

# Determination of Small Shear Modulus of Clayey Sand Using Bender Element Test

R. Sadeghzadegan, S. A. Naeini, A. Mirzaii

**Abstract**—In this article, the results of a series of carefully conducted laboratory test program were represented to determine the small strain shear modulus of sand mixed with a range of kaolinite including zero to 30%. This was experimentally achieved using a triaxial cell equipped with bender element. Results indicate that small shear modulus tends to increase, while clay content decreases and effective confining pressure increases. The exponent of stress in the power model regression analysis was not sensitive to the amount of clay content for all sand clay mixtures, while coefficient A was directly affected by change in clay content.

**Keywords**—Small shear modulus, bender element test, plastic fines, sand.

## I. INTRODUCTION

SHEAR modulus and damping ratio are among the important parameters that are widely used to evaluate the dynamic behavior of soil-structure systems. It is well known that seismic properties of soils are non-linear and vary appreciably with the amplitude of shear strain. Many researchers have marginally studied the strain dependency of these parameters in sands [1]-[3], clays [4]-[6] and mixed materials such as clayey sands [7].

The maximum shear modulus of soil occurs at very small shear strain. According to Fig. 1, the maximum shear modulus of soil is almost constant in very small strains.

Assuming that behavior of soil is linear at small strain level,  $G_0$  (small strain shear modulus) can be defined from the shear wave velocity as:

$$G_0 = \rho V_s^2 \quad (1)$$

where  $V_s$  and  $\rho$  are the shear wave velocity and density, respectively.

According to (1), small strain shear modulus ( $G_0$ ) can be computed from the shear wave velocity that can be measured by using of bender element test. Bender element (BE) testing is non-destructive and has become a standard method for the determination of the small-strain shear modulus. The test has been used extensively on laboratory samples due to its

reliability for small strain shear modulus calculation.

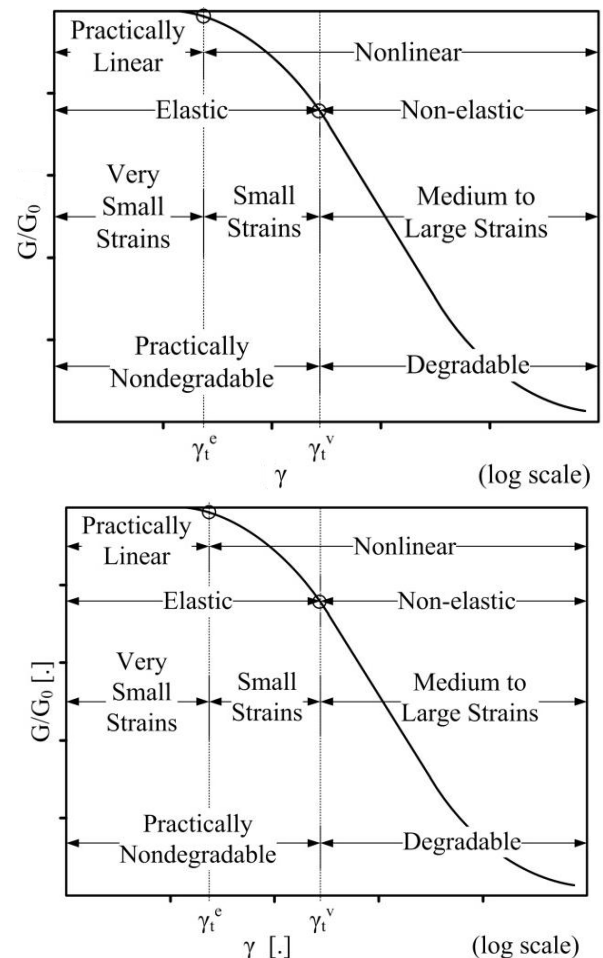


Fig. 1 Modulus reduction curve

The  $G_0$  of pure sands has been extensively investigated so far [8]–[10]. Moreover, a large number of researches have been done on small-strain shear modulus of clays [11]–[14]; however, small-strain behavior of mixed material such as clayey sand has been less noticed. In the other words, sands usually contain fine content which affects shear modulus of mixture. Lack of this kind of research is more obvious for sand with plastic fine compared to silty sands. Iwasaki and Tatsuka [15] found that small strain shear modulus of sand decreases quickly with increase in fine content for a constant void ratio.

