Research on satellite selection algorithm in ship positioning based on both geometry and geometric dilution of precision contribution

Pengfei Zhang(iD)

Abstract

With the networking of four Global Navigation Satellite Systems, the combination of multi-constellation applications has become an inevitable trend, and there will be more and more visible satellites that can be participated in ship positioning. However, the computational complexity increases sharply, which greatly improves the load capacity of the receiver's data processor and reduces the output frequency of the positioning result. To achieve the balance between positioning accuracy and computational complexity, a new fast satellite selection algorithm based on both of geometry and geometric dilution of precision contribution is proposed. Firstly, this article analyzes the geometry characteristics of the least visible satellites has minimum geometric dilution of precision that meet the positioning requirements and makes clear the layout of their elevation angles and azimuth angles. In addition, it derives the relationship of geometric dilution of precision and the visible satellites layout and gets geometric dilution of precision contribution of each satellite. Finally, based on the observation data of JFNG tracking station of the Multi-Global Navigation Satellite System (GNSS) Experiment trial network, the positioning error and the elapsed time of GPS/Beidou Satellite Navigation System and GPS/Beidou Satellite Navigation System/Russian Global Orbiting Navigation Satellite System (GLOANSS) are compared. Simulation results show that the algorithm solves the problem that there are a lot of matrix multiplications and matrix inversions in the traditional satellite selection algorithm, and the new algorithm can reduce computational complexity and increase receiver processing speed.

Keywords

GNSS, geometry, GDOP contribution, satellite selection, BDS, ship positioning

Introduction

When ships are sailing at sea, precise navigation and positioning not only determine the navigation efficiency of the ships but also the safety of the ships. The four Global Navigation Satellite Systems (GNSSs) that are currently on-orbit operation include the American GPS, the Russian GLONASS, the European Galileo system, and the Chinese Beidou Satellite Navigation System (BDS).[1] With the networking of four GNSSs, the number of navigation satellites in space has been increasing, over 100 satellites will circumnavigate the earth in the future. The number of visible satellites will increase while positioning with combination of multi-GNSS, which greatly improves the load

College of Mechatronic Engineering, North University of China, Taiyuan, China

Corresponding author:
Pengfei Zhang, College of Mechatronic Engineering, North University of China, No.3 Xueyuan Road, Taiyuan, Shanxi 030051, China.
Email: zhangpf@nuc.edu.cn

capacity of the receiver's data processor and reduces the output frequency of the positioning result, thus the issue of satellite selection is particularly important. Generally, the satellite selection algorithm is to select the fixed number of satellites whose constellation layout is better according to the Geometric Dilution of Precision (GDOP) in positioning calculation. Scholars at home and abroad have made a lot of research on the method of rapidly calculating GDOP and satellite selection algorithm in the multi-GNSS positioning. One general method is deduced from the positioning equations, which is obtained by performing a series of matrix operations on the observation matrix. The precision of this closed-form calculation is high, but the computational burden increases significantly with the number of visible satellites.[2,3] In recent years, machine learning methods such as genetic algorithm and support vector machine algorithm are used to solve GDOP calculation problems,[4–6] but they all have a common problem, that is, parameter selection problem. When the parameters are chosen properly, a good result can be obtained. All of these parameters are inputs based on experience, but there is not a very proper solution to this problem. At present, there are some mature methods of satellite selection, such as the best GDOP method, the maximum tetrahedron volume method, and the maximum orthogonal projection method.[7–9] How to balance the contradiction between computation and calculation accuracy is still a research hot spot. Firstly, this article summarizes the distribution of elevation angles and azimuth angles of the best geometry based on the geometry characteristics of the best visible satellites combination. Then, considering the contribution of each visible satellite to GDOP, a fast satellite selection algorithm based on geometry and GDOP contribution is proposed. Finally, the algorithm is simulated and analyzed with the actual measured data of Jiufeng Tracking station of Wuhan Province of China (JFNG) tracking station of the Multi-GNSS Experiment (MGEX) trial network.

