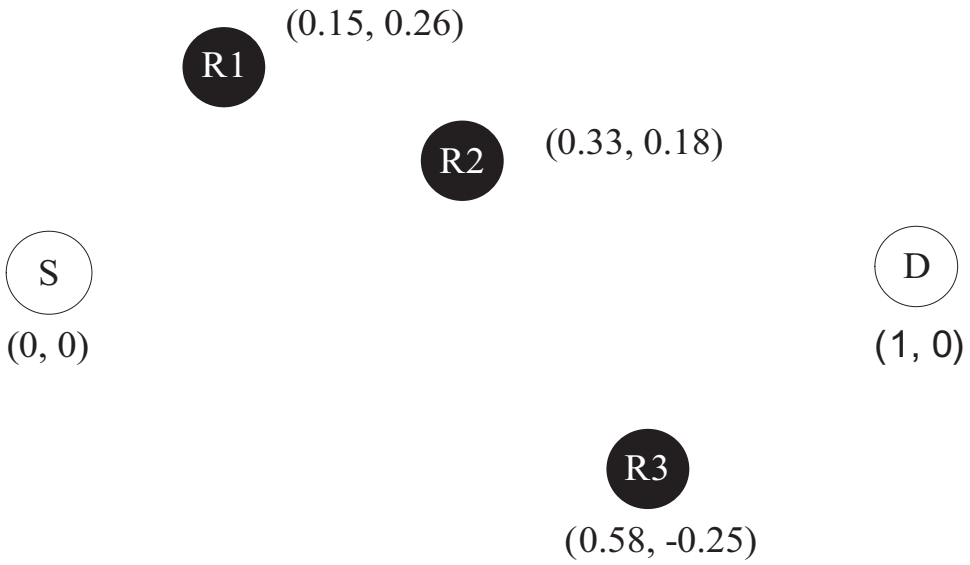


Figure 2.1: Illustration of 2-dimensional network model where distance between any two nodes denotes the normalized relay distance by the end-to-end distance.

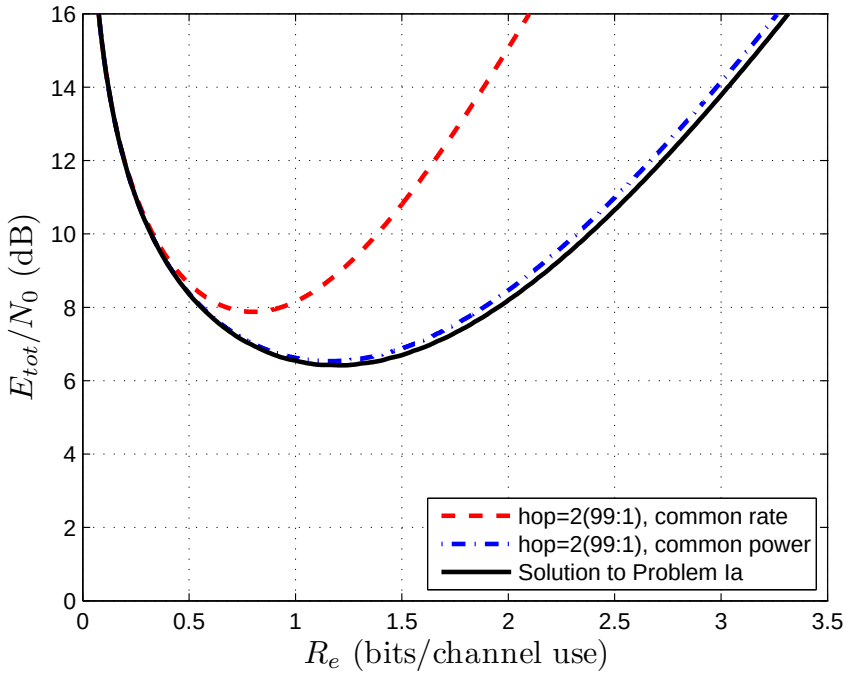


Figure 2.2: Normalized energy-bandwidth characteristic when $\gamma_c = 4.69$dB for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m.

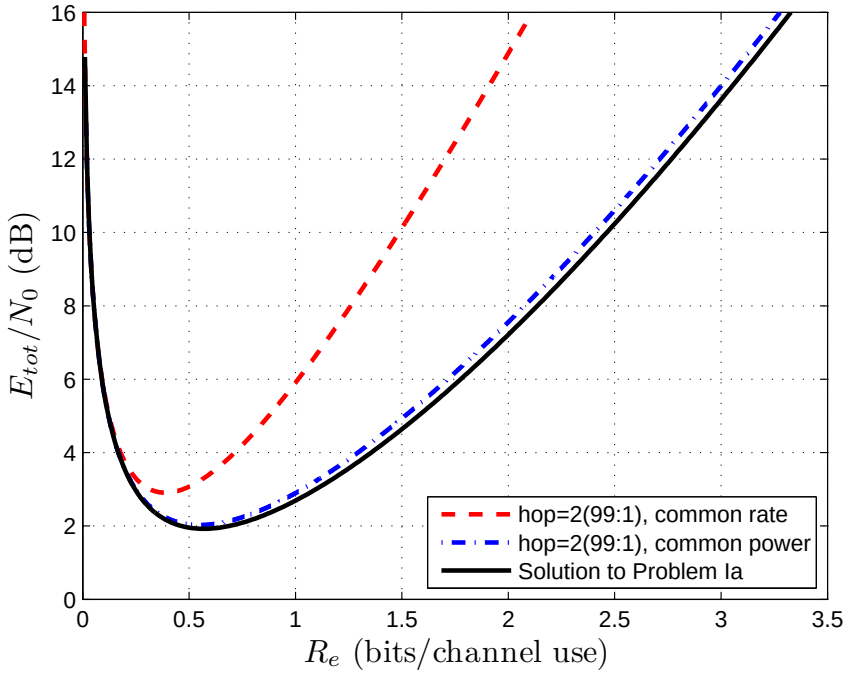


Figure 2.3: Normalized energy-bandwidth characteristic when $\gamma_c = -5.31$dB for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m.

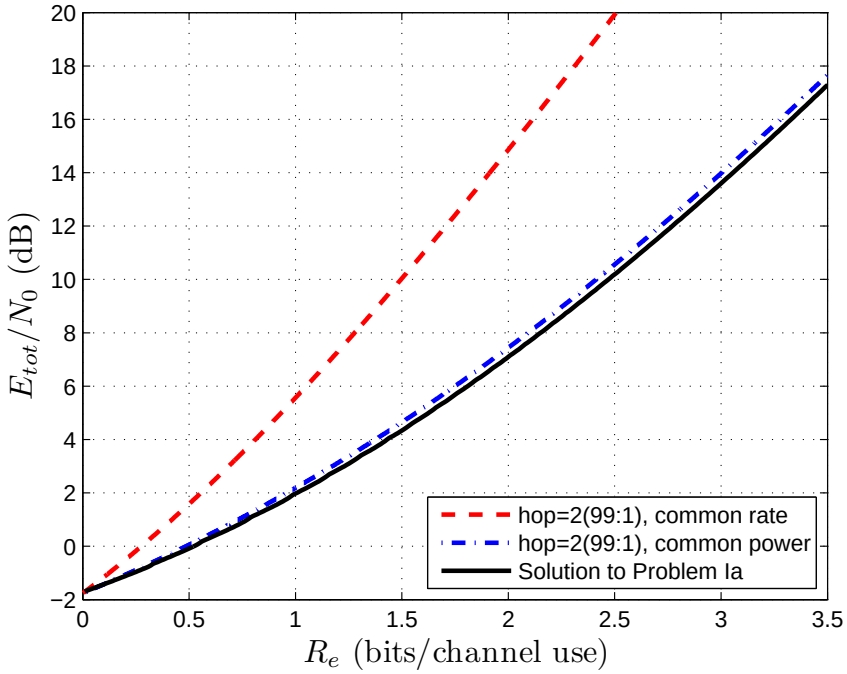


Figure 2.4: Normalized energy-bandwidth characteristic when $\gamma_c = 0$ for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m.

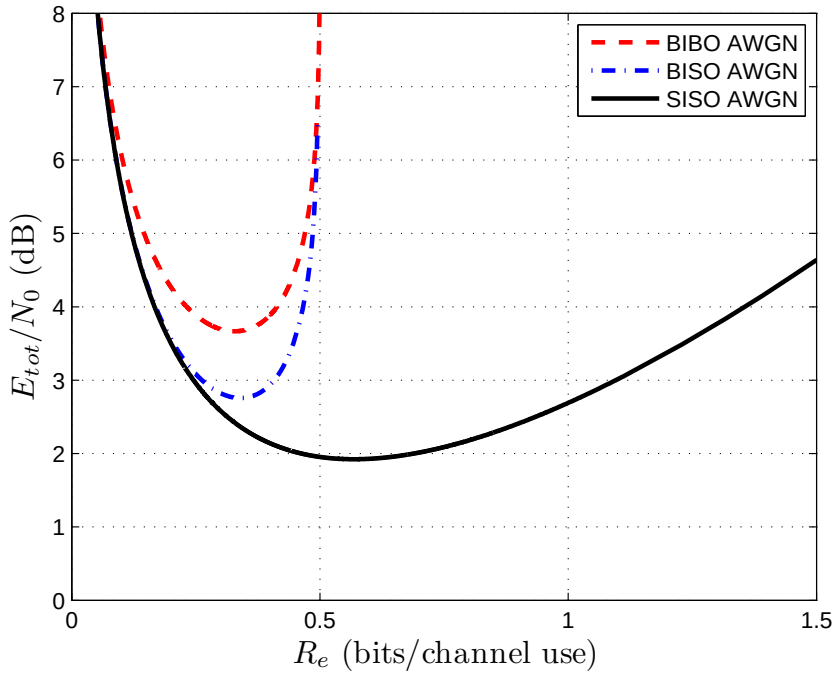
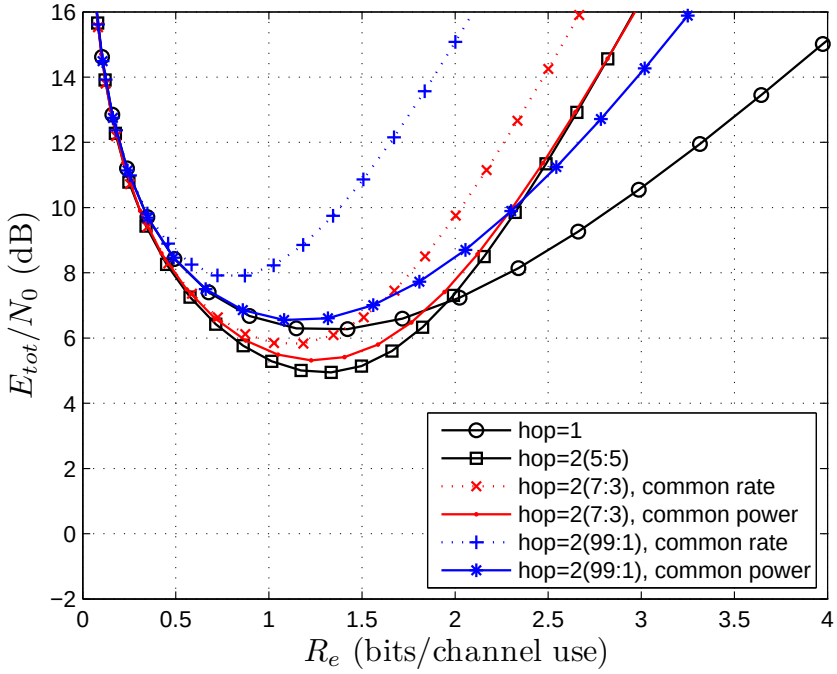
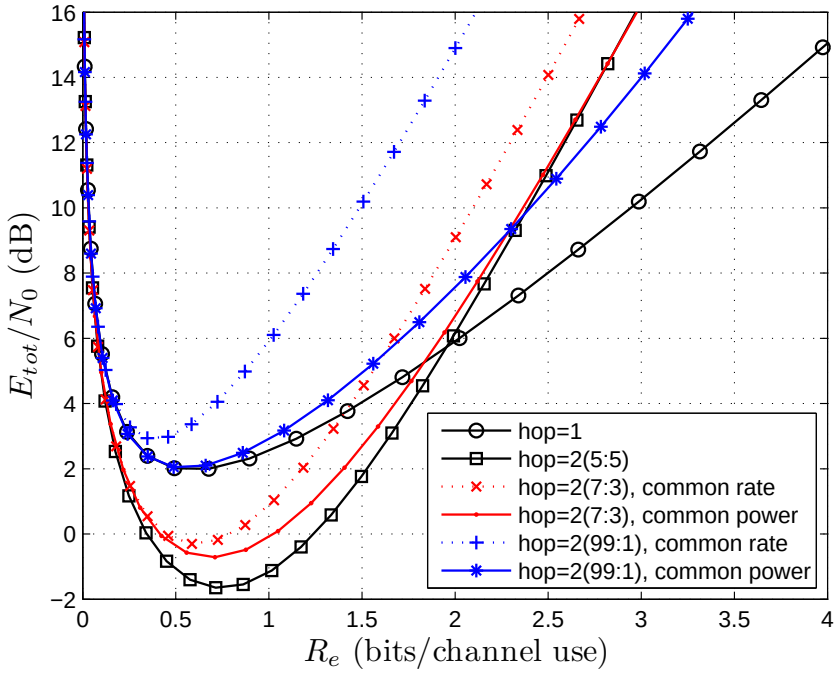


Figure 2.5: Normalized energy-bandwidth characteristic for various AWGN channel models when $\gamma_c = -5.31$dB for a multi-hop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 99 : 1$ at d_e=3000m.



(a) Normalized total energy consumption per information bit vs end-to-end rate with $\gamma_c = 4.69$dB



(b) Normalized total energy consumption per information bit vs end-to-end rate with $\gamma_c = -5.31$dB

Figure 2.6: Energy-bandwidth characteristic when $\gamma_c \neq 0$ with single hop ($k = 1$) and multi-hop ($k = 2$) for selected location of relays, $\alpha_1 : \alpha_2 = 5 : 5, 7 : 3$, and $99 : 1$ at d_e=3000m.

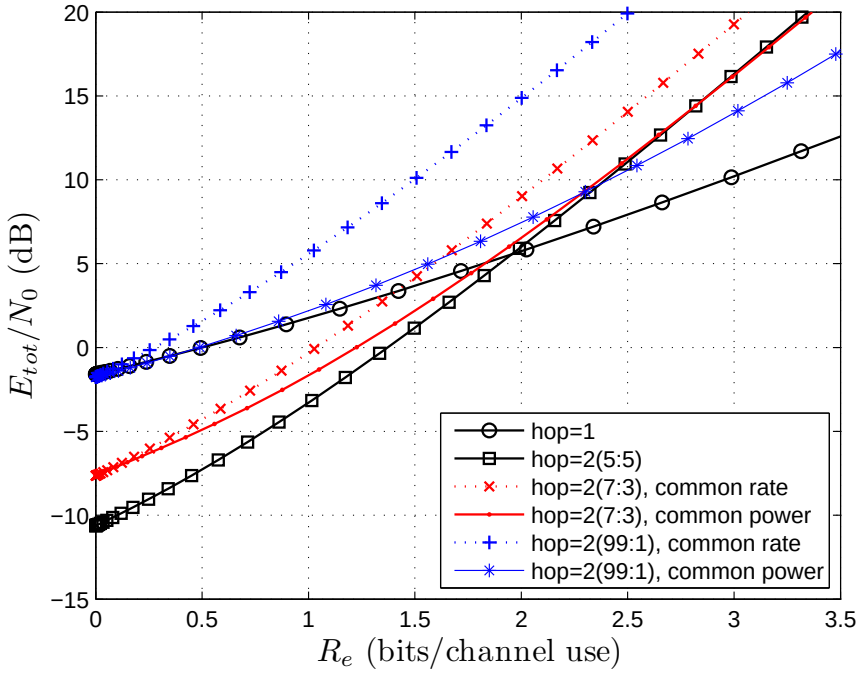


Figure 2.7: Normalized energy-bandwidth characteristic when $\gamma_c = 0$ with $k = 1$ single hop and $k = 2$ multi-hops for selected location of relays, $\alpha_1 : \alpha_2 = 5 : 5, 7 : 3$, and $99 : 1$ at d_e=3000m.

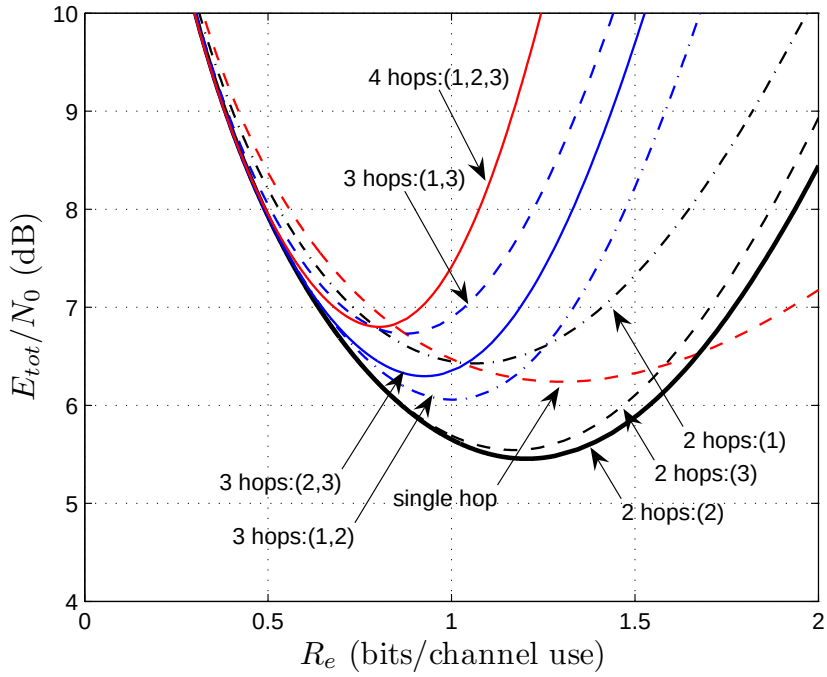


Figure 2.8: Normalized energy-bandwidth characteristic of 3 relays over AWGN channels when $\gamma_c = 4.69$dB at d_e=3000m.

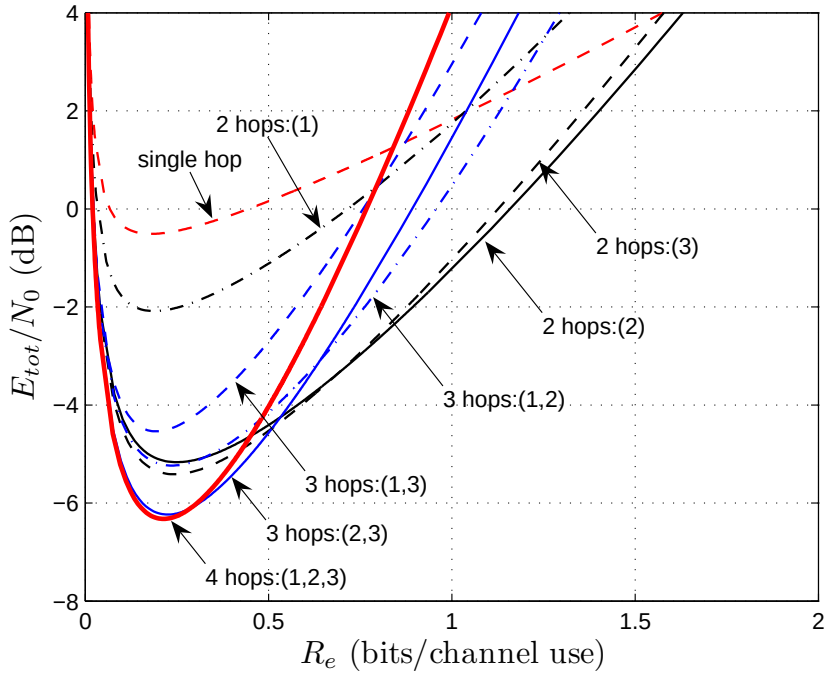


Figure 2.9: Normalized energy-bandwidth characteristic of 3 relays over AWGN channels when $\gamma_c = -17.36$dB at d_e=3000m.

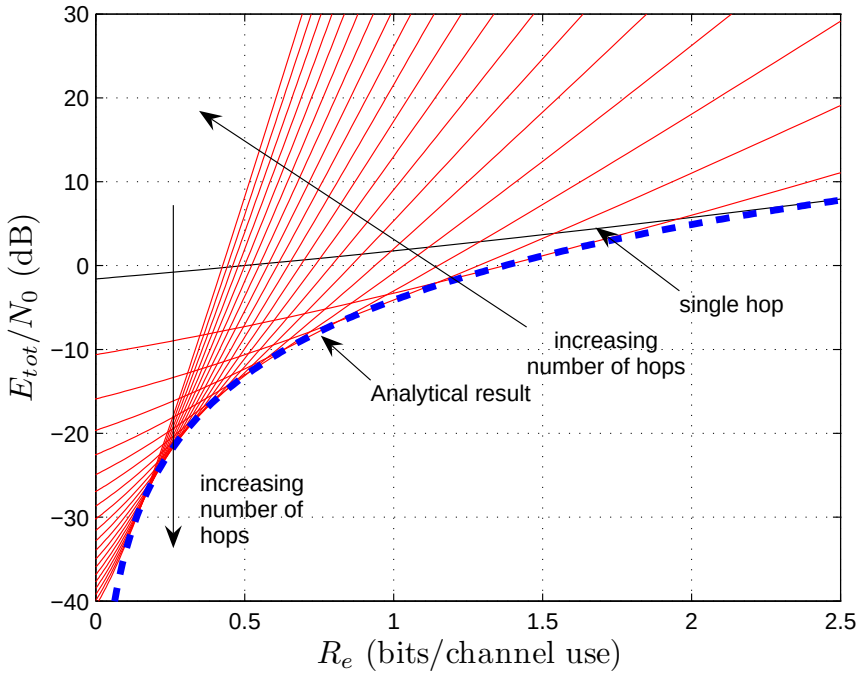


Figure 2.10: Normalized energy-bandwidth characteristic when $\gamma_c = 0$ for equi-spaced relays with $k = 1, ..., 20$, and the analytical result in (2.51) at d_e=3000m.

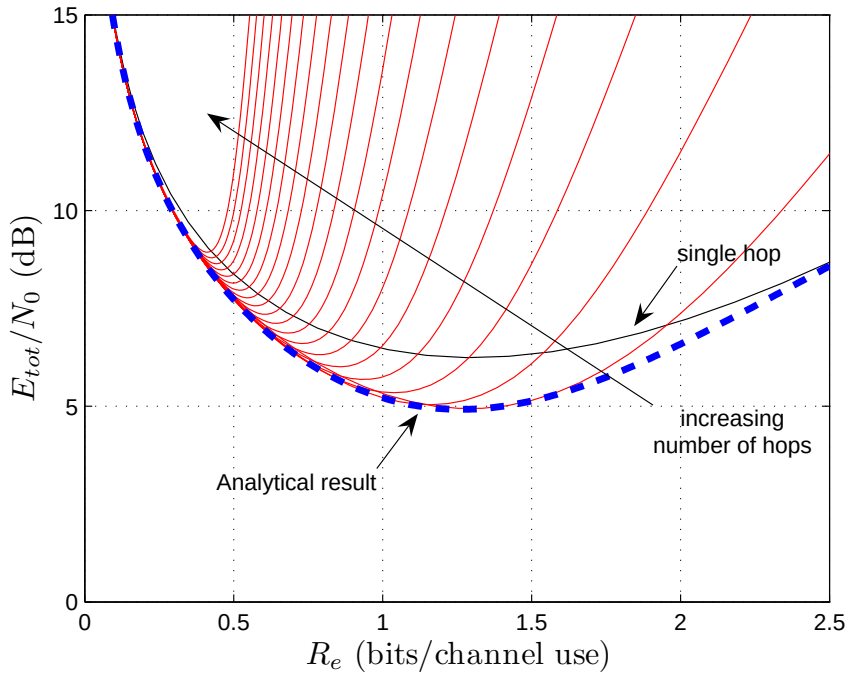


Figure 2.11: Normalized energy-bandwidth characteristic when $\gamma_c = 4.69$dB for equi-spaced relays with $k = 1, ..., 20$, and the analytical result in (2.51) at d_e=3000m.

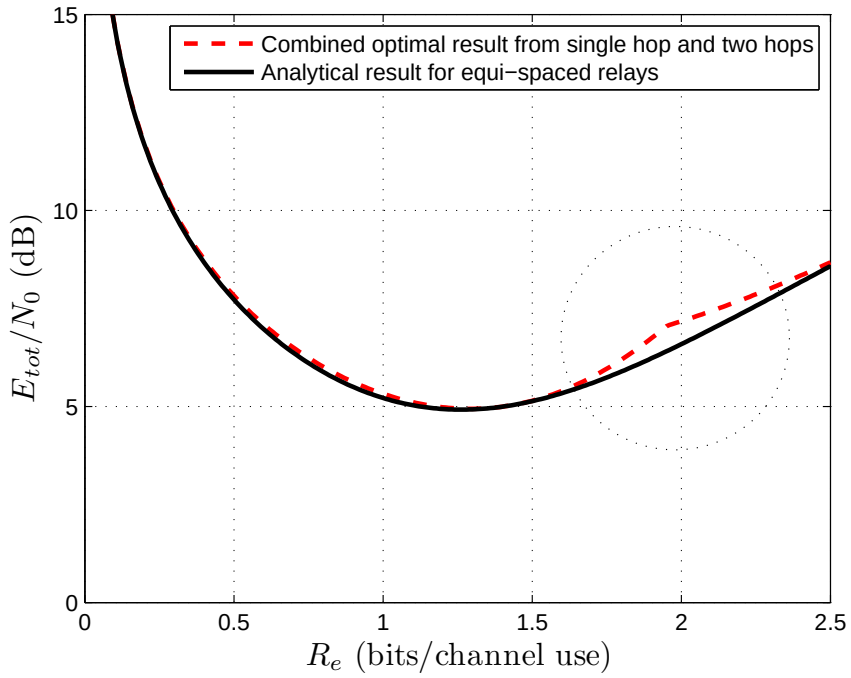


Figure 2.12: Energy-bandwidth characteristic comparison between the numerical optimization and equi-spaced relays case when $\gamma_c = 4.69$dB.

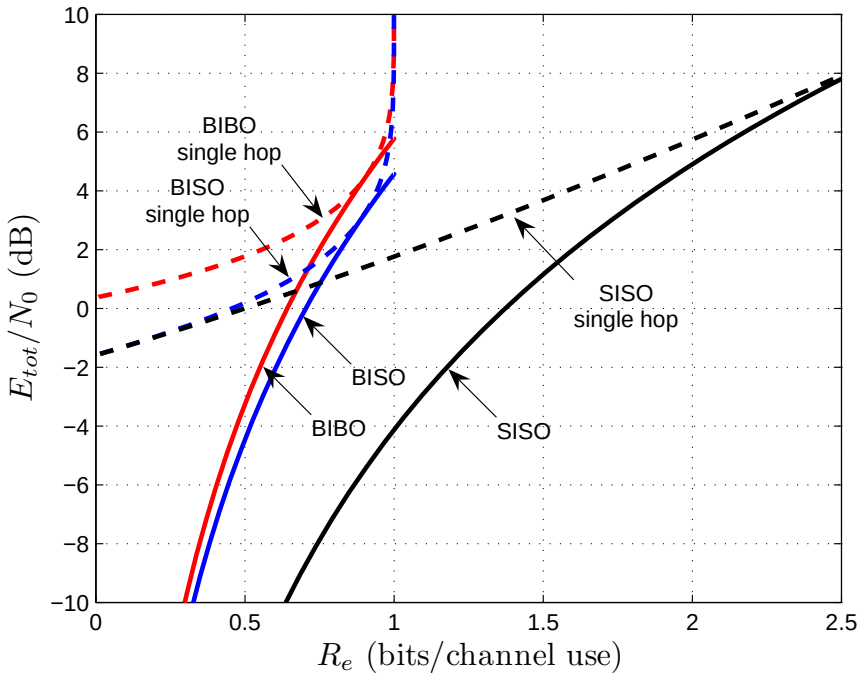


Figure 2.13: Normalized energy-bandwidth characteristic of the analytical result in (2.51) with the single hop case when $\gamma_c = 0$ for various AWGN channels at d_e=3000m.

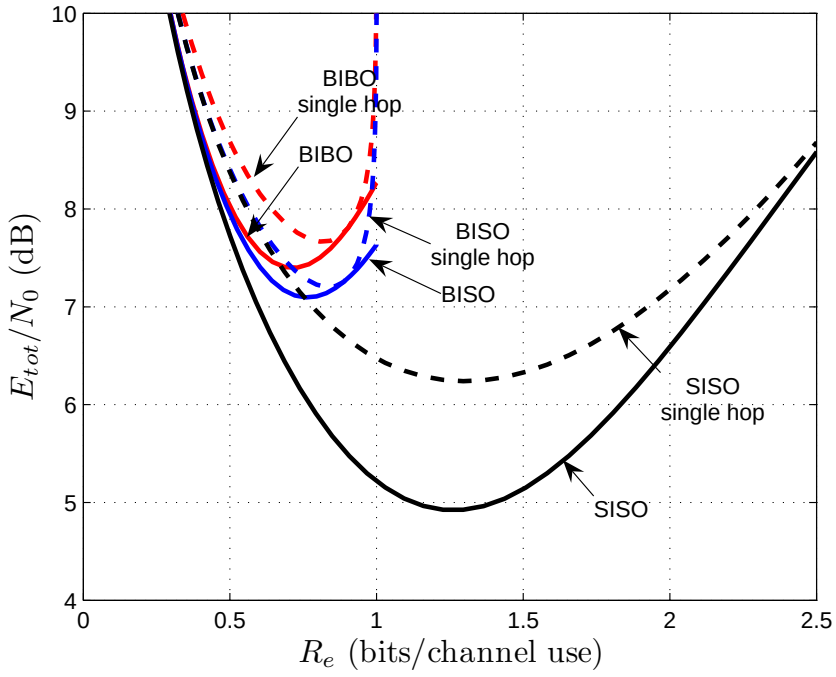


Figure 2.14: Normalized energy-bandwidth characteristic of the analytical result in (2.51) with the single hop case when $\gamma_c = 4.69$dB for various AWGN channels at d_e=3000m.