Wichtmann et al. [16] carried out a series of resonant column tests to investigate the effect of fine content on the shear modulus of quartz sand. They found that, by increasing

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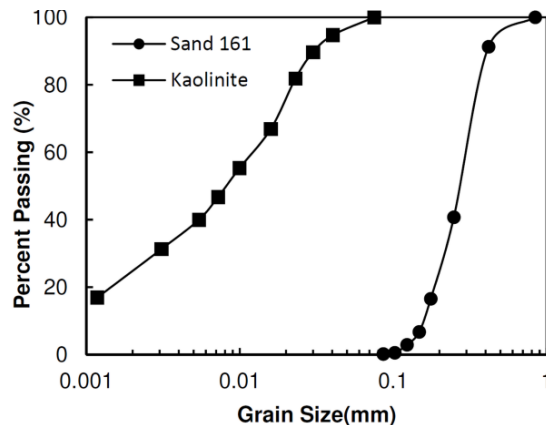
fine content up to 10%, the small-strain shear modulus,  $G_0$ , of the tested material decreased, and remained constant as the fine content further increased up to 20%. Utilizing bender element technique, Salgado et al. [17] showed a continuous reduction of small-strain modulus by increasing non plastic fine content up to 20%. They reported that for sands with 20% silt,  $G_0$  decreased 53% at relative density of 50% and confining pressure of 100 kPa. However, the effects of plastic fine content and their plasticity on the small shear modulus of sand have been less noticed.

Carraro et al. [18] conducted a series of bender element tests on the sand mixed with silt and Kaoline clay. They showed that  $G_0$  was affected by both the amount and plasticity of fine contents. They also found that, at similar relative density, small shear modulus of sand-clay mixture is generally higher than sand-silt mixture.

As discussed, despite the occurrence of many earthquake damages in sandy soils including fine contents, there is still a need of further research to determine the influence of clay content on the variation of small strain shear modulus of sands. This study represents the results of a laboratory testing program aimed to examine the effect of clay content on the small strain shear modulus of sand.

## II. MATERIAL AND TEST PROCEDURES

### A. Soil



(a)



(b)

Fig. 2 (a) Particle size distribution curves, and (b) microscopic images of tested materials

In the present study, Firozkooh sand No.161 was used as the base soil and kaolinite as the fine content. Firozkooh sand No.161 is uniform quartz sand with sub-angular to sub-rounded grains. Firozkooh sand was mixed with various

amount of kaolinite (i.e. 0%, 10%, 20%, and 30% by weight) to investigate the effect of plastic fine content on cyclic characteristics of sand-kaolinite mixtures. TABLE I tabulates the index properties of sand-kaolinite mixtures, and Fig. 2 shows the sieve analysis of each soil and microscopic images of tested materials.

TABLE I  
INDEX PROPERTIES OF SAND-KAOLINITE MIXTURE

Kaolinite content (%)	0	10	20	30
Specific gravity, $G_s$	2.673	2.665	2.643	2.626
Maximum void ratio, $e_{max}$	0.951	1.053	1.184	1.335
Minimum void ratio, $e_{min}$	0.563	0.467	0.393	0.399
Liquid limit (%)	-	12	17	22
Plasticity index (%)	N.P	N.P	5	11
Percentage <75 $\mu$ m	0.5	11	20	30
Unified classification	SP	SP-SC	SC	SC

### B. Triaxial Apparatus

A triaxial cell equipped with bender element device was utilized in this study, as illustrated in Fig. 3. The triaxial cell was equipped with a set of bender element receiver and transmitter located at bottom and top pedestals respectively. In each BE (i.e. bender element) test, a single sinusoidal signal was generated by an arbitrary function generator (AFG, 2005) at various frequencies, and the received signal was collected by a digital oscilloscope (Rigol, 1052E). The bender element system performance was calibrated using different dimensions of aluminum bars and system delays such as travel time in cable in order to determine the response time of the test setup [19].

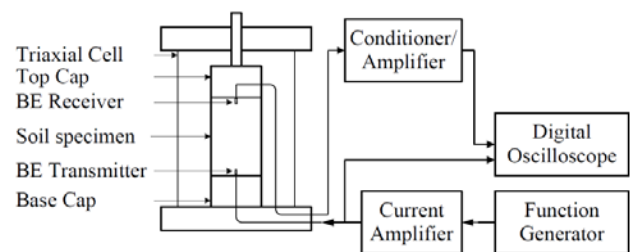


Fig. 3 Layout of modified triaxial apparatus

### C. Sample Preparation and Test Procedure

By using dry deposition method, cylindrical specimens, 50 mm in diameter and 100 mm in height, were prepared. Oven-dried sands were mixed by 0, 10, 20 and 30% of kaolinite. The samples were prepared in five equal layers, and zero falling head was used to prevent particle separation. To control the specified relative density of about 50%, the mold was hitted in a symmetrical pattern, as necessary. Initially, all samples were fully saturated by passing  $\text{CO}_2$ , de-aired water and applying a back pressure of 200 kPa for 2-3 hrs. A Skempton's B-value of 0.97 and larger was reached within this process. After saturation, specimens were consolidated under confining pressures of 50, 100, 200, 400, and 500 kPa. After the consolidation stage, the shear wave velocities of the samples were measured at various frequencies (i.e. from 1 to 20 kHz) of sinusoidal signals.

