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# Inquiry Based Learning: A Teaching and Parenting Opportunity

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## Abstract

In this paper, I discuss what appears to be a new perspective on inquiry based learning (IBL) by describing its parallels with parenting. IBL is a student-centered learning method involving collaborative work on carefully sequenced exercises, oral and written communication of solutions, and peer review. Students create their own knowledge and present their ideas, and the instructor acts as a facilitator. The parallels between IBL and parenting include a growth mindset, emphasizing process over outcome, learning from mistakes, learning how to get unstuck, and deconstructing tasks. IBL and parenting also involve similar social interactions, such as responding to difficult questions in the moment, being sensitive to body language, and active listening. I also discuss how both activities differ. By comparing IBL and parenting, instructors can understand both better; this can in turn make IBL seem like a natural type of instruction. Additionally, I describe opportunities for learning IBL that are particularly feasible for parents.

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**Keywords:** *inquiry based learning, parenting, growth mindset, mistakes, productive struggle, productive failure, active listening*

## 1. Introduction

Evidence supports the effectiveness of using inquiry based learning (IBL)<sup>1</sup> methods in the college mathematics classroom [10, 13]. Yet learning how to teach this way can be a significant change for an instructor.

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<sup>1</sup>We assume that the reader is unfamiliar with IBL and describe it in detail in Section 2.

Those who have full schedules, such as parents, may be reluctant to devote their time to learning new teaching methods. I have been using IBL teaching methods for a year, and I am also the mother of two children, ages three and five. The more I teach undergraduate mathematics with IBL and the more experience I gain as a parent, the more I notice similarities between the two.

The first goal of this paper is to list and explain the parallels between parenting and IBL. This way, we can better understand aspects of both activities. The second goal of this paper is to ease the transition to IBL. If IBL is compared to parenting, this may make it seem like a more natural way of teaching and therefore make it more understandable and easier to use. The third goal is to describe resources for instructors interested in learning to use IBL, ones that may be particularly accessible and attractive to parents.

We begin with an overview of the comparisons. The broad aims of parenting and IBL are similar: both involve helping others in a journey of discovery, while simultaneously developing their essential skills and independence. Parents are helping their children learn the skills needed to function in life until they are able to thrive on their own. The goals of IBL are for students to create their own knowledge, and also to develop their problem-solving, communication, and collaborative skills. The hope is that they can eventually perform these important skills proficiently without instructor guidance.

Apart from these broad goals, there are several other ways that IBL and parenting are similar, and we will explore these in depth in this paper. More specifically, we will examine three categories of parallels. The first category involves growth mindset, and includes emphasizing process over final results, making mistakes and learning from them, and learning how to make progress when one is stuck. The second category of parallels discusses how tasks must be deconstructed into smaller, more manageable ones in order to be learned. The third category deals with parent-child and instructor-student interactions and includes responding to difficult questions in the moment, being sensitive to body language, and active listening.

Many resources compare teaching in general with parenting. For example, Dweck likens the two by advising both parents and teachers to praise hard work and effort instead of talent and intelligence [5, pages 173-202]. Jenkins discusses how motivating children with rewards instead of punishment is similar to motivating students [9]. Barnas and Walker compare teaching styles to parenting styles and examine how these styles affect students [3, 15].

Su compares IBL and parenting in one part of the talk “Freedom through Inquiry” at the 2016 Inquiry Based Learning Forum [14]. Su uses a childhood experience with his mother to demonstrate how learning through inquiry supports the freedom to ask questions, to make mistakes, and to not worry about others’ expectations. This article appears to be the first resource that discusses at length the parallels between parenting and inquiry based learning in the college mathematics classroom.

This paper is organized as follows. Section 2 describes what IBL is, summarizes research on its effectiveness, and lists a handful of resources for learning how to use it. Sections 3, 4, and 5 explain the parallels between IBL and parenting, Section 6 addresses differences between the two activities, and Section 7 contains concluding remarks.

## 2. Background on IBL

### 2.1. What is IBL?

One way to describe IBL is as a form of instruction that follows two main principles, as Ernst writes in [7]: as much as possible, (1) *students* develop and explain new ideas and (2) *students* authorize the validity of ideas. The instructor, on the other hand, acts as a facilitator, guiding the topics of discussion, asking questions, and clarifying and summarizing ideas (and students also do these things). The goals of IBL include teaching content while developing students’ problem-solving, communication, and collaborative skills, and fostering their confidence and independence. To this end, students learn concepts by completing carefully sequenced exercises in which they work examples, experiment, ask questions, develop solutions, get feedback from their peers and instructor, and modify solutions based on feedback. For more information, the reader is encouraged to read Ernst and Hodge’s helpful descriptions in [7] and [8].

