

Determination of the Best Fit Probability Distribution for Annual Rainfall in Karkheh River at Iran

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Abstract—This study was designed to find the best-fit probability distribution of annual rainfall based on 50 years sample (1966-2015) in the Karkheh river basin at Iran using six probability distributions: Normal, 2-Parameter Log Normal, 3-Parameter Log Normal, Pearson Type 3, Log Pearson Type 3 and Gumbel distribution. The best fit probability distribution was selected using Stormwater Management and Design Aid (SMADA) software and based on the Residual Sum of Squares (R.S.S) between observed and estimated values. Based on the R.S.S values of fit tests, the Log Pearson Type 3 and then Pearson Type 3 distributions were found to be the best-fit probability distribution at the Jelogir Majin and Pole Zal rainfall gauging station. The annual values of expected rainfall were calculated using the best fit probability distributions and can be used by hydrologists and design engineers in future research at studied region and other region in the world.

Keywords—Log Pearson Type 3, SMADA, rainfall, Karkheh River.

I. INTRODUCTION

HYDROLOGISTS are not able to determine the time of hydrological phenomena occurrence such as rainfall in the nature but they can investigate previous events occurrence procedure and obtain their mean probability of occurrence. Calculation of hydrological phenomena such as rainfall or flood at different return periods can help to solve many problems in hydrology and to lessen the damage caused by them. Study on the past data of hydrological phenomena such as rainfall will help us to obtain some statistical parameters and then predicted future events in nature. The probability distributions are used in different fields of science in our life such as agricultural and engineering sciences.

Osati et al. reported that Pearson and log Pearson distributions were the best fit probability distribution for annual rainfall of Mazandaran and Golestan provinces in north of Iran [8]. Pearson type 3 and Log Pearson type 3 distributions were suitable for annual, seasonal and monthly precipitation in Japan studied by Sheng and Michio [9]. Waylen et al. found that the normal distribution was the best fit probability distribution for annual rainfall in Costa Rica [11]. Abdullah and Al-Mazroui offered that the normal distribution was the best fit probability distribution for annual rainfall 35 Region of the southwestern Saudi Arabia [3].

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Mohamed and Ibrahim found that Normal distribution is the best fit distribution for annual rainfall in Sudan country [7]. The predicting of rainfall values in futures using probability distributions have been studied by several researchers in the world [1], [2], [4]-[6], [10].

The main objective for this study is the analysis of the probability distribution of the annual rainfall in different stations across Karkheh river basin at Iran. Different probability distributions are compared in this paper to fit two rainfall gauge stations in Iran. Two rainfall gauge stations in Karkheh river basin in west of the Iran, located in the central and southern regions of the Zagros mountain range and its area is more than 50000 km², were selected with annually rainfall series during the period 1966 to 2015 in Karkheh River (Jelogir Majin and Pole Zal hydrometric stations which is upstream of Karkheh dam reservoir). These stations were shown in Fig. 1 with numbers 9 and 10. The data for these years were taken from Iran Water Resources Management Company (IWRMC).

II. METHODS

Basic concepts in statistics and probabilistic are probability distributions. The results of statistical experiments and their probabilities of occurrence are connected with probability distributions. Rainfall data from Karkheh river basin at Iran were evaluated with six probability distribution models to find the best fit model. At the first and before modelling of series, we need to have checks adequacy, accuracy and relevance conditions. The probability distribution models used include the normal (N), 2-parameter Log Normal (LN2), 3-parameter Log Normal (LN3), Pearson type III (P3), log-Pearson type III (LP3) and Gumbel (EVI) probability distributions.

The best fit probability distribution was selected using SMADA software and based on the R.S.S between observed and estimated values. The R.S.S value for each distribution calculates by (1).

$$R.S.S = \sqrt{\frac{\sum(Q_e - Q_o)^2}{n - m}} \quad (1)$$

Observed values, forecasted values, the numbers of data and the numbers of distribution parameter in above equation are Q_e and Q_o , n and m , respectively. The value of m is 2 for three distributions N, LN2, GEI, and it is 3 for P3, LP3, LN3 distributions.