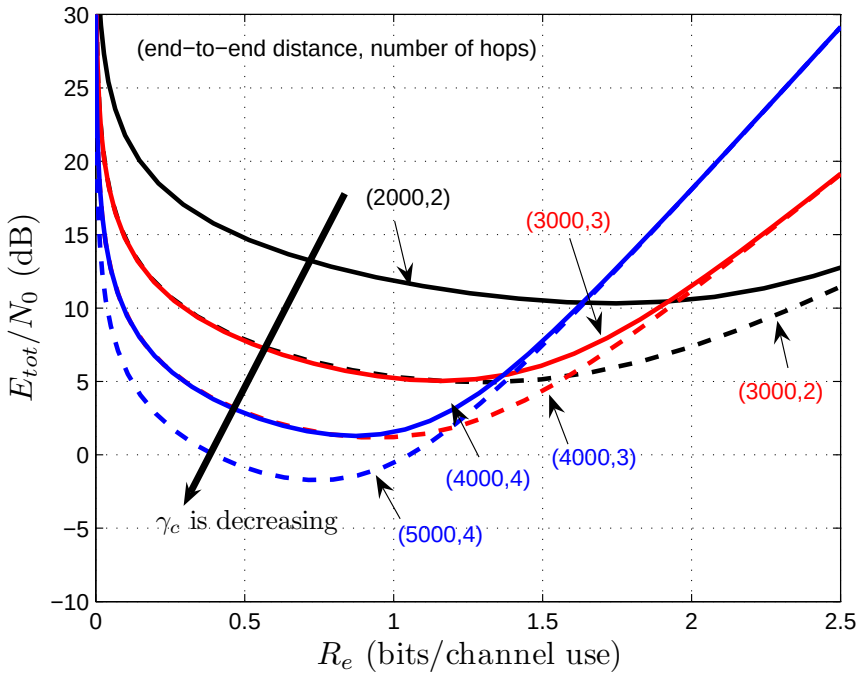


Figure 2.15: Normalized energy-bandwidth characteristic of different end-to-end distance and number of hops for equi-spaced relays when $E_p = 0.95\mu$J/symbol.

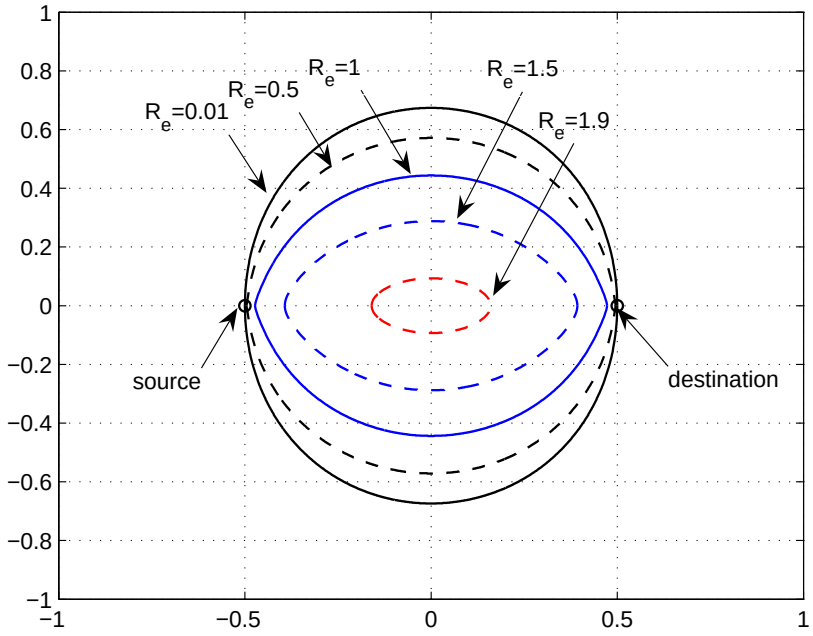


Figure 2.16: Feasible region of the relay location where a two hop network is more energy efficient than a single hop network for selected end-to-end rates.

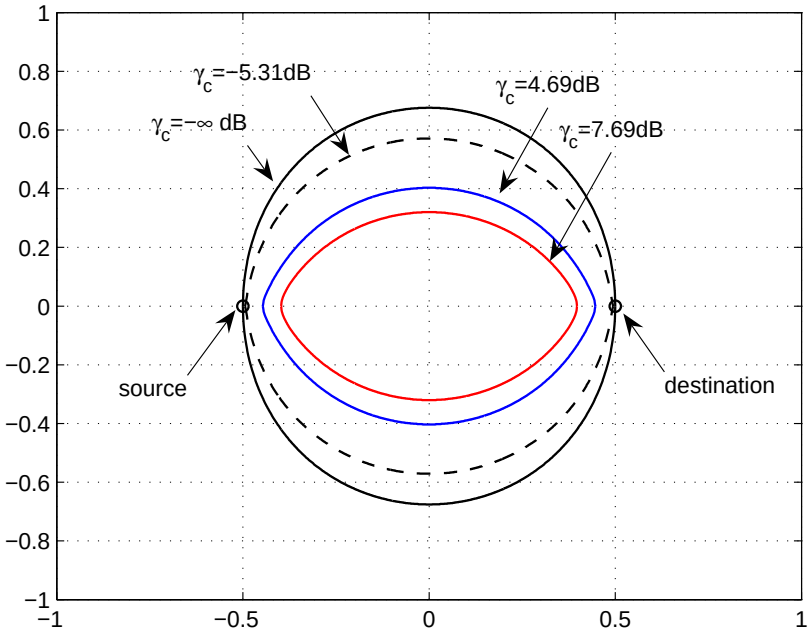


Figure 2.17: Feasible region of the relay location where a two hop network is better than a single hop network in terms of the minimum total energy consumption at the optimal end-to-end rate for selected receiver circuit processing energy.

CHAPTER 3

Minimum Energy-Per-Bit Wireless Multi-hop Networks with Spatial Reuse

In this chapter, a tradeoff between the total energy consumption-per-bit and the end-to-end rate under spatial reuse in wireless multi-hop network is developed and analyzed. The end-to-end rate of the network is the number of information bits transmitted (end-to-end) per channel use by any node in the network that is forwarding the data. In order to increase the bandwidth efficiency, spatial reuse is considered whereby simultaneous relay transmissions are allowed provided there is a minimum separation between such transmitters. The total energy consumption-per-bit includes the energy transmitted and the energy consumed by the receiver to process (demodulate and decoder) the received signal. The total energy consumption-per-bit is normalized by the distance between a source-destination pair in order to be consistent with a direct (single-hop) communication network. Lower bounds on this energy-bandwidth tradeoff are analyzed using convex optimization methods. For a given location of relays, it is shown that the total energy consumption-per-bit is minimized by optimally selecting the end-to-end rate. It is also demonstrated that spatial reuse can improve the bandwidth efficiency for a given total energy consumption-per-bit. However, at the rate that minimizes the total energy consumption-per-bit, spatial reuse does not provide lower energy consumption-per-bit compared to the case without spatial reuse. This is

because spatial reuse requires more receiver energy consumption at a given end-to-end rate. Such degraded energy efficiency can be compensated by varying the minimum separation of hops between simultaneous transmitters. In the case of equi-spaced relays, analytical results for the energy-bandwidth tradeoff are provided and it is shown that the minimum energy consumption-per-bit decreases linearly with the end-to-end distance.

3.1 Introduction

Multi-hop routing for wireless networks has garnered considerable interest recently. Energy optimization is a critical issue in the design of low-power wireless multi-hop networks. Typically, the analysis of such networks considers only the transmitted energy. However, the energy consumption of the receiver may not be negligible compared to the transmitter energy consumption. Thus, reducing the total energy consumption for end-to-end transmission is a critical design objective for such networks. In addition, efficient utilization of the scarce spectrum is important. In these multihop networks, as in point-to-point links, there is a fundamental tradeoff between energy efficiency and bandwidth efficiency.

Recently, research on the performance evaluation of multi-hop networks has focused on the problems of capacity and minimum transmission energy consumption [10]-[25]. Network capacity in random networks was analyzed to find the maximum throughput in both random and arbitrary networks [10]. The maximum achievable rate per bit is analyzed in [11]-[16] while in [15][21] the minimum transmission energy per bit is analyzed to measure the efficiency of transmission. A potential energy savings of multi-hop transmission results from reducing the distance between communicating nodes [25]. However, emphasis on transmission energy may not be an effective approach when a significant amount of energy relative to the transmission energy is consumed

in the receiver. In addition to energy consumption, bandwidth utilization needs to be considered. In [18] and [19], while receiver energy consumption is modeled and incorporated into the total energy consumption, bandwidth utilization is not considered. The minimum energy-bandwidth tradeoff with receiver energy consumption is analyzed in [31] without considering spatial reuse.

Because of the practical constraints in the design of radio devices, multi-hop transmission is usually accomplished with half duplex nodes, which leads to a loss in bandwidth efficiency. However, by reusing the time or frequency, increased utilization of radio resources can be anticipated, since more transmissions are involved. This, however, comes at the price of increased interference. In this chapter, we analyze the energy-bandwidth tradeoff with spatial reuse in a multi-hop network where nodes can be placed arbitrarily between a single source-destination pair over an additive white Gaussian noise (AWGN) channel. In this analysis, we formulate the overall performance in terms of how much energy will be consumed and how efficiently the network utilizes the bandwidth to deliver a single information bit using spatial reuse. The main outcome of this chapter is characterization of the tradeoff between energy efficiency $\frac{E_b}{N_0}$ and bandwidth efficiency R_e similar in nature to the tradeoff in a single hop (point-to-point) transmission over an AWGN channel determined by Shannon. Our work differs from previous work in two ways: (1) unlike the equi-distant assumption, relay nodes can be placed arbitrarily in a 2-dimensional plane with the requirement that the sum of each relaying distance should be equal or greater than the end-to-end distance, which makes it possible to compare the performance of different routing paths; and (2) unlike the typical transmission energy optimization, receiver energy consumption is included to find the minimum total energy consumption-per-bit and corresponding optimal end-to-end rate.

This chapter is organized as follows. In Section 3.2, the system model is described. Section 3.3 presents the performance measures considered in this work and the problem

formulation. Section 3.4 investigates lower bounds of the performance in terms of the end-to-end energy-bandwidth tradeoff. In Section 3.5, the special case of equi-spaced relays is considered. Numerical results are presented in Section 3.6, and finally Section 3.7 concludes the work.

3.2 System Model

In this section we describe the model we use for the network. We consider a stationary network that consists of a single source node and a single destination node, separated by a distance d_e, and $k-1$ relaying nodes located arbitrarily between the source and destination node, as shown in Fig. 3.1. In particular, we consider a decode-and-forward relaying protocol where relay node $j+1$ decodes the message sent from relay node j, re-encodes it using the same or a different codebook and then forwards the message to relay node $j+2$. We denote the relay distance of hop i from node $i-1$ to node i as d_i, given by $d_i = \alpha_i d_e$, where $0 < \alpha_i < 1$. Note that $\sum_{i=1}^{k} \alpha_i \geq 1$, which implies that not every relay is necessarily located on the line between the source and destination. Data from the source is forwarded to the destination through $k-1$ relays. The transmission at each hop is implemented using capacity-achieving codes with the same duration for each coded symbol for each link. For reliable communication, the rate of transmission R in information bits per channel use must be less than the capacity C. That is, $R < C(\gamma) \Leftrightarrow \gamma > g(R)$, where γ is the received energy per channel use-to-noise power spectral density ratio and $g(R) = C^{-1}(R)$ is the inverse of the channel capacity $C(\gamma)$. We assume that the channel capacity function $C(\gamma)$ is continuous, twice differentiable, and concave. For the case of an AWGN channel, the capacity and its inverse function are given by $C(\gamma) = \frac{1}{2}\log_2(1+2\gamma)$ and $g(R) = \frac{2^{2R}-1}{2}$, respectively.

We assume that the network operates with spatial reuse to facilitate simultaneous transmissions over the network. The bandwidth is reused by transmitters with a min-

imum separation of ρ hops between simultaneous transmitters. All interference from simultaneous transmitters is assumed to be modeled as Gaussian noise. Thus, it is important to determine which relays are selected to operate in the same transmission slot. We consider a transmission strategy whereby nodes are separated by at least 3 hops. This strategy avoids simultaneous transmissions by two nodes that are one hop from a neighboring receiving node. Hence, $3 \leq \rho \leq k$ and there are ρ non-overlapping transmission slots. Let S_i be the set of nodes that transmit simultaneously in slot i, $1 \leq S_i \leq \rho$. We assume the propagation of signals follows a η-th path-loss law model with distance. Thus, the received power P_r at node $j+1$ is given by $P_r = \delta P_{j,tx}/d_j^{\eta}$, where $P_{j,tx}$ is the transmitter output power of node j. The energy transmitted per channel use (modulation symbol) of node j, $E_{j,tx}$ can then be expressed as $E_{j,tx} = P_{j,tx}T_s$ for a modulation symbol duration T_s. Note that δ is a constant depending on the antenna characteristics. We set $\delta = 1$ throughout this chapter. In Fig. 3.1, as an example with $\rho = 3$, transmission by node 3 creates a certain amount of interference to node 1 and the destination, thus impairing the performance. The link signal-to-interference-and-noise ratio of the j-th link in the i-th transmission slot is then

$$\Gamma_j = \frac{E_{j,tx} d_j^{-\eta}}{N_0 + \sum_{l \in S_i, l \neq j} E_{l,tx} d_{l,j}^{-\eta}} = \frac{\gamma_j}{1 + \sum_{l \in S_i, l \neq j} \beta_{l,j} \gamma_l} \tag{3.1}$$

where $\gamma_j = \frac{E_{j,tx}}{N_0} d_j^{-\eta}$ and $\beta_{l,j} = \left(\frac{d_l}{d_{l,j}}\right)^{\eta} \neq \beta_{j,l}$. Note that $\beta_{l,j}$ denotes the relative distance ratio of the relaying distance to the interfering distance of link j in slot i. Therefore, the interfering distances from other simultaneous transmitters and the minimum separation of hops affects the possible rate of communication and the performance. The rate of transmission R_j is given by $R_j = C(\Gamma_j)$. We assume each node has the same receiver energy consumption, *i.e.*, E_p Joules are required to process a received coded symbol. In practice, the energy consumption per channel use depends on the power consumption and the processing time in the receiver. For some cases the power consumption of the

RF front end can dominate the energy consumption of the digital processing. In this case the receiver energy consumption can be expressed as a product of the receiver power consumption and the modulation symbol duration. Throughout this chapter, we assume E_p is constant.

3.3 Performance Measures

3.3.1 End-to-end rate

To measure the end-to-end bandwidth efficiency, we consider the number of information bits transmitted (end-to-end) per channel use transmitted by any node in the network that is forwarding the data. Unlike multi-hop transmission without spatial reuse, multi-hop transmission *with spatial reuse* suffers from interference from other active transmissions. As shown in Fig. 3.2, with spatial reuse there are ρ non-overlapping transmission slots and k hops for the end-to-end transmission. In contrast, for no spatial reuse there are k non-overlapping transmission slots and k hops.

Consider N total uses of the channel by the network. We assume that the same number of channel uses are assigned to the same transmission slot, i.e., N_i channel uses are assigned to the i-th slot for $i = 1, ..., \rho$. The number of information bits communicated in slot i by node $j \in S_i$ is $N_i R_j$. Let $R_{min,i}$ denote the minimum rate of all the transmissions in slot i. Then, the minimum number of information bits for the i-th slot is $N_i R_{min,i} = \min_{j \in S_i}(N_i R_j)$. With N_i channel uses, the j-th link can deliver $N_i R_j$ information bits while the network can only handle $N_i R_{min,i}$ information bits. Hence, $N_i R_{min,i}/R_j$ channel uses are needed to transmit $N_i R_{min,i}$ information bits. Maximizing the minimum end-to-end number of information bits yields the end-to-end

rate as follows

$$\begin{aligned} R_e &= \max_{\sum_{i=1}^{\rho} N_i = N} \min\left\{ \frac{N_i}{N} R_{min,i} \right\} \\ &= \max_{\sum_{i=1}^{\rho} q_i = 1} \min\{q_i R_{min,i}\} \end{aligned}$$

where $q_i = N_i/N$. The optimal solution to the above minimax problem can be easily obtained by letting $q_i R_{min,i} = q_j R_{min,j}$, which yields $q_i^* = R_{min,i}^{-1} / \sum_{j=1}^{\rho} R_{min,j}$. Therefore, the end-to-end rate with spatial reuse is

$$R_e = \left(\sum_{i}^{\rho} R_{min,i}^{-1} \right)^{-1} \text{ (bits/channel use)} \tag{3.2}$$

where ρ denotes the minimum separation of hops between simultaneous transmitters. Therefore, with spatial reuse, the end-to-end bandwidth utilization is enhanced by the spatial reuse factor ρ while the rate of each link is degraded by interference from simultaneous transmissions.

3.3.2 End-to-end energy consumption-per-bit

Consider the j-th link in slot i which is communicating at rate R_j. The energy consumption per information bit of link j, E_j, consists of transmission energy consumption and receiver energy consumption to process $N_i R_{min,i}/R_j$ received symbols. Thus, we have

$$E_j = \frac{\frac{N_i R_{min,i}}{R_j} \left(E_{j,tx} + E_p \right)}{N_i R_{min,i}} = \frac{E_{j,tx} + E_p}{R_j}. \tag{3.3}$$

Note that $E_{j,tx}$ and E_p can be normalized with d_e to be consistent with a single hop case [31] as follows:

$$\frac{E_{j,tx}}{N_0} d_e^{-\eta} = \frac{E_{j,tx}}{N_0} \alpha_j^{\eta} d_j^{-\eta} = \alpha_j^{\eta} \gamma_j \tag{3.4}$$

$$\gamma_c \triangleq \frac{E_p}{N_0} d_e^{-\eta} \tag{3.5}$$

where $\alpha_j = d_j/d_e$ is the normalized distance for link j. Our major interest in this chapter is to consider the overall energy consumed by all the nodes that forward the data across the network. In order to fairly represent the total energy consumption for different hops, we define the *normalized total energy consumption per information bit-to-noise power spectral density ratio* as

$$\frac{E_{tot}}{N_0} \triangleq \sum_{j=1}^{k} \frac{E_j}{N_0} d_e^{-\eta} = \sum_{j=1}^{k} \frac{\alpha_j^{\eta} \gamma_j + \gamma_c}{R_j}. \tag{3.6}$$

Note that, for low rates, the processing energy for the circuitry will dominate the total energy consumption, whereas for high rates the transmitted energy will dominate. In particular, when $\gamma_c = 0$, (3.6) denotes the sum of received energy per information bit in each link that is involved in the end-to-end transmission. This is consistent with a single link where the received energy is the usual performance measure as opposed to the transmitted energy.

3.3.3 Minimum energy consumption-per-bit with arbitrary transmission energy

For a given multi-hop routing path, the problem of minimizing total energy consumption-per-bit with arbitrary transmission energy of each link for a given end-to-end rate can

be formulated as the following optimization problem:

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \min_{\gamma_j} \quad & \sum_{j=1}^{k} \frac{\alpha_j^{\eta}\gamma_j + \gamma_c}{R_j} \\ \text{s.t.} \quad & R_j = C(\Gamma_j), j = 1, ..., k \\ & \sum_{i=1}^{\rho} R_{min,i}^{-1} = R_e^{-1} \end{aligned} \tag{3.7}$$

where the optimization variables are γ_j's for $j = 1, ..., k$. Due to the cross coupled terms of variables in the objective function, which can be seen in (3.1), it is difficult to prove the convexity of the optimization problem for the general case of number of hops and to find the optimal set of transmission energies for each link as well. Instead of finding the optimal solution to the general problem, we propose a common power strategy as a sub-optimal strategy, which was shown to provide a good result for the case without spatial reuse in [31]. We assume that concurrent transmitters in the same transmission slot transmit with the same energy, that is, $E_{j,tx} = E_{S_i,tx}$ for $j \in S_i$. With this assumption we obtain the following relations. For $j, l \in S_i$

$$\alpha_j^{\eta}\gamma_j = \alpha_l^{\eta}\gamma_l \tag{3.8}$$

$$\Gamma_j = \frac{E_{S_i,tx} d_j^{-\eta}}{N_0 + \sum_{l \in S_i, l \neq j} E_{S_i,tx} d_{q,j}^{-\eta}} = \frac{\gamma_j}{1 + \gamma_j \beta_j} \tag{3.9}$$

$$\beta_j = \sum_{l \in S_i, l \neq j} \left(\frac{d_j}{d_{l,j}} \right)^{\eta} \tag{3.10}$$

where β_j denotes the sum of relative distances of link j in slot i. The fundamental goal of this chapter is, for a given multi-hop routing path $\overline{\alpha} = (\alpha_1, ..., \alpha_k)$ and $\overline{\beta} = (\beta_1, ..., \beta_k)$, to find the optimal set of transmission energies $(E_{1,tx}, ..., E_{\rho,tx})$ that provides the minimum total energy consumption per information bit for a given end-to-end data rate R_e. We also focus on finding the global minimum total energy consumption and corresponding optimal end-to-end rate.

3.4 Minimum energy consumption-per-bit

3.4.1 Minimum energy-per-bit for a given end-to-end rate

We start with the optimization problem for a given end-to-end rate. This general problem of minimizing the total energy consumption-per-bit for a given end-to-end rate is given as

Problem I:

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \min_{E_{S_i,tx}} \quad & \sum_{m=1}^{k} \frac{E_{m,tx} + E_p}{R_m} \frac{d_e^{-\eta}}{N_0} \\ \text{s.t.} \quad & \textstyle\sum_{i=1}^{\rho} R_{min,i}^{-1} = R_e^{-1} \\ & E_{m,tx} = E_{S_i,tx} \\ & R_m = C(\Gamma_m) \text{ for } m \in S_i \end{aligned} \tag{3.11}$$

where the optimization variables are $E_{S_i,tx}$'s for $i = 1, ..., \rho$. Because the objective function contains the sum of decreasing convex functions and the constraint contains increasing concave functions, Problem I is not a convex optimization problem. However, by letting $x_m^{-1} = C(\Gamma_m)$, Problem I can be converted into the following problem.

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} = \min_{x_m} \quad & \sum_{m=1}^{k} x_m \left[\alpha_m^{\eta} h(x_m^{-1}) + \gamma_c\right] \\ \text{s.t.} \quad & \sum_{i=1}^{\rho} \dot{x}_i = R_e^{-1} \\ & \alpha_j^{\eta} h(x_j^{-1}) = \dot{\alpha}_i^{\eta} h(\dot{x}_i^{-1}) \end{aligned} \tag{3.12}$$

where

$$\gamma_m = h(x_m^{-1}) = \frac{g(x_m^{-1})}{1 - \beta_m g(x_m^{-1})} > 0 \tag{3.13}$$

$$I_i = \operatorname*{argmax}_{j \in S_i} x_j \tag{3.14}$$

$$\dot{x}_i \equiv x_{I_i} = \max_{j \in S_i} x_j = R_{min,i}^{-1} \tag{3.15}$$

$$\dot{\alpha}_i \equiv \alpha_{I_i}, \ \dot{\beta}_i \equiv \beta_{I_i}. \tag{3.16}$$

The form in (3.12) is still not a convex optimization problem due to the non-affinity of the second equality constraint. While numerical methods can provide the minimum energy consumption-per-bit for a given end-to-end rate, it is hard to obtain an optimal solution for arbitrary networks.

In summary, the original problem in (3.7) is simplified to Problem I in (3.12) by the assumption of equal transmission power for concurrent transmitters. This enables us to simplify the SINR relation in (3.1) to (3.9). While optimal solutions for both problems can be obtained by numerical methods, it is not easy to find optimal solutions when the number of nodes becomes large. In particular, Problem I is a nonconvex optimization problem even though it has been converted. Thus, instead of finding the optimal solution of Problem I, a lower bound to Problem I can be obtained by eliminating the equal power constraint in (3.12), which is applicable to any arbitrary networks with a large number of nodes. This leads to the following convex problem.

Lower bound of Problem I:

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} \geq \min_{x_m} \quad & \sum_{m=1}^{k} x_m \left[\alpha_m^{\eta} h(x_m^{-1}) + \gamma_c\right] \\ \text{s.t.} \quad & \sum_{i=1}^{\rho} \dot{x}_i = R_e^{-1} \end{aligned} \tag{3.17}$$

where the optimization variables are x_m's for $m = 1, ..., k$. For (3.17) to be a convex problem, it is required that $h(x) = \dfrac{g(x)}{1 - \beta g(x)}$ be an increasing convex function for $0 < g(x) < 1/\beta$ and $\beta > 0$ [31], which can be easily shown. Therefore, the optimal parametric solution to the lower bound of Problem I can be obtained from the Karush-Kuhn-Tucker (KKT) conditions.

Parametric solution to lower bound of Problem I: For $m = 1, ..., k$, let $x_m^{-1} = C(\Gamma_m)$. For a given Lagrange multiplier λ, and $\overline{\alpha}$ and $\overline{\beta}$, $\dot{x}_i^*$ can be determined by solving the following equation from its Lagrangian function for $\dot{x}_i$, $i = 1, ..., \rho$

$$\dot{\alpha}_i^{\eta} \left[\frac{h'(\dot{x}_i^{-1})}{\dot{x}_i} - h(\dot{x}_i^{-1})\right] \;=\; \gamma_c - \lambda \tag{3.18}$$

and $x_j^* \neq \dot{x}_i$ for $j \in S_i$ can be determined by solving the following equation for x_j

$$\alpha_m^\eta \left[\frac{h'(x_m^{-1})}{x_m} - h(x_m^{-1}) \right] = \gamma_c \tag{3.19}$$

where $0 < x_m^{-1} < C(1/\beta_m)$ for $m = 1, ..., k$ and $\gamma_c \geq 0$. Since the function $f(u) = uh'(u) - h(u)$ in the left hand side of (3.18) and (3.19) is increasing with $u > 0$ and $f(0) = 0$, it is required that $\lambda \leq \gamma_c$. Hence, the optimal γ_m^* is obtained from x_m^*, which also provides

$$R_e = \left(\sum_{i=1}^{\rho} \dot{x}_i^* \right)^{-1} \leq \left(\sum_{i=1}^{\rho} C^{-1}(1/\dot{\beta}_i) \right)^{-1} \tag{3.20}$$

where it should be noticed that the end-to-end rate is bounded, since the minimum link rate of each slot i is bounded by the maximum achievable SINR. The resulting minimum total energy consumption-per-bit is lower bounded by

$$\frac{E_{tot}(\overline{\alpha}, R_e)}{N_0} \geq \sum_{m=1}^{k} x_m^* \left[\alpha_m^\eta h(1/x_m^*) + \gamma_c \right] \tag{3.21}$$

$$= \sum_{i=1}^{\rho} \left[\dot{\alpha}_i^\eta h' \left(\frac{1}{\dot{x}_i^*} \right) + \lambda \dot{x}_i^* + \sum_{\substack{j \in S_i \\ j \neq i}} \alpha_j^\eta h' \left(\frac{1}{x_j^*} \right) \right].$$

The parametric solution to the lower bound of Problem I is obtained from the following procedure using numerical methods:

Step 1. Fix λ.

Step 2. Solve (3.18) for $\dot{x}_i$.

Step 3. For $x_j \neq \dot{x}_i$ with $j \in S_i$, solve (3.19) for x_j.

Step 4. Repeat the above steps for $i = 1, ..., \rho$

Step 5. Determine R_e and $\frac{E_{tot}}{N_0}$ from (3.20) and (3.21).

By varying λ we can determine the tradeoff between E_{tot}/N_0 and R_e. Using (3.13), (3.18) can be simplified to

$$\begin{aligned}&\dot{\alpha_i}^{\eta}(\dot{x}_i^{-1}g'(\dot{x}_i^{-1}) - g(\dot{x}_i^{-1}) + \dot{\beta}_i g(\dot{x}_i^{-1})^2)\\ &= (\gamma_c - \lambda)(1 - 2\dot{\beta}_i g(\dot{x}_i^{-1}) + (\dot{\beta}_i)^2 g(\dot{x}_i^{-1})^2).\end{aligned} \tag{3.22}$$

Note that when there is no significant interference, *i.e.,* $\dot{\beta}_i \ll 1$, (3.22) can be approximated as

$$\dot{\alpha_i}^{\eta}\dot{x}_i^{-1}g(\dot{x}_i^{-1}) + \left[2\dot{\beta}_i(\gamma_c - \lambda) - \dot{\alpha_i}^{\eta}\right] g(\dot{x}_i^{-1}) = \gamma_c - \lambda. \tag{3.23}$$

For the case of an AWGN channel, we have $g(x) = C^{-1}(x) = 0.5(2^{2x} - 1)$, which leads to $h(x_m^{-1}) = \frac{(2^{2/x_m} - 1)}{2 - \beta_m(2^{2/x_m} - 1)}$. Therefore, it is straightforward to obtain the solution to the lower bound of Problem I for an AWGN channel. In particular, the solution to (3.22) for an AWGN channel can be approximated as

$$\frac{2\ln 2}{\dot{x}_i^*} \approx \left[\mathcal{W}\left(\frac{\theta_1(1 + \dot{\beta}_i) - 1}{e} e^{\theta_1 \dot{\beta}_i}\right) + 1 - \theta_1 \dot{\beta}_i\right] \tag{3.24}$$

where $\theta_1 = 2\dot{\alpha_i}^{-\eta}(\gamma_c - \lambda)$, and $\mathcal{W}(\cdot)$ is the principal branch of the Lambert W-function, for which $\mathcal{W}(x) \geq -1$ [34]. Note that when there is no interference, *i.e.,* $\beta_i = 0$, the approximated solution (3.24) is the same as the result of [31].

3.4.2 Minimum energy-per-bit for an optimal end-to-end rate

Now consider the minimum total energy consumption-per-bit by optimally selecting the end-to-end rate for a given multi-hop routing path as follows.

Problem II:

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} = \min_{E_{j,tx}} \quad & \sum_{j=1}^{k} \frac{E_{j,tx} + E_p}{R_j} \frac{d_e^{-\eta}}{N_0} \\ \text{s.t.} \quad & R_j = C(\Gamma_j), j = 1, ..., k \\ & E_{j,tx} = E_{S_i,tx} \text{ for } j \in S_i \end{aligned} \tag{3.25}$$

where the optimization variables are $E_{j,tx}$'s. Note that in (3.11) and (3.25), both $E_{j,tx}$ and $E_{S_i,tx}$ are the same, but in (3.25) our goal is to find the global minimum energy-per-bit over the rates by removing the constraint for the given rate related to the first condition related to ρ in (3.11). Thus, $E_{j,tx}$ is preferred over $E_{S_i,tx}$.

As in Problem I, (3.25) cannot be converted into an equivalent convex optimization problem. However, a lower bound of Problem II can be obtained from the unconstrained convex optimization problem of (3.17)

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} \geq \min_{x_m} \left(\sum_{m=1}^{k} x_m \left[\alpha_m^{\eta} h(x_m^{-1}) + \gamma_c \right] \right). \tag{3.26}$$

Solution to lower bound of Problem II: Let x_m^* for $m = 1, ..., k$ be the solution to

$$\alpha_m^{\eta} \left[\frac{h'(x_m^{-1})}{x_m} - h(x_m^{-1}) \right] = \gamma_c. \tag{3.27}$$

The solution to the lower bound of Problem II is then as follows

$$\frac{E_{tot}(\overline{\alpha}, R_e^*)}{N_0} \geq \sum_{m=1}^{k} x_m^* \left[\alpha_m^{\eta} h(1/x_m^*) + \gamma_c \right] \tag{3.28}$$

$$= \sum_{m=1}^{k} \alpha_m^{\eta} h' \left(\frac{1}{x_m^*} \right) \tag{3.29}$$

where (3.28) can be obtained from (3.21) by substituting $\lambda = 0$. For the case of an

AWGN channel, when $\beta \ll 1$, the solution to (3.27) is approximated as

$$\frac{2\ln 2}{x_m^*} \approx \left[\mathcal{W}\left(\frac{\theta_2(1+\beta_m)-1}{e}e^{\theta_2\beta_m}\right)+1-\theta_2\beta_m\right] \tag{3.30}$$

where $\theta_2 = 2\alpha_m^{-\eta}\gamma_c$. Notice that when there is no interference, *i.e.,* $\beta_m = 0$, the approximated solution (3.30) is the same as the result of [31]. In particular, when the circuit processing energy is ignored, substituting $\gamma_c = 0$ to (3.27) yields $w(u) \triangleq ug'(u) - g(u)(1-\beta_m g(u))^2 = 0$, where $u = x_m^{-1}$. It can be easily shown that $w(u)$ is increasing in u for $0 < g(u) < 1/\beta_m$ and $w(0) = 0$. Therefore, we obtain the optimal solution $1/x_m^* = 0$ and, correspondingly, the optimal minimum total energy consumption-per-bit for the case of $\gamma_c = 0$.

