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Trajectory of asteroid 2017 SB20 within the CRTBP

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Abstract. Regular monitoring the trajectory of asteroids to a future time is a necessity, because the variety of known probably unsafe near-Earth asteroids are increasing. The analysis is perform to avoid any incident or whether they would have a further future threat to the Earth or not. Recently a new Near Earth Asteroid (2017 SB20) has been observed to cross the Earth orbit. In view of this we obtain the trajectory of Asteroid in the circular restricted three body problem with radiation pressure and oblateness. We examine nature of Asteroid's orbit with Lyapunov Characteristic Exponents (LCEs) over a finite intervals of time. LCE of the system confirms that the motion of asteroid is chaotic in nature. With the effect of radiation pressure and oblateness the length of curve varies in both the planes. Oblateness factor is found to be more perturbative than radiation pressure. To see the precision of result obtain from numerical integration we show the error propagation and the numerical stability is assured around the singularity by applying regularized equations of motion for precise long-term study.

Keywords. Celestial mechanics—CRTBP—radiation—oblateness—asteroid—trajectory—chaos.

1. Introduction

In recent years, the study of Near Earth Objects (NEOs) is a most celebrated problem among the researchers. NEOs may be an asteroid or comet whose trajectory intersects the orbit of the Earth. The importance arises as Near Earth Asteroids (NEAs) are of very small size that fly by near the Earth and has the potential to collide or a close approach to the Earth that they may represent a potential impact threat. It is well known that the motions of most of the bodies including the NEOs in the planetary system are basically of chaotic nature (Knežević 1996). Recent papers on different aspects and motions of small bodies including NEAs or comets such as Morais & Namouni (2013) studied the motions of an asteroids which is an retrograde resonance with Jupiter and Saturn. Galushina *et al.* (2017) studied the dynamics of an asteroids - companions to Venus.

Nowadays small bodies mission are under concept studies of space agencies such as the National Aeronautics and Space Administration (NASA), European Space Agency (ESA) and Japan Aerospace Exploration Agency (JAXA). Over the many years several dynamical systems consisting of two body, few bodies or n-body problem have been investigated and proposed in order to understand and explain the orbital behavior of realistic celestial systems. Among the n-body problem the simplest and most extensively studied model is the restricted three-body problem (RTBP) (Szebehely 1967). To obtain the approximate solution for the sets of equations numerical techniques are being used. Abouelmagd *et al.* (2014) has applied Lie-Series to integrate the system of equations in the Earth–Moon system.

Lyapunov Characteristic Exponents (LCEs) is a basic technique, to understand the behavior of a dynamical system (Benettin *et al.* 1980; Wolf *et al.* 1985; Skokos 2010). The chaotic or regular nature of an orbit is indicated by the value of maximal LCE. It measures the rate of exponential divergence of perturbed initial conditions with respect to each other, in terms of some convenient metrics in the phase space of state vectors Kneževic & Ninkovic (2005). The nature of orbit is regular if the LCE is always zero. The larger the value of LCE, the greater the rate of exponential divergence which indicates the chaotic nature of orbits for long intervals of time.

In this paper, we consider the Sun–Earth–Asteroid system with radiation pressure and oblateness. We choose 2017 SB20 asteroid as an infinitesimal mass as it was close approached to the Earth on October 11, 2017. The initial conditions for the numerical integration of the model are taken from JPL on a particular epoch of close approach of asteroid 2017 SB20. We obtain trajectories in the physical as well as in regularized plane using Levi-Civita transformation. The LCE confirms that the motions are of chaotic nature in both the planes. Oblateness coefficient considered in the model and has a significant impact compared to the radiation pressure on the traced trajectory of NEOs. To check the accuracy of the computations use the error propagation which is obtained under control in case of physical plane whereas in regularized plane the error increases with time. It confirms that the regularized equations are useful for numerical integration when the infinitesimal mass is nearer to the primaries. Otherwise the trajectory computed using the governing equations of motion throughout the computation than trajectories will continue after singularity, because the Runge-Kutta integrator jump above the singularity point which affects the accuracy of the results. So the regularized equations should be used near singularities.

The paper is organized as follows: In Section 2 we formulate a dynamical model in the planar circular restricted three-body planetary system accounting radiation pressure of the bigger primary (the Sun) and oblateness of the Earth. Section 3 describes the LC-regularization of the dynamical model. In Section 4, we provide the numerical propagation of local error of the system during computations. Section 5 concludes the research work.