### III. TEST RESULTS

THE RESULTS OF SHEAR VELOCITY MEASUREMENTS FOR SATURATED CONDITION ARE SUMMARIZED IN

TABLE II. Fig. 4 shows an example of transmitted single sinusoidal wave and the received response. The travel time of the shear wave can be measured by means of several methods including first arrival, cross correlation, peak to peak, phase detection analysis, or wave length analysis [20]–[23].

As shown in Fig. 4, the first arrival time of the received wave appeared to be a suitable method for understudying soil and was used in calculations. Bender element tests were performed at various frequencies (from 1 to 20 kHz) of sinusoidal signals to examine which frequency resulted in a clear received signal. According to Fig. 4 for a clean sand and sand with 30% of clay, the results appear to demonstrate that transmission of waves with a frequency of 3 kHz led to a clear received shear wave. This was evident in all the samples with different fine contents considered in this study and appeared to be in good agreement with the observations of other researchers in the past [19], [24].

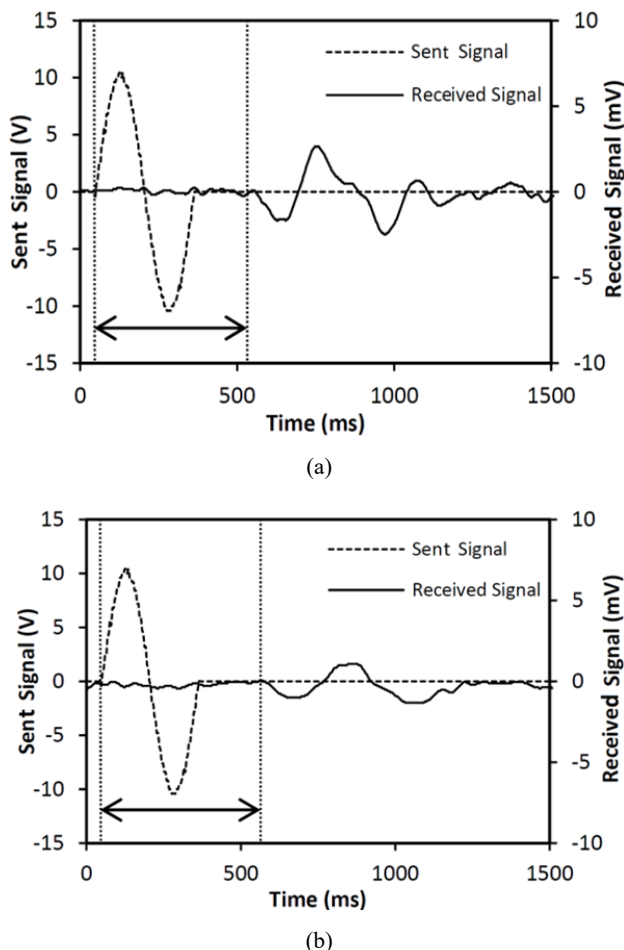


Fig. 4 Example of received shear wave in (a) clean sand and (b) sand with 30% clay

According to the results and amount of available data in the literature, the void ratio and effective confining stress are among the key affecting factors on the small-strain shear modulus in saturated soils. Various empirical equations were developed for  $G_0$  based on these two factors [25], [26] usually given in the form of:

$$G_0 = A \cdot F(e) \cdot \left(\frac{\sigma'}{P_a}\right)^n \quad (2)$$

where  $A$  and  $n$  are the fitting parameters,  $F(e)$  is the void ratio function,  $\sigma'$  is the effective stress, and  $P_a$  is the reference pressure and is often assumed to be equal to atmospheric pressure.

Various functions of void ratio for  $F(e)$  were suggested in the literature, among the widely used equations is [15], [27]:

$$F(e) = \frac{(2.17-e)^2}{1+e} \quad (3)$$

Equation (3) was used to calculate the corresponding values of  $F(e)$  for saturated tests results. Fig. 6 plots the variation of  $G_0/F(e)$  versus clay content and appears to show that the ratio of  $G_0/F(e)$  is almost linearly decreased with increasing clay content. This trend was evident for the range of effective confining pressures considered.