There are multitudinous ways in which an instructor can incorporate IBL methods into their teaching, and the methods used depend on the students, the subject, and the instructor’s teaching style. Here are some examples of how an IBL course can be structured. I emphasize that these are only examples and that there are numerous other possible implementations.

In one possible format, the instructor hands out a set of exercises as class begins each day, and students work in groups of three or four to complete them. The exercises are accessible enough that students do not need extensive directions from the instructor to start working on them. There are a variety of possibilities for these exercises, such as routine ones that develop basic skills or conceptual ones with problem statements that lead to different interpretations and solutions. The instructor walks around the room to answer questions and makes comments on student work. After students have written solutions and ideas, the students and instructor discuss these as a whole class, and the discussion is based on the work that students have produced. During this discussion period, the instructor clarifies concepts and summarizes main ideas.

In another version students work from a set of IBL notes and deliver daily presentations of exercises from them. A set of IBL notes is similar to other mathematics textbooks in that it contains definitions, examples, theorems, and exercises. However, in IBL notes, examples and theorems are presented as exercises for the *reader*. The material in the notes can follow a sequence similar to the following: a definition is stated, and the reader is asked to determine whether specific examples satisfy a definition and prove related statements. For further clarification of what IBL notes are like, an excerpt from Morrow's abstract algebra notes [12] are provided in Appendix A. The students work on exercises from the notes before class. They present their ideas/solutions during class time. Then their peers and the instructor give feedback, students modify their work based on this feedback, and the instructor clarifies and summarizes important ideas. Finally, students submit written solutions of these exercises to be graded by the instructor.

It is also important to note that many instructors mix IBL methods with lecture or other activities. Yoshinobu describes how there are different levels of IBL instruction [17]. For example, a professor may lecture but also pose questions using think-pair-share: the professor asks a question, students think about it for a short period of time, students share their ideas with a partner for a short period of time, and the whole class discusses these ideas. Alternatively an instructor may use the group work format mentioned above for half of the time and lecture during the other half.

## *2.2. Research about the effectiveness of IBL*

Research supporting the effectiveness of IBL is compelling. In one study, Laursen and her colleagues [10, 11] compared IBL and non-IBL mathematics courses at four universities. They analyzed outcomes of students in a hundred course sections. Their measures included a survey about learning gains, a survey about attitudes and beliefs about mathematics, and grades in subsequent courses. In the survey about learning gains, IBL students reported higher gains than their non-IBL peers in mathematical thinking and understanding, confidence, persistence, a positive attitude about mathematics, working with others, seeking help, and appreciating different perspectives. Furthermore, IBL students generally earned the same or better grades in later courses as their non-IBL peers.

Laursen and her team also disaggregated data by gender and achievement level. The results about female students were remarkable. In non-IBL courses, women reported lower learning gains in mathematical thinking and understanding, confidence, persistence, and a positive attitude about mathematics, as compared to their male classmates. However, women who took IBL courses reported learning gains in these areas similar to their male classmates. IBL men reported approximately the same results in these areas as non-IBL men. In the survey about attitudes and beliefs, students answered pre- and post-course questions regarding confidence in doing mathematics. Women in non-IBL courses stated that their confidence decreased from pre- to post-course, whereas their male classmates reported smaller decreases. On the other hand, women in IBL courses reported that their confidence increased from pre- to post-course, and improved even more than their male classmates' confidence. In summary, women who took an IBL course reported higher learning gains and confidence than their non-IBL female peers, and there was no loss to those of men who had taken an IBL course. Finally, grades in subsequent courses of both non-IBL and IBL women were similar to those of their male classmates.

The results about less well-prepared students were also notable. Students with a prior GPA of less than 2.5 who took an IBL course improved their course grades, on average, by 0.3-0.4 grade points in subsequent courses (on a four-point scale). However, the grades of their non-IBL, less well-prepared classmates decreased on average.

A study by Rasmussen, Kwon, Allen, Marrongelle, and Burtch [13] suggests that IBL students perform the same as non-IBL peers on procedural tasks, but perform better on conceptual ones. They compared differential equations courses taught with traditional lecture approaches to those taught with IBL. Students in both types of courses were assessed with routine problems (ones that tested basic skills typically taught in differential equations), as well as with more conceptual problems. On the routine problems, students in the IBL courses performed approximately the same as those who received traditional lecture, so both groups learned the same routine skills. However, on the conceptual problems, IBL students scored significantly higher than the lecture students.