Relationship between GDOP and geometry for multi-GNSSs

For GNSS, the accuracy of the user's position obtained by the receiver depends on the geometry of visible satellites and the pseudo-range error. Where the geometry of that is reflected by GDOP, and the smaller the value, the higher the positioning accuracy. After the time-space coordinates are unified, the GDOP of multi-GNSSs and its observation matrix H can be expressed as[10]

$$\text{GDOP} = \sqrt{\text{trace}(H_n{}^\mathrm{T}H_n)^{-1}} \tag{1}$$

Supposing that n_i is the visible satellites number of the ith system, and the total number of visible satellites in the multi-GNSSs is

$$n = n_{\text{GPS}} + n_{\text{GLONASS}} + n_{\text{GAL}} + n_{\text{BDS}} \tag{2}$$

H_n is an $n \times 7$ dimensional matrix, and H_i represents the first three columns of the observation matrix of the ith system, where $i = $ GPS, GLONASS, GAL, BDS, thus

$$H_n = \begin{bmatrix} H_{\text{GPS}} & 1_{\text{GPS}} & 0_{\text{GPS}} & 0_{\text{GPS}} & 0_{\text{GPS}} \\ H_{\text{GLO}} & 0_{\text{GLO}} & 1_{\text{GLO}} & 0_{\text{GLO}} & 0_{\text{GLO}} \\ H_{\text{GAL}} & 0_{\text{GAL}} & 0_{\text{GAL}} & 1_{\text{GAL}} & 0_{\text{GAL}} \\ H_{\text{BDS}} & 0_{\text{BDS}} & 0_{\text{BDS}} & 0_{\text{BDS}} & 1_{\text{BDS}} \end{bmatrix} \tag{3}$$

where subscripts GPS, GLONASS, Galileo System (GAL), and BDS represents GPS, GLONASS, Galileo system, and BDS. In the last four columns of H_n, 1_i and 0_i are respectively the i-dimensional column vectors, which are used to solve four different system clock differences

$$\begin{bmatrix} H_i & 1_i \end{bmatrix} = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & 1 \\ a_{x2} & a_{y2} & a_{z2} & 1 \\ a_{x3} & a_{y3} & a_{z3} & 1 \\ a_{x4} & a_{y4} & a_{z4} & 1 \end{bmatrix} \tag{4}$$

where a_{xi}, a_{yi}, and a_{zi} are related to the elevation and azimuth angle of the visible satellite.

Assuming that position of the ith visible satellite in East–North–Up (ENU) coordinate system is (x_{Li}, y_{Li}, z_{Li}), then the elevation angle EL_i and the azimuth angle A_i can be expressed as follows

$$EL_i = \tan^{-1}\left(\frac{z_{Li}}{\sqrt{x_{Li}^2 + y_{Li}^2}} \right) \tag{5}$$

$$A_i = \tan^{-1}\left(\frac{x_{Li}}{y_{Li}} \right) \tag{6}$$

The matrix in equation (4) can be rewritten as

$$\begin{bmatrix} H_i & 1_i \end{bmatrix} = \begin{bmatrix} \cos EL_1 \sin A_1 & \cos EL_1 \cos A_1 & \cos EL_1 & 1 \\ \cos EL_2 \sin A_2 & \cos EL_2 \cos A_2 & \cos EL_2 & 1 \\ \cos EL_3 \sin A_3 & \cos EL_3 \cos A_3 & \cos EL_3 & 1 \\ \cos EL_4 \sin A_4 & \cos EL_4 \cos A_4 & \cos EL_4 & 1 \end{bmatrix} \tag{7}$$

From the formula above and the formula of GDOP calculation, as can be seen that the GDOP value is only related to the elevation angle and azimuth angle of the visible satellite, so we can combine the elevation angle, azimuth angle, and geometry of visible satellites to research on satellite selection algorithm.[11,12] In addition, from the best geometry satellite selection algorithm, it can be seen that the key problem is the GDOP calculation.