2. Dynamical model

We consider the Sun, the Earth, and an infinitesimal mass i.e. an Asteroid (2017 SB20) in the planar circular restricted three-body problem (CRTBP). We consider the bigger primary, as a source of radiation and smaller primary as oblate spheroid. Both the primaries are moving in the circular orbits under the gravitational influence of each other around the common center of mass. The infinitesimal body is moving in the plane defined by two moving bodies without disturbing their motions. Since the Sun is a source of radiation, the resultant force of the Sun on the infinitesimal mass is obtained using the solar data as (Martyusheva *et al.* 2015). The constant values of the Sun used in calculation of radiation pressure are solar mass ($M_\odot = 1.989 \times 10^{30}$ kg); radius ($R_\odot = 695700$ km); luminosity ($L_\odot = 3.86 \times 10^{26}$ W); solar constant ($S_\odot = 1.372 \times 10^3$ W m^{-2}) and Radius of the Earth ($r = 6371$ km) and velocity of light ($c = 2.998 \times 10^5$ km s^{-1}).

This force acts straight along heliocentric line-vector of asteroid and adversely to the Sun gravitation:

$$F = F_\odot^{grav}(1 - \beta), \tag{1}$$

where

$$\beta = \left(\frac{F_\odot^{rad}}{F_\odot^{grav}}\right) = \left(\frac{L_\odot}{4\pi c}\right)\frac{1}{GM_\odot}\frac{A}{m}, \tag{2}$$

is the radiation factor, $F_\odot^{rad} = \frac{L_\odot}{4\pi r^2 c}A$, $F_\odot^{grav} = \frac{GM_\odot m}{r^2}$ and m is the mass of an asteroid. Now following the assumption of restricted three-body problem in the rotating system, the equations of motion in the rotating coordinate system, in non-dimensional form, can be written as Tiwary & Kushvah (2015):

$$\ddot{x} - 2n\dot{y} = \frac{\partial U}{\partial x}, \tag{3}$$

$$\ddot{y} + 2n\dot{x} = \frac{\partial U}{\partial y}, \tag{4}$$

$$U = n^2\left(\frac{(x^2 + y^2)}{2}\right) + \frac{(1-\mu)(1-\beta)}{r_1} + \frac{\mu}{r_2} + \frac{\mu A_2}{2r_2^3}, \tag{5}$$

where U is the pseudo-potential, $\mu = 3.00351 \times 10^{-6}$ is the mass parameter for the Sun–Earth system, $n = \sqrt{1 + \frac{3A_2}{2}}$ is the perturbed mean motion, $A_2 = 0.0034$ is the oblateness coefficient of the Earth and r_1 and r_2 are the position vectors of the infinitesimal mass from the larger and smaller primaries, respectively. The explicit formulation and discussions of n and A_2 can be referred in Srivastava *et al.* (2017).

Now, we discuss the dynamics of the Asteroid (2017 SB20) in the frame of CRTBP which fly by the Earth harmlessly on October 11, 2017 at 8.9 times distance to the Moon, at a speed of 7.2 km/s. The initial data for position and velocity are taken from the JPL website (on October 11, 2017). We transform the orbital data obtained from the webpage in the frame of CRTBP and obtain the initial conditions to integrate the equations of motion.

$$\begin{aligned} x_0 &= 0.994641225299951, \\ y_0 &= 1.920100100283277 \times 10^{-2}, \\ \dot{x}_0 &= -0.1931500636749737, \\ \dot{y}_0 &= -0.1095432875923946. \end{aligned} \tag{6}$$

The trajectory of the Asteroid on Integrating equations (3-4) with the above initial values for a time span of 45 years we obtain the trajectory of the asteroid (as

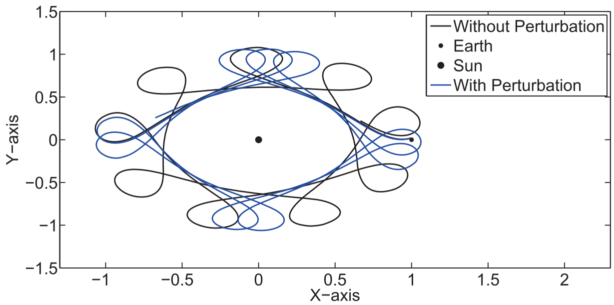

Figure 1. Trajectory of an Asteroid 2017 SB20 in the circular restricted three body problem with time span upto 45 years.