### *2.3. Resources for learning IBL methods*

While research suggests that teaching via IBL is worthwhile, it can appear challenging since it may be different from how one has been teaching. The Academy of Inquiry Based Learning (AIBL) runs a summer workshop to help instructors, at all stages in their careers, learn how to use IBL [1]. I attended one at DePaul University in June 2017, facilitated by Danielle Champney, Gulden Karakok, Brian Katz, and T. Kyle Petersen. At the workshop we learned what IBL is, different ways to structure an IBL course, and how to manage day-to-day operations. We also made plans to teach a target course using IBL, and were supported in this effort through an email listserv during the year following the workshop.

I was particularly attracted to the workshop because it was manageable time-wise and financially for me, the mother of two preschool-aged children in full-time daycare. The workshop runs for four days during the summer, when a parent may have a more flexible schedule. It is held at three different locations throughout the United States, and AIBL offers travel grants and pays for food and lodging. The workshop requires no preparation beyond deciding on your target course and gathering relevant materials. Yet it can have long-term effects on a career. For example it can provide an entrance into the scholarship of teaching and learning. In other words, I believe it is an efficient and inexpensive professional development opportunity with long-term benefits.

For those who cannot attend the workshop, AIBL has a mentoring program that pairs beginning users with experienced ones [2]. I worked with an excellent mentor, T. Kyle Petersen, when I used IBL for the first time in the

spring of 2017. It was very helpful to have regular conversations about how to structure my course, how to run daily operations, and how to deal with issues as they arose.

Much of what is written in this paper is based on what I learned at the IBL workshop and from working with my mentor, and how I have personally interpreted those ideas and carried them out in my own teaching. A similar comment applies to the ideas about parenting. There is a broad spectrum of possibilities for both activities, but I hope that much of what I write resonates with the reader.

### **3. Growth Mindset**

The first category of parallels relates to growth mindset. Growth mindset, a term introduced by Stanford psychology professor Carol Dweck, broadly speaking, is “...based on the belief that your basic qualities are things you can cultivate through your own efforts. Although people may differ in every which way—in their initial talents and aptitudes, interests, or temperaments—everyone can change and grow through application and experience” [5, page 7]. In opposition to this is a fixed mindset, which is the view that one’s skills and potential are fixed and cannot be significantly changed through practice.

We often take a growth mindset approach to parenting. We believe that, in our children, are biological urges to learn communication, motor, and cognitive skills. That is, we believe that inherent in our children is a potential to learn these skills, and we will do whatever we can to help them learn and, moreover, have fun doing so. In the IBL workshop, we discussed how we can take a similar attitude towards our students. While the urge to understand mathematics may not be a biological one, the analogy with child-rearing is strong: instead of assuming that students cannot learn something, we assume that they have not *yet* learned it. Thus, we will work with students to make the mathematical discovery process as enjoyable and productive a journey as possible.

Three components involved in the growth mindset approach, in both parenting and IBL, are an emphasis on process over outcome, learning from mistakes, and learning how to get unstuck. We explore these in the following sections.



### 3.1. *Emphasis on process over outcome*

Children learn everyday skills through a long multi-phase process that can often take days, months, or years. For example, consider how a child learns to walk. A child starts by pulling up onto furniture and supporting themselves while taking small steps, or the parent may hold the child's hands and guide them as they<sup>2</sup> walk. The child will practice this for weeks or months, with gradually decreasing parental intervention, before they can eventually take steps on their own. They will fall often when first walking on their own, but within a year or two, they will become fully proficient. The child is not focused on the outcome—proficient walking—but rather on the pleasure and stimulation involved in each stage of the process. It is exciting to be able to move around in a new way, it is stimulating to feel new sensations in the legs and feet, and it is fun to laugh as the parent holds their hands and moves with them. All the while, the parent carefully watches the child and helps when necessary.

There are three aspects to note. First the process of learning consists of many phases. Secondly there are rewards from the process itself (as mentioned above). Finally, the focus is predominately on the child's actions rather than on those of the parent, and the parent offers feedback and support *in reaction to* those actions.