III. RESULT AND DISCUSSION

After checking of our studied rainfall data using run test and water resources council (WRC) tests, we found that all data have adequacy, accuracy and relevance conditions and also there is no outlier data. Therefore, these data are suitable for estimation and predicting. The plotting of the forecasted rainfall against return period is presented in Figs. 2-5. As in Tables I-IV, the R.S.S values of this series for six common probability distributions can be seen. In these tables, Q_2 , Q_3 , etc. show the amount of rainfall with different return periods. According to Figs. 2-5 and Tables I-IV, P3 and LP3 have the maximum fitting for annual maximum and mean rainfall in two stations studied, Jelogir Majin and Pole Zal stations, the minimum R.S.S value for annual maximum and mean precipitation are 1.90, 1.74, 1.69, 1.53, 1.71, 1.69, 1.35, and 1.09, respectively. So, they are the best distribution for estimating rainfall data in studied region.

IV. CONCLUSION

In this study, annual rainfall data of Karkheh River at

Jelogir Majin and Pole Zal stations was plotted against their hydrologic years and six probability distributions. The expected values of rainfall estimates calculated using the best fit probability distributions at the rainfall gauging stations might be used by engineers to design of safely hydrologic projects in region. Also, these calculations and predicting may be used to influence decisions relating to local economics and hydrologic safety systems. Performances of the probability distributions were assessed using the R.S.S and absolute differences between predicted and observed data. The following conclusions were drawn from the study:

- The LP3 and P3 distributions had the highest fit for annual mean and maximum rainfall in Jelogir Majin and Pole Zal gauging stations. So, the lowest R.S.S values for annual maximum and mean rainfall are 1.90, 1.74, 1.69, 1.53, 1.71, 1.69, 1.35, and 1.09, respectively.
- The LP3 distributions are acceptable distribution types for representing statistics of rainfall in Karkheh River at Iran with the P3 distribution as a potential alternative.