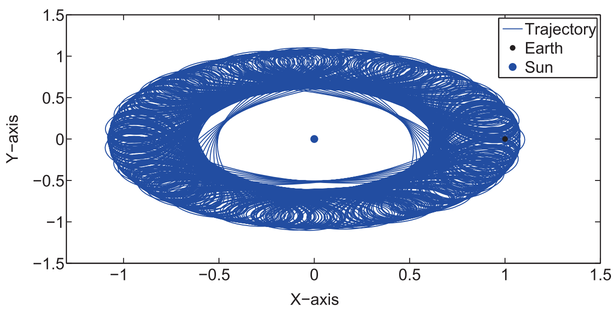

Figure 2. Trajectory of the Asteroid 2017 SB20.

Table 1. Effect of radiation pressure and oblateness on the trajectory in the physical plane for 45 years.

β	A_2	Length of trajectory (au)
0	0.0000	18.28639554
0	0.0034	19.08977951
1×10^{-9}	0.0000	18.28639549
1×10^{-9}	0.0034	19.08977950
5×10^{-9}	0.0000	18.28639527
5×10^{-9}	0.0034	19.08977946
1×10^{-8}	0.0000	18.28639500
1×10^{-8}	0.0034	19.08977941
5×10^{-8}	0.0000	18.28639283
5×10^{-8}	0.0034	19.08977900
1×10^{-7}	0.0000	18.28639011
1×10^{-7}	0.0034	19.08977848
5×10^{-7}	0.0000	18.28636840
5×10^{-7}	0.0034	19.08977435

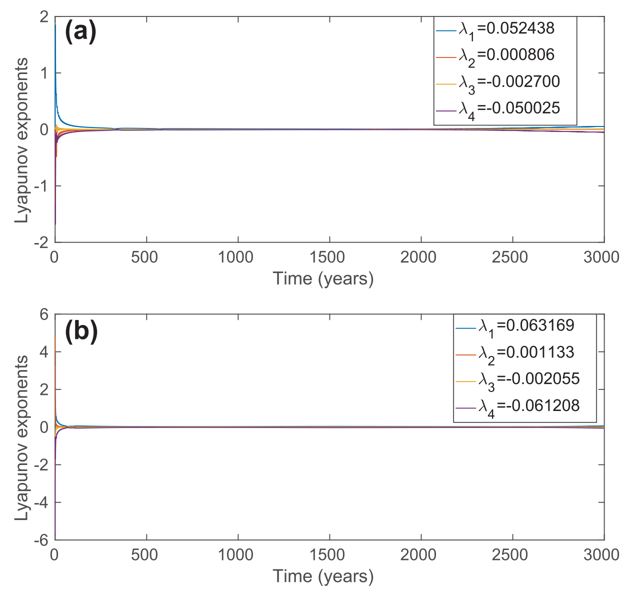

Figure 3. Lyapunov exponents (a) in classical model (b) with perturbation.

depicted in Fig. 1). Here we consider small time scale to analyze the differences in the trajectories obtain in physical and regularized plane. In Fig. 1, the curves with black colour depicts the trajectory in classical model and blue curve shows the trajectory with the effect of radiation pressure ($\beta = 5 \times 10{-}7$) and oblateness of the Earth ($A_2 = 0.0034$). The nature of trajectory remains same but the length of curve increases with perturbation. In Fig. 2 the trajectory for longer time span of 3000 years are also shown which proves that the motion is of chaotic nature. It also shows that the motion of the asteroid is bounded as we see in the zero-velocity curve (ZVC). We show the effect of radiation pressure in tabular form (Table 1). The radiation pressure of an asteroid depends on the ratio of its area and mass $\left(\frac{A}{m}\right)$. To see the influence of radiation pressure, we choose the value of β from 1×10^{-9} to 5×10^{-7} (Martyusheva *et al.* 2015). With the increasing value of radiation pressure the length of trajectory covered in the same duration is decreasing. The chaotic nature of orbits are confirmed on computation of LCE using the method well described in Wolf *et al.* (1985). We show the maximum LCE in Fig. 3 in which frame labels (a) is for classical case and (b) is for model with perturbation respectively. We use following initial values for the computation of trajectory and LCE. Different colours in the figures indicate the maximal Lyapunov exponents values. With the effect of radiation pressure and oblateness the chaotic values increases, corresponding values are indicated in the figures.