We see these three aspects in an IBL course. Solving a problem consists of multiple phases, and the overall process could consist of the following steps: students work on an exercise individually or in groups, share with peers and the instructor to get feedback, modify their ideas, and complete an assessment. For example, in an IBL course based on student presentations, students may write a draft proof before class and present it in class. Their peers give feedback, with input from the instructor, and the students decide whether the proof is acceptable. This part may involve a lengthy discussion. After the students have decided that a solution is valid, they then write up a final proof that is submitted to the instructor. As with a child learning to walk, there are pleasures and rewards in the process itself rather than on the final outcome. Students may feel satisfaction from creating ideas that are validated by their classmates. They may feel camaraderie in working with

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<sup>2</sup>In many parts of this paper, the pronouns they/them/theirs are used instead of she/her/hers and he/him/his, for gender neutrality.

each other to solve a problem. They may gain insight from explaining an idea to a peer, either through specific feedback from the peer or because it can be clarifying to articulate an idea to another person. Lastly, the focus is on the students' ideas throughout the process, and the instructor offers guidance in reaction to those, just as the focus is on a child's actions in the child-rearing scenario, and the parent responds to those.

Dweck clarifies in a follow-up article to *Mindset* [5] that having a growth mindset is not simply about effort. Rather, the goal of having a growth mindset is learning, so really a growth mindset is about effort and also improving from it [4]. Thus, having a growth mindset must include strategies such as learning from mistakes and learning how to get unstuck.

### 3.2. *Making mistakes*

Two ways to manage making mistakes in both parenting and IBL include building an environment supportive of mistakes and reflecting on them for learning. As parents, we create a supportive environment by assuming that children will repeatedly make mistakes, and when they occur, we have a mild or amused reaction. We then focus on dealing with the mistake and giving advice for the future. For example, when my son was learning to drink from a cup, I expected him to have spills, and when he did spill, I would say, "Whoopsie!" in a lighthearted way and then clean it up. Then I would offer advice to prevent future spills, such as, "Now we know that we have to keep the cup away from the edge of the table." There are also social interactions that help create an environment that supports mistakes, and we elaborate on those in Section 5.

Creating a classroom environment built on trust where students feel comfortable making mistakes, and then using mistakes for learning, are fundamental parts of IBL. Some instructors use the terms *productive struggle* or *productive failure* to describe the process of learning from mistakes. An instructor can create a supportive environment by using student presentations (as described above), group work, or some type of communal format where students are expected to share their attempts at solutions. These activities can be low-stakes if grading is gentle. A course grade may even measure productive failure and the risks that students take in displaying their mistakes publicly. Yoshinobu explains how students in his course presented twice during the semester about their own productive failure and what they learned from it,

and this was a portion of their course grade [16]. Ernst also describes the importance of creating classroom opportunities for productive failure and productive struggle (he makes a distinction between the two) [6].

Parents create a supportive environment for their children where they can experiment and fail safely; they also make sure to help them learn from their mistakes. IBL similarly encourages instructors to use mistakes for student learning. As mentioned above, Yoshinobu required that students describe what insight they gained from their mistakes. Additionally, after students share their ideas in an IBL setting, classmates and the instructor offer feedback, and then students modify their work, perhaps multiple times. During this process, the instructor may take the opportunity to highlight important things to remember for the future. For example, if a student is multiplying elements of a non-abelian group and is not careful about the order of elements, then we may say something like, “I am glad that this solution reminded us that, when multiplying group elements, we need to be careful about the order of elements. It is so natural to not have to worry about that, because in many examples (the integers, real numbers, etc.), the operation is commutative.”

### *3.3. Learning how to get unstuck*

We may aid our children in three ways when they are stuck: we resist the temptation to fix the situation ourselves, draw on our own experience to offer possible solutions, and keep the children and students motivated. For example, when my son was learning to put on his shoes, he got the back of the shoe folded under the heel. The first few times this happened, I took off the shoe for him and put it back on correctly. This kept happening, so I stopped myself from fixing it for him and instead coached him in a few ways, by imagining what I would do in that situation. I told him to take off the shoe and start over, to force his heel in and hope that the back of the shoe straightened out, or to pull on the tab on the back of the shoe so that the heel slid in comfortably. All the while, I motivated him by praising any progress made.

We can take a similar approach to helping our students who are stuck. The often communal format of an IBL course is designed in such a way that a student who is stuck can seek help from their peers or the instructor during class time. The instructor’s first temptation may be to tell the students how

to solve a problem, but the aim is to let student work guide the discussion. Thus, we might offer possible solutions by asking: “Why did you choose to...?”, “What is the definition of...?”, “What is your ultimate goal?”, “Could you explain this line?” We may also tell them to work with an example or ask them if they proved similar statements in the past that may be good references. With these suggestions, we as instructors are putting ourselves in the students’ position, thinking of ideas we would use to make progress, and modeling those ideas for our students. Also, we motivate the student by praising what they have done so far and by assuming that if the instructor and student keep trying, then eventually they will find a solution.