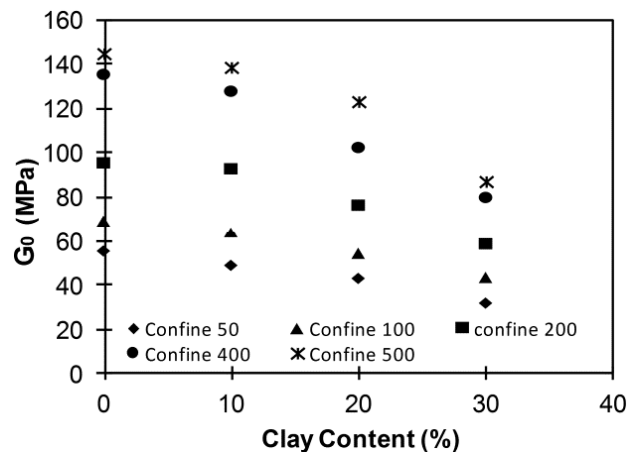


Fig. 5 Variation of  $G_0$  versus clay content in saturated tests

The fitting parameters  $A$  and  $n$  in (2) were determined with the plot of  $G_0/F(e)$  versus normalized effective confining pressure (i.e.  $\sigma'/P_a$ ) as illustrated in Fig. 7. It was observed that, at a given normalized effective confining pressure, the ratio of  $G_0/F(e)$  decreased by increasing the amount of fine content. The parameters  $A$  and  $n$  were best fitted according to the data in Fig. 7 for each of the sand-clay mixtures and were summarized in TABLE III. Accordingly, it was found that the parameter  $n$  was almost insensitive to the changes in clay content and had an approximate value of 0.42, while the parameter  $A$  was decreased within the increase of clay content.

TABLE II  
BENDER ELEMENT TESTS RESULTS IN SATURATED CONDITION

Confining Pressure (kPa)	50			100			200			400			500		
Kaolinite content (%)	e	$V_s$ (m/s)	$G_0$ (MPa)	e	$V_s$ (m/s)	$G_0$ (MPa)	e	$V_s$ (m/s)	$G_0$ (MPa)	e	$V_s$ (m/s)	$G_0$ (MPa)	e	$V_s$ (m/s)	$G_0$ (MPa)
0	0.761	191	55.4	0.759	214	69.6	0.756	251	95.9	0.749	298	135.7	0.745	308	145.3
10	0.767	182	48.9	0.763	206	64.1	0.758	248	93.2	0.749	290	128.1	0.747	302	139.1
20	0.798	171	43.0	0.794	192	54.3	0.788	228	76.8	0.778	263	102.8	0.772	288	123.7
30	0.879	151	31.9	0.873	176	43.4	0.864	204	58.7	0.852	238	80.3	0.841	248	87.8

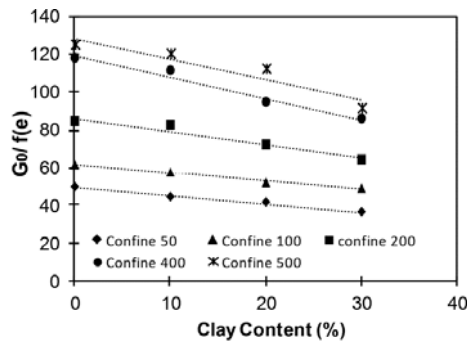


Fig. 6 Variation of  $G_0/F(e)$  versus clay content in saturated tests

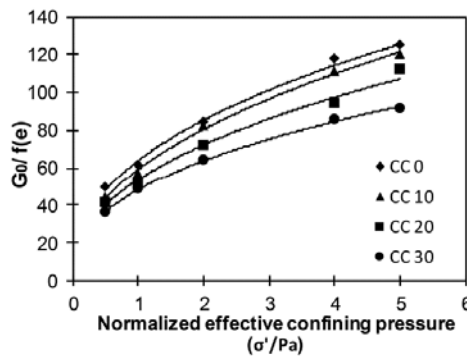


Fig. 7 Variation of  $G_0/F(e)$  versus effective confining pressure in saturated state

TABLE III  
VALUES OF PARAMETERS A AND N IN ALL SAND-CLAY MIXTURES

Clay Content (%)	A	n
0	63.7	0.41
10	58.4	0.44
20	51.7	0.43
30	48.0	0.42

#### IV. CONCLUSION

A series of bender elements tests were conducted on saturated isotropically consolidated specimen of clean sand and sand containing different amount of plastic fine content, ranges from 0 to 30%. The effects of clay content and effective confining pressure were both studied on the small strain shear modulus of sand.

The small shear modulus tended to decrease while the quantity of clay increased from Bender Element tests. The exponent of stress in the power model regression analysis was not sensitive to the amount of clay content for all sand clay

mixtures, while coefficient A was directly affected by change in clay content.

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