Solution to Problem II with $\gamma_c = 0$:

$$\begin{aligned}\frac{E_b(\overline{\alpha}, R_e^*)}{N_0} &= \lim_{1/x_m^*\to 0}\left(\sum_{m=1}^{k}\alpha_m^{\eta}x_m^* h(1/x_m^*)\right)\\ &= \ln 2\cdot\sum_{m=1}^{k}\alpha_m^{\eta}.\end{aligned}$$

This implies that the minimum energy consumption-per-bit occurs at $R_e = 0$ when only the transmission energy is considered. Hence, without receiver energy consumption, multi-hop routing with spatial reuse achieves the same minimum total energy consumption per information bit as multi-hop routing without spatial reuse in [31].

3.5 Performance of equi-distant multi-hop routing

In this section we derive the optimal end-to-end rate and number of hops to minimize the overall energy consumption-per-bit of equi-spaced relays between the source and the destination. We also investigate how the number of hops and the end-to-end distance affect the performance. The optimal solutions are obtained by decoupling the joint

optimization problem in rate and number of hops into two sub-problems. We first obtain the optimal number of hops as a function of R_e. Then, the optimal value of R_e is derived [18][38].

Consider the case of equi-spaced relays using spatial reuse with minimum separation between simultaneous transmitters. In practice, depending on the relay position in the network, each link SINR can vary even if the same transmission energy level is allowed in each link. However for the purpose of investigating the impact of the end-to-end rate and number of hops, we assume that each link experiences the same amount of interference. This requires $k \geq 2\rho$, since at least one interferer must exist in each transmission slot. Also, the cochannel interference is assumed to be independent of the number of hops by considering the major interference from the immediate neighboring concurrent transmitter, that is, $\beta = (\rho - 1)^{-\eta}$. As such, each link is assumed to communicate at the same rate. Let Γ be the SINR at each link. The end-to-end rate is then

$$R_e \;=\; \frac{R}{\rho} = \frac{C(\Gamma)}{\rho} \leq \frac{C(\beta^{-1})}{\rho} \tag{3.31}$$

and the received SNR of each link is $\gamma = \frac{g(\rho R_e)}{1-\beta g(\rho R_e)} \triangleq h(\rho R_e)$, where $g(\cdot)$ denotes the inverse of the channel capacity function and $0 < g(\rho R_e) < 1/\beta$, since $\gamma > 0$. The total energy consumption per information bit is then

$$\frac{E_{tot}(k, R_e)}{N_0} = \sum_{i=1}^{k} \frac{k^{-\eta}\gamma + \gamma_c}{\rho R_e} = \frac{k^{1-\eta}h(\rho R_e) + k\gamma_c}{\rho R_e}. \tag{3.32}$$

With transformation of the variables $k = e^{-x}$ and $\rho R_e = e^{-y}$, the problem of minimizing the total energy consumption-per-bit can be converted into an equivalent optimization problem

$$\frac{E_{tot}(x^*, y^*)}{N_0} = \min_{x \in \mathbf{R}, y \in \mathbf{R}} e^{y} \left[e^{(\eta-1)x} h(e^{-y}) + \gamma_c e^{-x} \right] \tag{3.33}$$

where $h(e^{-y}) = \frac{g(e^{-y})}{1-\beta g(e^{-y})}$.

Proposition 3.1. The objective function $E_{tot}(x,y)/N_0$ is convex in x, y for $x, y \in \mathbf{R}$ if $g(u)$ satisfies the following conditions

$$g(u)g'(u) + u\left[g''(u)g(u) - g'(u)^2\right] > 0. \tag{3.34}$$

Proof: See Appendix B.1. ■

Therefore, if the sufficient conditions are satisfied, the optimization problem is a convex optimization problem and thus any locally optimal solution is guaranteed to be a globally optimal solution. For the case of an AWGN channel, it can be easily verified that the objective function in (3.33) is convex (see Appendix B.1). Other channels of interest are the binary input AWGN channel and the binary input hard decision AWGN channel, whose channel capacity functions [39] are given by

$$\begin{aligned} C_{BISO}(\gamma) &= 1 - \int_{-\infty}^{\infty} g(x - \sqrt{2\gamma}) \log_2(1 + e^{-2x\sqrt{2\gamma}}) dx \\ C_{BIBO}(\gamma) &= 1 - H_2(Q(\sqrt{2\gamma})) \end{aligned}$$

where $g(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ and $H_2(x) = -x\log_2(x) - (1-x)\log_2(1-x)$ is the binary entropy function. Numerical methods can verify the sufficient condition for convexity of the above two channels. Note that interference from other transmissions is regarded as additional Gaussian noise.

3.5.1 Optimization of the number of hops

For a given end-to-end rate, treating the number of hops as a continuous variable, we optimize the total energy consumption-per-bit over x:

$$\frac{E_{tot}(x^*, y)}{N_0} = \min_{x \in \mathbf{R}} e^y \left[e^{(\eta-1)x} h(e^{-y}) + \gamma_c e^{-x}\right]. \tag{3.35}$$

Setting the derivative with respect to x equal to zero yields the optimal number of hops for a given end-to-end rate as follows

$$k^* = e^{-x^*} = \left[\frac{\eta - 1}{\gamma_c} \frac{g(e^{-y})}{1 - \beta g(e^{-y})}\right]^{1/\eta}. \tag{3.36}$$

The minimum total energy consumption-per-bit at the optimal number of hops is then

$$\frac{E_{tot}(k^*, R_e)}{N_0} = \left[\frac{\eta - 1}{\gamma_c}\right]^{1/\eta - 1} \frac{\eta}{\rho R_e} \left[\frac{g(\rho R_e)}{1 - \beta g(\rho R_e)}\right]^{1/\eta}. \tag{3.37}$$

3.5.2 Optimization of the end-to-end rate

The problem of determining the optimal end-to-end rate is formulated by substituting (3.36) into (3.33) which results in

$$\frac{E_{tot}(x^*, y^*)}{N_0} = \min_y \left(\eta \left[\frac{\eta - 1}{\gamma_c}\right]^{\frac{1}{\eta} - 1} \left[\frac{e^{\eta y} g(e^{-y})}{1 - \beta g(e^{-y})}\right]^{1/\eta}\right).$$

Setting the derivative with respect to y to be equal to zero yields $\eta g(e^{-y})(1 - \beta g(e^{-y})) = e^{-y} g'(e^{-y})$. Let u^* be the solution to $\eta g(u)[1 - \beta g(u)] = u g'(u)$. The optimal end-to-end rate is then

$$R_e^* = \frac{u^*}{\rho} \tag{3.38}$$

and the optimal number of hops at the optimal rate is obtained by substituting (3.38) into (3.36),

$$\begin{aligned} k^*(R_e^*) &= \left[\frac{\eta - 1}{\gamma_c} \frac{g(u^*)}{1 - \beta g(u^*)}\right]^{1/\eta} \\ &= \left[\frac{g(u^*)}{1 - \beta g(u^*)}\right]^{1/\eta} \left[\frac{E_p}{N_0} \frac{1}{\eta - 1}\right]^{-1/\eta} d_e. \end{aligned} \tag{3.39}$$

The resulting normalized minimum total energy consumption-per-bit at the optimal rate is then

$$\begin{aligned}\frac{E_{tot}(k^*, R_e^*)}{N_0} &= \eta \left[\frac{\eta - 1}{\gamma_c}\right]^{\frac{1}{\eta}-1} \frac{1}{u^*} \left[\frac{g(u^*)}{1 - \beta g(u^*)}\right]^{1/\eta} \\ &= f_c(\eta) \left[\frac{E_p}{N_0}\right]^{1-1/\eta} d_e^{1-\eta}. \end{aligned} \tag{3.40}$$

where

$$f_c(\eta) = \frac{1}{u^*} \left[\frac{g(u^*)}{1 - \beta g(u^*)}\right]^{1/\eta} \left[\frac{\eta}{(\eta - 1)^{1-1/\eta}}\right] \tag{3.41}$$

which is a function of the channel and the propagation loss exponent. From (3.6), the unnormalized minimum total energy consumption-per-bit is given by

$$\frac{E_{tot}^U(k^*, R_e^*)}{N_0} = \frac{E_{tot}(k^*, R_e^*)}{N_0} d_e^{\eta} = f_c(\eta) \left[\frac{E_p}{N_0}\right]^{1-1/\eta} d_e. \tag{3.42}$$

From the above results it is observed that the optimal rate depends only on the relative distance ratio β if the optimal number of hops satisfies the constraint $k^* \geq 2\rho$, which grows linearly with d_e, as in the case without spatial reuse. This is because the transmitted energy becomes dominant as d_e increases, which implies that more hops yields more energy efficiency for small γ_c. Thus, as γ_c becomes small, networks with more hops can achieve the minimum energy consumption-per-bit, as will be shown in the numerical results. In addition, there are two key observations for the minimum normalized total energy consumption-per-bit: it (i) increases with the $\frac{\eta-1}{\eta}$ power of the processing energy and (ii) decreases with $d_e^{1-\eta}$. Therefore, we can conclude that the actual unnormalized minimum total energy consumption-per-bit increases linearly with d_e from the relation between the normalized total energy consumption-per-bit and the end-to-end distance in (3.6). Note that the optimal energy constant in (3.41) depends on the channel capacity, the path-loss exponent, and the relative distance ratio. For

different input/output AWGN channels (*e.g.* binary input or binary output), the resulting energy consumption constants are given in Table 3.1 along with the loss incurred relative to the AWGN channel for $\rho = 3$.

3.6 Numerical Results

In the numerical results, unless otherwise specified, we assume that the path-loss exponent is $\eta = 4$, the channel is an AWGN, the minimum separation of hops between simultaneous transmissions is $\rho = 3$, the noise power spectral density $N_0 = -174\text{dBm/Hz}$, and the circuit processing energy is $E_p = 0.095\mu\text{J/symbol}$ which at an end-to-end distance of $d_e = 3\text{km}$ yields $\gamma_c = -5.31\text{dB}$.

In Fig. 3.4, the normalized energy-bandwidth tradeoffs are plotted, where results are drawn based on numerical optimization with the same transmission energy, the lower bound of Problem I, the numerical result with arbitrary transmission energy, and the optimal result without spatial reuse in [31], respectively. The distance ratio vector is $\overline{\alpha} = (0.3, 0.25, 0.15, 0.3)$ and $\gamma_c = -5.31\text{dB}$. The lower bound on the energy-per-bit with the same transmission energy constraint in the same slot is shown to be very tight to the actual energy-per-bit with the same constraint. The arbitrary transmission energy and the case without spatial reuse are drawn together to observe the effect of transmission energy restriction and the performance loss from spatial reuse. At low rates the energy consumption due to the transmitter alone decreases while the total energy consumption increases due to the energy needed for processing. At high rates, however, the total energy consumption increases as the transmitted energy increases. Thus, such characteristics yield a nonzero optimal rate that minimizes the total energy consumption-per-bit, similar to the result in [31]. It is also seen that the end-to-end bandwidth utilization is limited, because the rate of communication in each slot is bounded by cochannel interference, as indicated in (3.20).

In Fig. 3.5 and Fig. 3.6, the normalized energy-bandwidth tradeoff with a transmission energy constraint is plotted for different processing energies, $\gamma_c = -17.36$dB and $\gamma_c = 0$ ($-\infty$dB) where the distance ratio vectors are given as $\overline{\alpha} = (0.3, 0.25, 0.15, 0.3)$ for $k = 4$ hops and $\overline{\alpha} = (0.3, 0.2, 0.1, 0.25, 0.15)$ for $k = 5$ hops. As the processing energy decreases, the optimal rate minimizing the total energy consumption-per-bit decreases, because the transmitted energy is more dominant as γ_c becomes small. In particular, when $\gamma_c = 0$, the minimum total energy consumption-per-bit occurs at $R_e = 0$, which verifies the result of Problem II with $\gamma_c = 0$. In addition, when $\gamma_c = 0$, it shows that a multi-hop network with more hops is more energy-efficient. This is because more relays reduce the distance between hops, which further contributes to reducing the transmitted energy.

In order to show that different selection of relays causes different amounts of interference because of the different distances from transmitters to the receivers, we consider a wireless network with 3 relays as an example, where the locations of the nodes are given in Fig. 3.3. Using the lower bound to the solution of Problem I, we compare the performance of two possible multihop routes in Fig. 3.7. It is observed based on the bound that route 2 is better than route 1 at low rates while at high rates route 1 is better, because route 2 suffers more from interference as the transmission energy increases. An efficient algorithm that can find the optimal route for a given rate is currently unknown when the number of nodes is large. However, the insight into which route is better based on bounds is very useful.

Fig. 3.8 depicts the impact of the minimum separation of simultaneous transmitters when the relays are placed equidistantly for $\gamma_c = -5.31$dB at $d_e = 3$km. Instead of the result in Section 3.5 where the same interference is assumed in the same transmission slot, lower bounds to the solution of Problem I are used. Note that as ρ increases, fewer simultaneous transmissions are used and $\rho = k$ corresponds to the case without spatial reuse. At low rates, the energy efficiency is improved with fewer simultaneous

transmissions. However, at high rates, an optimal selection of ρ is necessary, because more spatial reuse can cause a significant amount of interference whereas less spatial reuse reduces the bandwidth utilization. Hence, it should be noted that whether spatial reuse can improve the bandwidth utilization and contribute to enhancing the energy efficiency depends on the desired end-to-end rate.

In Fig. 3.9 and Fig. 3.10, lower bounds on the normalized energy-bandwidth tradeoff are plotted for $\gamma_c = -17.36$dB and $\gamma_c = 0$ to further explore the impact of processing energy. In Fig. 3.9 where $\gamma_c > 0$, it is seen that a multi-hop transmission without spatial reuse is more energy efficient at low rates. This is because the receiver energy consumption without spatial reuse, $\gamma_c R_e^{-1}$, is less than that with spatial reuse, $\gamma_c(R_e^{-1} + \sum_{R_j \neq R_{min,i}} R_j^{-1})$, for $i = 1, ..., \rho$ at a given end-to-end rate. However, as shown in Fig. 3.10 for $\gamma_c = 0$, more spatial reuse yields more energy efficiency at low rates, because: (1) only the transmission energy is considered, and (2) less transmission energy is required for more spatial reuse transmission to produce the same end-to-end rate before it suffers from severe cochannel interference. For short end-to-end distances such as $d_e = 3000$m in Fig. 3.9, it is seen that at high rates the spatial reuse case cannot outperform the no spatial reuse case, even if the minimum separation is adjusted. However, when d_e becomes large, such as $d_e = 30$km in Fig. 3.11, it is observed that the spatial reuse case is better than the no spatial reuse case at high rates. This verifies that, although both cases suffer from bandwidth efficiency loss due to half duplex property, for large end-to-end distance, increased utilization of resources is expected by spatial reuse.

To investigate the relation between the optimal number of hops and the end-to-end distance or equivalently normalized processing energy, we plot the normalized energy-bandwidth tradeoff for the equi-distant relays case in Fig. 3.12 and Fig. 3.13 with the same interference assumption as in Section 3.5. Since we assume that at least one interferer must exist in each transmission slot, $k \geq 6$ hops are considered for $\rho = 3$. In order to assess the performance for very low processing energy, cases with

$\gamma_c = -45.31$dB are compared with cases for $\gamma_c = -5.31$dB. When $\gamma_c = -5.31$dB, the minimum energy consumption-per-bit occurs at $k = 6$, which is the smallest number of hops when $\rho = 3$. This implies that when the normalized processing energy consumption is not negligible, a multi-hop network with less hops is more energy efficient. On the contrary, when the normalized receiver energy consumption is very low but not equal to 0, e.g. $\gamma_c = -45.31$dB, a multi-hop network with more hops is more energy efficient compared to a multi-hop network with less hops. This can be identified from the result in (3.39) that as γ_c increases, the optimal number of hops decreases. Correspondingly, the minimum energy consumption-per-bit also increases as d_e decreases, which verifies the results in (3.40). It is also observed that, due to the limited achievable SINR, the end-to-end bandwidth utilization is limited and converges to $C(\beta^{-1})/\rho$, because its immediate neighboring simultaneous transmitter is a major source of interference. Note that when multiple interferers are considered, the end-to-end rate converges to $(\sum_{\rho} C^{-1}(1/\dot{\beta}_i))^{-1}$, as described in (3.20).

Performance comparisons for various AWGN channel models are presented in Fig. 3.14, where $k = 6$ equally spaced hops are considered. Input constrained AWGN channels, binary input with hard decision (BIBO AWGN) and binary input/unquantized output AWGN (BISO AWGN), show a bounded bandwidth utilization at $R_e = 1/3$ bits/channel, and degraded performance compared to the unquantized input/output AWGN channel (SISO AWGN) due to their inherently limited bandwidth utilization. This can be identified in Table 3.1, where the optimal normalized energy consumption constants are provided for different channel models. It is seen that the optimal energy consumption per information bit decreases as bandwidth is utilized efficiently through the channel.

3.7 Conclusion

This chapter establishes the minimum energy-bandwidth tradeoff for wireless multi-hop networks where relays can be placed arbitrarily between the source and destination. The end-to-end rate is first introduced to capture the end-to-end bandwidth utilization, in which spatial reuse is used by restricting the minimum number of hops between simultaneous transmitters. Normalized total energy consumption-per-bit for a multi-hop network, which includes the transmission energy and required energy for processing at each receiver, is considered to be consistent with a single hop network. A parametric solution for lower bounds on the minimum total energy consumption per information bit is derived by solving an equivalent convex optimization problem. An optimal end-to-end rate exists from the relation of the transmission and processing energy at a given end-to-end rate. When the normalized processing energy is small, the optimal end-to-end rate decreases due to the dominance of the transmitted energy consumption. In particular, when the receiver processing energy is not included, multi-hop routing with spatial reuse shows the same performance as multi-hop routing without spatial reuse. It is shown that spatial reuse only contributes to enhancing bandwidth utilization, not reducing energy consumption-per-bit. Instead the energy efficiency can be improved by varying the minimum separation of hops between simultaneous transmitters, which is related to adjusting the resulting cochannel interference.

The chapter also establishes a general relationship between the number of hops or equivalently minimum energy consumption and the end-to-end distance by approximating cochannel interference as being equal for each link. It is shown that the normalized minimum energy consumption-per-bit decreases with $1 - \eta$ power of the end-to-end distance, which indicates that the unnormalized energy consumption-per-bit increases with the end-to-end distance. In addition, the optimal number of hops is linearly proportional to the end-to-end distance, implying that for large end-to-end distance, more hops achieves the minimum energy consumption.

In conclusion, there exists an optimal transmission strategy that provides the best performance at a given energy-bandwidth tradeoff by considering simultaneous transmissions and the number of hops. Therefore, in order to minimize the energy consumption-per-bit at a given end-to-end rate, the number of hops and the minimum separation of simultaneous transmitters should be considered jointly.

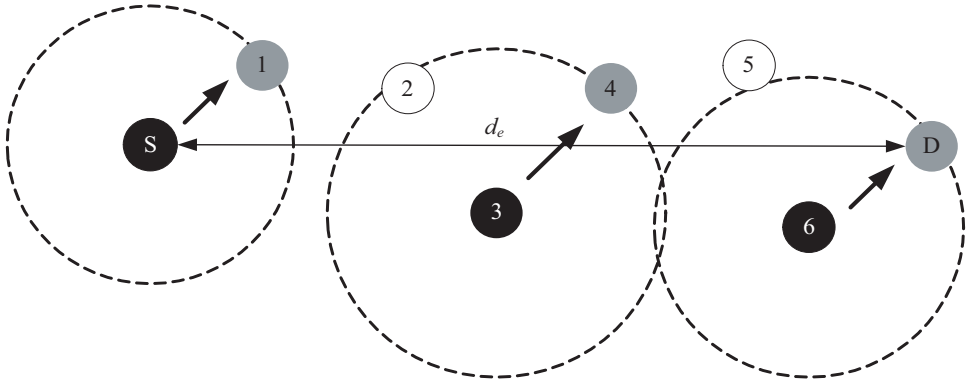
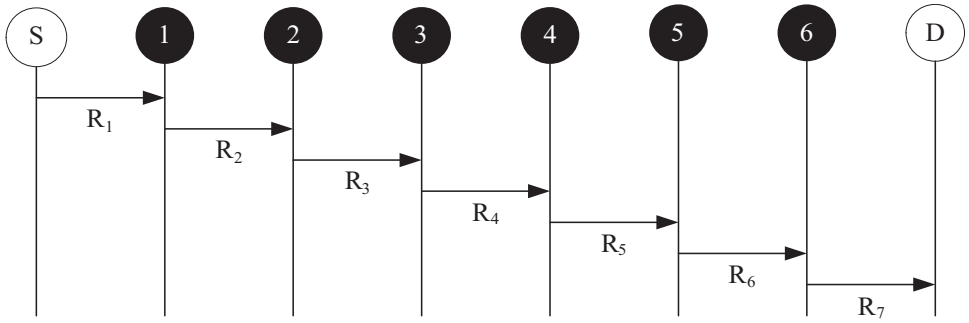
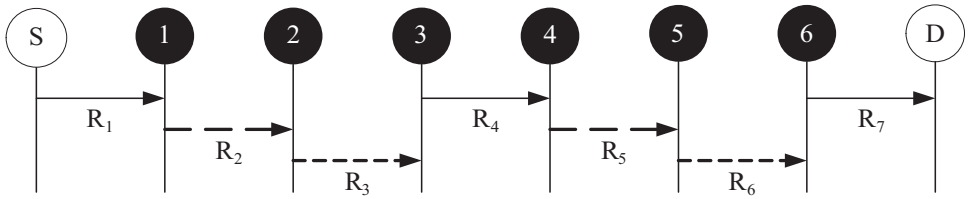


Figure 3.1: Illustration of network model with spatial reuse, $\rho = 3$.



(a) Multi-hop communication without spatial reuse



(b) Multi-hop communication with spatial reuse

Figure 3.2: Illustrative example of multi-hop transmission scenario.

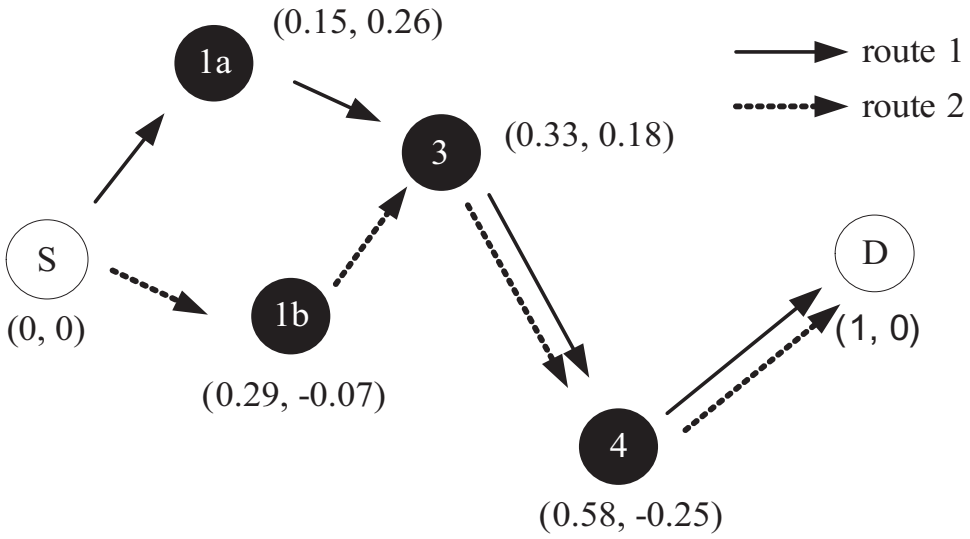


Figure 3.3: Example of two possible routes in 2-dimensional network model where distance between any two nodes denotes the normalized relay distance by the end-to-end distance with spatial reuse, $\rho = 3$.

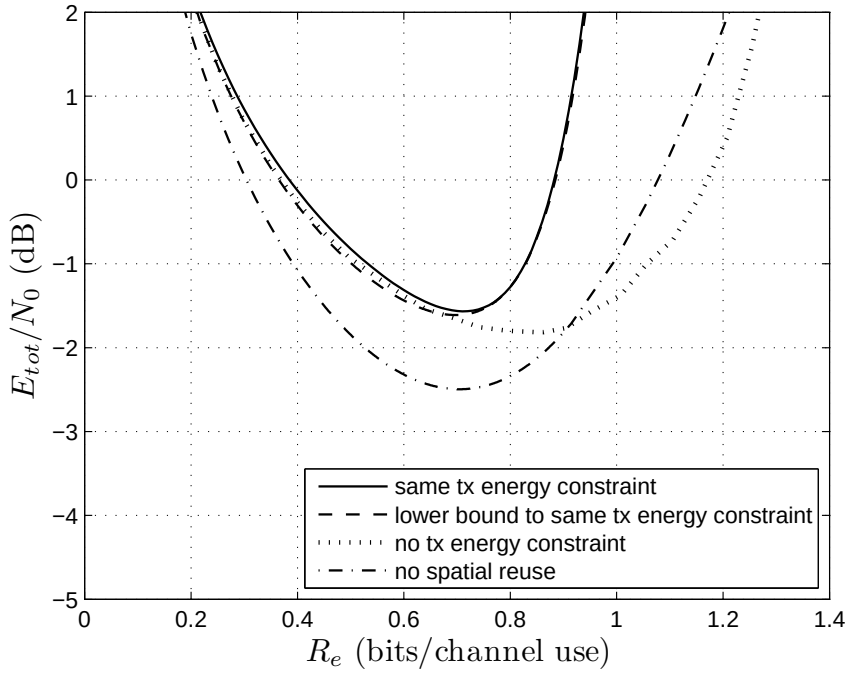


Figure 3.4: Comparison of normalized energy-bandwidth tradeoff when $\gamma_c = -5.31$dB with $k = 4$ hops for selected location of relays at $d_e = 3000$m and $\rho = 3$.

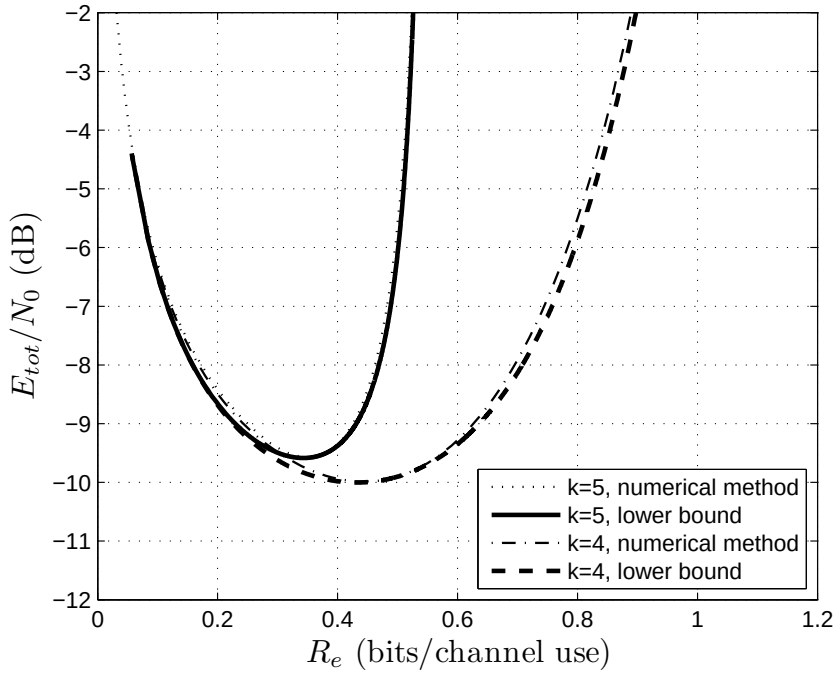


Figure 3.5: Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot when $\gamma_c = -17.36$dB with $k = 4$ and $k = 5$ hops for selected location of relays at $d_e = 3000$m and $\rho = 3$.

Table 3.1: Optimal energy consumption constants for various AWGN channels

Type	Energy constant			Loss (dB)		
	$\eta = 2$	$\eta = 3$	$\eta = 4$	$\eta = 2$	$\eta = 3$	$\eta = 4$
SISO	3.27	2.83	2.25	-	-	-
BISO	3.35	3.03	2.55	-0.10	-0.29	-0.54
BIBO	4.34	3.60	2.86	-1.23	-1.04	-1.04

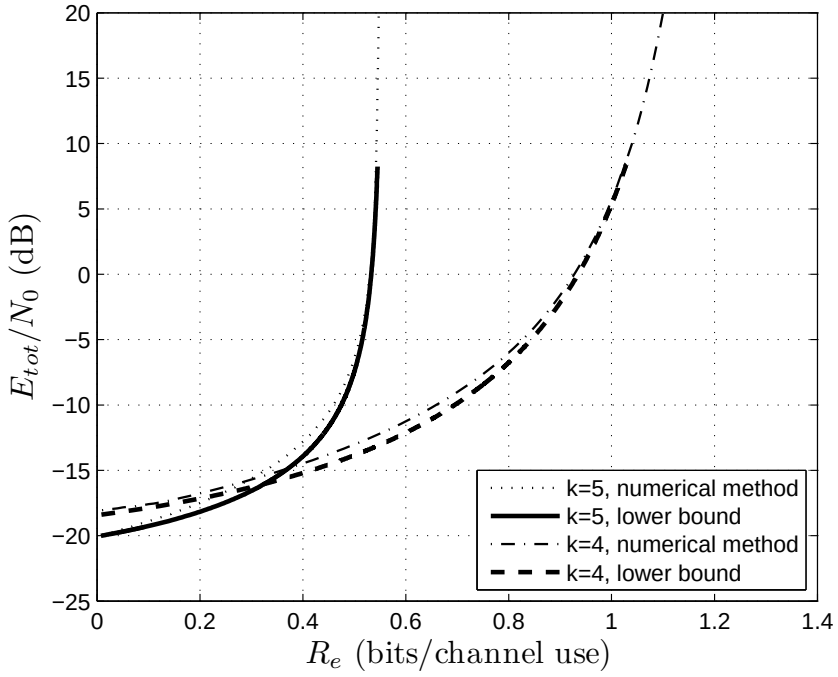


Figure 3.6: Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot when $\gamma_c = 0$ with $k = 4$ and $k = 5$ hops for selected location of relays at $d_e = 3000$m and $\rho = 3$.