Additionally, to manage frustration due to being stuck on a problem, an instructor can use an idea suggested to me by T. Kyle Petersen (personal communication, 2017). Students and instructor discuss and agree on a set amount of time that students should spend preparing for class (one to two hours, for example). Before class, students do preliminary work on a list of problems for the agreed-upon amount of time, and any ideas related to the problem are considered acceptable. At the beginning of class, the instructor quickly checks this preliminary work and gives full credit for trying something. This way, the work that the students must do is clear, they will likely do the work because they get credit for it, and they may develop strategies for solving problems that they initially find difficult.

#### **4. Deconstruction of Tasks**

When we are teaching a child a new skill, we deconstruct it into fine detail, communicate the details in a carefully laid-out sequence, and highlight subtleties. For example, when teaching my son to hang up his coat, I had to first think in detail about the steps involved. Then I explained as follows. “First pick up your coat, then walk over to the coat rack. Then grab the hood with two hands, and fit the top over the hook.” Throughout this process, my son did a good part of the work, but I guided his hands at certain points. Additionally, I highlighted certain parts of the process, by saying things such as “It is helpful if you grab the part of the hood that normally sits above your forehead, and hang that part on the hook”. This approach is helpful in the mathematics classroom as well.

As mentioned in Section 2.1, instructors and students often (but not necessarily) work from a set of IBL notes structured as follows. After a definition

is stated, students determine whether specific examples satisfy the definition, and then they prove related statements. For example, in Morrow's notes [12], a student is introduced to the definition of a group, and then asked to determine whether several pairings of sets and operations are groups, including the integers and real numbers under addition and multiplication, the set of integers modulo  $n$  ( $n$  is a positive integer) under modular addition and multiplication, the set of symmetries of the regular pentagon under composition (students having been introduced to modular integers and symmetries previously), and matrices under multiplication. Then students are introduced to the definition of an abelian group and asked to determine whether certain groups are abelian. Finally, students prove or disprove related statements. See Appendix A for an excerpt from [12] that includes these exercises.

Similar to helping a child learn how to hang up their coat, the process of learning what a group is has been deconstructed into fine detail and arranged in a careful sequence that gradually introduces more complexity. The first step of grabbing the coat and walking to the coat rack is the most straightforward. In learning the definition of a group, we also start with something more straightforward, namely, with sets and operations that are very familiar (integers and real numbers under the usual addition and multiplication). We then incorporate more complexity: the child must grab the hood and then fit it over the hook in such a way that the coat stays on the hook, and this requires more concentration and effort. With the definition of a group, we work with increasingly more complicated objects: for example we work with sets whose elements are familiar, but the operation is not (the set of integers modulo  $n$  under modular multiplication and addition). Next we consider sets whose elements are not numbers and whose operations are more complicated (symmetries of the regular pentagon under composition and matrices under multiplication). Then we incorporate a new idea when determining whether a group operation is commutative.

Similar to how the parent highlights the subtlety of hanging the top part of the hood on the hook, there are subtleties that the instructor may wish to ask about, including whether the elements of a group are required to be numbers or whether the operation is important (for example, the real numbers under addition form a group, but the real numbers under multiplication do not since zero has no multiplicative inverse). In a traditional lecture course, similar examples may be discussed, but the instructor would be leading the discussion, whereas with IBL, the students themselves are making inferences

about the subtleties of the definition of a group (that elements are not necessarily numbers, that the operation can determine whether a group is formed, that the operation is not necessarily commutative, etc.).

## **5. Parent-Child and Instructor-Student Interactions**

In order to create an environment where children/students feel comfortable making mistakes and where they feel supported when they are struggling, we have to carefully and sensitively interact with each other. Here are some ways that I personally do this: responding to difficult questions honestly and constructively, being sensitive to body language, and practicing active listening.

IBL and parenting both put us in the position of having to respond in the moment to difficult questions. For example, my son asked me if he was going to die, and I had to quickly think of a way to respond that was truthful but reassuring. I told him that he will die, but not for a very long time, and that I will take very good care to make sure of that. When children ask questions to which we do not know the answer, then we may tell them so, and ask whether we can figure it out together. If we are giving feedback to a student on work they have produced, but they have made mistakes, then we aim to be truthful but still encouraging. For example, we will pause and try to think how we can work with what they have produced (if possible). If we cannot see how to use any of their ideas, then we may aim to say something along the lines of “It is good that you tried this, but I am not sure I see what do from here, honestly—maybe I am missing something. But what does come to my mind is. . . . Does this help us make progress?” And if we are simply not sure how to answer their questions or how to suggest a path forward, we may tell them this, and say that we should find a way to make progress together.