3. Levi-Civita regularization

In this section, Levi-Civita (LC) regularization is applied on the dynamical model discussed in Section 2

This regularization is important during numerical computation of trajectory when it is close proximity to the primaries. Indeed, a collision between any two objects is marked by the fact that their distance becomes zero. The problem of singularities plays an important role under conceptual, computational and physical aspects. During close encounters the velocity of small body increases and to compensate for the infinite increase of the velocity during close approaches the regularization theory is introduced. In this regard Levi-Civita regularization was first time introduced for removing the collisions in the two body problem Levi-Civita (1904). Later on many researchers (Érdi 2004; Celletti *et al.* 2011; Lega *et al.* 2011; Roman & Szücs-Csillik 2014) applied this transformation to the restricted three-body problem by translating the origin of the coordinate system to one of the primaries. It is based on suitable change of coordinates with the introduction of fictitious time (Celletti *et al.* 2011).

The regularization methods play important role in the analytic treatment of collision trajectories applied for the long term studies of the motion of the celestial bodies (Waldvogel 2006; Celletti *et al.* 2011; Roman & Szücs-Csillik 2012).

For the regularization of the governing equations (3–4) of CRTBP we consider the location of primaries at $(-\mu, 0)$ and $(1-\mu, 0)$. Here we apply the transformations around the smaller primaries with radiation effect in the model. The major orbital perturbations of the infinitesimal mass in CRTBP occurs when it encounters around the smaller primary. The change of coordinates for removing the singularities around the Earth are introduced by the parametric coordinates (u, v) through the expression

$$x = u^2 - v^2 + 1 - \mu, \tag{7}$$

$$y = 2uv. \tag{8}$$

We define regularized time s in the regularized plane which is related to the ordinary time t through the expression

$$dt = R ds, \tag{9}$$

$$R = u^2 + v^2 = \sqrt{(x - 1 + \mu)^2 + y^2} \tag{10}$$

The dynamical model given in equation (3–4) are transformed by considering the above transformations. In this regard thoroughly explanation of the transformation from the governing equations of motion to the regularized equations of motion may be seen in Celletti *et al.* (2011). We obtain the regularized equations of motion including the radiation pressure term (β) and oblateness coefficient (A_2) as

$$u'' = \left[\frac{1}{R}\left(u' + v' - \frac{\mu}{2} - \frac{3\mu A_2}{4R^2}\right) + \frac{R}{2}\left(n^2(R + 1 - \mu) - \frac{(1-\mu)(1-\beta)(R+1)}{r_1^3}\right)\right] u + 2nRv', \tag{11}$$

$$v'' = \left[\frac{1}{R}\left(u' + v' - \frac{\mu}{2} - \frac{3\mu A_2}{4R^2}\right) + \frac{R}{2}\left(n^2(R - 1 + \mu) - \frac{(1-\mu)(1-\beta)(R-1)}{r_1^3}\right)\right] v - 2nRu', \tag{12}$$

where the $'$ symbol denotes the differentiation with respect to regularized time s and $r_1 = \sqrt{(u^2 - v^2 + 1)^2 + (2uv)^2}$. The initial conditions given in physical plane are accordingly transformed in the regularized plane using the transformation formulation (Lega *et al.* 2011). If $x < 1-\mu$, then the values of transformed coordinates are $u = \frac{y}{2v}$ and $v = \sqrt{\frac{R-(x-1+\mu)}{2}}$, or if $x \geq 1 - \mu$, then $u = \sqrt{\frac{R+(x-1+\mu)}{2}}$ and $v = \frac{y}{2u}$. The velocity component transforms as $u' = \frac{1}{2}(u\dot{x} + v\dot{y})$ and $v' = \frac{1}{2}(u\dot{y} - v\dot{x})$.

$$\begin{aligned} u_0 &= 0.0853761907344269, \\ v_0 &= 0.1124493892129707, \\ u'_0 &= -0.0144042462294134, \\ v'_0 &= 0.006183609035766365. \end{aligned} \tag{13}$$

The trajectories of an asteroid in the regularized plane for shorter and longer period are shown in Figs. 4 and 5 respectively. The black and blue curves, show the trajectories of the asteroid within regularized CRTBP classical model and with perturbations respectively. With the inclusion of oblateness, the trajectory deviates when it comes near to the Earth. From these trajectories, we observe that the nature of motions of an infinitesimal mass remains same as in the physical plane. The trajectory depicted in Fig. 5 resembles with the zero velocity curve in regularized case (Szebehely 1967).