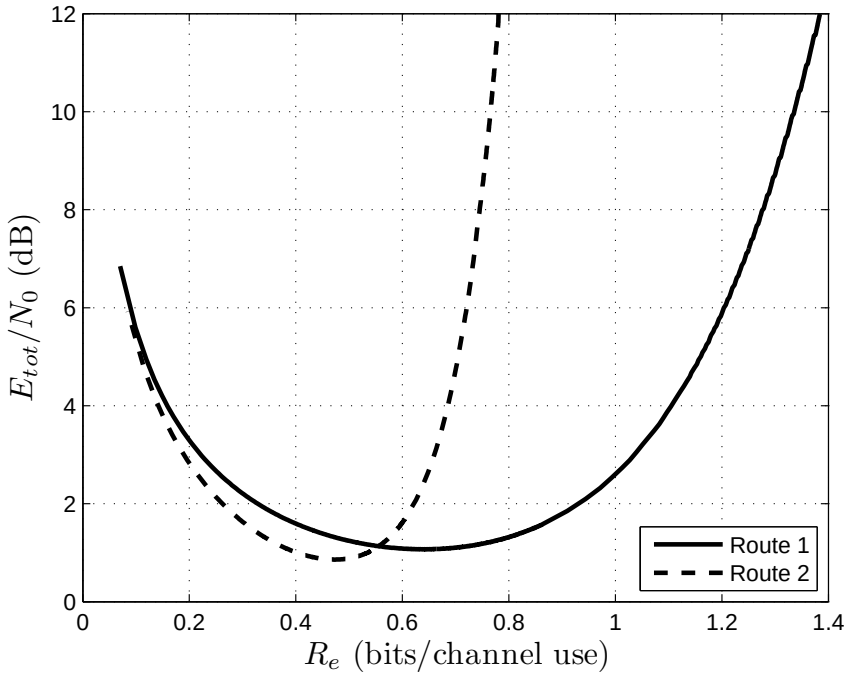


Figure 3.7: Comparison of lower bounds of normalized energy-bandwidth tradeoff for two different routes: $k = 4$ hops for selected location of relays when $\gamma_c = -5.31$dB at $d_e = 3000$m.

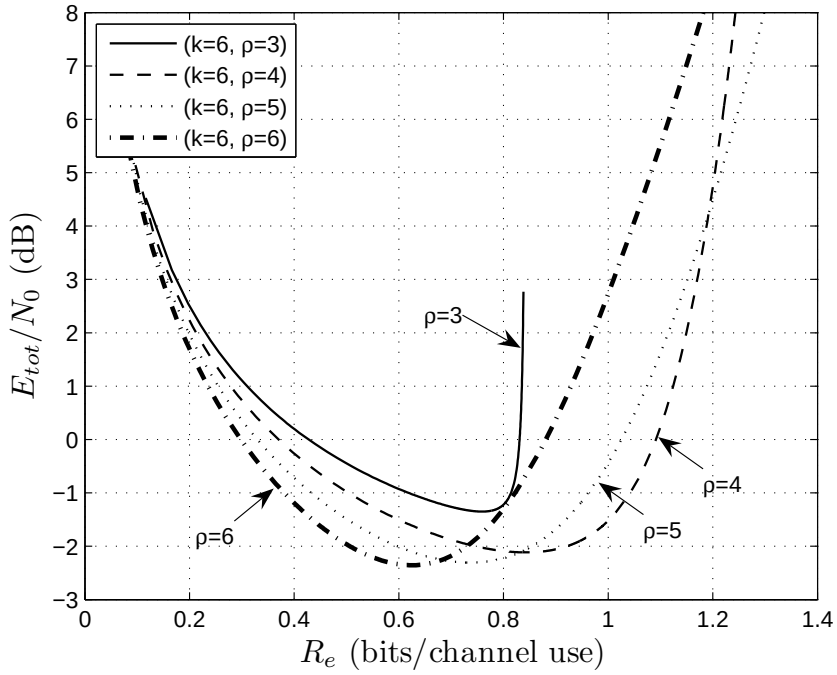


Figure 3.8: Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot: $k = 6$ hops for selected minimum separation of simultaneous transmitters when $\gamma_c = -5.31$dB for equi-distant relays at $d_e = 3000$m.

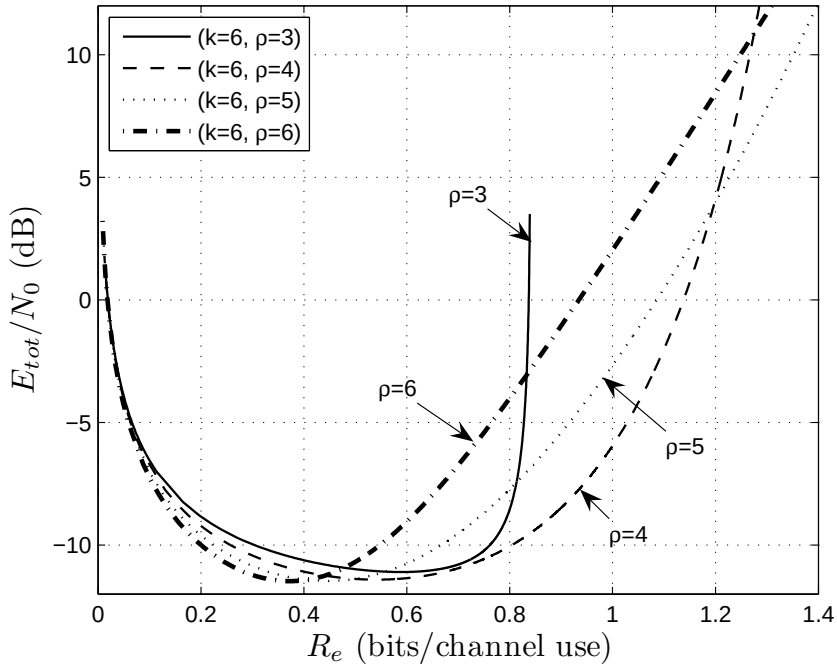


Figure 3.9: Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot: $k = 6$ hops for selected minimum separation of simultaneous transmitters when $\gamma_c = -17.36$dB for equi-distant relays at $d_e = 3000$m.

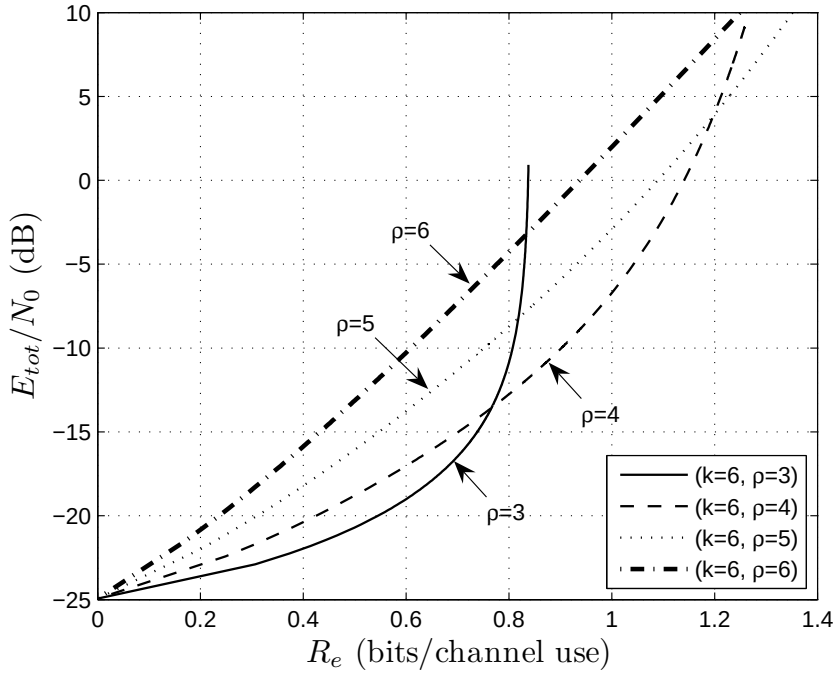


Figure 3.10: Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot: $k = 6$ hops for selected minimum separation of simultaneous transmitters when $\gamma_c = 0$ for equi-distant relays $d_e = 3000$m.

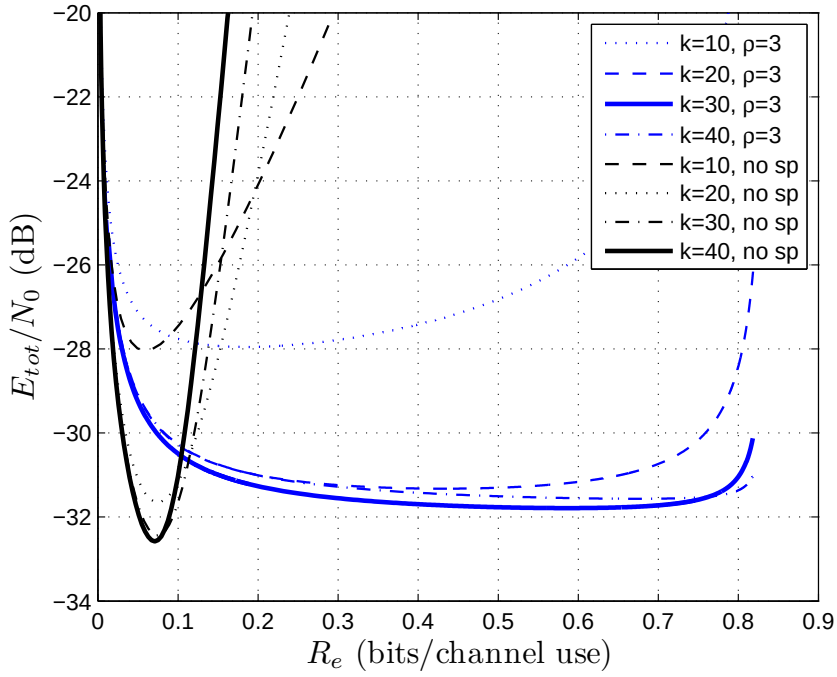


Figure 3.11: Lower bound of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot when $\rho = 3$ and $\gamma_c = -45.31$dB for equi-distant relays $d_e = 30$km.

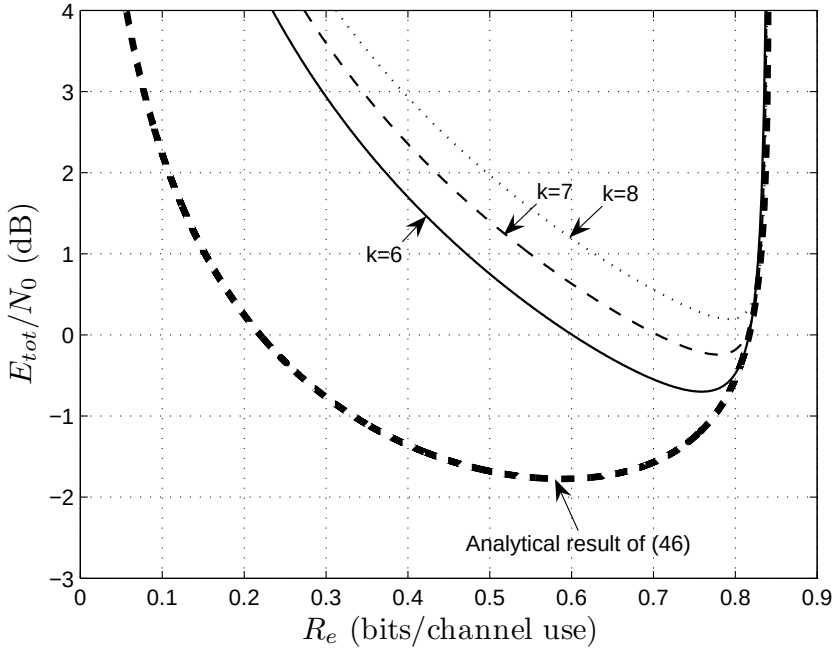


Figure 3.12: Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot for selected number of hops, $k = 6, 7, 8$ when $\gamma_c = -5.31$dB for equi-distant relays $d_e = 3$km and $\rho = 3$.

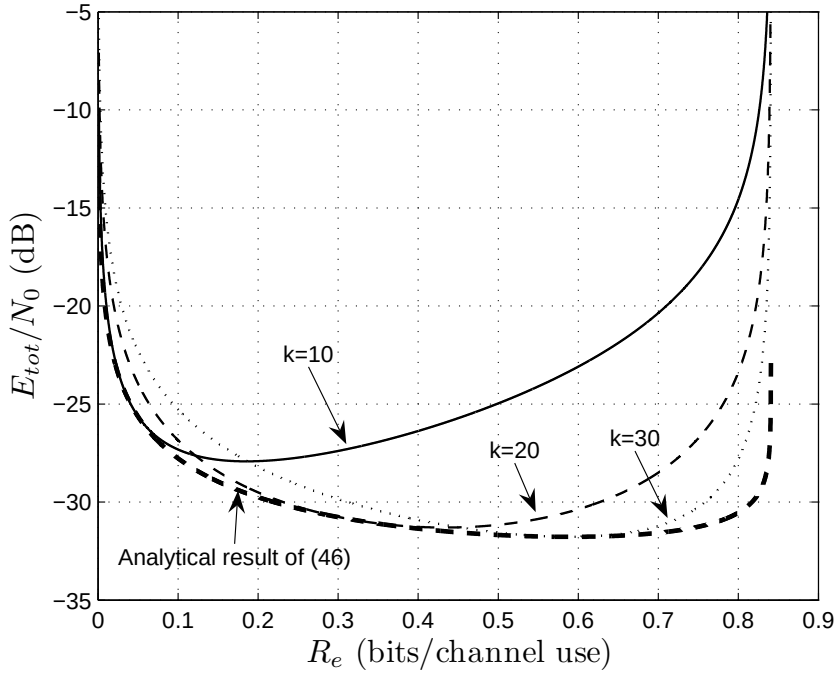


Figure 3.13: Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot for selected number of hops, $k = 10, 20, 30$ when $\gamma_c = -45.31$dB for equi-distant relays $d_e = 3$km and $\rho = 3$.

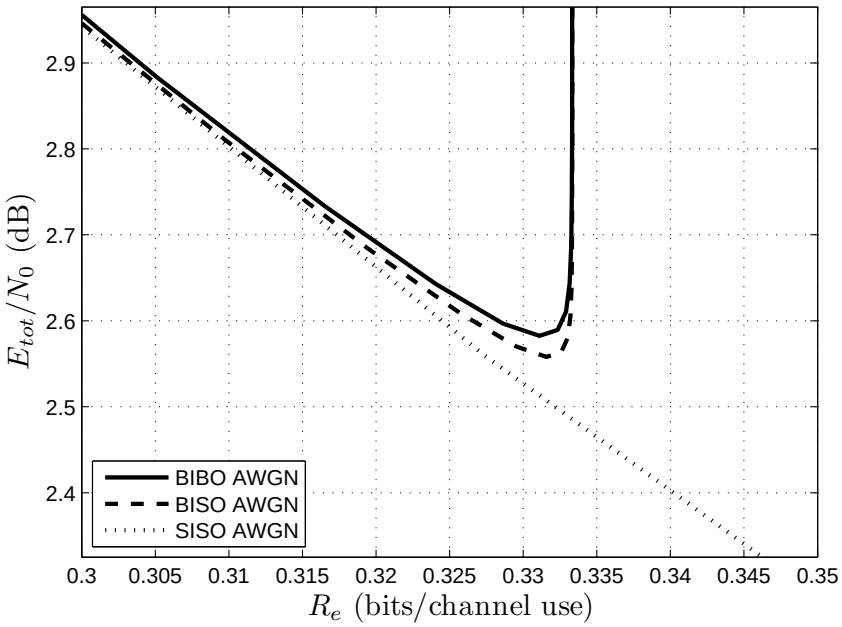


Figure 3.14: Comparison of normalized energy-bandwidth tradeoff with transmission energy constraint in the same slot for various AWGN channel models when $\gamma_c = -5.31$dB and $k = 6$ hops for equi-distant relays $d_e = 3$km and $\rho = 3$.

CHAPTER 4

Energy-Bandwidth Tradeoff in Multi-hop Wireless Networks with Reed-Solomon Codes

The energy-bandwidth tradeoff of an additive white Gaussian noise channel is characterized for multi-hop wireless networks with a single source-destination pair. A total energy consumption model takes into account both the energy radiated by the transmitters and the energy consumed by the circuits to process the received signals. The channel coding and modulation is that Reed-Solomon codes are used with either BPSK or orthogonal modulation. A modified threshold model is used for the successful packet reception from the error correcting capability of channel code and symbol error probability of modulation scheme. Then the end-to-end throughput is defined as the number of information bits successfully delivered (end-to-end) per coded symbol transmitted across the entire network. The performance depends on the location of relays, the energy transmitted and the data rate used by each relay. Optimum strategies that minimize the total energy consumption-per-bit for a given end-to-end throughput are found. In particular, it is shown that BPSK modulation can achieve higher bandwidth efficiency while at low rates orthogonal modulation achieves higher energy efficiency. Also to examine the impact of overhead to the overall costs when packet retransmission allowed, the minimum total energy consumptions per bit for a specific rate are compared. When the relays are equally spaced, a closed-form expression for the energy-

bandwidth tradeoff is provided and the minimum energy consumption-per-bit over all possible end-to-end rates is obtained as well. Low rates require more receiver energy consumption because the receiver is on for a longer time while high rates incur high transmission energy. Therefore at low rates routes with more hops achieve the minimum energy consumption-per-bit while at high rates less hops achieve the minimum energy consumption-per-bit.

4.1 Introduction

Multihop routing for wireless networks has recently garnered much attention due to its potential for reducing energy consumption and extending the effective communication range. The potential for energy-savings of multihop networks results from reducing the average distance between communicating relays and plays an important role in energy-limited wireless environments. In addition, efficient utilization of the available bandwidth is an important design issue in wireless networks. However, these two fundamental goals of minimum energy consumption and maximum bandwidth utilization cannot be achieved simultaneously. Therefore this motivates the need to analyze and design protocols for wireless networks which provide the best possible energy-bandwidth tradeoff.

Previous research has taken different approaches to analyzing wireless multihop relay network [8]-[16]. The study of wireless networks using information theory has led to new achievable rates [8]-[20] as well as several relaying protocols for the maximum throughput [47]-[10]. In [21], the energy-bandwidth tradeoff is analyzed based on equispaced relays in a linear network without considering the receiver energy consumption at the receivers. The energy efficiency of cooperative multihop networks over an orthogonal AWGN channel is studied in [16]. Although various results have been derived, they have something in common: namely they focus on the energy radiated from the trans-

mitters. Consideration only on transmitted energy is quite reasonable for typical long range communications because total energy consumption is dominated by transmission energy. However, for short range communications such as sensor networks, the situation may be different because the energy dissipation to process the received signal at the receiver is not negligible compared to the energy transmitted. In [18]-[19], the receiver energy consumption is considered for minimizing the total energy consumption but the bandwidth efficiency is not analyzed. In addition to the total energy consumption, the end-to-end achievable bandwidth efficiency also needs to be taken into account. The end-to-end bandwidth efficiencies of different routing schemes are analyzed to find a sub-optimal routing path with high bandwidth efficiency in a linear network without considering the receiver energy consumption [26]. In [31], the tradeoff between the total energy consumption-per-bit including circuit processing energy dissipation and the end-to-end rate is studied in arbitrary 2-dimensional networks with the assumption of channel capacity achieving codes between relays.

In this chapter, we consider the tradeoff between the total energy consumption-per-bit and the end-to-end throughput in wireless multihop networks where relays can be placed arbitrarily between a single source-destination pair when practical channel coding and modulation are used. The problem and solution we consider is limited to multihop transmissions based on point-to-point communication and interference-free scheduling which requires perfect synchronization among nodes and geographical information of all relays in the network. The end-to-end throughput measures how efficiently the network utilizes the bandwidth to deliver one information bit successfully by considering the codeword error probability of Reed-Solomon code with BPSK or orthogonal modulation. The total energy consumption-per-bit captures the energy transmitted and the energy required to process the received symbols to forward data. For an arbitrary distribution of relays we find the optimal rate and the power level of each link that minimizes the total energy required for a given end-to-end throughput.

Also we find the end-to-end throughput that minimizes the total energy consumption-per-bit and further compare the performance in order to observe the additional resources required when packet retransmission is allowed. When the receiver energy consumption is ignored, unlike channel capacity achieving codes, the rate that minimizes the total energy consumption-per-bit is not zero. This is because at low rates the energy-per-coded bit becomes small which leads to the increase of error probability. However, the code is not able to compensate for the increase of error probability with the increase in error correction capability.

In summary, the main contribution of this chapter is two fold. First, we propose a framework to analyze the tradeoff between total energy consumption and end-to-end bandwidth utilization for practical coding and modulation schemes. In particular, the impact of the relay's distribution in 2-dimensional networks are quantified to provide the best route for a given distribution of relays unlike the typical analysis in linear networks. In addition, for the successful packet reception, a modified threshold model is used to provide the highest available code rate for a given SNR level. This can be computed from error correcting capability of channel code and symbol error probability of modulation scheme. Secondly, we show that there is a minimum total energy consumption-per-bit for a given route that can be obtained by the optimal selection of the end-to-end throughput depending on transmission scheme.

The rest of this chapter is organized as follows. We describe the system model in Section 4.2, and provide the performance measures and the problem formulation in Section 4.3. In Section 4.4, we evaluate the performance in terms of the total energy consumption-per-bit and end-to-end throughput for BPSK and orthogonal modulation. In Section 4.5, the special case of equi-spaced relays in linear networks is considered. We provide numerical results in Section 4.6 and summarizes the results in Section 4.7.

4.2 System Model

Consider a wireless network where a source node communicates with a destination node, separated by a distance d_e, through a multi-hop route with the information traversing through $k-1$ relay nodes that are arbitrarily located between the source and the destination. We denote the relay distance of hop i as d_i and let $\alpha_i = d_i/d_e$ where $0 < \alpha_i < 1$ and $\overline{\alpha} = (\alpha_1, ..., \alpha_k)$.

In the i-th link transmission, an (N, K_i) Reed-Solomon code is assumed where the elements of the code words are selected from an alphabet of $M = 2^L$ symbols. Thus L information bits are mapped into one of the M symbols. The block length N, the number of information symbols K_i, and the error correcting capability t are related by

$$N = M - 1 = 2^L - 1 \tag{4.1}$$

$$K_i = 1, 2, ..., N-1 \tag{4.2}$$

$$t = \lfloor 0.5(N - K_i) \rfloor. \tag{4.3}$$

A RS encoded symbol is mapped into L-BPSK or 2^L-ary orthogonal signals. Note that for L-BPSK modulation, NL channel uses are required to transmit K_iL information bits and each modulated symbol has energy E_s, thus an L-BPSK modulated signal has energy LE_s. For 2^L-ary orthogonal modulation, $N2^L$ channel uses are required for K_iL information bits and an orthogonally modulated signal has energy E_s. The rate of transmission R_i in information bits per channel use is then

$$R_i = \begin{cases} \frac{K_i}{N} \text{ (bits/channel use), BPSK} \\ \frac{K_iL}{N2^L} \text{ (bits/channel use), orthogonal modulation.} \end{cases} \tag{4.4}$$

Then the modulated symbols are transmitted through the channel and each relay is assumed to be in the far-field region of the corresponding transmitter. The received

power $P_{r,i}$ in link i in the far-field is modeled as

$$P_{r,i} = \frac{\beta}{d_i^{\eta}} P_{out}, \quad d_i > 1$$

where P_{out} is the transmitter output power and η is a path-loss exponent (typically between 2 and 4), and β is a constant from the antenna characteristics. For simplicity we assume $\beta = 1$.

The received symbols are then demodulated with a symbol error probability of the i-th link, $P_{M,i}$. For L-BPSK modulation, the symbol error probability P_M is given by

$$P_M = 1 - (1 - p)^L \tag{4.5}$$

where p is the bit error probability of BPSK which is given by $p = Q(\sqrt{2E_s/N_0}) = Q(\sqrt{2\gamma})$ when the received SNR per channel use is $\gamma = E_s/N_0$. For 2^L-ary orthogonal modulation, the symbol error probability P_M can be expressed using an upper bound by the union bound as follows

$$P_M < 2^L Q\left(\sqrt{\frac{E_s}{N_0}}\right). \tag{4.6}$$

For RS codes with bounded distance decoding, the decoder can decode correctly only if t or fewer error occurs. Hence the probability of not decoding correctly is given by

$$P_{e,i} = \sum_{j=t+1}^{N} \binom{N}{j} P_{M,i}^j (1 - P_{M,i})^{N-j} \tag{4.7}$$

where $P_{M,i}$ denotes a channel encoded symbol error probability of the i-th link [39]. The throughput of the i-th link, S_i is then given by

$$S_i = R_i(1 - P_{e,i}). \tag{4.8}$$

Regarding the forwarding strategy, a simple decode-and-forward protocol is considered in which relay i decodes the message sent from relay $i-1$, re-encodes it, and then forwards the message to relay $i+1$. We consider link connectivity only between intermediate neighboring relays, and thus any cooperative relaying scheme to exploit cooperative diversity is not allowed.

We assume that each node has the same circuitry which requires circuit energy E_p Joules to process a single received symbol for L-BPSK modulation, and thus totally NLE_p Joules are required. However, for 2^L-ary orthogonal modulation, $N2^LE_p$ Joules are totally consumed. In practice, the circuit processing energy depends on a transceiver design and architecture, and coding and modulation techniques. We assume E_p is a constant throughout this chapter. We note that the total energy consumed per information bit is the energy consumed per coded bit, E_p, divided by the rate of the code. As such, very low rates require a large amount of energy per information bit to process the received signal. This leads to an optimal rate that minimizes the total energy. High rates require high transmitted energy per information bit. Low rates require high receiver processing energy per information bit.

We assume that each node operates in half-duplex mode, hence it cannot transmit and receive simultaneously. Thus each transmission needs to be scheduled to avoid conflicts. Inter-link interference is not considered here since in this chapter we assume that the network operates without spatial reuse. Perfect synchronization is assumed to be achieved among the nodes, hence each relay node transmits in its own slot (time or frequency) without any interference.

4.3 Performance Measures and Problem Formulation

4.3.1 End-to-end Rate

To measure the end-to-end bandwidth efficiency, we consider the number of information bits transmitted (end-to-end) per channel use transmitted by any node in the network that is forwarding the data. The end-to-end throughput S_e without spatial reuse is given by $S_e = \left(\sum_{j=1}^{k} S_j^{-1}\right)^{-1}$ (bits/channel use) where S_j is the throughput on the hop j and k denotes the number of hops [31].

4.3.2 End-to-end Energy Consumption

To measure the total energy consumption per information bit, we consider the energy transmitted and the energy required for processing the received symbol at the receiver. The total energy consumption per information bit, E_t, is the ratio of total consumed energy divided by the total number of information bits transferred over the network, which can be expressed as

$$E_t = \frac{\text{total energy for transmitting and processing}}{\text{total number of information bits}}.$$

For fair comparison purposes with the standard results on energy required for reliable communication in [31], the energy is normalized by the end-to-end propagation loss. We define the normalized total energy consumption per information bit-to-noise power spectral density ratio as

$$\frac{E_{tot}}{N_0} = \frac{E_t}{N_0} d_e^{-\eta}$$

and the normalized energy transmitted on the i-th link is defined as

$$\frac{E_{i,tx}}{N_0} d_e^{-\eta} = \frac{E_{i,tx}}{N_0} \alpha_i^{\eta} d_i^{-\eta} = \alpha_i^{\eta} \gamma_i$$

where $E_{i,tx}$ is the energy transmitted per coded symbol and $\gamma_i = \frac{E_{i,tx}}{N_0} d_i^{-\eta}$ denotes the received energy per coded symbol-to-noise power spectral density ratio. Similarly a normalized circuit processing energy γ_c is defined as $\gamma_c \triangleq \frac{E_p}{N_0} d_e^{-\eta}$.

4.3.3 Minimum energy consumption per bit

For L-BPSK modulation, since it requires NLE_s and NLE_p Joules for transmitting and processing KL information bits, the total energy consumption per information bit for L-BPSK modulation is

$$E_t = \frac{NLE_s + NLE_p}{KL} = \frac{NE_s + NE_p}{K} = \frac{E_s + E_p}{R} \text{ (J/information bit)} \tag{4.9}$$

where $R = K/N$ for L-BPSK. However, for 2^L-ary orthogonal modulation, it requires NE_s and $N2^L E_p$ Joules for KL information bits, we have

$$E_t = \frac{NE_s + N2^L E_p}{KL} = \frac{E_s}{2^L R} + \frac{E_p}{R} \text{ (J/information bit).} \tag{4.10}$$

where $R = KL/N2^L$ for 2^L-ary orthogonal modulation.

Therefore, for a given relay distribution $\overline{\alpha} = (\alpha_1, ..., \alpha_k)$, the problem of minimizing the total energy consumption per information bit with arbitrary transmission energy and rate of each link for a given end-to-end throughput S_e can be formulated as the following optimization problem

Problem Ia:

$$\begin{aligned}\frac{E_{tot}(\overline{\alpha}, S_e)}{N_0} = \min_{\substack{(\gamma_1,...,\gamma_k)\\(R_1,..,R_k)}} \quad & \sum_{i=1}^{k} \frac{\alpha_i^{\eta}\gamma_i}{F_i S_i} + \frac{\gamma_c}{S_i} \\ \text{s.t.} \quad & S_e^{-1} = \textstyle\sum_{i=1}^{k} S_i^{-1} \\ & S_i = R_i(1 - P_{e,i}) \\ & F_i = \begin{cases} 1, \text{ BPSK} \\ 2^L, \text{ orthogonal modulation.} \end{cases}\end{aligned} \tag{4.11}$$

where the optimization variables are γ_i and R_i for $i = 1, ..., k$, and F_i results from the bandwidth characteristics of each modulation which can be identified in (4.9) and (4.10). While numerical methods can determine the minimum energy consumption-per-bit for a given end-to-end throughput, it is hard to obtain an optimal solution for arbitrary networks.

In this chapter, we provide a threshold model for successful packet reception where the code rate can be expressed as a function of symbol error probability from the error correcting capability of channel code and symbol error probability of modulation scheme, which makes the optimization problem with a single variable. For this threshold model, we have

$$P_{e,i} = \begin{cases} 0, \ t > NP_{M,i} \\ 1, \ t \leq NP_{M,i}. \end{cases}$$

Then from the parameter relations of the RS codes in (4.3), we have

$$\frac{K_i}{N} \leq 1 - 2P_{M,i} \tag{4.12}$$

and from (4.4) and (4.8),

$$S_i = R_i \leq \begin{cases} 1 - 2P_{M,i}, \text{ BPSK} \\ (1 - 2P_{M,i})\frac{L}{2^L}, \text{ orthogonal modulation.} \end{cases} \tag{4.13}$$

where $0 \leq P_M < 1/2$. With this bounded code rate, the optimization problem in (4.11) can be expressed as

Problem Ib:

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, S_e)}{N_0} = \min_{(\gamma_1,..,\gamma_k)} \quad & \sum_{i=1}^{k} \frac{\alpha_i^{\eta}\gamma_i}{F_i R_i} + \frac{\gamma_c}{R_i} \\ \text{s.t.} \quad & S_e^{-1} = \textstyle\sum_{i=1}^{k} R_i^{-1} \\ & S_i = R_i = \begin{cases} 1 - 2P_{M,i}, \text{ BPSK} \\ (1 - 2P_{M,i})\frac{L}{2^L}, \text{ orthogonal modulation.} \end{cases} \\ & F_i = \begin{cases} 1, \text{ BPSK} \\ 2^L, \text{ orthogonal modulation.} \end{cases} \end{aligned} \tag{4.14}$$

where $P_{M,i}$ denotes the symbol error probability of the i-th link and the optimization variable is γ_i for $i = 1, ..., k$. Our fundamental goal of this chapter is, for a given routing path $\overline{\alpha} = (\alpha_1, ...\alpha_k)$, to find the optimal set of transmission energies $(E_{1,.tx}, ..., E_{k,tx})$ that provides the smallest total energy consumption per information bit for a given end-to-end throughput S_e. We also focus on the global minimum of total energy consumption and the corresponding optimal end-to-end throughput.