Since we are often reacting to others’ actions in parenting and IBL, both activities require us to be sensitive to body language. When we perceive discomfort or confusion, we work to identify the source and address the problem. With parenting, we are naturally inclined to be aware of, to interpret, and to respond to a child’s body language, largely because they are learning how to express their needs in words. When a child is quiet or angry and we are confused about why, we may ask them what is wrong. If they do not respond,

we may offer possible reasons (“Are you hungry?” “Are you angry at me for doing...?” etc.), until we can identify what they need.

In teaching, since we see our students for a limited amount of time and we do not interact with them as intimately as with our children, we may have to be particularly sensitive to body language. For example, blank looks on students’ faces when we ask whether they understand may need to be addressed. It may be that they are confused but do not feel comfortable saying so or cannot articulate why, so we work to identify issues. For example, a fellow IBL workshop participant, Sarah Bockting-Conrad suggests that we may ask students to use their thumbs to indicate their level of understanding: thumbs up means they do understand, thumbs down means they are very confused, thumbs in the middle means they are somewhat confused (personal communication, 2017). If a significant portion indicates that they are very or somewhat confused, then we can talk more about the topic by asking specific questions, such as whether they would like to review an idea or look at another example.

Finally, active listening is helpful in both IBL and parenting. With IBL, an instructor is often responding directly to what students say, so the instructor must listen to and fully understand what they are saying. One method is to listen without interrupting, and if needed, ask questions and rephrase to check understanding. I also do this with my children. For example, I may ask them how their school day went. They often need to search for the words to describe this, so I wait patiently as they do so or ask them about specific activities, such as what they had for lunch or whether they went outside. If they tell me that they played on the playground, then I ask them who they played with, what they played, etc.

## **6. Differences Between IBL and Parenting**

The ways IBL and parenting differ include the level of instructor/parent intervention, student/child resistance, and depth of connection. We describe these briefly.

First, a parent intervenes and gives direct guidance more often than an IBL instructor. For example, when helping a child hang up their coat, the parent may guide the child’s hands with their own if the child is having difficulty. However, in an IBL classroom, the instructor’s aim is to help the students

create their own knowledge and to help students learn to validate ideas. Thus, if a student is having difficulty, the instructor will often first allow time for classmates to offer suggestions. If progress is yet to be made, then the instructor may ask guiding questions, instead of providing a solution.

Second, instructors may have to manage student resistance to IBL. Sources of resistance include reluctance to share ideas with peers and instructor due to fear of criticism and frustration due to being stuck on a problem. (An instructor can use the ideas in Section 5 to create a supportive environment to manage students' fear of criticism and suggestions in Section 3.3 to manage frustration due to being stuck.) Parents may also have to handle children's resistance to doing things on their own, but I believe that resistance from students can be a more significant issue.

Finally, there are deep and complicated emotional bonds and, often, biological urges that push parents to raise their children in the most effective way that they can. While many instructors care deeply about their students' learning and work very hard to be effective, the relationship between an instructor and student is simpler and biological urges are certainly not present.

## 7. Concluding remarks

Although there are ways in which IBL instruction and parenting differ, we have discovered several ways in which they are comparable. One goal of discussing these similarities has been to clarify the many aspects that comprise parenting and IBL. Another goal is for the similarities to show that IBL is a natural and accessible type of instruction.

IBL is doable for everyone, regardless of experience or the courses one teaches. There are many resources for learning how to use it and a large community that supports instructors (the AIBL and the IBL Special Interest Group of the Mathematical Association of America are two examples in North America). Furthermore, preparing to teach with IBL is similar to preparing any new course, but it is a method that can be applied beyond a single subject and to all of one's teaching.

Personally, I have found my work with IBL to be quite fulfilling. It is difficult to separate how much parenting has helped me understand the above parallels from how much IBL has helped me understand them. However, since



becoming a parent and teaching with IBL, I recognize the value of mistakes, more comfortably admit my own errors, and have noticed a decrease in my own anxiety about mathematics or not knowing an answer. I have shifted my focus outwards, and as a result, I am more sensitive to others and respond more thoughtfully. At the same time, I am more honest in my responses since I feel more confident that I can be constructive. Breaking down mathematical concepts into accessible, more manageable tasks has helped me better understand the material that I teach.