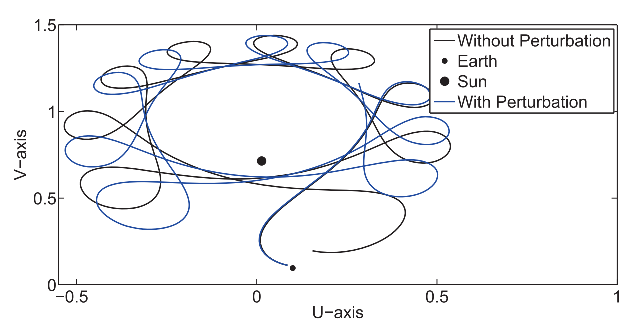

Figure 4. Trajectory of an Asteroid 2017 SB20 in the regularized plane with time span upto 45 years.

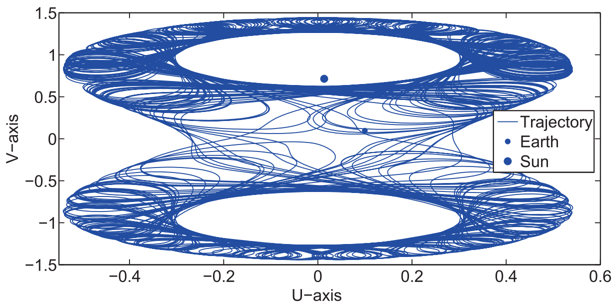

Figure 5. Trajectory of an Asteroid 2017 SB20 in the regularized plane for longer time span of 3000 years.

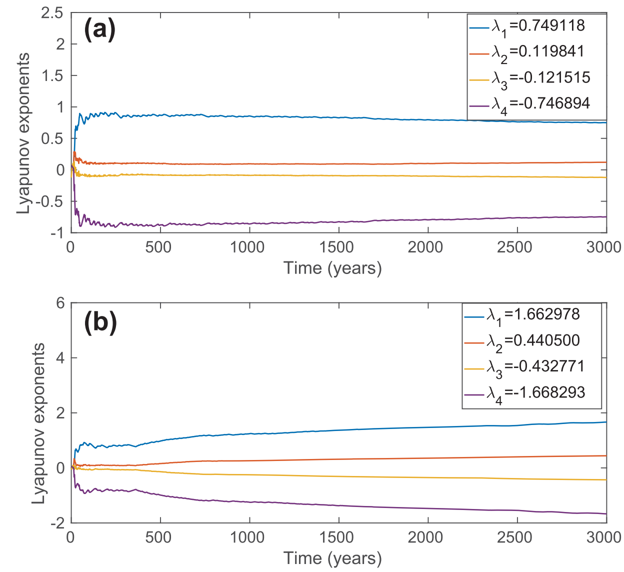

Figure 6. Lyapunov exponents using regularized equations of motion (a) without perturbation, (b) with perturbation.

Table 2. Effect of radiation pressure and oblateness on the trajectory in the regularized plane for 65 years.

β	A_2	Length of trajectory (au)
0	0.0000	9.473535081
0	0.0034	10.10390607
1×10^{-9}	0.0000	9.473535022
1×10^{-9}	0.0034	10.10390622
5×10^{-9}	0.0000	9.473534788
5×10^{-9}	0.0034	10.10390681
1×10^{-8}	0.0000	9.473534494
1×10^{-8}	0.0034	10.10390756
5×10^{-8}	0.0000	9.473532148
5×10^{-8}	0.0034	10.10391350
1×10^{-7}	0.0000	9.473529215
1×10^{-7}	0.0034	10.10392094
5×10^{-7}	0.0000	9.473505755
5×10^{-7}	0.0034	10.10398039

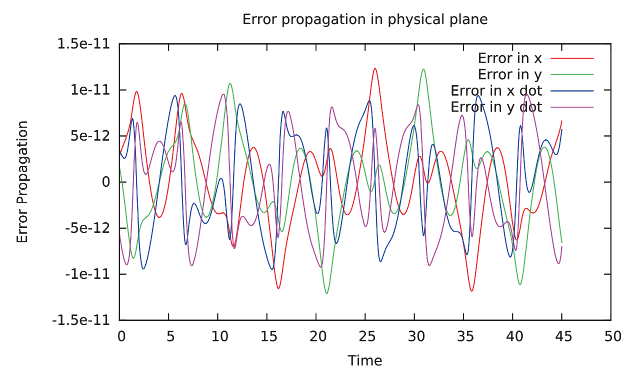

Figure 7. Error propagation in physical plane.