4.4 Performance Evaluation

In this section we investigate the energy-bandwidth tradeoff for the case of BPSK and orthogonal modulation. For each case we approximate the symbol error probability to obtain closed form solutions for arbitrary networks. Before proceeding this section,

we summarize what is done for the problem simplification as follows. Problem Ia is the original optimization problem which contains two variables regarding SNR and code rate but it is hard to obtain optimal solutions when the number of relays becomes large. Thus, Problem Ia is simplified to Problem Ib, a single variable optimization problem, by using threshold model where the maximum achievable rate is found from the characteristics of channel code and modulation scheme. However it is still hard to find optimal solutions when the number of relays becomes large. As such, we need to approximate the symbol error probability which enables us to find optimal analytical solutions for arbitrary networks.

4.4.1 BPSK

Using the fact that $Q(x) \leq 0.5 \exp(-0.5x^2)$, the symbol error probability P_M can be approximated as

$$P_M \approx L \cdot p \leq \frac{L}{2} e^{-\gamma}. \tag{4.15}$$

where since $P_M < 1/2$, we have $\gamma > \ln L$. Therefore, from (4.13), we have

$$R = \frac{K}{N} \leq 1 - 2P_M \approx 1 - Le^{-\gamma} \text{ (information bits/channel use)} \tag{4.16}$$

Then the optimization problem for k multihops can be written as

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, S_e)}{N_0} = \min_{(\gamma_1,..,\gamma_k)} \quad & \sum_{i=1}^{k} \frac{\alpha_i^{\eta}\gamma_i + \gamma_c}{1 - Le^{-\gamma_i}} \\ \text{s.t.} \quad & S_e^{-1} = \textstyle\sum_{i=1}^{k} (1 - Le^{-\gamma_i})^{-1} \\ & \gamma_i > \ln L \end{aligned} \tag{4.17}$$

where the optimization variables are γ_i's. The above optimization problem can be converted into an equivalent convex optimization problem as shown below. Let $1 -$

$Le^{-\gamma} = x^{-1}$ and thus we have $\gamma = \ln\left(\frac{Lx}{x-1}\right)$.

Problem IIa:

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, S_e)}{N_0} = \min_{(x_1,..,x_k)} \quad & \sum_{i=1}^{k} x_i \left[\alpha_i^{\eta} \ln\left(\tfrac{Lx_i}{x_i-1}\right) + \gamma_c\right] \\ \text{s.t.} \quad & S_e^{-1} = \textstyle\sum_{i=1}^{k} x_i \\ & x_i > 1. \end{aligned} \tag{4.18}$$

The objective function $f(x) = x \ln\left(\frac{x}{x-1}\right)$ is easily shown to be convex for $x > 1$ because its second order derivative with respect to x is $f''(x) = \frac{1}{(x-1)^2 x} > 0$ for $x > 1$. Then the optimal solution can be obtained using the Lagrangian function of (4.18)

$$L(x_1, ..., x_k, \lambda) = \sum_{i=1}^{k} x_i \left[\alpha_i^{\eta} \ln\left(\frac{Lx_i}{x_i - 1}\right) + \gamma_c\right] - \lambda \left[\sum_{i=1}^{k} x_i - S_e^{-1}\right] \tag{4.19}$$

where $x_i > 1$ for all i. Therefore the Karush-Kuhn-Tucker (KKT) conditions give the following parametric solution which can be evaluated by numerical techniques,

$$\lambda \;=\; \alpha_i^{\eta} \left[\ln\left(\frac{Lx_i}{x_i - 1}\right) - \frac{1}{x_i - 1}\right] + \gamma_c \tag{4.20}$$

where $x_i > 1$ for all i. The solution to the above equation can be obtained as

$$x_i^* = \frac{\mathcal{W}_{-1}(-e^{a-1})}{1 + \mathcal{W}_{-1}(-e^{a-1})} \tag{4.21}$$

where $a = (\lambda - \gamma_c)\alpha_i^{-\eta} - \ln L$ and $\mathcal{W}_{-1}(x)$ denotes the alternate branch of the Lambert W-function, for which $\mathcal{W}(x) < -1$. Therefore the solution to Problem IIa is as follows:

$$\begin{aligned} \gamma_i &= \ln\left(\frac{Lx_i^*}{x_i^* - 1}\right) && (4.22) \\ S_e^{-1} &= \sum_{i=1}^{k} x_i^* && (4.23) \\ \frac{E_{tot}(\overline{\alpha}, S_e)}{N_0} &= \sum_{i=1}^{k} x_i^* \left[\alpha_i^{\eta} \ln\left(\frac{Lx_i^*}{x_i^* - 1}\right) + \gamma_c\right] && (4.24) \end{aligned}$$

.

4.4.2 Orthogonal Modulation

Using the union bound, the symbol error probability P_M can be upper bounded

$$P_M < MQ(\sqrt{\frac{E_s}{N_0}}) \leq 0.5Me^{-0.5\gamma} = 2^{L-1}e^{-0.5\gamma} \tag{4.25}$$

where since $P_M < 1/2$, we have $\gamma > 2L\ln 2$. Therefore, from (4.13), we have

$$R = \frac{KL}{N2^L} \leq \frac{(1-2P_M)L}{2^L} \approx \frac{(1-2^Le^{-0.5\gamma})L}{2^L} \text{ (information bits/channel use)} \tag{4.26}$$

Then the optimization problem for k multihops can be written as

$$\begin{aligned} \frac{E_{tot}(\overline{\alpha}, S_e)}{N_0} = \min_{(\gamma_1,..,\gamma_k)} \quad & \sum_{i=1}^{k} \frac{\alpha_i^{\eta}\gamma_i + 2^L\gamma_c}{(1-2^Le^{-0.5\gamma_i})L} \\ \text{s.t.} \quad & S_e^{-1} = \textstyle\sum_{i=1}^{k} \left[L2^{-L}(1-2^Le^{-0.5\gamma_i})\right]^{-1} \\ & \gamma_i > 2\ln 2^L \end{aligned} \tag{4.27}$$

where the optimization variables are γ_i's. The above optimization problem can be converted into an equivalent convex optimization problem as shown below. Let $(1 - 2^Le^{-0.5\gamma}) = x^{-1}$ and thus we have $\gamma = 2\ln\left(\frac{2^Lx}{x-1}\right)$.

Problem IIb:

$$\begin{aligned}\frac{E_{tot}(\overline{\alpha}, S_e)}{N_0} = \min_{(x_1,..,x_k)} \quad & \sum_{i=1}^{k} L^{-1} x_i \left[2\alpha_i^{\eta} \ln\left(\tfrac{2^L x_i}{x_i - 1}\right) + 2^L \gamma_c\right] \\ \text{s.t.} \quad & S_e^{-1} = \tfrac{2^L}{L} \textstyle\sum_{i=1}^{k} x_i \\ & x_i > 1.\end{aligned} \tag{4.28}$$

Since we have shown that the objective function is convex for $x > 1$, the optimal solution can be obtained using the Lagrangian function of (4.28)

$$L(x_1, ..., x_k, \lambda) = \sum_{i=1}^{k} \frac{x_i}{L} \left[2\alpha_i^{\eta} \ln\left(\frac{2^L x_i}{x_i - 1}\right) + 2^L \gamma_c\right] - \lambda \left[\frac{2^L}{L} \sum_{i=1}^{k} x_i - S_e^{-1}\right] \tag{4.29}$$

where $x_i > 1$ for all i. Therefore the Karush-Kuhn-Tucker (KKT) conditions give the following parametric solution which can be evaluated by numerical techniques,

$$\lambda = 2^{-L} \left(2\alpha_i^{\eta} \left[\ln\left(\frac{2^L x_i}{x_i - 1}\right) - \frac{1}{x_1 - 1}\right] + 2^L \gamma_c\right) \tag{4.30}$$

where $x_i > 1$ for all i. The solution to the above equation can be obtained as

$$x_i^* = \frac{\mathcal{W}_{-1}(-e^{a-1})}{1 + \mathcal{W}_{-1}(-e^{a-1})} \tag{4.31}$$

where $a = 2^{L-1}(\lambda - \gamma_c)\alpha_i^{-\eta} - \ln 2^L$ and $\mathcal{W}_{-1}(x)$ denotes the alternate branch of the Lambert W-function, for which $\mathcal{W}(x) < -1$. Therefore the solution to Problem IIb is as follows:

$$\gamma_i = 2 \ln\left(\frac{2^L x_i^*}{x_i^* - 1}\right) \tag{4.32}$$

$$S_e^{-1} = \frac{2^L}{L} \sum_{i=1}^{k} x_i^* \tag{4.33}$$

$$\frac{E_{tot}(\overline{\alpha}, S_e)}{N_0} = \sum_{i=1}^{k} L^{-1} x_i^* \left[2\alpha_i^{\eta} \ln\left(\frac{2^L x_i^*}{x_i^* - 1}\right) + 2^L \gamma_c\right]. \tag{4.34}$$

4.4.3 Parametric Solution to the Minimum Energy-Bandwidth Tradeoff

From the results of previous sections, we can obtain the solution to Problem I and II from the following procedure using numerical methods:

Step 1. Fix λ.

Step 2. Solve (4.21) or (4.31) for x_i for $i = 1, ..., k$.

Step 3. Determine γ_i, S_e and E_{tot}/N_0 and from (4.22) and (4.32).

By varying λ we can determine the optimal tradeoff between E_{tot}/N_0 and S_e.

4.5 Performance of Equidistant Multi-hop Routing

Consider the case of equi-spaced relays where $\alpha_i = 1/k$. In this case the same link SNR, γ is achieved and thus each link communicates at the same rate.

1) BPSK: From (4.17), the end-to-end throughput S_e is then

$$S_e = \frac{R}{k} = \frac{1 - Le^{-\gamma}}{k} \tag{4.35}$$

and the total energy consumption per information bit is

$$\frac{E_{tot}(k, S_e)}{N_0} = \frac{k^{1-\eta}\gamma + k\gamma_c}{1 - Le^{-\gamma}} = \frac{k^{-\eta} \ln\left(\frac{L}{1-kS_e}\right) + \gamma_c}{S_e} \tag{4.36}$$

With a transform of variables $k = e^{-x}$ and $S_e = e^{-y}$, the minimum total energy consumption can be converted into an equivalent optimization problem

$$\begin{aligned} \frac{E_{tot}(x^*, y^*)}{N_0} = \min_{x,y \in \mathbf{R}} \quad & e^{\eta x + y} \ln\left(\tfrac{L}{1-e^{-x-y}}\right) + \gamma_c e^{y} \\ \text{s.t.} \quad & x + y > 0 \end{aligned} \tag{4.37}$$

where the constraint results from the relation that $0 < S_e < 1/k$, that is, $e^{-y} < e^x$.

2) Orthogonal modulation: From (4.27), the end-to-end throughput S_e is then

$$S_e = \frac{R}{k} = \frac{L2^{-L}(1 - 2^L e^{-\gamma/2})}{k} \tag{4.38}$$

and the total energy consumption per information bit is

$$\frac{E_{tot}(k, S_e)}{N_0} = \frac{k^{1-\eta}\gamma + k\gamma_c}{L(1 - 2^L e^{-\gamma/2})} = \frac{k^{-\eta} 2 \ln\left(\frac{2^L}{1-k2^L S_e/L}\right) + 2^L \gamma_c}{2^L S_e} \tag{4.39}$$

With a transform of variables $k = Le^{-x}$ and $2^L S_e = e^{-y}$, the minimum total energy consumption can be converted into an equivalent optimization problem

$$\frac{E_{tot}(x^*, y^*)}{N_0} = \min_{x,y \in \mathbf{R}} \quad 2L^{-\eta} e^{\eta x + y} \ln\left(\frac{2^L}{1-e^{-x-y}}\right) + 2^L \gamma_c e^y \tag{4.40}$$

$$\text{s.t.} \qquad x + y > 0. \tag{4.41}$$

3) Minimum total energy consumption: From the previous sections, we can rewrite the minimum of the total energy consumption per information bit as

Problem III:

$$\frac{E_{tot}(x^*, y^*)}{N_0} = \min_{x,y \in \mathbf{R}} \quad a_1 e^{\eta x + y} \ln\left(\frac{a_2}{1-e^{-x-y}}\right) + a_3 \gamma_c e^y \tag{4.42}$$

$$\text{s.t.} \qquad x + y > 0. \tag{4.43}$$

where for BPSK, $a_1 = a_3 = 1$ and $a_2 = L$. For orthogonal modulation, $a_1 = 2L^{-\eta}$ and $a_2 = a_3 = 2^L$. The convexity of the objective is shown in Appendix C. Then from the complementary slackness in the Karush-Kuhn-Tucker (KKT) conditions [38], we can derive that the Lagrangian multiplier is equal to 0 since $x + y > 0$ which makes the optimization straightforward.

4) Optimization of the number of hops: For a given end-to-end throughput, treating

the number of hops as a continuous variable, we optimize the total energy consumption over x

$$\frac{E_{tot}(x^*, y)}{N_0} = \min_x \left[a_1 e^{\eta x + y} \ln \left(\frac{a_2}{1 - e^{-x-y}} \right) + \gamma_c e^y \right]. \tag{4.44}$$

By setting the derivative to zero, we obtain

$$e^{-x^*} = be^y \tag{4.45}$$

where

$$b = 1 - L \cdot \exp \left(\mathcal{W}_{-1} \left(\frac{-1}{\eta L e^{-1/\eta}} \right) + \frac{1}{\eta} \right) \tag{4.46}$$

The optimal number of hops for a given end-to-end throughput is given by for the case of BPSK

$$k^*(S_e) = be^y = bS_e^{-1} \tag{4.47}$$

and for the case of orthogonal modulation respectively

$$k^*(S_e) = Lbe^y = \frac{Lb}{2^L} S_e^{-1}. \tag{4.48}$$

Then the total energy consumption at the optimal number of hops is given by

$$\frac{E_{tot}(x^*, y)}{N_0} = a_1 b^{-\eta} \ln \left(\frac{a_2}{1 - b} \right) \cdot e^{(1-\eta)y} + a_3 \gamma_c e^y. \tag{4.49}$$

5) Optimization of the end-to-end throughput: The problem of determining the optimal end-to-end throughput is formulated by substituting (4.45) into (4.42), which

results in

$$\frac{E_{tot}(x^*, y^*)}{N_0} = \min_y \left[a_1 b^{-\eta} \ln\left(\frac{a_2}{1-b}\right) \cdot e^{(1-\eta)y} + a_3 \gamma_c e^y \right] \tag{4.50}$$

Setting the derivative with respect to y equal to zero, we obtain

$$e^{-y^*} = \left[\frac{a_3 \gamma_c}{a_1 b^{-\eta} (\eta - 1) \ln\left(\frac{a_2}{1-b}\right)} \right]^{1/\eta} \tag{4.51}$$

The optimal number of hops $k^*(S_e^*)$ for the case of BPSK is obtained by substituting (4.51) into (4.45),

$$k^*(S_e^*) = be^{y^*} = \left[\frac{\gamma_c}{a_1(\eta - 1) \ln\left(\frac{a_2}{1-b}\right)} \right]^{-1/\eta} \tag{4.52}$$

$$= \left[\frac{1}{a_1(\eta - 1) \ln\left(\frac{a_2}{1-b}\right)} \cdot \frac{E_p}{N_0} \right]^{-1/\eta} d_e \tag{4.53}$$

and for the case of orthogonal modulation,

$$k^*(S_e^*) = Lbe^{y^*} = L \left[\frac{a_3 \gamma_c}{a_1(\eta - 1) \ln\left(\frac{a_2}{1-b}\right)} \right]^{-1/\eta} \tag{4.54}$$

$$= L \left[\frac{a_3}{a_1(\eta - 1) \ln\left(\frac{a_2}{1-b}\right)} \cdot \frac{E_p}{N_0} \right]^{-1/\eta} d_e. \tag{4.55}$$

The resulting minimum total energy consumption is then

$$\frac{E_{tot}(k^*, S_e^*)}{N_0} = \left[a_1 b^{-\eta} \ln\left(\frac{a_2}{1-b}\right) \right]^{1/\eta} [1 + a_3(\eta - 1)] \left[\frac{\gamma_c}{\eta - 1} \right]^{1-1/\eta} \tag{4.56}$$

$$= \left[a_1 b^{-\eta} \ln\left(\frac{a_2}{1-b}\right) \right]^{1/\eta} \frac{1 + a_3(\eta - 1)}{(\eta - 1)^{1-1/\eta}} \left[\frac{E_p}{N_0} \right]^{1-1/\eta} d_e^{1-\eta} \tag{4.57}$$

From the above results we observe that the optimal rate decreases as d_e^{-1} and the optimal number of hops grows linearly with d_e. In addition, there are two key observations for

the minimum normalized total energy consumption: it (i) grows as the $\frac{\eta-1}{\eta}$ power of the processing energy and (ii) decreases as $d_e^{1-\eta}$. Therefore we can conclude that the actual unnormalized minimum total energy consumption increases linearly with d_e from the relation between the normalized total energy consumption and the end-to-end distance.

4.6 Numerical Results

In the results we assume the path-loss exponent $\eta = 4$, end-to-end distance $d_e =$ 3000m, noise power spectral density $N_0 = -174$dBm/Hz and circuit processing energy $E_p = 0.293\mu$J/symbol ($\gamma_c = -5.31$dB) unless otherwise specified.

In Fig. 4.2 - Fig. 4.4, we plot the normalized energy-bandwidth tradeoffs for a two hop network where the ratio of link distances are 6 : 4 with different processing energies, γ_c= -∞, -5.31, and -15.31 in dB scale respectively. This is done based on numerical optimization of Problem Ia in (4.11) and Ib in (4.14), and approximate parametric solutions of each modulation scheme in (4.22) and (4.32). Energy-bandwidth tradeoff of soft-input/soft-output AWGN channel is plotted for comparison using the result of [31] as well. Problem Ib where a modified threshold model is assumed without any approximation for the symbol error probability is shown to be tight to Problem Ia which is the actual energy-per-bit as γ_c becomes large. Because the symbol error probabilities are approximated as shown in (4.15) and (4.25), there are performance gaps in Problem IIa and IIb where parametric solutions are used. However it is shown that our approximate parametric solution tracks well the actual energy-bandwidth tradeoff characteristic. At low rates the energy consumption due to the transmitter alone decreases because of increasing error protection with lower rate codes, while the overall energy consumption increases due to the energy needed for processing and the increased error probability. In particular, when $\gamma_c = 0$, unlike channel capacity achieving codes, the minimum energy-per-bit is not achieved as $S_e \to 0$ because of the increased error probability. At

high rates the energy consumption increases due to the increase in required transmitted energy for high rate communication. Such characteristics lead to an optimal nonzero rate that minimizes the total energy consumption-per-bit. The results also show that at low rates, orthogonal modulation achieves better energy efficiency while at high rates, BPSK modulation achieves higher bandwidth efficiency.

In Fig. 4.5, we compare the normalized energy-bandwidth tradeoffs of BPSK and orthogonal modulation for $\gamma_c = -5.31$dB when packet retransmissions are allowed using numerical optimization of Problem Ia. To incorporate the impact of retransmission, we assume the geometric distribution for the successful packet distribution with $p = 1 - P_e$ in (4.7) and the average number of retransmission is then $1/p$. A packet is retransmitted until it is successfully decoded. The overhead in terms of energy consumption and channel uses is considered. It is observed in both modulation schemes that there are a slight performance loss due to retransmission. Note that this small loss may result from the small number of hops, and thus when the number of relays becomes large, the performance loss accumulated at each hop can be significantly large.

To show that different selection of relays affect the energy-bandwidth tradeoff, we consider a wireless network with 2 relays as an example where the locations of the nodes are given as shown in Fig. 4.1. Using the approximate solutions in (4.22) and (4.32), we compare the performance of three possible multihop routes for $\gamma_c = -5.31$dB in Fig. 4.6. It is observed from these figures that optimal routing is done with three hops using all available nodes as relays at low rates while at high rates, two hops using relay 2 outperforms the other routes regardless of the modulation. This is because fewer hops require less energy than more hops while at low rates the reveres is true, which can be also verified for the linear, equi-spaced network as discussed next. The efficient algorithm which can find the optimal route for a given rate is currently unknown when the number of nodes is large. However the insight into which route is better based on approximated solutions is very useful.

In Fig. 4.7, the normalized energy-bandwidth characteristics for the equi-spaced relays from the result of (4.49) are shown for BPSK and orthogonal modulation for $\gamma_c = -5.31$dB. The analytical result treating k as a continuous variable gives a very good approximation to the minimum energy consumption-per-bit as a function of rate. These figures also illustrate the result that at high rates fewer hops should be used whereas at low rates a larger number of hops is better.

4.7 Conclusion

In this chapter, we considered interference-free communication through multi-hop routing where relays can be arbitrarily placed between a single source-destination pair. Instead of channel capacity achieving codes, we introduced a modified threshold model where a practical Reed-Solomon channel code with BPSK and orthogonal modulation scheme are considered for the successful packet retransmission. Based on this model, we formulated the normalized total energy consumption-per-bit and the end-to-end throughput to transmit information bits successfully for the optimal transmission energy and rate on each hop. The results showed that there is a performance gap due to approximations on the symbol error probability but its overall characteristic tracks well the actual energy-bandwidth tradeoff. We showed that orthogonal modulation yields better energy-bandwidth tradeoff at low rates due to its improved energy efficiency whereas BPSK modulation yields better energy-bandwidth tradeoff at high rates. Unlike the channel capacity achieving codes, when receiver processing energy is not taken into account, the minimum total energy consumption-per-bit is not achieved as the end-to-end throughput becomes small. It is shown that the total energy consumption-per-bit can be minimized by optimally choosing the rate, determined from the location of relays and the end-to-end distance. Therefore, by comparing the total energy consumption-per-bit for different location of relays, a routing path which achieves the

best energy-bandwidth tradeoff can be obtained. We also showed that by optimizing jointly over the end-to-end throughput and number of hops for equi-spaced relays, the optimal number of hops and the optimal end-to-end throughput can be found. In the future, practical cooperative strategies or MIMO technologies can be taken into account to improve the bandwidth utilization.

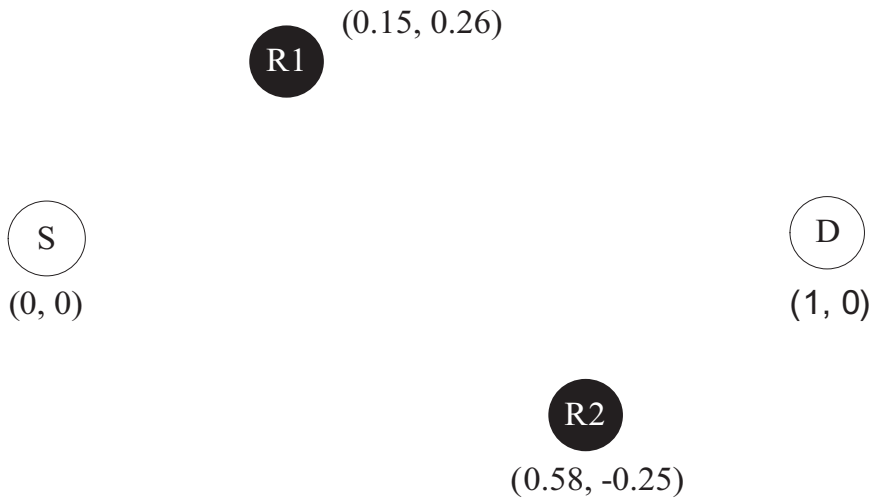


Figure 4.1: Illustration of 2-dimensional network model where distance between any two nodes denotes the normalized relay distance by the end-to-end distance.

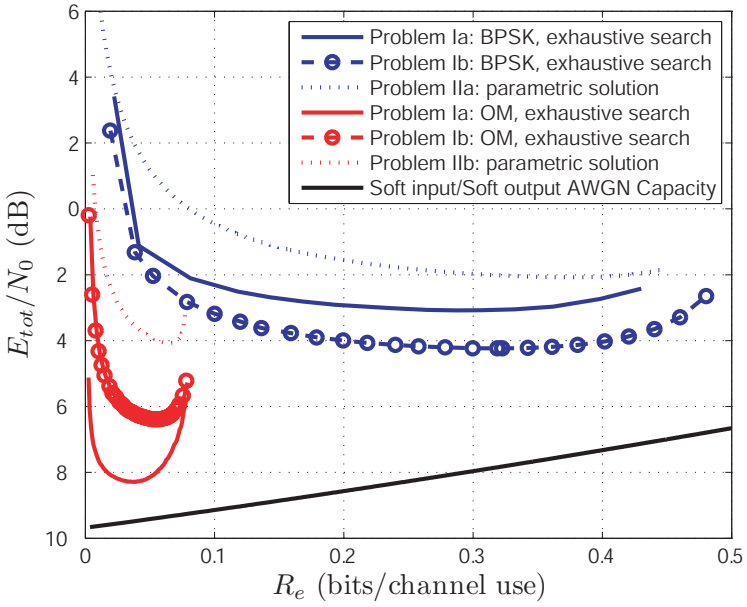


Figure 4.2: Comparison of energy-bandwidth tradeoff for practical modulation with $L = 5$ when $\gamma_c = 0$ for a multihop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 6 : 4$ at $d_e = 3000m$.

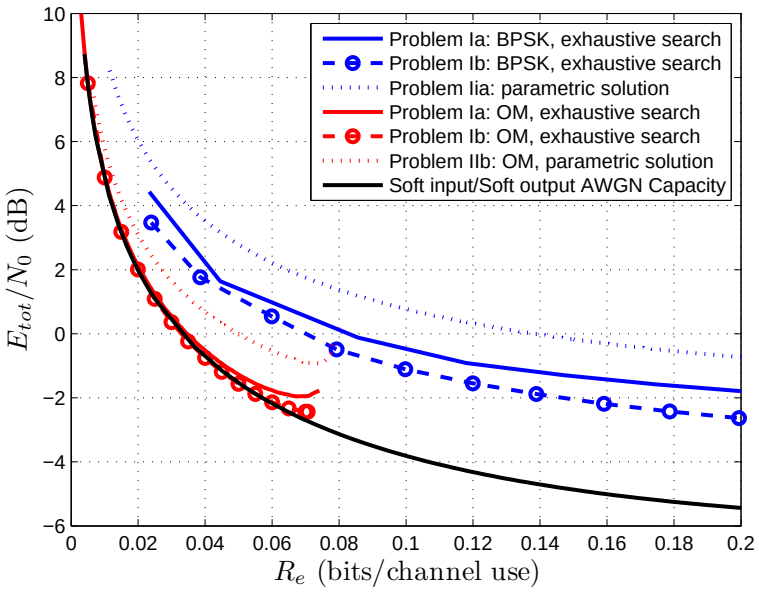


Figure 4.3: Comparison of energy-bandwidth tradeoff for practical modulation with $L = 5$ when $\gamma_c = -15.31$dB for a multihop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 6 : 4$ at $d_e = 3000m$.

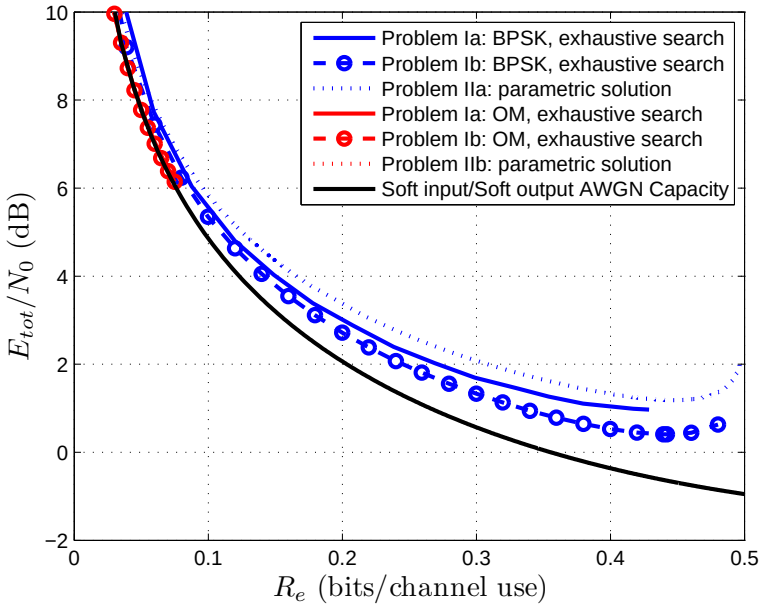
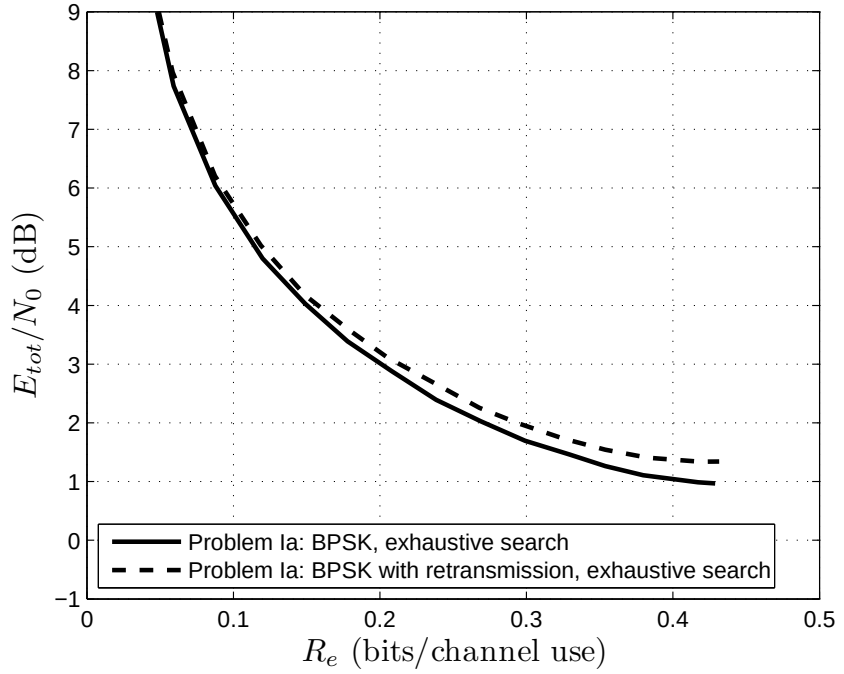
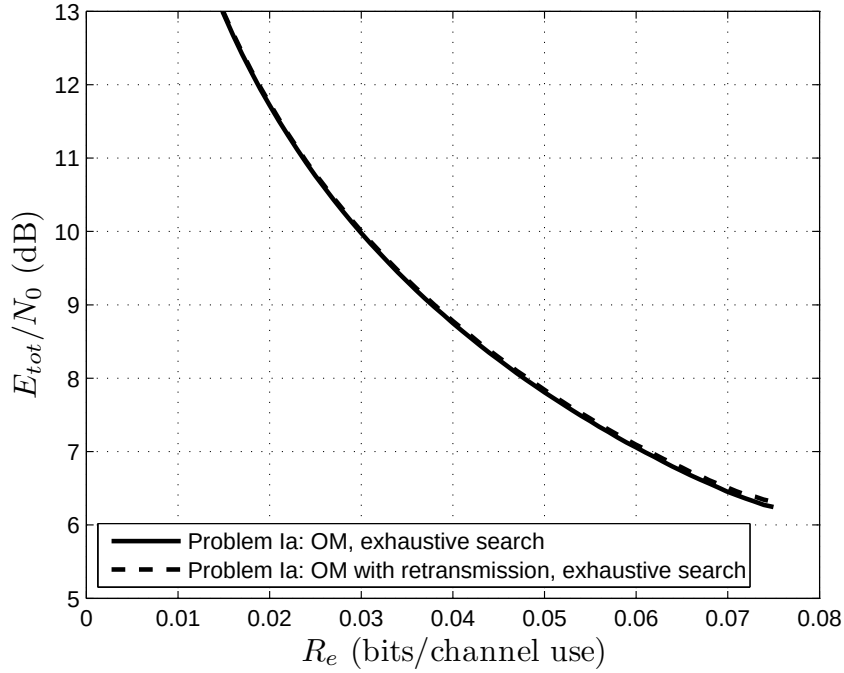


Figure 4.4: Comparison of energy-bandwidth tradeoff for practical modulation with $L = 5$ when $\gamma_c = -5.31$dB for a multihop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 6 : 4$ at $d_e = 3000m$.