I plan to learn about and teach with IBL methods indefinitely, in all of my courses. I am particularly interested in using IBL methods in introductory courses, such as intermediate algebra and calculus. Additionally, I would like to explore research in child development regarding growth mindset, making mistakes, learning to get unstuck, and responding to children's body language and difficult questions. It will be illuminating to see how this research relates to IBL instruction.

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## A. Sample IBL materials

The following excerpt is from Morrow’s IBL notes *Introduction to Abstract Algebra* [12], available for free through the *Journal of Inquiry Based Learning in Mathematics*. The excerpt begins with the definition of a group, asks the reader to work on exercises applying this definition, then poses exercises that involve proofs.

The notes contain definitions and comments, but the majority of the text consists of exercises to be worked by the reader (the items with Arabic numerals). The exercises begin with number 5. Problems 1-4 help students understand the definition of a binary operation and are not included here. An exercise labelled “(ex)” provides an example that the reader uses to understand a definition or theorem. Since (ex) exercises are more straightforward, students may compare solutions to them in small groups and deliver presentations on more complex exercises, ones that involve proofs. Additionally, there are footnotes that contain comments for the instructor that are not included in the student version.

**Definition:** A *group* is a set  $G$  together with a binary operation  $*$  on  $G$  satisfying the following:

- I. The operation  $*$  is associative.
- II. There is an element in  $G$  which is an identity for  $*$ .
- III. Every element in  $G$  has an inverse with respect to  $*$  in  $G$ .

We denote the group by  $\langle G, * \rangle$ .

We refer to the set  $G$  as the *underlying set* of the group  $\langle G, * \rangle$ . (However if the specific operation is clear from the context, or is not important in the context, we sometimes simply write  $G$  instead of  $\langle G, * \rangle$  for the group, and speak of “the group  $G$ ”.)

Note that we assume all the familiar properties of the operations on  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$  and  $\mathbb{C}$ , and you may use these freely in all that follows.

5. (ex) Which of the following are groups? If not, explain why not.
  - (a)  $\langle \mathbb{Z}, + \rangle$ .
  - (b)  $\langle \mathbb{Z}, - \rangle$ .
  - (c)  $\langle \mathbb{Z}, \times \rangle$ .
  - (d)  $\langle \mathbb{Z}, \div \rangle$ .
  - (e)  $\langle \mathbb{R}^+, \times \rangle$ . ( $\mathbb{R}^+$  denotes the set of positive real numbers.)
  - (f) The set of symmetries of a regular pentagon with operation composition.
  - (g)  $\mathbb{Z}_6$  with operation addition mod 6.
  - (h)  $\mathbb{Z}_6$  with operation multiplication mod 6.
  - (i)  $\mathbb{Z}_6 \setminus \{0\}$  with operation multiplication mod 6.
  - (j)  $\mathbb{Z}_5 \setminus \{0\}$  with operation multiplication mod 5.

6. Prove that the following is, or is not a group, as appropriate.  
The set  $S = \mathbb{R} \setminus \{1\}$  with operation defined by  $a * b = a + b - ab$  for all  $a$  and  $b$  in  $S$ . (On the right side of the equation, the operations are the usual addition and multiplication in  $\mathbb{R}$ .)
7. Prove that the following is, or is not a group, as appropriate.  
The set  $M_2(\mathbb{R})$  of all 2 by 2 matrices, with real numbers as entries, and operation matrix multiplication.

**Definitions:**

- A group  $\langle G, * \rangle$  is said to be *abelian* if  $*$  is commutative.
  - We say a group is *finite* if the underlying set contains finitely many elements. We say a group is infinite if the underlying set contains infinitely many elements.
  - For a finite group  $G$ , the *order of  $G$*  is the number of elements in  $G$ .
8. (ex) Provide at least two examples of abelian groups.
  9. (ex) Refer back to question 5. Identify the finite groups in that question, and for each of these state the order of the group.
  10. Provide at least two examples of non-abelian groups. For one of these, prove that the group is non-abelian.
  11. Suppose  $\langle G, * \rangle$  is a group, with  $s, t$  and  $u$  in  $G$ . Prove or disprove as appropriate: If  $s * t = u * s$ , then  $t = u$ .

**More Notation:** For convenience, instead of using “ $*$ ” to denote the group operation, we often use multiplicative notation as follows:

- In place of  $a * b$  write  $ab$ .
- Denote an inverse of  $a$  (the existence of which is ensured by the group axioms), by  $a^{-1}$ .
- Let  $a^1$  denote  $a$ , and for  $n \in \mathbb{N}$ , with  $n > 1$  define  $a^n$  to be  $aa^{n-1}$ .