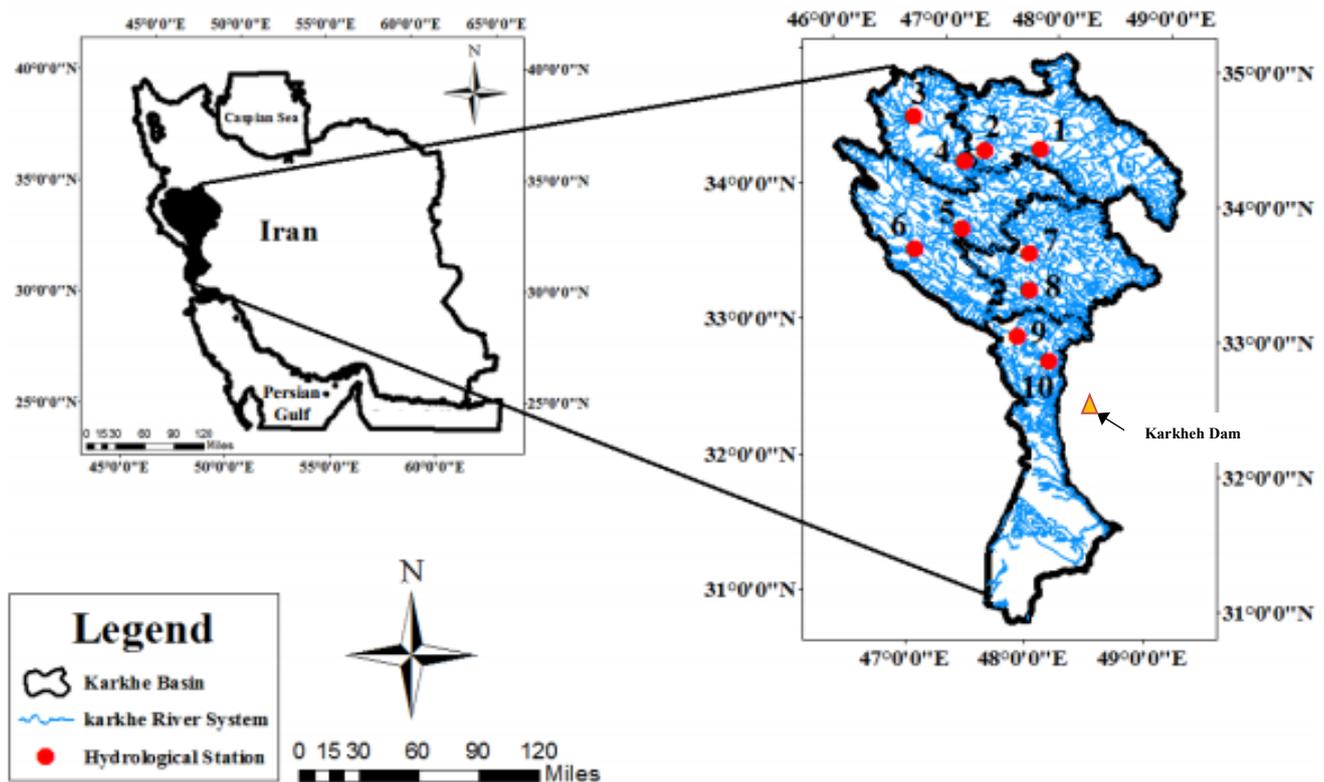


Fig. 1 The study area position

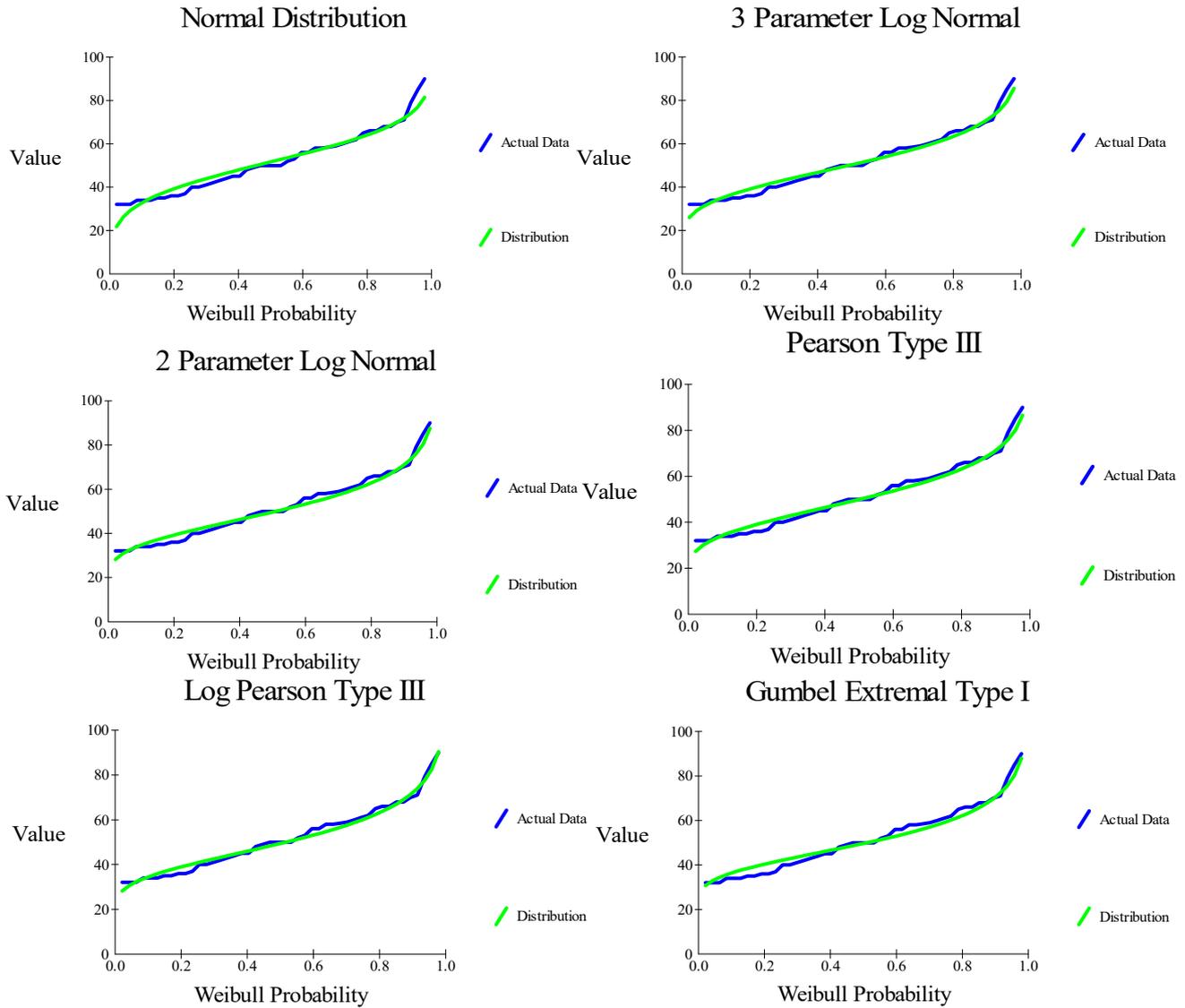


Fig. 2 Observational and predicted annual maximum precipitation values of distributions by SMADA in Jelogir Majin station

TABLE I
 ANNUAL MAXIMUM PRECIPITATION WITH DIFFERENT RETURN PERIOD (MM) IN JELOGIR MAJIN STATION

Return Period (year)	Probability Distribution					
	Normal	2 Parameters Log Normal	3 parameters Log Normal	Pearson Type III	Log Pearson Type III	Gumbel Extremal Type I
Q ₂₀₀	89.53	101.96	97.55	99.35	107.31	104.12
Q ₁₀₀	85.87	95.12	91.94	93.37	99.24	96.43
Q ₅₀	81.86	88.15	86.10	87.15	91.19	88.72
Q ₂₅	77.40	80.99	79.93	80.60	83.08	80.94
Q ₁₀	70.50	71.04	71.04	71.22	72.05	70.47
Q ₅	64.02	62.82	63.36	63.21	63.18	62.17
Q ₃	57.98	56.02	56.75	56.41	55.99	55.59
Q ₂	51.64	49.67	50.32	49.92	49.42	49.65
RSS	2.93	1.93	2.07	1.90	1.74	2.35