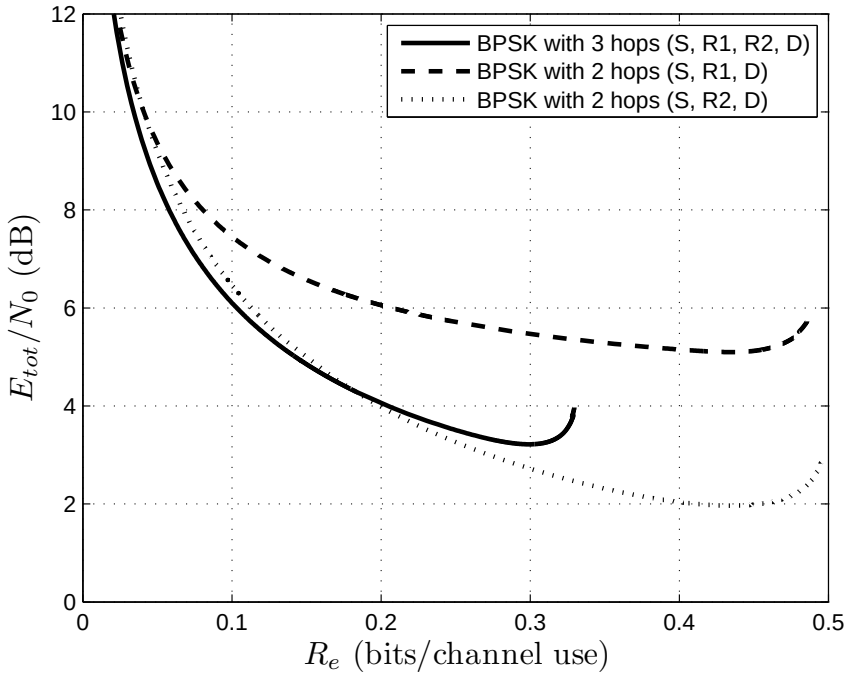


(a) BPSK modulation

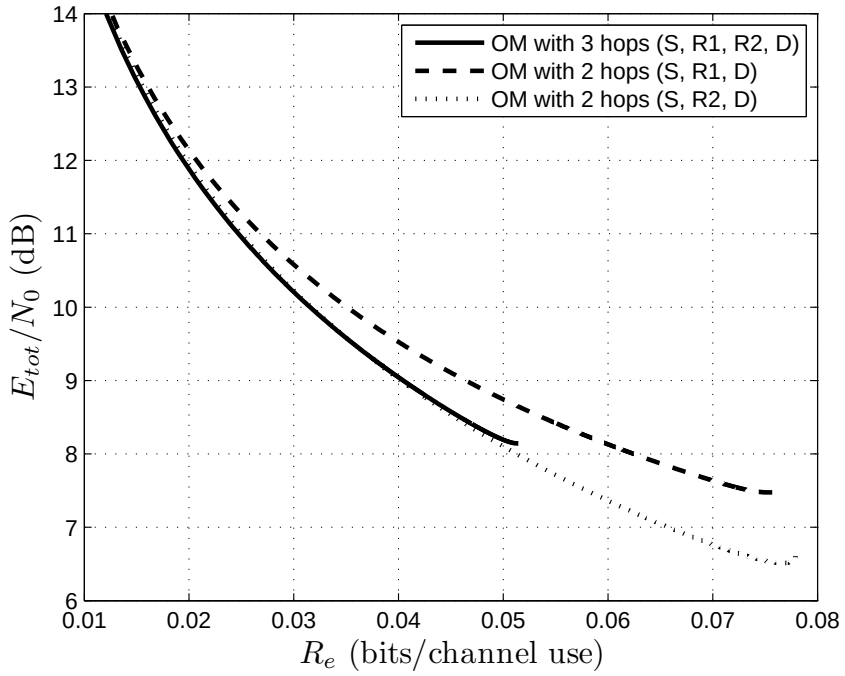


(b) Orthogonal modulation

Figure 4.5: Comparison of energy-bandwidth tradeoff between the case without retransmission and with retransmission for BPSK modulation with $L = 5$ when $\gamma_c = -5.31$dB for a multihop ($k = 2$) with selected location of relays, $\alpha_1 : \alpha_2 = 6 : 4$ at $d_e = 3000m$.

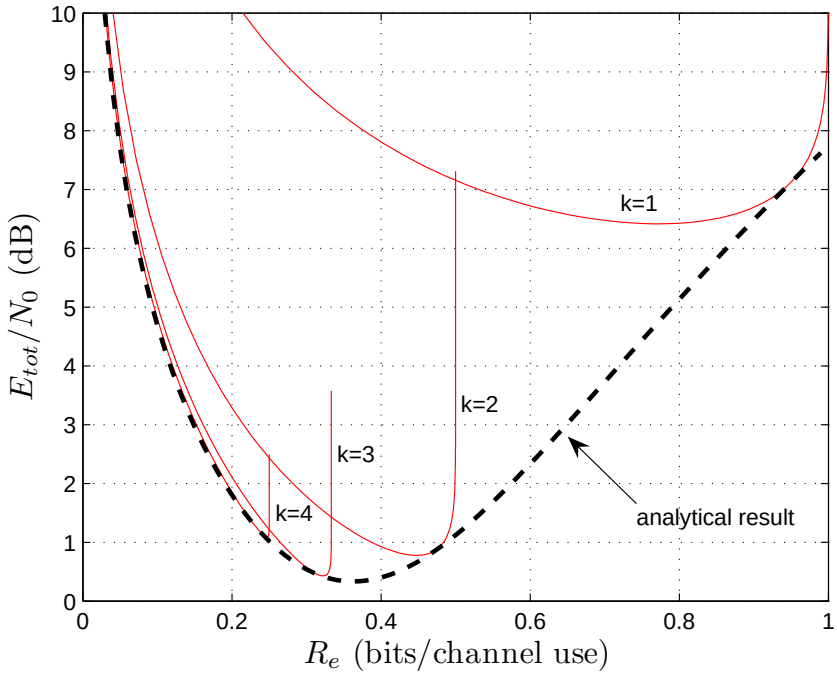


(a) BPSK modulation

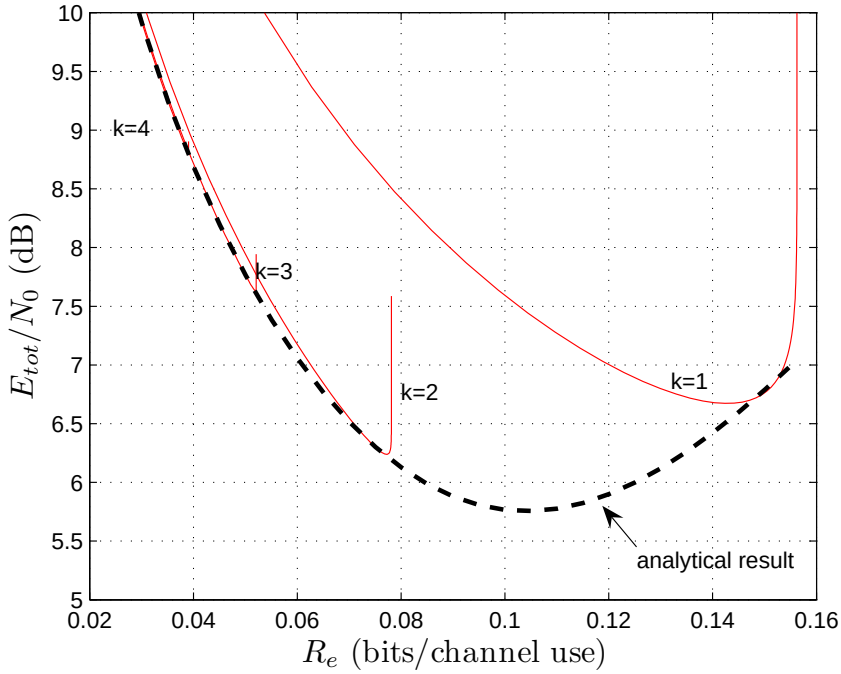


(b) Orthogonal modulation

Figure 4.6: Comparison of normalized energy-bandwidth characteristic of 2 relays over AWGN channels when $\gamma_c = -5.31$dB at d_e=3000m.



(a) BPSK modulation



(b) Orthogonal modulation

Figure 4.7: Normalized energy-bandwidth tradeoff with $L = 5$ when $\gamma_c = -5.31$dB for equi-spaced relays with $k = 1, 2, 3, 4$ and the analytical result in (4.49) at $d_e = 3000m$.

CHAPTER 5

On the Energy-Bandwidth Tradeoff for AWGN Relay Channels

This chapter investigates the energy-bandwidth tradeoff of various relaying strategies over the AWGN channel. The total energy consumption per information bit includes the receiver circuit processing energy at the relay and destination as well as the transmitted energy. With the cooperation of the source and relay, the end-to-end bandwidth efficiency can be improved at the cost of receiver processing energy. We consider both decode-and-forward and amplify-and-forward strategies. Energy-bandwidth tradeoffs for the decode-and-forward and amplify-and-forward relaying strategies are compared. It is shown that for low rates, the case without cooperation outperforms the case with cooperation for both strategies. However when the energy required for processing at the receiver is ignored, it is helpful to cooperate for the amplify-and-forward scheme while the result depends on the desired rate for the decode-and-forward scheme. It is also shown that the amplify-and-forward scheme does not achieve the minimum energy per bit at low SNR due to its noise propagation characteristic. Bounds on the minimum total energy consumption are derived and compared with the result without cooperation.

5.1 Introduction

Wireless ad-hoc networks consist of a number of nodes where the communication between nodes takes place through several intermediate nodes and forms a multi-hop network. In such systems, high bandwidth efficiency is of great importance to deliver information bits across the network. In addition, the total energy consumption in energy constrained networks is an important consideration because nodes have limited power to transmit data. Thus minimizing the energy consumption should be considered. Therefore the energy-bandwidth tradeoff needs to be analyzed in wireless ad hoc networks for an efficient channel utilization and energy consumption.

The range and capacity of wireless networks can be increased by employing relays[42][43]. In [44][20], user cooperative protocols for relay networks have been investigated to exploit spatial diversity. In [45], upper and lower bounds on the outage capacity and the ergodic capacity were established for wireless relay channels where nodes operates in a time-division manner. The capacity of the Gaussian degraded relay channel and an upper bound for the capacity of the Gaussian relay channel were established in [46]. Also motivated by energy constraints, recent work has also investigated the saving in transmission energy using relaying [12][14]. In [15], bounds on the capacity of AWGN relay channels were used to establish bounds on the minimum energy per bit. While focusing on the transmission energy is reasonable for long range communication, energy dissipation in the receiver to process the received signal is not negligible for short range communication. In [31], the circuit energy consumption was incorporated to minimize the total energy consumption while considering the bandwidth efficiency simultaneously

.

In this chapter, we consider the tradeoff between the total energy consumption and the end-to-end rate in wireless relay networks where nodes can be placed arbitrarily between source-destination pair and operate in a time-division manner. Specifically by taking into account the circuit energy consumption, we characterize the overall cost

in terms of the end-to-end channel uses and the total energy consumption to deliver a single information bit to the destination. Depending on whether nodes cooperate or not, we study bounds on the channel capacity for the AWGN channel. Building on these we provide bounds on the total energy consumption per information bit for the case of decode-and-forward (DF) and amplify-and-forward (AF) schemes.

The rest of this chapter is organized as follows. We describe the system model and problem formulation in Section 5.2. We present bounds on total energy per bit using capacity bounds for the decode-and-forward scheme in Section 5.3 and the amplify-and-forward scheme in Section 5.4. Section 5.5 contains the numerical results and Section 5.6 contains our conclusions.

5.2 System Model and Problem Formulation

We consider the two discrete-time AWGN relay channel models depicted in Fig. 5.1 where the source and destination are separated by a distance, d_e and the relay is located arbitrarily between the source and destination. We denote the distance between the source and relay as d_1 given by $d_1 = \alpha_1 d_e$, and the distance between the relay and destination as $d_2 = \alpha_2 d_e$ where $0 < \alpha_i < 1$ for $i = 1, 2$ with the constraint $\sum_{i=1}^{2} \alpha_i \geq 1$. The transmitted signal is attenuated with a power decay law in each transmission. Thus, the received power P_r along the distance d is given by

$$P_r = \frac{\delta}{d^{\eta}} P_{out}$$

where P_{out} is the transmitter output power, η is a path-loss exponent (typically taking values between 2 and 4), and δ is a constant depending on the antenna characteristics. We set $\delta = 1$ throughout this chapter. Furthermore we assume that each node cannot transmit and receive simultaneously.

Consider the relay channel with cooperation in Fig. 5.1(a). The system considered

has two states. In state 1 the source node transmits a signal to the destination or the relay. In state 2 the relay transmits as well as the source in the case of cooperation. The received signals at the relay and at the destination in state 1 are given by

$$Y_{R,1} = \sqrt{d_1^{-\eta}} X_{S,1} + Z_{R,1}$$

and

$$Y_{D,1} = \sqrt{d_e^{-\eta}} X_{S,1} + Z_{D,1}.$$

The received signal at the destination in state 2 is given by

$$Y_{D,2} = \sqrt{d_e^{-\eta}} X_{S,2} + \sqrt{d_2^{-\eta}} X_{R,2} + Z_{D,2}$$

where $X_{S,i}$ and $Y_{D,i}$ are the signals transmitted by the source and received by the destination in state i and $X_{R,2}$ is the signal transmitted by the relay. The noises $Z_{R,1}$ and $Z_{D,i}$ are assumed to be independent white Gaussian noise with power $\sigma^2 = N_0/2$. In the relay channel without cooperation, Fig. 5.1(b), the received signal at the relay during the relay reception, state 1, is given by

$$Y_{R,1} = \sqrt{d_1^{-\eta}} X_{S,1} + Z_{R,1}$$

and the received signal at the destination during the relay transmission, state 2, is given by

$$Y_{D,2} = \sqrt{d_2^{-\eta}} X_{R,2} + Z_{D,2}.$$

Suppose that in a total of N channel uses, the channel is used N_1 times in state 1 and $N_2 = N - N_1$ times in state 2. We assume the following per-symbol power constraints:

(i) $E[X_{S,i}^2] \leq E_1$ and (ii) $E[X_{R,2}^2] \leq E_2$ where $X_{S,i}$ and $X_{R,2}$ are the transmitted signals from the source at state i and relay. Depending on the relaying strategy, the problem is to find the end-to-end rate, R_e which is defined as the number of information bits transmitted per channel use across the relay network [31].

We are also interested in finding the actual energy consumption per information bit which includes the transmitted and processing energy at the receiver. For comparison purposes with the standard results on energy required for reliable communication in [31], the energy is normalized by the end-to-end propagation loss. Therefore the total energy consumption per information bit, E_t, is the ratio of total consumed energy divided by the total number of information bits transferred using N channel uses which can be expressed as

$$E_t = \frac{\text{total energy for transmitting and processing}}{\text{total number of information bits}}.$$

We assume that each node has the same circuitry which requires circuit energy E_p Joules to process a single received coded symbol and E_p is constant throughout this chapter. Also we define the normalized total energy consumption per information bit-to-noise power spectral density ratio as

$$\frac{E_{tot}}{N_0} = \frac{E_t}{N_0} d_e^{-\eta}$$

where the normalized energy transmitted on the i-th link is defined as

$$\frac{E_i}{N_0} d_e^{-\eta} = \frac{E_i}{N_0} \alpha_i^{\eta} d_i^{-\eta} = \alpha_i^{\eta} \gamma_i$$

and $\gamma_i = \frac{E_i}{N_0} d_i^{-\eta}$ denotes the received energy per coded symbol-to-noise power spectral density ratio. Similarly a normalized circuit processing energy γ_c is defined as $\gamma_c \triangleq \frac{E_p}{N_0} d_e^{-\eta}$.

Our goal is, for a given relay location, to find the optimal set of transmission energies E_1 and E_2 that yields the minimum total energy consumption for a given end-to-end rate. However, this is, in general, a nonconvex optimization problem. Thus, in this chapter we consider a suboptimal strategy, common power scheme, in which every node transmits at the same energy, that is, $E_1 = E_2$ or equivalently $\alpha_1^\eta \gamma_1 = \alpha_2^\eta \gamma_2$ [31].

5.3 Decode and Forward Relaying

5.3.1 Without Cooperation

Consider a typical decode-and-forward (DF) multihop protocol where each decoding is done based on the transmission from its immediate neighbor node [31]. The end-to-end rate R_e is given as

$$R_e = \max_{0 \le q \le 1} \min\{qI(X_{S,1}; Y_{R,1}), \bar{q}I(X_{R,2}; Y_{D,2})\} \tag{5.1}$$

where

$$\begin{aligned} I(X_{S,1}; Y_{R,1}) \le R_1 &= \frac{1}{2}\log_2\left(1 + \frac{2E_1 d_1^{-\eta}}{N_0}\right), \\ I(X_{R,2}; Y_{D,2}) \le R_2 &= \frac{1}{2}\log_2\left(1 + \frac{2E_2 d_2^{-\eta}}{N_0}\right) \end{aligned} \tag{5.2}$$

and

$$q = N_1/N,\ \bar{q} = 1 - q = N_2/N.$$

Note that the equalities in (5.2) are achieved with the Gaussian input distribution which is assumed throughout the chapter. The end-to-end rate R_e is maximized with respect to a by setting $qR_1 = \bar{q}R_2$, which yields $q^* = \frac{R_1^{-1}}{R_1^{-1}+R_2^{-1}}$. Hence we have

$$R_e \;=\; \frac{1}{R_1^{-1} + R_2^{-1}}. \tag{5.3}$$

The corresponding total energy consumption per information bit is expressed as

$$\begin{aligned} E_t &= \frac{N_1^* E_1 + N_1^* E_p + N_2^* E_2 + N_2^* E_p}{N R_e} \\ &= \frac{E_1 + E_p}{R_1} + \frac{E_2 + E_p}{R_2} \end{aligned} \tag{5.4}$$

and the normalized total energy consumption is then

$$\frac{E_{tot}}{N_0} = \sum_{i=1}^{2} \frac{\alpha_i^{\eta} \gamma_i + \gamma_c}{R_i} \tag{5.5}$$

where $\gamma_i = E_i d_i^{-\eta}/N_0$ denotes the received signal energy per channel use-to-noise power spectral density ratio at i-th link. For the case of a decode-and-forward without cooperation, the optimal energy-bandwidth tradeoff is derived in [31].

5.3.2 Upper Bound with Cooperation

Consider a decode-and-forward relaying of a Gaussian relay channel where the destination decodes using block 1 and block 2, and the source node cooperates with the relay transmission. The end-to-end rate R_e is upper-bounded by [45][46]

$$\begin{aligned} & R_e \le R_e^+ \\ & = \max_{0 \le q \le 1} \min\{ q I(X_{S,1}; Y_{R,1}, Y_{D,1}) + \bar{q} I(X_{S,2}; Y_{D,2} | X_{R,2}), \\ & q I(X_{S,1}; Y_{D,1}) + \bar{q} I(X_{S,2}, X_{R,2}; Y_{D,2}) \} \end{aligned}$$

where

$$\begin{aligned} & I(X_{S,1}; Y_{R,1}, Y_{D,1}) \le R_1 = \frac{1}{2} \log_2 \left(1 + 2(1 + \alpha_1^{\eta}) \gamma_1\right) \\ & I(X_{S,2}; Y_{D,2} | X_{R,2}) \le R_2(\rho) = \frac{1}{2} \log_2 (1 + 2(1 - \rho^2) \alpha_1^{\eta} \gamma_1) \end{aligned}$$

and

$$I(X_{S,1};Y_{D,1}) \le R_3 = \frac{1}{2}\log_2(1+2\alpha_1^{\eta}\gamma_1)$$

$$I(X_{S,2},X_{R,2};Y_{D,2}) \le R_4(\rho)$$

$$= \frac{1}{2}\log_2(1+2\alpha_1^{\eta}\gamma_1+2\gamma_2+2\rho\sqrt{\alpha_1^{\eta}\gamma_1\gamma_2})$$

and we define

$$\rho = \frac{E[X_{S,2}X_{R,2}]}{\sqrt{E[X_{S,2}^2]E[X_{R,2}^2]}}.$$

For any ρ, $0 \le \rho \le 1$, since $R_2 < R_3 < R_1$ and $R_2 < R_3 < R_4$, the minimax solution for q is obtained by solving $qR_1 + \bar{q}R_2(\rho) = qR_3 + \bar{q}R_4(\rho)$, which yields $q^*(\rho) = \frac{R_4(\rho)-R_2(\rho)}{R_1-R_3+R_4(\rho)-R_2(\rho)}$. Hence we have

$$R_e^+ = \max_{0\le\rho\le1} \frac{R_1R_4(\rho) - R_2(\rho)R_3}{R_1 + R_4(\rho) - R_2(\rho) - R_3}. \tag{5.6}$$

The corresponding total energy consumption per information bit is expressed as

$$\begin{aligned} E_t(\rho) &\ge \frac{N_1^*E_1 + 2N_1^*E_p + N_2^*E_1 + N_2^*E_2 + N_2^*E_p}{NR_e^+(\rho)} \\ &= \frac{E_1 + (1-q^*(\rho))E_2 + 2E_p}{R_e^+(\rho)} \end{aligned} \tag{5.7}$$

and the normalized total energy consumption is then lower-bounded by

$$\begin{aligned} \frac{E_{tot}(\rho)}{N_0} \ge \frac{E_{tot}^+(\rho)}{N_0} =& \frac{\alpha_1^{\eta}\gamma_1 + 2\gamma_c}{R_e^+(\rho)} \\ &+ \frac{R_1 - R_3}{R_1R_4(\rho) - R_2(\rho)R_3}\alpha_2^{\eta}\gamma_2. \end{aligned} \tag{5.8}$$

Note that the optimum value of ρ maximizing the end-to-end rate may be different from the value of ρ minimizing the total energy consumption per information bit which can be seen in (5.6) and (5.7).

5.3.3 Lower Bound with Cooperation

The lower bound is obtained using a special case of Theorem 7 in [43] that yields

$$\begin{aligned} R_e \geq R_e^- = \max_{0\leq q\leq 1} \min\{ & qI(U;Y_{R,1}) + qI(X_{S,1};Y_{D,1}|U) \\ & + \bar{q}I(X_{S,2};Y_{D,2}|X_{R,2}), \\ & qI(X_{S,1};Y_{D,1}) + \bar{q}I(X_{S,2},X_{R,2};Y_{D,2})\} \end{aligned}$$

where the relay decodes part of the message which is represented by U and cooperatively sends information with the source to help the destination decode the previous message, and the destination decodes the current message. Consider

$$\begin{aligned} & I(U;Y_{R,1}) + I(X_{S,1};Y_{D,1}|U) \\ = \; & h(Y_{R,1}) - h(Y_{R,1}|U) + h(Y_{D,1}|U) - h(Y_{D,1}|X_{S,1},U) \\ = \; & \frac{1}{2}\log_2\left(1+\frac{E_1 d_1^{-\eta}}{\sigma^2}\right) + h(Y_{D,1}|U) - h(Y_{R,1}|U) \end{aligned}$$

where $\sigma^2 = N_0/2$. Note that $h(Y_{R,1}) = h(aX_{S,1}+Z_{R,1}) = 0.5\log_2(2\pi e(E_1 d_1^{-\eta}+\sigma^2))$ and $h(Y_{D,1}|X_{S,1},U) = h(bX_{S,1}+Z_{D,1}|X_{S,1},U) = h(Z_{D,1}) = 0.5\log_2(2\pi e\sigma^2)$ where $a = \sqrt{d_1^{-\eta}}$ and $b = \sqrt{d_e^{-\eta}}$. We now find an upper bound on $h(Y_{D,1}|U) - h(Y_{R,1}|U)$. Since we have the following relation such that

$$\begin{aligned} \frac{1}{2}\log_2(2\pi e\sigma^2) & \leq h(Y_{R,1}|U,X_{S,1}) \\ & \leq h(Y_{R,1}|U) \leq h(Y_{R,1}) \\ & = \frac{1}{2}\log_2(2\pi e(E_1 d_1^{-\eta}+\sigma^2)) \end{aligned}$$

there exists a constant $0 \leq \beta \leq 1$ such that

$$\begin{aligned} h(Y_{R,1}|U) &= \frac{1}{2}\log_2(2\pi e(\beta E_1 d_1^{-\eta} + \sigma^2)) \\ &\leq h(Y_{R,1}) = \frac{1}{2}\log_2(2\pi e(E_1 d_1^{-\eta} + \sigma^2)). \end{aligned}$$

Note that $\frac{a}{b} = \alpha_1^{-\eta} > 1$ since $0 < \alpha_1 \leq 1$. From [15], $h(Y_{R,1}|U) = h(aX_{S,1} + Z_{R,1}|U)) = h(aX_{S,1} + Z_{D,1}|U) > h(bX_{S,1} + Z_{D,1}|U) = h(Y_{D,1}|U)$ and thus,

$$\begin{aligned} I(U;Y_{R,1}) + I(X_{S,1};Y_{D,1}|U) &< \frac{1}{2}\log_2(1 + \frac{E_1 d_1^{-\eta}}{\sigma^2}) \\ &= I(X_{S,1}|Y_{R,1}) \end{aligned}$$

which leads to the capacity of degraded relay channel as follows [45][46].

$$\begin{aligned} R_e^- = \max_{0\leq q\leq 1} \min\{&qI(X_{S,1};Y_{R,1}) + \bar{q}I(X_{S,2};Y_{D,2}|X_{R,2}), \\ &qI(X_{S,1};Y_{D,1}) + \bar{q}I(X_{S,2},X_{R,2};Y_{D,2})\} \end{aligned}$$

Therefore the tightest lower bound of decode-forward relaying channel is the capacity of degraded relay channel. The end-to-end rate and total energy consumption per information bit can be obtained using the upper bound case in (5.6) and (5.7).

5.3.4 Minimum energy per bit

Next consider a special case where the source and relay transmit at the same power. Since each node transmits at the same power level, we have $\alpha_1^\eta \gamma_1 = \alpha_2^\eta \gamma_2$. For the case of the degraded relay channel, we have

$$\begin{aligned} I(X_{S,1};Y_{R,1}) \leq R_1 &= \frac{1}{2}\log_2(1 + 2\alpha_1^{-\eta}\alpha_2^\eta\gamma_2) \\ I(X_{S,2};Y_{D,2}|X_{R,2}) \leq R_2(\rho) &= \frac{1}{2}\log_2(1 + 2(1-\rho^2)\alpha_2^\eta\gamma_2) \end{aligned}$$

and

$$I(X_{S,1};Y_{D,1}) \leq R_3 = \frac{1}{2}\log_2(1+2\alpha_2^{\eta}\gamma_2)$$

$$I(X_{S,2},X_{R,2};Y_{D,2}) \leq R_4(\rho)$$
$$= \frac{1}{2}\log_2(1+2\alpha_2^{\eta}\gamma_2(1+\alpha_2^{-\eta}+\rho\alpha_2^{-\eta/2})).$$

The resulting total energy consumption per information bit is then

$$\frac{E_{tot}(\rho)}{N_0} \leq \frac{E_{tot}^{+}(\rho)}{N_0} = \frac{\alpha_2^{\eta}\gamma_2(2R_1+R_4(\rho)-R_2(\rho)-2R_3)}{R_1R_4-R_2R_3} + \frac{2\gamma_c}{R_e^{-}(\rho)}.$$

When $\gamma_c = 0$, it can be easily shown that the total energy consumption per information bit is non-decreasing in γ, and hence the minimum total energy per information bit of degraded relay channel for a given ρ is as follows.

$$\begin{aligned}\left(\frac{E_{tot}}{N_0}\right)_{min} &= \lim_{R_e\to 0}\frac{E_{tot}^{+}(\rho)}{N_0} \\ &= \lim_{\gamma\to 0}\frac{\alpha_{max}^{\eta}\gamma(2R_1+R_4(\rho)-R_2(\rho)-2R_3)}{R_1R_4(\rho)-R_2(\rho)R_3} \\ &= \ln 2\frac{\alpha_1^{\eta}-2(\alpha_1\alpha_2)^{\eta}+2\alpha_2^{\eta}+\rho^2(\alpha_1\alpha_2)^{\eta}+\rho\alpha_1^{\eta}\alpha_2^{\eta/2}}{1+\alpha_2^{\eta}-(\alpha_1\alpha_2)^{\eta}+\rho^2(\alpha_1\alpha_2)^{\eta}+\rho\alpha_2^{\eta/2}}.\end{aligned} \tag{5.9}$$

Similarly for the case of the upper bound of the capacity,

$$\begin{aligned}I(X_{S,1};Y_{R,1},Y_{D,1}) &\leq R_1 \\ &= \frac{1}{2}\log_2\left(1+2(1+\alpha_1^{\eta})\left(\frac{\alpha_2}{\alpha_1}\right)^{\eta}\gamma\right)\end{aligned}$$

and hence the minimum total energy per information bit is given as follows.

$$\begin{aligned}\left(\frac{E_{tot}}{N_0}\right)_{min} &= \lim_{R_e\to 0}\frac{E_{tot}^{-}(\rho)}{N_0} \\ &= \ln 2\frac{\alpha_1^{\eta}+2\alpha_2^{\eta}+\rho^2(\alpha_1\alpha_2)^{\eta}+\rho\alpha_1^{\eta}\alpha_2^{\eta/2}}{1+\alpha_1^{\eta}+\alpha_2^{\eta}+\rho^2(\alpha_1\alpha_2)^{\eta}+\rho\alpha_2^{\eta/2}(1+\alpha_1^{\eta})}.\end{aligned} \tag{5.10}$$

For the case without cooperation, we have [31]

$$\left(\frac{E_{tot}}{N_0}\right)_{min} = \lim_{R_e \to 0} \frac{E_{tot}}{N_0} = \ln 2(\alpha_1^\eta + \alpha_2^\eta).$$

5.4 Amplify and Forward Relaying

In this section, we investigate a relaying scheme where the relay amplifies its past received signal and forwards it to the destination, so called amplify-and-forward (AF) scheme.