It is important to note that we have simply introduced some notation; the operation “multiplication” in a group is NOT in general familiar old multiplication. Take care when working in an arbitrary group not to take for granted properties of exponents that are familiar from working with the real numbers. So for example, in the next two problems you may *not* assume that  $a^m a^n = a^{m+n}$ , nor that  $(a^m)^n = a^{mn}$ .

12. If  $a \in G$  and  $n \in \mathbb{N}$  then both  $(a^n)^{-1}$  and  $(a^{-1})^n$  have unambiguous interpretations in terms of the definitions already given. Prove that these two are in fact equal.
13. Prove that if  $G$  is a group, with  $a$  in  $G$ , then  $(a^{-1})^{-1} = a$ .

You have shown in problem 12 that  $(a^n)^{-1}$  and  $(a^{-1})^n$  have unambiguous meanings, and are in fact equal. The symbol  $a^{-n}$  on the other hand is not automatically defined by the definitions already given. It is convenient to define  $a^{-n}$  as simply another notation for  $(a^n)^{-1}$  and  $(a^{-1})^n$ :

**Definition:** In a group  $G$  with  $a \in G$ , we define  $a^{-n}$  to be  $(a^n)^{-1}$ . Also we define  $a^0$  to be the identity,  $e$ . As we said, we cannot simply assume that exponents will have the same properties in an arbitrary group as they do when working with real numbers. Some familiar properties of exponents for real numbers are in fact false in certain groups. The next two problems establish two basic principles that *do* apply in an arbitrary group.<sup>3</sup>

14. Suppose  $G$  is a group, with  $a \in G$ . Prove that  $a^m a^n = a^{m+n}$  for all integers  $m$  and  $n$ .
15. Suppose  $G$  is a group, with  $a \in G$ . Prove that  $(a^m)^n = a^{mn}$  for all integers  $m$  and  $n$ .
16. Suppose  $G$  is a group, with  $a, b$  and  $x$  in  $G$ . If  $x = a^{-1}b$ , can we conclude that  $xa = b$ ? Either prove this conclusion true, or provide a counterexample.
17. Suppose  $G$  is a group, with  $a$  and  $b$  in  $G$ . Prove that if  $ab = e$ , then  $ba = e$ . Use this to prove that if  $G$  is a group, with  $a$  and  $b$  in  $G$  and  $ab = e$ , then  $a$  is the inverse of  $b$ .
18. Prove or disprove, as appropriate: In a group, inverses are unique. (More precisely: if  $G$  is a group, and if  $a \in G$ , then there is a unique inverse for  $a$  in  $G$ .)
19. Prove or disprove, as appropriate: Suppose  $G$  is a group, with  $a, b$  and  $c$  in  $G$ . If  $ac = bc$ , then  $a = b$ .<sup>4</sup>

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<sup>3</sup>For time considerations the instructor might tell students to skip the following two problems.

<sup>4</sup>Once students have completed this problem, I introduce the term “right cancellation”, and comment that “left cancellation” is also valid.

20. Prove or disprove, as appropriate: If  $G$  is a group, with  $a$  and  $b$  in  $G$ , then  $(ab)^2 = a^2b^2$ .
21. Prove or disprove, as appropriate: If  $G$  is a group, with  $a$  and  $b$  in  $G$ , then  $(ab)^{-1} = a^{-1}b^{-1}$ .
22. Prove or disprove, as appropriate: If  $G$  is a group, with  $a$  and  $b$  in  $G$ , then  $(ab)^{-1} = b^{-1}a^{-1}$ .

Historically, the central focus of abstract algebra was the solution of equations. The following problem gives an indication of the connection:

23. Suppose  $G$  is group, with  $a$  and  $b$  in  $G$ . Consider the equation  $ax = b$ .
  - (a) Prove that  $a^{-1}b \in G$ .
  - (b) Prove by substituting that  $x = a^{-1}b$  is a solution for the equation.
  - (c) Prove that  $x = a^{-1}b$  is the *only* solution for the equation  $ax = b$ ; that is, this solution is *unique*.<sup>5</sup>

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<sup>5</sup>I used to include this question in the form “If  $G$  is a group, then for all  $a$  and  $b$  in  $G$ , there is a *unique* solution in  $G$  for the equation  $ax = b$ ”. However I have found it effective to get to the subtleties more quickly by asking it as in this problem.