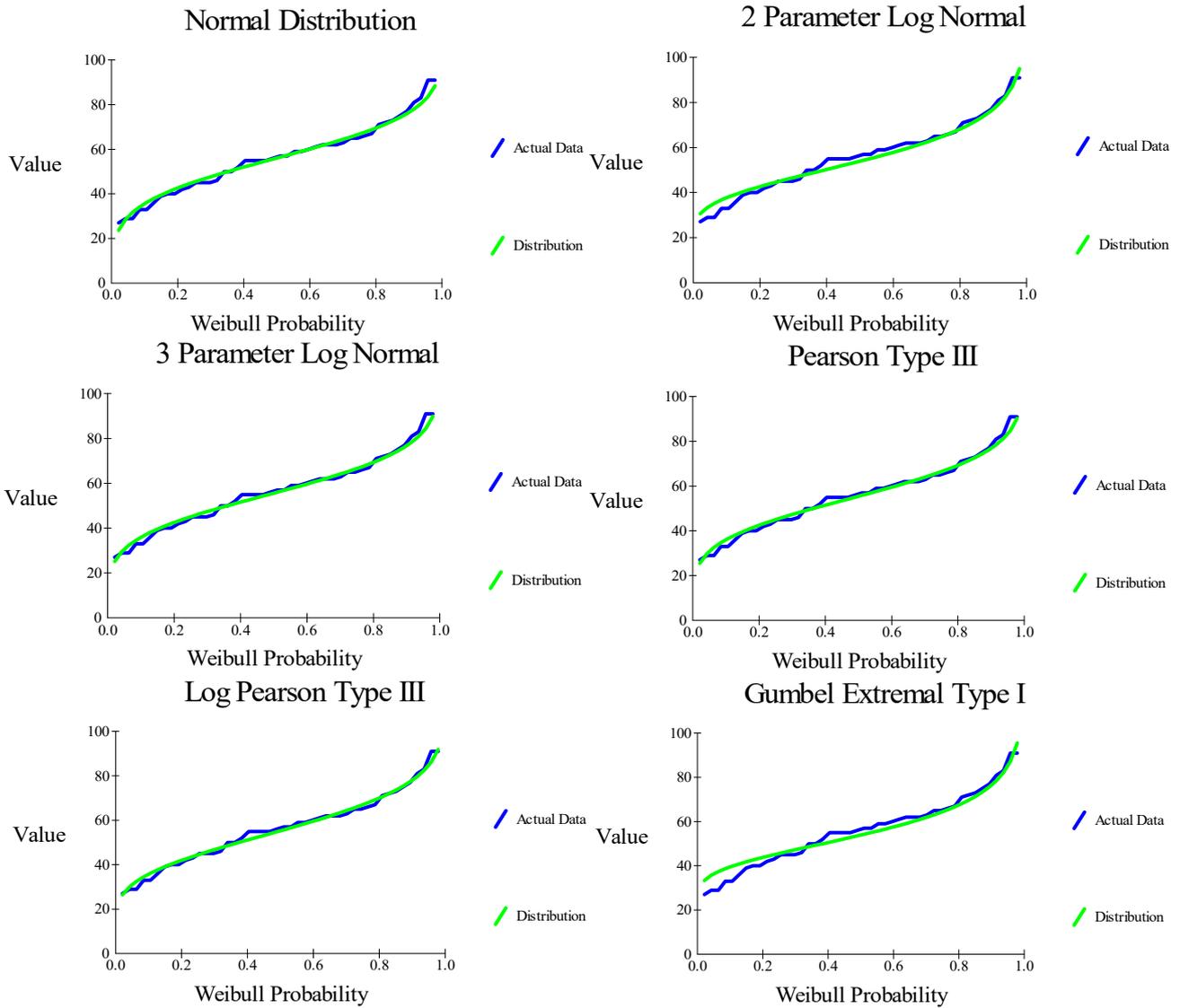


Fig. 3 Observational and predicted annual maximum precipitation values of distributions by SMADA in Pole Zal station

TABLE II
 ANNUAL MAXIMUM PRECIPITATION WITH DIFFERENT RETURN PERIOD (MM) IN POLE ZAL STATION

Return Period (year)	Probability Distribution					
	Normal	2 Parameters Log Normal	3 parameters Log Normal	Pearson Type III	Log Pearson Type III	Gumbel Extremal Type I
Q ₂₀₀	97.17	110.66	99.84	100.50	101.78	112.99
Q ₁₀₀	93.19	103.22	95.26	95.78	97.27	104.65
Q ₅₀	88.84	95.66	90.33	90.71	92.28	96.28
Q ₂₅	84.00	87.90	84.94	85.18	86.69	87.85
Q ₁₀	76.52	77.11	76.78	76.85	78.02	76.49
Q ₅	69.49	68.20	69.33	69.29	69.98	67.49
Q ₃	62.95	60.82	62.55	62.67	62.67	60.35
Q ₂	56.07	53.92	55.60	55.48	55.26	53.91
RSS	1.81	2.38	1.79	1.69	1.53	3.07