5.4.1 With Cooperation

Consider the following relaying scheme for AWGN channel. In state 1, the message is encoded and $X_{S,1}$ is transmitted. The destination receives $Y_{D,1} = \sqrt{d_e^{-\eta}} X_{S,1} + Z_{D,1}$, and the relay receives $Y_{R,1} = \sqrt{d_1^{-\eta}} X_{S,1} + Z_{R,1}$. In state 2, the source sends $X_{S,2} = X_{S,1}$ and the relay transmits $X_{R,2} = aY_{R,1}$, where $a = \sqrt{\frac{E_2}{E_1 d_1^{-\eta} + \sigma^2}}$ is chosen to satisfy the per-symbol energy constraint of the relay. The destination receives $Y_{D,2} = \sqrt{d_e^{-\eta}} X_{S,1} + \sqrt{d_2^{-\eta}} a \left(\sqrt{d_1^{-\eta}} X_{S,1} + Z_{R,1} \right) + Z_{D,1}$ and starts decoding the message using $Y_{D,1}$ and $Y_{D,2}$. Note that the relay amplifies the received signal without any further operation, and thus the relay transmission requires the same number of channel uses as the source transmission which leads to $q = 0.5$, a fixed same fractional block length. Then the achievable rate is given by

$$\begin{aligned} R_e &= \frac{1}{2} I(X_{S,1}, X_{S,2}; Y_{D,1}, Y_{D,2}) = \frac{1}{2} I(X_{S,1}; Y_{D,1}, Y_{D,2}) \\ &= \frac{1}{4} \log_2 \left(1 + \frac{d_e^{-\eta} E_1}{\sigma^2} + \frac{\left(\sqrt{d_e^{-\eta}} + a\sqrt{(d_1 d_2)^{-\eta}} \right)^2 E_1}{\sigma^2 (1 + a^2 d_2^{-\eta})} \right) \\ &= \frac{1}{4} \log_2 \left(1 + 2\alpha_1^\eta \gamma_1 + \frac{(2\gamma_1 + 4\gamma_1^2) \left[\sqrt{\alpha_1^\eta} + \sqrt{\frac{2\gamma_2}{1+2\gamma_1}} \right]^2}{1 + 2\gamma_1 + 2\gamma_2} \right) \end{aligned}$$

where a is the scaling factor at the relay which is given by

$$a = \sqrt{\frac{E_2}{E_1 d_1^{-\eta} + \sigma^2}} = \sqrt{\frac{2\gamma_2 d_2^{\eta}}{1 + 2\gamma_1}}.$$

The corresponding total energy consumption per information bit is expressed as

$$\begin{aligned} E_t &= \frac{N_1 E_1 + N_1 E_p + N_2 E_1 + N_2 E_2 + N_2 E_p}{N R_e} \\ &= \frac{2E_1 + 2E_p + E_2}{2R_e} \end{aligned}$$

where $N_1 = N_2 = N/2$ and the normalized total energy consumption is then

$$\frac{E_{tot}}{N_0} = \frac{\alpha_1^{\eta}\gamma_1 + \gamma_c + 0.5\alpha_2^{\eta}\gamma_2}{R_e} \tag{5.11}$$

5.4.2 Without Cooperation

Next consider an amplify-and-forward relaying scheme where there is no direct connection between the source and destination. Similar to the previous section, the achievable rate is given by

$$\begin{aligned} R_e &= \frac{1}{2} I(X_{S,1}; Y_{D,2}) = \frac{1}{4} \log_2 \left(1 + \frac{a^2 (d_1 d_2)^{-\eta} E_1}{\sigma^2 (1 + a^2 d_2^{-\eta})} \right) \\ &= \frac{1}{4} \log_2 \left(1 + \frac{4\gamma_1 \gamma_2}{1 + 2\gamma_1 + 2\gamma_2} \right) \end{aligned} \tag{5.12}$$

where $a = \sqrt{\frac{E_2}{E_1 d_1^{-\eta} + \sigma^2}} = \sqrt{\frac{2\gamma_2 d_2^{\eta}}{1+2\gamma_1}}$. The corresponding total energy consumption per information bit is expressed as

$$E_t = \frac{N_1 E_1 + N_2 E_2 + N_2 E_p}{N R_e} = \frac{E_1 + E_2 + E_p}{2R_e} \tag{5.13}$$

where $N_1 = N_2 = N/2$ and the normalized total energy consumption is then

$$\frac{E_{tot}}{N_0} = \frac{\alpha_1^{\eta}\gamma_1 + \alpha_2^{\eta}\gamma_2 + \gamma_c}{2R_e} \tag{5.14}$$

5.4.3 Minimum energy per bit

Next consider a special case where the source and relay transmit using the same power, $\alpha_1^{\eta}\gamma_1 = \alpha_2^{\eta}\gamma_2$. Hence the end-to-end rate with cooperation becomes

$$\begin{aligned} R_e &= \frac{1}{4}\log_2\left(1 + 2\alpha_1^{\eta}\gamma_1 + \frac{(2\gamma_1 + 4\gamma_1^2)\left[\sqrt{\alpha_1^{\eta}} + \sqrt{\lambda}\right]^2}{1 + 2(1+\beta)\gamma_1}\right) \\ &\geq \frac{1}{4}\log_2\left(1 + 2\alpha_1^{\eta}\gamma_1 + \frac{2\alpha_1^{\eta}\gamma_1 + (\alpha_1^{\eta} + \beta + 2\sqrt{\alpha_1^{\eta}\beta})4\gamma_1^2}{1 + 2(1+\beta)\gamma_1}\right) \\ &= R_e^{-}. \end{aligned}$$

where $\beta = \alpha_1^{\eta}\alpha_2^{-\eta}$ and $\lambda = \frac{2\beta\gamma_1}{1+2\gamma_1}$. Thus we obtain

$$\begin{aligned} \gamma_1 &= \frac{c(1+\beta) - 2\alpha_1^{\eta} + \sqrt{[c(1+\beta) - 2\alpha_1^{\eta}]^2 + 4c\lambda_1}}{4\lambda_1} \\ &\approx \frac{c(1+\beta)}{2(\alpha_1^{\eta}(2+\beta) + \beta + 2\sqrt{\alpha_1^{\eta}\beta})}. \end{aligned} \tag{5.15}$$

where $c = 2^{4R_e^{-}} - 1$ and $\lambda_1 = \alpha_1^{\eta}(2+\beta) + \beta + 2\sqrt{\alpha_1^{\eta}\beta}$. Then the lower bound on the end-to-end rate leads to a upper bound on the total energy consumption per information bit as follows

$$\frac{E_{tot}}{N_0} \leq \frac{1.5\alpha_1^{\eta}\gamma_1 + \gamma_c}{R_e^{-}}.$$

Therefore the normalized total energy consumption per information bit is approximated as

$$\frac{E_{tot}}{N_0} \approx \frac{1}{R_e^-}\left(\frac{3}{4}\frac{(\alpha_1^\eta+\alpha_2^\eta)(2^{4R_e^-}-1)}{1+\alpha_1^\eta+2\alpha_2^\eta+2\sqrt{\alpha_2^\eta}}+\gamma_c\right). \tag{5.16}$$

When $\gamma_c = 0$, the minimum energy per bit is approximated as

$$\left(\frac{E_{tot}}{N_0}\right)_{min} \approx 3\ln 2\frac{\alpha_1^\eta+\alpha_2^\eta}{1+\alpha_1^\eta+2\alpha_2^\eta+2\sqrt{\alpha_2^\eta}}. \tag{5.17}$$

Note that when $\gamma_c = 0$, the achievable rate is not a concave function of SNR; it is concave for high SNR and convex for low SNR. It can be interpreted that as the transmission energy is decreased, the received signal at the relay becomes too noisy and hence the relay transmission largely contains information about the received noise at the relay. Such characteristic yields that the minimum energy per bit does not occur at $R_e \to 0$ although $\lim_{R_e\to 0}\frac{E_{tot}}{N_0} = 1.5\ln 2$.

For the case without cooperation, we have

$$R_e = \frac{1}{4}\log_2\left(1+\frac{4\alpha_1^\eta\alpha_2^{-\eta}\gamma_1^2}{1+2(1+\alpha_1^\eta\alpha_2^{-\eta})\gamma_1}\right) \tag{5.18}$$

and thus

$$\begin{aligned}\gamma_1 &= \frac{(1+\alpha_1^\eta\alpha_2^{-\eta})c+\sqrt{(1+\alpha_1^\eta\alpha_2^{-\eta})^2c^2+4\alpha_1^\eta\alpha_2^{-\eta}c}}{4\alpha_1^\eta\alpha_2^{-\eta}}\\ &\geq \frac{1+\alpha_1^\eta\alpha_2^{-\eta}}{2\alpha_1^\eta\alpha_2^{-\eta}}c\end{aligned}$$

where $c = 2^{4R_e}-1$. Therefore the normalized total energy consumption per information bit without cooperation is given by

$$\frac{E_{tot}}{N_0} \geq \frac{(\alpha_1^\eta+\alpha_2^\eta)(2^{4R_e}-1)+\gamma_c}{2R_e} \tag{5.19}$$

and when $\gamma_c = 0$,

$$\left(\frac{E_{tot}}{N_0}\right)_{min} \approx 2\ln 2(\alpha_1^\eta + \alpha_2^\eta). \tag{5.20}$$

Note that when $\gamma_c = 0$, the minimum energy per bit is not achieved when $R_e \to 0$ as the case with cooperation. In addition, $\lim_{R_e \to 0} \frac{E_{tot}}{N_0} = \infty$ unlike the case with cooperation which converges to $1.5 \ln 2$. The intuitive reason is that as the transmission energy reduces, both schemes apparently suffer from the noisy signal propagation but even a small amount of side information from the source can be useful, which results in more energy efficiency for the case of cooperation.

5.5 Numerical Results

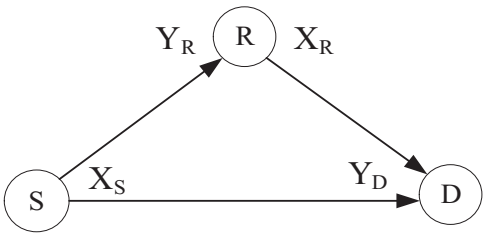
In the results we assume the path-loss exponent $\eta = 4$, end-to-end distance $d_e = 3000$m, noise power spectral density $N_0 = -174$dBm/Hz and circuit processing energy $E_p = 0.095\mu$J/symbol ($\gamma_c = -5.31$dB) unless otherwise specified.

In Fig. 5.2 and Fig. 5.3 where a two hop network is assumed with the ratio of link distances equal to 7:3, energy-bandwidth tradeoffs of the forward-and-decode and amplify-and-forward schemes are compared for various transmission power selections, that is, (i) arbitrary E_1 and E_2, (ii) $E_1 = E_2$ (common power), and (iii) $E_2 = 0.4E_1$. For case (i) the result is obtained by brute force optimization and for case (ii) and (iii), the results are used for the decode-and-forward scheme from (5.8), amplify-and-forward scheme from (5.11) and (5.14). It is observed that there is an optimal non-zero rate minimizing the total energy consumption as in [31]. Note that the performance gap between arbitrary power case and common power case with amplify-and-forward scheme is larger than the gap with decode-and-forward scheme. This is because the relay in the amplify-and-forward scheme scales its power level based on the transmitted signal from the source. In particular when the relay is close to the destination, arbitrary

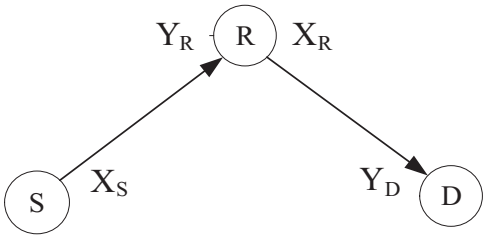
transmission power scheme is more energy efficient than common transmission power scheme. This further implies that transmission power adaptation is necessary especially for the amplify-and-forward scheme. Also notice that there is a significant performance loss in the degraded relay channel in Fig. 5.2

In Fig. 5.4 and Fig. 5.5, we plot the normalized energy-bandwidth tradeoff for different location of relays to see the impact of relay location. In the amplify-and-forward scheme, when the relay is close to the source where the received signal strength is relatively good, cooperative strategy shows better energy efficiency at high rates while noncooperative strategy does at low rates. However when the relay is close to the destination, cooperative strategy is inferior to noncooperative strategy since the source consumes more transmission energy to achieve the same end-to-end rate as noncooperative strategy. When it comes to the comparison between two forwarding schemes, the decode-and-forward scheme outperforms the amplify-and-forward scheme at high rates regardless of node cooperation. This is because the amplify-and-forward scheme has a fundamental limit in terms of the bandwidth utilization unlike the forward-and-decode scheme. Also note that as the relay is closer to the source, upper and lower bounds of the decode-and-forward scheme are very close because the loss by the degraded channel becomes small.

In Fig. 5.6 and Fig. 5.7, we plot the normalized energy-bandwidth tradeoff when the receiver processing energy is not taken into account. For the amplify-and-forward scheme, the relay location does not incur significant performance gaps between cooperative and noncooperative schemes unlike the case with receiver energy consumption. However for the decode-and-forward scheme, the relay position determines the usefulness of node cooperation. Note that the minimum energy per bit of the amplify-and forward scheme does not occur as $R_e \to 0$ because its achievable rate is a convex function at low rates but a concave function at high rates as shown in Fig. 5.8. The approximations of the energy-bandwidth tradeoff from (5.16) and (5.19) are plotted in



(a) Relay channel model with cooperation.



(b) Relay channel model without cooperation

Figure 5.1: Illustration of relay channel.

Fig. 5.9. Our results approximate well except at low rates and the minimum energy per bit at $R_e \to 0$ is relatively close to the true minimum.

5.6 Conclusion

The chapter investigated the energy-bandwidth tradeoff for wireless multi-hop networks where relays can be placed arbitrarily between the source and destination. With emphasis on the circuit energy consumption and node cooperation, we derived bounds on the total energy consumption per information bit using the capacity of the Gaussian channel for the decode-and-forward and amplify-and-forward scheme. In particular, the generalized block Markov lower bound on capacity of the relay channel are used to show that the tightest lower bound is the capacity of the degraded relay channel in time-division mode. For the low rate region where the circuit energy dominates the overall energy consumption, noncooperative relaying strategy outperforms cooperative strategy regardless of its forwarding method, and in particular the amplify-and-forward scheme is useful in this region since it does not need to process the received signal at the relay. However at high rates the decode-and-forward scheme outperforms the amplify-and-forward scheme which shows limited bandwidth utilization. Therefore, depending on the location of relay and desired end-to-end throughput based on the available network energy budget, both forwarding strategy and node cooperation should be used.

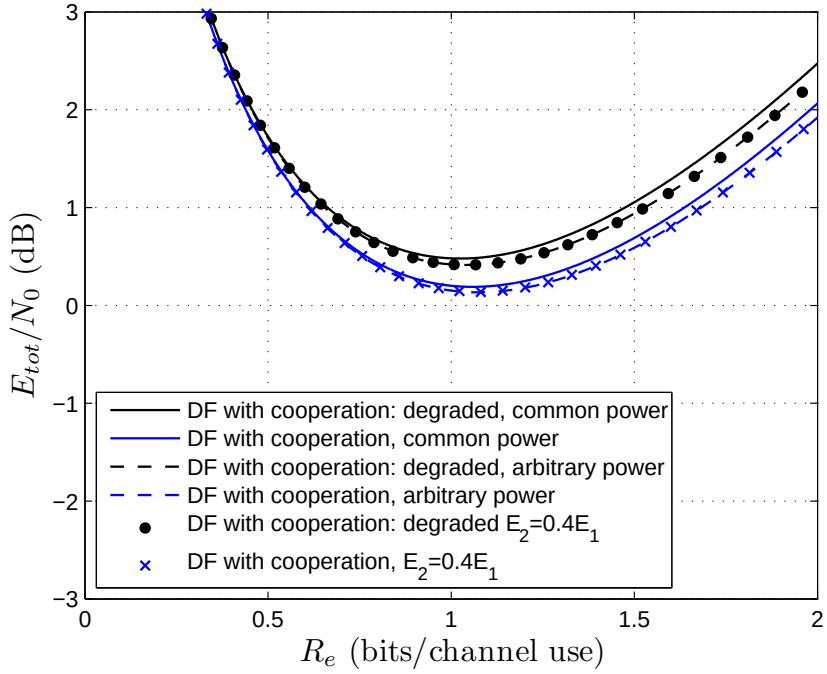


Figure 5.2: Comparison of the decode-and-forward scheme when $\gamma_c = -5.31$dB with a selected location of relay, $\alpha_1 : \alpha_2 = 7 : 3$ for a common power scheme.

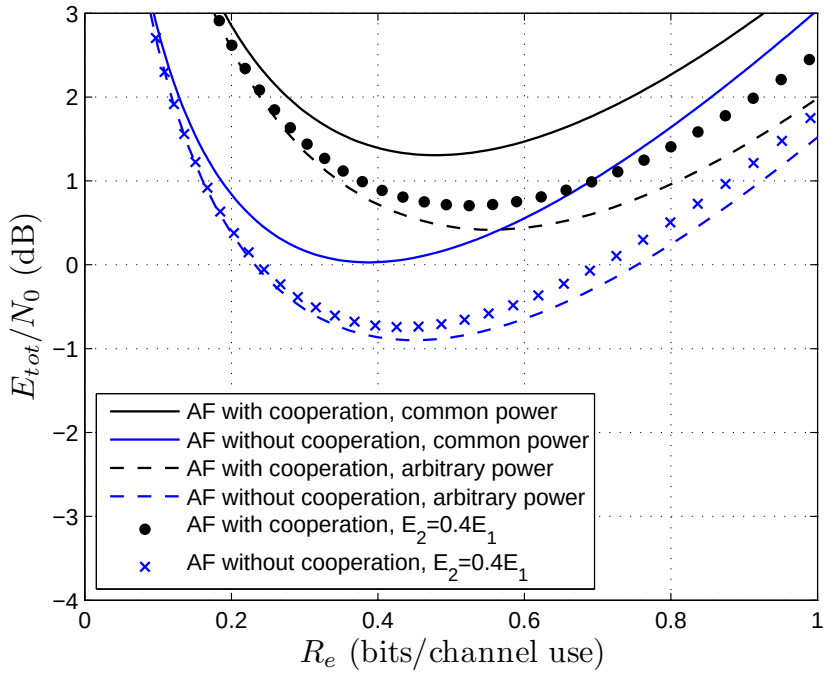


Figure 5.3: Comparison of the amplify-and-forward scheme when $\gamma_c = -5.31$dB with a selected location of relay, $\alpha_1 : \alpha_2 = 7 : 3$ for a common power scheme.

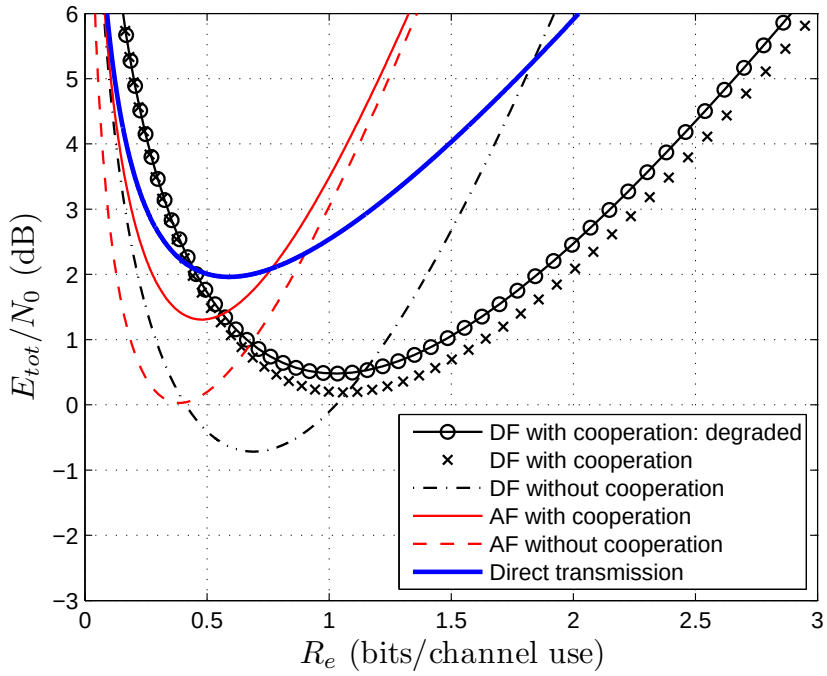


Figure 5.4: Energy-bandwidth tradeoff when $\gamma_c = -5.31$dB with single hop and a selected location of relay, $\alpha_1 : \alpha_2 = 7 : 3$ for a common power scheme.

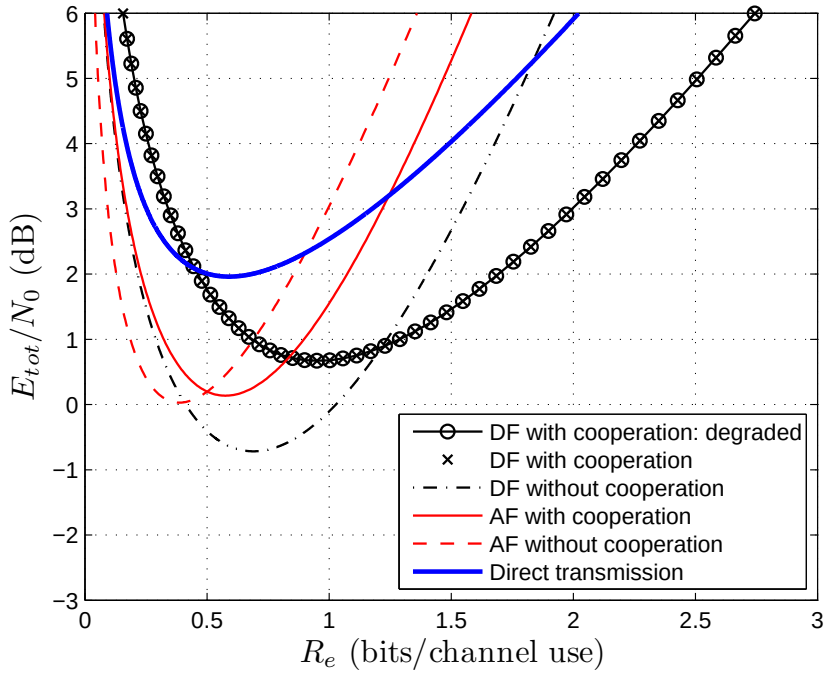


Figure 5.5: Energy-bandwidth tradeoff when $\gamma_c = -5.31$dB with single hop and a selected location of relay, $\alpha_1 : \alpha_2 = 3 : 7$ for a common power scheme.

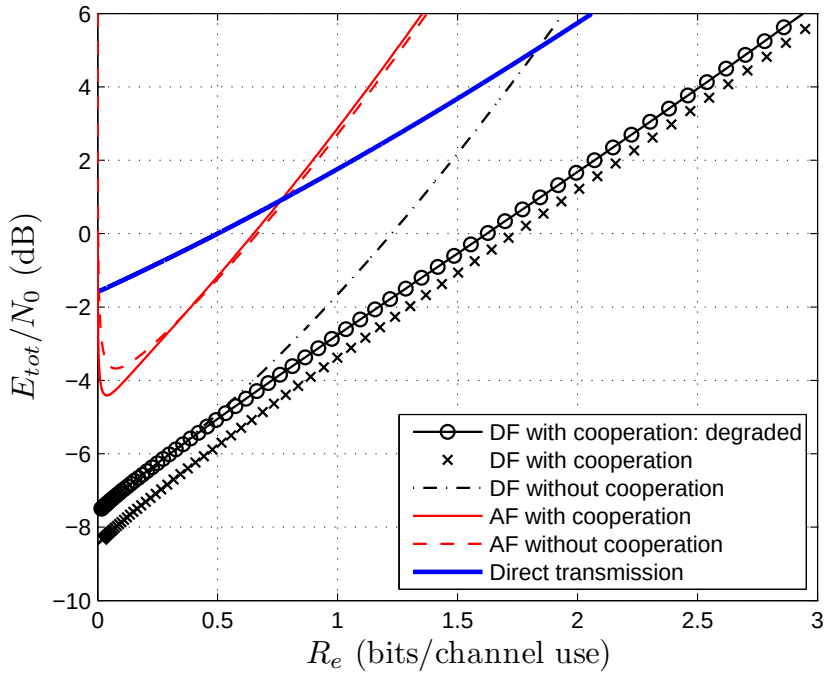


Figure 5.6: Energy-bandwidth tradeoff when $\gamma_c = 0$ with single hop and a selected location of relay, $\alpha_1 : \alpha_2 = 7 : 3$ for a common power scheme.

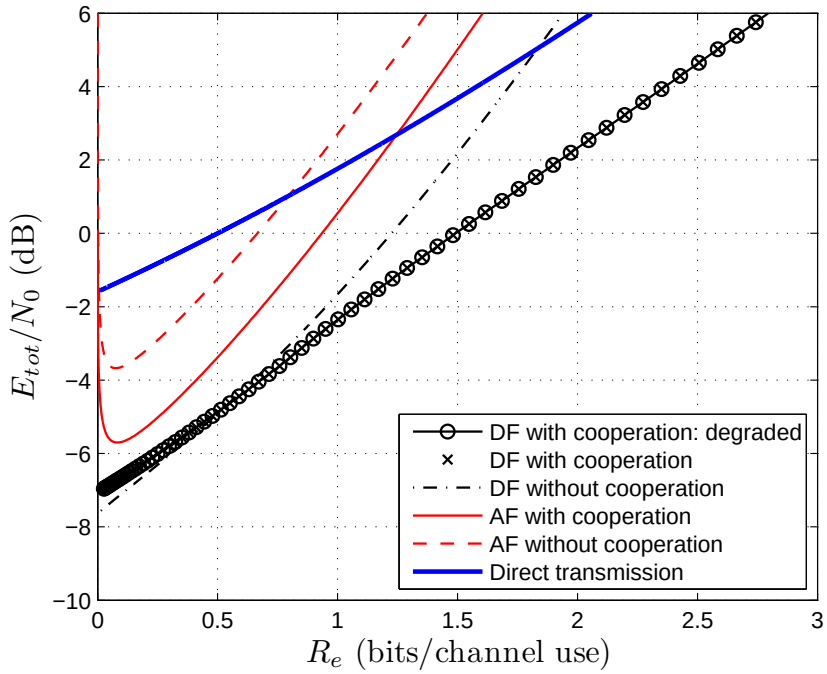


Figure 5.7: Energy-bandwidth tradeoff when $\gamma_c = 0$ with single hop and a selected location of relay, $\alpha_1 : \alpha_2 = 3 : 7$ for a common power scheme.

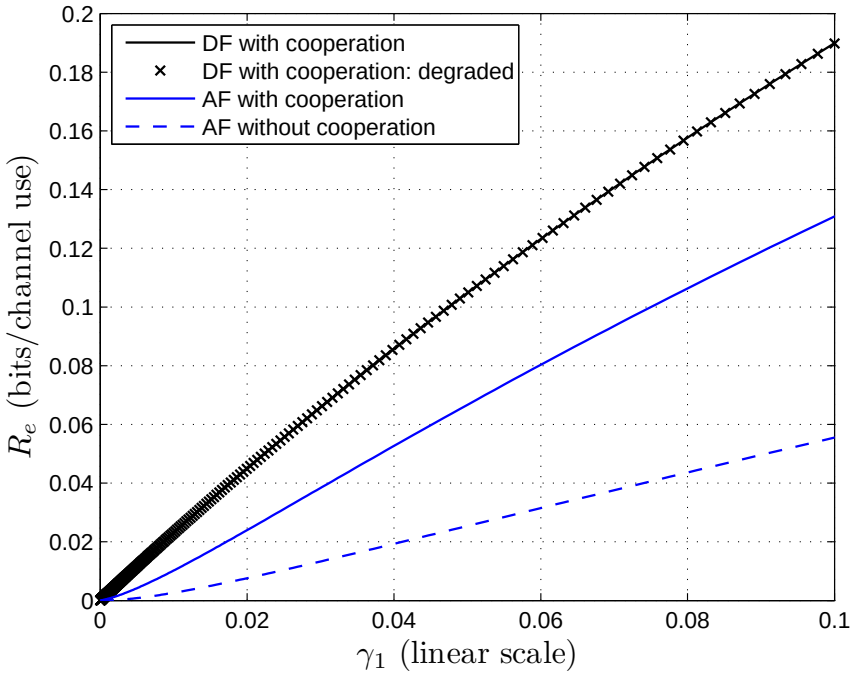


Figure 5.8: Comparison of end-to-end rates with a selected location of relay, $\alpha_1 : \alpha_2 = 3 : 7$ for a common power scheme.

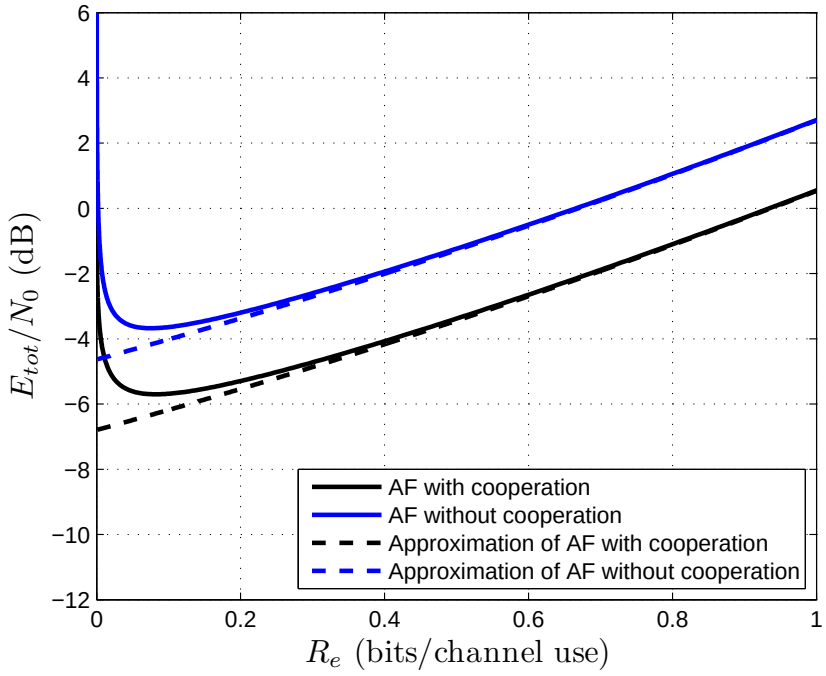


Figure 5.9: Comparison of energy-bandwidth tradeoffs for amplify-and-forward scheme when $\gamma_c = 0$ with a selected location of relay, $\alpha_1 : \alpha_2 = 3 : 7$ for a common power scheme.