With the effect of radiation factor and oblateness the length of curve varies (as shown in Table 2), in the regularized case the length of curve decreases with the increase of radiation pressure whereas in case of oblateness it increases. In this case we found that the time taken to reach at the same place as in physical plane is more (65 years) due to stretching of time in regularized case. The motion will be slowed down near the singularities.

The LCE computed using the regularized equations of motion confirms that these equations are only useful during close encounters. Fig. 6 shows Lyapunov exponents without perturbation in frame (a) whereas frame (b) is with perturbation effect taken into consideration. The value of maximum Lyapunov exponents increases from the physical plane. With the perturbation effect the chaoticity increases in regularized case.

4. Error propagation

Error propagation of numerical integration for the governing equations of motion are of order 10^{-11} (Fig. 7) whereas in case of regularized equations of motion it increases as distance between infinitesimal mass and primaries are increase due to time span. In the first frame of Fig. 8 for short time the local error are approximately of order 10^{-12} but for time greater than 30 years error propagation increases as shown in second frame of Fig. 8. For time grater than 38 years it appeared large which could be due to floating point round of error. The order of error also confirms that near singularities it preferable to use regularized equations of motion which gives the precise trajectory.

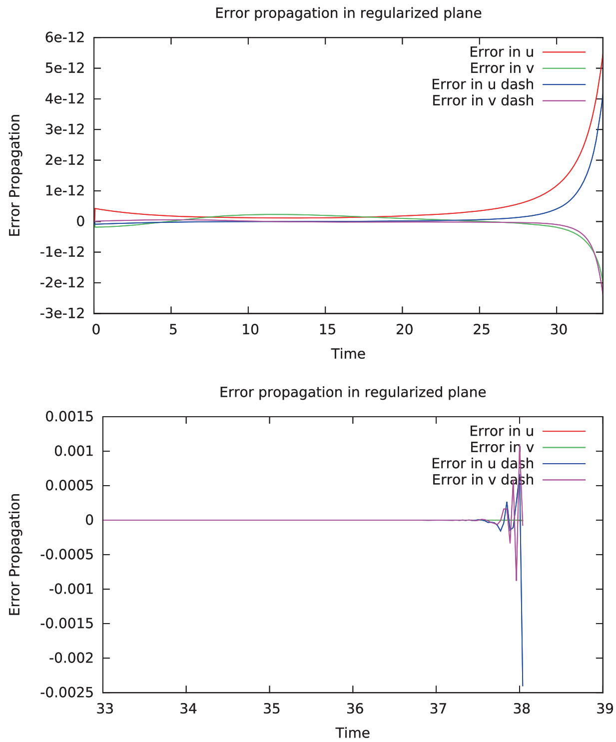

Figure 8. Error propagation in regularized plane.

5. Conclusions

In this work, we have examined the dynamics of a Near Earth Asteroid (2017 SB20) as an infinitesimal mass in the frame of circular restricted three body problem (CRTBP) in physical and regularized planes. We have numerically integrated the equations of motion and obtain the trajectory of asteroid in both the planes. We have found that the nature of orbits remains same in both the cases. The structure of the motion resembles with the zero velocity curve for longer time duration it cannot cross the energy level. Lyapunov exponents confirms the chaotic nature of orbits. With the effect of radiation pressure the trajectory traced by infinitesimal mass in both the planes reduces, whereas in case of oblateness coefficient the length of curves decreases in physical plane but increases in regularized plane. Overall from this study it is found that for the motion of NEOs the oblateness coefficient is more dominant as compare to radiation pressure. From LCE it confirms that the chaoticity in regularized case is more than that of the physical plane. Also the chaoticity increases with perturbation effects. We conclude through the error propagation of numerical integration that we cannot use the regularized equations of motion for long time as error increases. The regularized equations of motion can only be used when the infinitesimal mass is very nearer to the primary. This study may be further implied for the numerical integration of long term evolution of similar Asteroids trajectory.

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