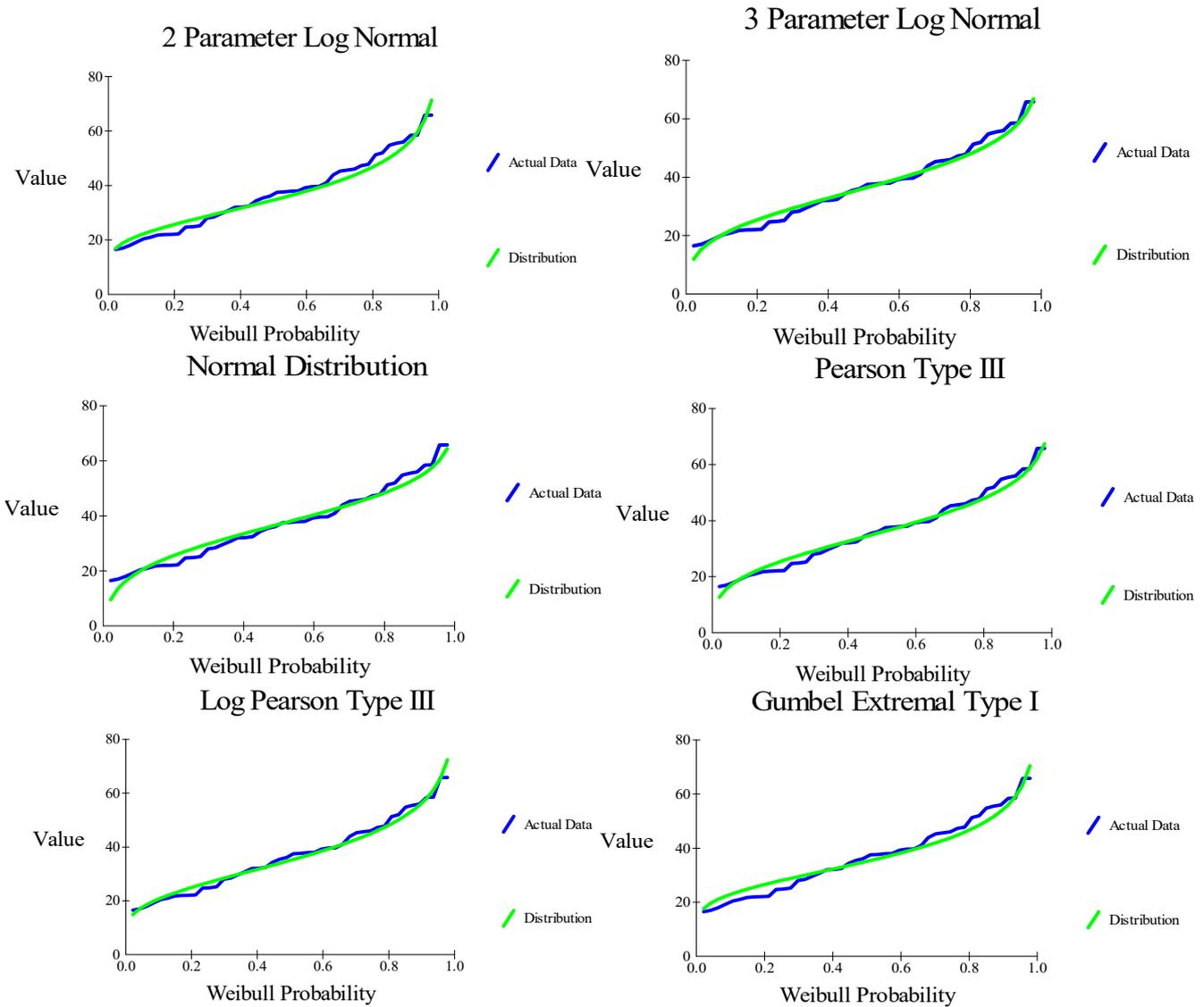


Fig. 4 Observational and predicted annual mean precipitation values of distributions by SMADA in Jelogir Majin station

TABLE III
 ANNUAL MEAN DISCHARGE WITH DIFFERENT RETURN PERIOD (M³/S) IN JELOGIR MAJIN STATION

Return Period (year)	Probability Distribution					
	Normal	2-parameter Log Normal	3-parameter Log Normal	Pearson Type III	Log Pearson Type III	Gumbel Extremal Type I
Q ₂₀₀	71.82	86.58	76.42	77.50	86.29	85.25
Q ₁₀₀	68.44	79.24	71.97	72.82	79.77	78.17
Q ₅₀	64.75	71.92	67.26	67.88	73.06	71.08
Q ₂₅	60.65	64.58	62.19	62.59	66.08	63.91
Q ₁₀	54.29	54.67	54.68	54.80	56.27	54.27
Q ₅	48.33	46.75	48.00	47.93	48.09	46.63
Q ₃	42.77	40.41	42.08	41.90	41.32	40.57
Q ₂	36.93	34.68	36.15	35.93	35.03	35.10
RSS	2.16	2.21	1.98	1.71	1.69	2.44