CHAPTER 6

Conclusions

This final chapter summarizes the contributions of the dissertation and highlights numerous areas for further research.

6.1 Summary of Contributions

In this dissertation, we have created a framework for analyzing the energy-bandwidth tradeoff in energy-constrained wireless multi-hop networks where relays can be placed arbitrarily between a source-destination pair in 2-dimensional plane. By jointly considering the energy transmitted and processed at relays, some interesting results are obtained, such as that there is an optimal rate which minimizes the total energy consumption and increasing number of hops may not always be energy efficient. We also studied how spatial reuse and cooperative relaying strategies can be useful for the greater benefits gained by simultaneous transmission and node cooperation to improve the end-to-end bandwidth efficiency, and how practical limits on the energy-bandwidth tradeoff can be close to the fundamental limits by considering the successful packet reception.

Specifically, in Chapter 2, we formulated the normalized total energy consumption and the end-to-end bandwidth utilization to transmit information bits for the optimal transmission energy and rate on each hop and two practical communication schemes:

(1) common rate scheme and (2) common power scheme. The results showed that the common power case yields better energy-bandwidth tradeoff than the common rate case due to the rate adaptation in each link, while both cases have the same asymptotic performance in the power-limited regime when the circuit processing energy is not taken into account. In addition, we showed that an irregular distribution of relays with common power scheme can outperform equi-spaced relays at high end-to-end rates, which implies reducing the number of hops is more energy-efficient. We also showed that the total energy consumption can be minimized by optimally choosing the rate, determined from the location of relays and the end-to-end distance. Therefore, by comparing the total energy consumption for different location of relays, a routing path which achieves the best energy-bandwidth tradeoff can be obtained. In addition, we showed that by optimizing jointly over the end-to-end rate and number of hops for equi-spaced relays, the optimal number of hops and the optimal end-to-end rate can be found.

In Chapter 3, we introduced spatial reuse for multi-hop routing transmissions, where spatial reuse is used by restricting the minimum number of hops between simultaneous transmitters. A parametric solution for lower bounds on the minimum total energy consumption per information bit is derived by solving an equivalent convex optimization problem. When normalized processing energy is small, optimal end-to-end rate decreases due to the dominance of the transmitted energy consumption. In particular when the receiver processing energy is not included, multi-hop routing with spatial reuse shows the same performance as multi-hop routing without spatial reuse. It is shown that spatial reuse only contributes to enhancing bandwidth utilization, not reducing energy consumption-per-bit. Instead the energy efficiency can be improved by varying the minimum separation of hops between simultaneous transmitters, which is related to adjusting the resulting cochannel interference. This chapter also established a general relationship between the number of hops or equivalently minimum energy consumption,

and end-to-end distance by approximating cochannel interference as equal for each link.

Then in Chapter 4, instead of channel capacity achieving codes, we introduced a modified threshold model where a practical Reed-Solomon channel code with BPSK and orthogonal modulation scheme are considered for the successful packet retransmission. Based on this model, we formulated the normalized total energy consumption-per-bit and the end-to-end throughput to transmit information bits successfully for the optimal transmission energy and rate on each hop. The results showed that there is a performance gap due to approximations on the symbol error probability but its overall characteristic is well tracking of the actual energy-bandwidth tradeoff. We showed that orthogonal modulation yields better energy-bandwidth tradeoff at low rates due to its improved energy efficiency whereas BPSK modulation yields better energy-bandwidth tradeoff at high rates. In addition, we showed that unlike the channel capacity achieving codes, when receiver processing energy is not taken into account, the minimum total energy consumption-per-bit is not achieved as the end-to-end throughput becomes small.

Finally in Chapter 5, we derived bounds on the total energy consumption per information bit using the capacity of the Gaussian channel for the decode-and-forward and amplify-and-forward scheme. In particular, the generalized block Markov lower bound on capacity of the relay channel are used to show that the tightest lower bound is the capacity of the degraded relay channel in time-division mode. For the low rate region where the circuit energy dominates the overall energy consumption, noncooperative relaying strategy outperforms cooperative strategy regardless of its forwarding method, and in particular the amplify-and-forward scheme is useful in this region since it does not need to process the received signal at the relay. However at high rates the decode-and-forward scheme outperforms the amplify-and-forward scheme which shows limited bandwidth utilization. Therefore, depending on the location of relay and desired end-to-end throughput based on the available network energy budget, both forwarding strategy and node cooperation should be used.

6.2 Future Research

There are a variety of fruitful areas for future research on cooperative diversity and related topics. We have mentioned many issues in earlier chapters, but we repeat some of the larger and more important ones here.

- **Non-static network:** There are a number of modeling and analytical considerations that can be further exploited in the context of multi-hop routing in 2-dimensional network. Our analysis has assumed a static network and interference-free scheduling which presumes perfect synchronization among all nodes and knowledge of the overall network characteristics at all nodes. However, for a dynamically changing network with time varying wireless channel characteristics, we need to find an optimal energy-bandwidth tradeoff in a decomposed or distributed way since only part of the network needs to be reconfigured which makes distributed algorithms more efficient. In our future work, it would be interesting to find more efficient distributed algorithms so that the complete recomputation of the optimal energy-bandwidth tradeoff may not be required when the topology changes.

- **Practical cooperative forwarding with temporal or spectral diversity:** We saw that cooperative relaying may be very beneficial depending on the target end-to-end rates and forwarding strategies. We focused in the latter part of this dissertation on scenarios in which only amplify-and-forwarding and decode-and-forwarding strategies are employed. In more general channel environments, the channel may exhibit variations across time and frequency as well as space. Since there are many effective methods for exploiting both temporal and spectral diversity, it is important to address whether, and to what extent, it is useful to additionally exploit spatial diversity through cooperative diversity. For the relaying based on node collaboration, the framework can be extended to more

complicated space-time codes, which leads to another tradeoff between diversity and multiplexing. It will be interesting to investigate how many nodes should participate in reliability enhancement and throughput boosting. Due to the local information exchange required for node collaboration and the necessity of routing in a networking scenario, the tradeoff between diversity and multiplexing will be different from that in a traditional MIMO system.

- **Multiple source-destination pairs:** Throughout the dissertation we have assumed a single source-destination pair in the network in which interference-free scheduling is possible. However, in real networks, there are tremendous traffic across the network from different source-destination pairs that may result in severe interference. As such, there will be two significant considerations: (1) load-balancing scheduling to coordinate confliction from several source-destination pairs, that is, how to prevent specific relays from being overused, and (2) the best candidate relays for a single source-destination pair may not be the best choice for multiple source-destination pairs because the selfish selection may degrade the overall network performance due to the unwanted excessive interference. Future work in this area should focus on finding the optimal interference-aware scheduling algorithms for load balancing and the best energy-bandwidth tradeoff.

APPENDICES

APPENDIX A

Convexity and Solutions of Chapter 2

A.1 Convexity and Solution of Problem Ia

Proposition A.1. Suppose $f : \mathbf{R} \to \mathbf{R}$ is increasing and concave. Let g denote its inverse, *i.e.*, $g(f(x)) = x$. Then $y = g(x)$ is an increasing convex function.

Since $g(f(x)) = x$, we have $f'(g(x))g'(x) = 1$. Hence the first derivative with respect to x is given by

$$g'(x) = \frac{1}{f'(g(x))} > 0$$

since $f'(x) > 0$. Further the second derivative with respect to x is given by

$$g''(x) = -\frac{f''(g(x))g'(x)}{[f'(g(x))]^2} > 0$$

since $g'(x) > 0$ and $f''(x) < 0$. Therefore $g(x)$ is an increasing convex function. ■

Proposition A.2. The function $y = x(c_1 g(x^{-1}) + c_2)$ is convex in x for $x \geq 0$ and $c_1 > 0, c_2 \geq 0$ if $g(x)$ is increasing convex.

Proof: The first order partial derivative with respect to x is given by

$$\frac{\partial y}{\partial x} = c_1 g(x^{-1}) - c_1 \frac{g'(x^{-1})}{x} + c_2$$

and further the second order derivative with respect to x is given by

$$\frac{\partial^2 y}{\partial x^2} = \frac{c_1 g''(x^{-1})}{x^3} > 0$$

since $g''(x) > 0$ for all x. ■

For a given location of relays, the energy optimization problem for a given end-to-end rate R_e from (2.4) is

$$\begin{aligned} \min_{(\gamma_1,..,\gamma_k)} \quad & \sum_{i=1}^{k} \frac{\alpha_i^{\eta}\gamma_i + \gamma_c}{C(\gamma_i)} \\ \text{s.t.} \quad & \textstyle\sum_i C(\gamma_i)^{-1} = R_e^{-1} \end{aligned} \tag{A.1}$$

where the optimization variables are γ_i's. Let $x_i = \frac{1}{C(\gamma_i)}$ and $C^{-1}(x) = g(x)$. Then the optimization problem can be converted into an equivalent convex optimization problem

$$\begin{aligned} \min_{(x_1,..,x_k)} \quad & \sum_{i=1}^{k} x_i \left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c\right] \\ \text{s.t.} \quad & \textstyle\sum_i x_i = R_e^{-1} \end{aligned} \tag{A.2}$$

where the optimization variables are x_i's and the equality constraint is affine.

Proposition A.3. The function $f(x_1, ..., x_k) = \sum_{i=1}^{k} h(x_i) = \sum_{i=1}^{k} x_i \left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c\right]$ is convex in x_i for $x_i \geq 0$ for $i = 1, ..., k$ and $\gamma_c \geq 0$.

Proof: To prove convexity of $f(x_1, ..., x_k)$, it is sufficient to prove convexity of $h(x_i) = x_i \left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c\right]$ for any i since a non-negative sum of convex functions is convex. From (A.1), $g(x) = C^{-1}(x)$ is increasing convex since the channel capacity function $C(x)$ is increasing concave. Then using the result from (A.2), $h(x_i)$ is convex for $x_i > 0$. ■

The optimal solution can be obtained using the Lagrangian function of (A.2)

$$L(x_1, ..., x_k, \lambda) = \sum_{i=1}^{k} x_i \left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c\right] - \lambda(\sum_i x_i - R_e^{-1})$$

where $x_i \geq 0$ for all i. Therefore the Karush-Kuhn-Tucker (KKT) conditions give the following parametric solution which can be evaluated by numerical techniques,

$$\lambda = \alpha_i^{\eta} \left(g(x_i^{-1}) + x_i \frac{\partial g(x_i^{-1})}{\partial x_i} \right) + \gamma_c \tag{A.3}$$

$$= \alpha_i^{\eta} \left(g(x_i^{-1}) - \frac{1}{x_i} g'(x_i^{-1}) \right) + \gamma_c \tag{A.4}$$

$$\sum_{i=1}^{k} x_i = \frac{1}{R_e} \tag{A.5}$$

where $x_i \geq 0$ for all i.

A.2 Convexity and solution of Problem Ib

For a given location of relays, the energy optimization problem is

$$\min_{(\gamma_1,..,\gamma_k)} \quad \sum_{i=1}^{k} \frac{\alpha_i^{\eta} \gamma_i + \gamma_c}{C(\gamma_i)} \tag{A.6}$$

where the optimization variables are γ_i's. Let $x_i = \frac{1}{C(\gamma_i)}$ and $C^{-1}(x) = g(x)$. Then the optimization problem can be converted into an equivalent convex optimization problem from (A.3)

$$\min_{(x_1,..,x_k)} \quad \sum_{i=1}^{k} x_i \left[\alpha_i^{\eta} g(x_i^{-1}) + \gamma_c \right] \tag{A.7}$$

where the optimization variables are x_i's.

Setting the first derivative to zero yields

$$\frac{g'(1/x_i)}{x_i} - g(1/x_i) = \alpha_i^{-\eta} \gamma_c \tag{A.8}$$

where $x_i \geq 0$ for all i. Therefore from the optimal solution of the above equation, we

obtain the optimal SNR for i-th hop

$$\gamma_i^* = g(1/x_i^*) = C^{-1}(1/x_i^*) \tag{A.9}$$

where $x_i^* \geq 0$ for all i.

For the case of the AWGN channel, from (A.8) we obtain

$$\frac{\ln 2}{x_i^*} e^{2\ln 2/x_i^*} - \frac{1}{2}(e^{2\ln 2/x_i^*} - 1) = \alpha_i^{-\eta}\gamma_c$$

where $x_i^* \geq 0$ for all i. Substituting $2\ln 2/x_i^* = u_i^* + 1$ yields

$$u_i^* = \frac{2\ln 2}{x_i^*} - 1 = \mathcal{W}\left(\frac{2\alpha_i^{-\eta}\gamma_c - 1}{e}\right) \tag{A.10}$$

$$\frac{1}{x_i^*} = C(\gamma_i^*) = \frac{1}{2}\log_2(1 + 2\gamma_i^*) \tag{A.11}$$

$$\gamma_i^* = \frac{1}{2}\left[\exp\left(\mathcal{W}\left(\frac{2\alpha_i^{-\eta}\gamma_c - 1}{e}\right) + 1\right) - 1\right]. \tag{A.12}$$

Using (A.12), it is straightforward to obtain the optimal end-to-end rate and the minimum total energy consumption in (2.13). Further using $u_i = 1 + \mathcal{W}\left(\frac{2\alpha_i^{-\eta}\gamma_c - 1}{e}\right)$, we obtain $e^{u_i} - 1 = u_i e^{u_i} - 2\alpha_i^{-\eta}\gamma_c$ which yields the second part of the minimum total energy consumption in (2.13).

A.3 Proof of Property 2.1

Property 2.1 $\left(\frac{E_{tot}}{N_0}\right)_{\gamma_c=0} = \frac{E_b}{N_0}$ is strictly increasing in $R_e > 0$.

Proof: Suppose that for a given R_e', $\{\gamma_1', ..., \gamma_k'\}$ is the set of received SNR that achieves the rate at each hop R_i' and the end-to-end received energy per information bit E_b'/N_0. Now suppose we are allowed to have $\lambda R_e'$ where $\lambda > 1$. Then it is easy to see that the rate at each hop has increased by λ from the fact that $\frac{1}{\lambda R_e'} = \sum_{i=1}^{k} \frac{1}{\lambda R_i'}$. Since this new received SNR γ_i resulting from the increased rate of each hop should only be

greater than γ_i' and $\frac{\gamma_i}{C(\gamma_i)}$ in (2.3) for $i = 1, ..., k$ is an increasing function on γ_i, E_b/N_0 is strictly increasing in R_e. ■

A.4 Conditions for convexity of Problem III

Proposition 2.1 The function $f(x, y) = e^{\eta x+y} g(e^{-x-y}) + \gamma_c e^y$ is convex in x, y for $x, y \in \mathbf{R}$ if $g(u)$ satisfies the following conditions for $u \geq 0$

1. $a' = u^2 g''(u) - ug'(u) + g(u) > 0$.

2. $a'c' - (b')^2 > 0$ where $b' = \eta u g'(u) - u^2 g''(u) - ng(u)$ and $c' = u^2 g''(u) + \eta^2 g(u) + (1 - 2\eta)ug'(u)$.

To prove convexity of $f(x, y)$ in x and y, it is sufficient to prove convexity of $f_1(x, y) = e^{\eta x+y} g(e^{-x-y})$ since $f_2(y) = \gamma_c e^y$ is convex in y for $\gamma_c \geq 0$ and a non-negative sum of convex functions is convex. The second order partial derivative of $f_1(x, y)$ with respect to x is given by

$$\frac{\partial^2 f_1}{\partial x^2} = e^{y+\eta x} \left[g(u) - ug'(u) + u^2 g''(u) \right] \triangleq a$$

and with respect to y

$$\frac{\partial^2 f_1}{\partial y^2} = e^{y+\eta x} \left[\eta^2 g(u) + (1 - 2\eta)ug'(u) + u^2 g''(u) \right] \triangleq c$$

where $u = e^{-x-y} \geq 0$. Finally with respect to x and y, we have

$$\frac{\partial^2 f_1}{\partial x \partial y} = e^{y+\eta x} \left[\eta g(u) - \eta u g'(u) + u^2 g''(u) \right] \triangleq b.$$

Therefore, the Hessian of $f(x, y)$ is given by

$$\mathbf{H} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}.$$

A 2×2 matrix of the form $\mathbf{H}$ with $a > 0$ is positive definite if and only if $\Delta = ac - b^2 > 0$ (Schur's complement condition [38]). Then for $u \geq 0$ and $\gamma_c \geq 0$, it is sufficient for its Hessian to be positive definite that the following conditions be satisfied for any $\gamma_c \geq 0$.

1. $a' = u^2 g''(u) - ug'(u) + g(u) > 0$.
2. $a'c' - (b')^2 > 0$, where $b' = \eta u g'(u) - u^2 g''(u) - ng(u)$ and $c' = u^2 g''(u) + \eta^2 g(u) + (1 - 2\eta)ug'(u)$.

■

For the case of the AWGN channel it is sufficient to show that $g(u) = e^u - 1$ satisfies the conditions. For the first condition, we have

$$a' = u^2 e^u - ue^u + e^u - 1 > 0, \text{ for } u \geq 0$$

since a' is increasing with $u \geq 0$ and $f_1(0) = 0$. For the second condition, we obtain $b' = (\eta - \eta u + u^2)e^u - \eta$ hence

$$a'c' - (b')^2 = (\eta - 1)^2 u e^u (e^u - u - 1) > 0 \text{ for } u \geq 0.$$

Therefore the function $E_{tot}(x, y)/N_0$ for the AWGN channel is convex in $x, y \in \mathbf{R}$.

APPENDIX B

Convexity of Chapter 3

B.1 Condition for convexity

Proposition 3.1 The function $f(x,y) = e^{(\eta-1)x+y}h(e^{-y}) + \gamma_c e^{-x+y}$ is convex in x and y for $x \in \mathbf{R}$ and $y \in \mathbf{R}$ if $g(u)$ satisfies the following conditions for $u \geq 0$ and $\beta \geq 0$

$$g(u)g'(u) + u\left[g''(u)g(u) - g'(u)^2\right] > 0 \tag{B.1}$$

where $h(u) = \frac{g(u)}{1-\beta g(u)} > 0$ for $1 - \beta g(u) > 0$.

Proof: To prove convexity of $f(x,y)$ in x and y, it is sufficient to prove convexity of $f_1(x,y) = e^{(\eta-1)x+y}h(e^{-y})$ since $f_2(x,y) = \gamma_c e^{-x+y}$ is convex in x, y for $\gamma_c \geq 0$ and the non-negative sum of convex functions is convex. The second order partial derivative of $f_1(x,y)$ with respect to x is given by

$$\frac{\partial^2 f_1(x,y)}{\partial x^2} = (\eta-1)^2 e^{(\eta-1)x+y}h \triangleq a > 0$$

and with respect to y

$$\frac{\partial^2 f_1(x,y)}{\partial y^2} = e^{(\eta-1)x+y}\left[h - e^{-y}h' + e^{-2y}h''\right] \triangleq c$$

Finally, with respect to x and y, we have

$$\frac{\partial^2 f_1(x,y)}{\partial x \partial y} = (\eta-1)e^{(\eta-1)x+y}\left[h - e^{-y}h'\right] \triangleq b$$

where h, h', and h'' denote $h(e^{-y})$,$h'(e^{-y})$, and $h''(e^{-y})$, respectively. Therefore, the Hessian of $f_1(x,y)$ is given by

$$\mathbf{H} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}.$$

A 2×2 matrix of the form $\mathbf{H}$ with $a > 0$ is positive definite if and only if $\Delta = ac - b^2 > 0$ (Schur's complement condition [38]). Let $u = e^{-y} \geq 0$. Then, for $u \geq 0$, its Hessian is given by

$$\Delta = uh(u)h'(u) + u^2\left[h(u)h''(u) - h'(u)^2\right] \tag{B.2}$$

where

$$h(u) = \frac{g(u)}{1-\beta g(u)} > 0 \tag{B.3}$$

$$h'(u) = \frac{g'(u)}{(1-\beta g(u))^2} > 0 \tag{B.4}$$

$$h''(u) = \frac{g''(u) - \beta g(u)g''(u) + 2\beta g'(u)^2}{(1-\beta g(u))^3} \tag{B.5}$$

since $g(u)$ is increasing convex [31] and $1-\beta g(u) > 0$. Therefore $\Delta > 0$ if the following condition is satisfied for $u \geq 0$ and $g(u) < 1/\beta$ with $\beta \geq 0$

$$\begin{aligned} &u\left[1-\beta g(u)\right]\left[g(u)g'(u) + u(g''(u)g(u) - g'(u)^2)\right] \\ &+ \beta u^2 g(u)g'(u)^2 > 0 \end{aligned}$$

Therefore, for any $\beta \geq 0$, it is sufficient for the Hessian to be positive definite if

$g(u)g'(u) + u\left[g''(u)g(u) - g'(u)^2\right] > 0.$ ■

APPENDIX C

Convexity of Chapter 4

C.1 Convexity of Problem III

For the minimum energy consumption, we need to show the convexity of the objective function in (4.42) which can be shown using the proposition V.1 in [31]. The function $\frac{E_{tot}(x,y)}{N_0} = e^{\eta x+y} g(e^{-x-y}) + \gamma_c e^y$ is convex if $g(u) = -\ln(1-u)$ satisfies the following conditions for $0 < u < 1$:

1. $a' = u^2 g''(u) - u g'(u) + g(u) > 0$.
2. $a'c' - (b')^2 > 0$ where $b' = \eta u g'(u) - u^2 g''(u) - n g(u)$ and $c' = u^2 g''(u) + \eta^2 g(u) + (1-2\eta) u g'(u)$.

For the first condition, we have

$$a' = \frac{u^2}{(1-u)^2} - \frac{u}{1-u} - \ln(1-u). \tag{C.1}$$

By letting $\frac{u}{1-u} = v > 0$ for $0 < u < 1$, we have

$$a' = v^2 - v + \ln(1+v) = f(v). \tag{C.2}$$

It can be shown easily that $f(v)$ is increasing for $v > 0$ because $f(0) = 0$, which leads to $a' = f(v) > 0$. For the second condition, we have

$$b' = \eta v - v^2 - \eta \ln(1+v) \tag{C.3}$$

$$c' = v^2 + \eta^2 \ln(1+v) + (1-2\eta)v. \tag{C.4}$$

Therefore we obtain

$$a'c' - (b')^2 = v\,(n-1)^2\,[(v+1)\ln(1+v) - v] = h(v). \tag{C.5}$$

It can be easily shown that $h(v)$ is increasing for $v > 0$ since $h(0) = 0$, which results in $a'c' - (b')^2 = h(v) > 0$. Therefore the optimization problem in (4.42) is a convex optimization problem.

BIBLIOGRAPHY

BIBLIOGRAPHY

[1] C. E. Shannon, "A mathematical theory of communication," Bell Systems Techincal Journal, 27:379-423, 623-656, Jul.-Oct. 1948.

[2] C. Berrou and A. Glavieux and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding: Turbo codes," in *Proc. of IEEE International Conference on Communications (ICC)*, pp. 1064-1070, 1993.

[3] R. G. Gallager, "Low-density parity-check codes," in *IRE Transactions on Information Theory*, vol. 9(1), pp. 666-680, Jan. 1962.

[4] R. G. Gallager, *Low-Density Parity Check Codes*, MIT Press, Cambridge, MA, 1963.

[5] D. MacKay, "Good error correcting codes based on very sparse matrices," *IEEE Trans. Inform. Theory*, vol. 45(2), pp.399-431, Mar. 1999.

[6] T. M. Cover, "Broadcast channels," *IEEE Trans. Inform. Theory*, vol. 18(1), pp.2-14, Jan. 1972.

[7] D. Slepian and J. K. Wolf, "A coding theorem for multiple access channels with correlated sources," Bell Systems Techincal Journal, 52:1037-1076, 1973.

[8] P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46(2), pp.388-404, Mar. 2000.

[9] F. Xue, L.-L. Xie, and P. R. Kumar, "The transport capacity of wireless networks over fading channels," *IEEE Trans. Inform. Theory*, vol. 51(3), pp.834-847, Mar. 2005.

[10] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation," *IEEE Trans. Inform. Theory*, vol. 50(5), pp.748-767, May. 2004.

[11] S. Verdú, "On channel capacity per unit cost," *IEEE Trans. Inf. Theory*, vol. 36(5), pp. 1019-1030, Sept. 1990.

[12] V. Rodoplu and T. H. Meng, "Bits-per-Joule capacity of energy-limited wireless networks," *IEEE Trans. Wireless Commun.*, vol. 6(3), pp. 857-865, March 2007.

[13] S. Verdú, "Spectral efficiency in the wideband regime," *IEEE Trans. Inf. Theory*, vol. 48(6), pp. 1319-1343, June 2002.

[14] E. Uysal-Biyikoglu and A. E. Gamal, "On adaptive transmission for energy efficiency in wireless data networks," *IEEE Trans. Inf. Theory*, vol. 50(12), pp. 3081-3094, Dec. 2004.

[15] A. El Gamal, M. Mohseni and S. Zahedi, "Bounds on capacity and minimum energy-per-bit for AWGN relay channels," *IEEE Trans. Inf. Theory*, vol. 52(4), pp. 1545-1561, Apr. 2006.

[16] J. Gomez-Vilardebo and A. I. Perez-Neira, "Bounds on maximum rate-per-energy for orthogonal AWGN multiple relay channels," *IEEE Trans. Wireless Commun.*, vol. 7(11), pp. 4238-4247, Nov. 2008.

[17] S. Cui, A. J. Goldsmith, and A. Bahai, "Energy-constrained modulation optimization," *IEEE Trans. Wireless Commun.*, vol. 4(5), pp. 2349-2360, Sep. 2005.

[18] T. Shu, M. Krunz, and S. Vrudhula, "Joint optimization of transmission power-time and bit energy efficiency in CDMA wireless sensor networks," in *IEEE Trans. Wireless Commun.*, vol. 5(3), pp. 3109-3118, Nov. 2006.

[19] S. Cui, R. Madan, and A. J. Goldsmith, and S. Lall, "Cross-layer energy and delay optimization in small-scale sensor networks," *IEEE Trans. Wireless Commun.*, vol. 6(10), pp. 3688-3699, Oct. 2007

[20] G. Kramer, M. Gastpar and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," in *IEEE Trans. Inf. Theory*, vol. 51(9), pp. 3037-3063, Sep. 2005.

[21] M. Sikora, J. N. Laneman, M. Haenggi, D. J. Costello, and T. E. Fuja, "Bandwidth and power efficient routing in linear wireless networks," *Joint Special Issue of IEEE Trans. Inf. Theory and IEEE Trans. Networking*, vol. 52, pp. 2624-2633, June 2006.

[22] M. Sikora, J. N. Laneman, M. Haenggi, D. J. Costello, and T. E. Fuja, "On the optimum number of hops in linear wireless networks," *Proc. IEEE Information Theory Workshop*, 2004, p. 165-169

[23] Ö. Oyman and S. Sandhu, "A Shannon-theoretic perspective on fading multihop networks," in *Proc. Conf. Information Sciences and Systems*, 2006, p.525-530.

[24] Ö. Oyman and S. Sandhu, "Non-ergodic power-bandwidth tradeoff in linear multihop wireless networks," in *Proc. IEEE International Symposium on Information Theory*, 2006, p.1514-1518.

[25] A. J. Goldsmith and S. B. Wicker, "Design challenges for energy-constrained ad hoc wireless networks," *IEEE Wireless Commun. Mag.*, vol. 9(4), pp. 8-27, Aug. 2002.

[26] D. Chen, M. Haenggi, and J. N. Laneman, Distributed spectrum-efficient routing algorithms in wireless networks, in *IEEE Trans. Wireless Commun.*, vol. 7(12), pp. 5297 - 5305, Dec. 2008.

[27] W. E. Stark, H. Wang, A. Worthen, S. Lafortune, and D. Teneketzis "Low-energy wireless communication network design," *IEEE Wireless Commun.*, pp. 60-72, Aug. 2002.

[28] H. Wang, "Analysis and design of efficient wireless networks," University of Michigan Ph.D. Thesis, 2004.

[29] C. Bae and W. E. Stark, "Energy and bandwidth efficiency in wireless networks," in *Proc. International Conference on Communications, Circuits and Systems*, 2006, pp.1297-1302. 2006.

[30] C. Bae and W. E. Stark, "A tradeoff between energy and bandwidth efficiency in wireless networks," in *Proc. IEEE Military Communications Conference*, 2007, pp.1297-1302.

[31] C. Bae and W. E. Stark, "End-to-end energy-bandwidth tradeoff in multi-hop wireless networks," *IEEE Trans. Inf. Theory*, vol. 55(9), pp. 4051-4066, Sep. 2009.

[32] A. Agustin and J. Vidal, "Amplify-and-forward cooperation under interference-limited spatial reuse of the relay slot," in *IEEE Trans. Wireless Commun.*, vol. 7(5), pp. 1952-1962, May 2008.

[33] O. Gurewitz, A. de Baynast, and E. K. Knightly, "Coopertative strategies and achievable rates for tree networks with optimal spatial reuse," in *IEEE Trans. Inf. Theory*, vol. 53(10), pp. 3596-3614, Oct. 2007.

[34] R. Corless, G. Gonnet, D. Hare, D. Jeffrey, and D. Knuth, "On the Lambert-W function," in *Advances in Computational Mathematics*, vol. 5(1), pp. 329-359, 1996.

[35] N. Blaunstein and C. Christodoulou, *Radio propagation and adaptive antennas for wireless communication links*, Hoboken, NJ: John Wiley & Sons, 2007

[36] T.S. Rappaport, *Wireless Communications: Principles and Practice*, Englewood Cliffs, NJ: Prentice Hall, 1996

[37] S. L. Howard, C. Schlegel, and K. Iniewski, "Error control coding in low-power wireless sensor networks:when is ECC energy-efficient?," in *EURASIP Journals on Wireless Communication and Networking*, vol.2006, pp.1-14, 2006.

[38] S. Boyd and L. Vandenberghe, *Convex Optimization*, New York: Cambridge Univ. Press, 2004

[39] J. G. Proakis, *Digital Communications*, 4th ed. New York: McGraw-Hill, 2000

[40] M. A. M. Vieira, C. N. Coelho Jr., D. C. da Silva Jr., and J. M. da Mata, "Survey on wireless sensor network devices," in *In Proc. IEEE Conference on Emerging Technologies and Factory Automation*, Sep. 2003.

[41] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE Journal on Selected Areas in Communications*, vol. 25, pp.379-389, Feb. 2007.

[42] E. C. van der Meulen, "Three-terminal communication channels," *Adv. Appl. Probab.*, vol. 3, pp. 120-154, 1971.

[43] T. Cover and A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. IT-25(5), pp. 572-584, Sep. 1979.

[44] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50(12), pp. 3062-3080, Dec. 2004.

[45] A. Høst-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Trans. Inf. Theory*, vol. 51(6), pp. 2020-2040, June 2005.

[46] M. A. Khojastepour, A. Sabharwal, and B. Aazhang, "On capacity of Gaussian 'cheap' relay channel," in *Proc. IEEE Global Communications Conf. (GLOBECOM'03)*, San Francisco, CA, Dec. 2003.

[47] P. Gupta and P. R. Kumar, "Towards an information theory of large networks: an achievable rate region," *IEEE Trans. Inf. Theory*, vol. 49(8), pp. 1877-1894, Aug. 2003