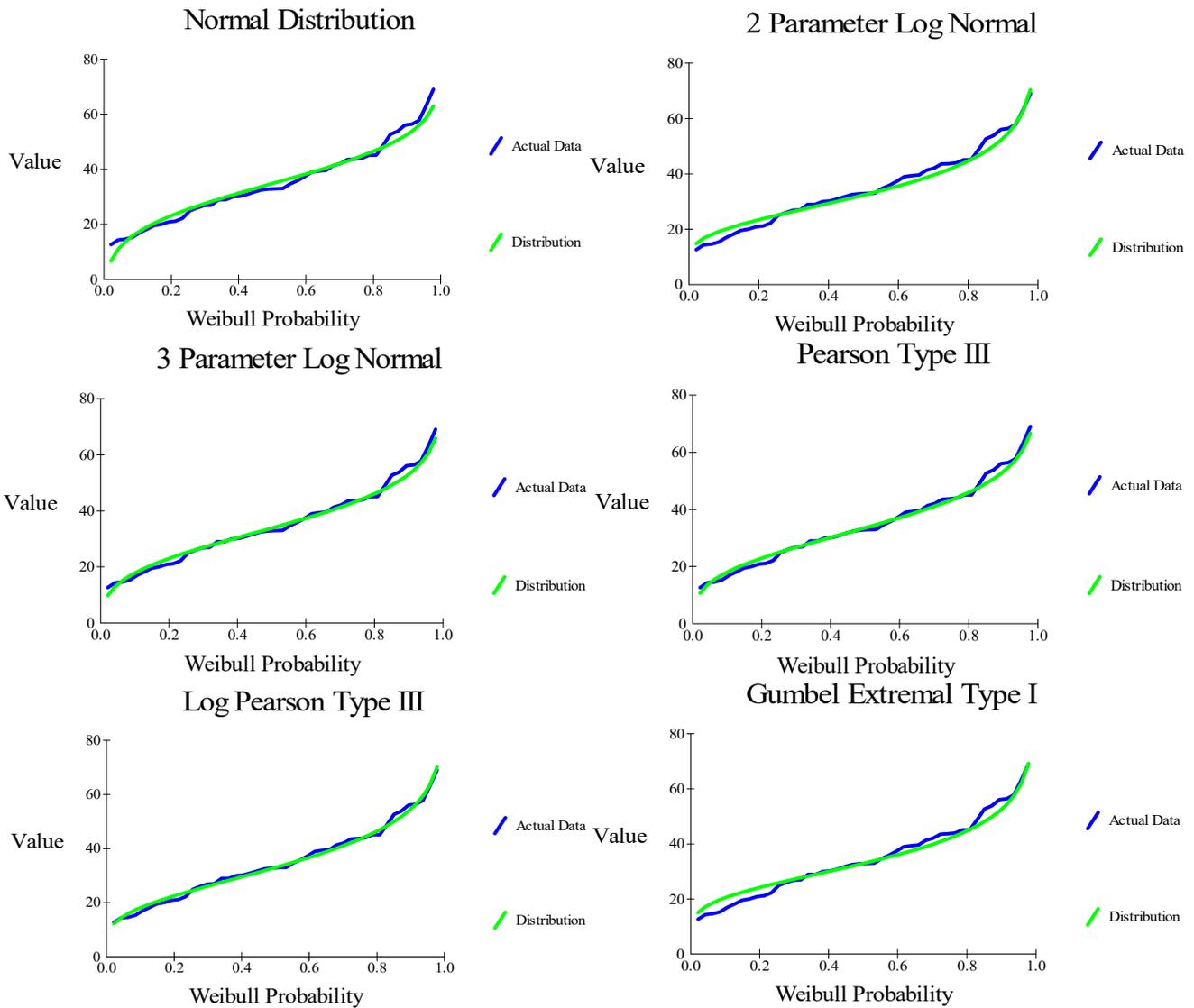


Fig. 5 Observational and predicted annual Mean Precipitation values of distributions by SMADA in Pole Zal station

TABLE IV
 ANNUAL MEAN PRECIPITATION WITH DIFFERENT RETURN PERIOD (M³/S) IN POLE ZAL STATION

Return Period (year)	Probability Distribution					
	Normal	2 Parameters Log Normal	3 parameters Log Normal	Pearson Type III	Log Pearson Type III	Gumbel Extremal Type I
Q ₂₀₀	70.52	86.91	76.26	77.58	82.81	84.27
Q ₁₀₀	67.06	78.97	71.45	72.49	76.93	77.02
Q ₅₀	63.28	71.12	66.38	67.14	70.72	69.75
Q ₂₅	59.08	63.31	60.96	61.45	64.12	62.42
Q ₁₀	52.57	52.87	53.02	53.16	54.57	52.54
Q ₅	46.46	44.65	46.03	45.93	46.39	44.73
Q ₃	40.78	38.14	39.90	39.67	39.49	38.52
Q ₂	34.80	32.32	33.83	33.55	32.99	32.92
RSS	2.03	1.96	1.45	1.35	1.09	2.09

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