GDOP contribution

The value of GDOP varies with the number of satellites chosen for positioning calculation from visible satellites.

However, there is a certain regularity between the change of GDOP value and the number of satellites.

Assuming that H_n is an observation matrix of n satellites, let $H_n = [h_1, h_2, \cdots, h_n]^T$, the observation matrix that the ith satellite is removed from the n satellites is H_{n-1}^i, and the relationship between the two matrices is as follow[13]

$$H_n^T W_n H_n = H_{n-1}^{i}{}^T W_{n-1}^i H_{n-1}^i + h_i^T w_i h_i \qquad (8)$$

Recording $(H_n^T W_n H_n)^{-1}$ as G_n, then

$$GDOP^2 = \text{trace} G_n \qquad (9)$$

From Sherman–Morrison formula, it can be obtained as follows

$$\begin{aligned} G_{n-1}^i &= (H_{n-1}^{i}{}^T W_{n-1}^i H_{n-1}^i)^{-1} \\ &= (H_n^T W_n H_n - h_i^T w_i h_i)^{-1} \\ &= G_n + G_n h_i^T w_i (1 - h_i G_n h_i^T w_i)^{-1} h_i G_n \end{aligned} \qquad (10)$$

where $(1 - h_i G_n h_i^T w_i)$ is a scalar recorded as K_i, then

$$\begin{aligned} GDOP_{n-1}^{i}{}^2 &= \text{trace} G_{n-1}^i \\ &= \text{trace} G_n + \text{trace}(G_n h_i^T w_i h_i G_n / K_i) \\ &= GDOP_n^2 + \text{trace}(G_n h_i^T w_i h_i G_n / K_i) \end{aligned} \qquad (11)$$

From the formula (11), it can be seen that the ith satellite contribution to GDOP of all of n visible satellites is

$$\Delta G = \text{trace}(G_n h_i^T w_i h_i G_n / K_i) \qquad (12)$$

To determine the sign of $\text{trace}(G_n h_i^T w_i h_i G_n / K_i)$, making singular value decomposition of H_{n-1}^i, then

$$H_{n-1}^i = USV^T \qquad (13)$$

where U and V are orthogonal matrices of $(n-1) \times (n-1)$ and $(m+3) \times (m+3)$, respectively, and S is a diagonal matrix of $(n-1) \times (m+3)$.

Substituting formula (13) into formula (8) and making the same orthogonal variation on both sides of the formula, then using the Sherman–Morrison formula[14]

$$\begin{aligned} &\{V^T (H_n^T W_n H_n) V\}^{-1} \\ &= \left\{ V^T \left((USV^T)^T W_{n-1}^i USV^T \right) V + V^T h_i^T w_i h_i V \right\}^{-1} \\ &= (Z + v w_i v^T)^{-1} = Z^{-1} - \frac{1}{1 + v^T Z^{-1} w_i v} Z^{-1} v w_i v^T Z^{-1} \end{aligned} \qquad (14)$$

where $Z = S^T W_{n-1}^i S = \text{diag}(z_{11}, z_{22}, \cdots, z_{(m+3) \times (m+3)})$, $|v = V^T h_i^T = [v_1 \quad v_2 \quad \cdots \quad v_{(m+3)}]^T$.

Orthogonal transform does not change the trace of the matrix, so

$$\begin{aligned} &\text{trace}\left((H_{n-1}^{i}{}^T W_{n-1}^i H_{n-1}^i)^{-1} \right) = \text{trace}(Z^{-1}) \\ &= \text{trace}\left\{ \left(V^T (H_n^T W_n H_n) V \right)^{-1} \right\} + \text{trace}(k v_z w_i v_z^T) \\ &= \text{trace}\left((H_n^T W_n H_n{}^n)^{-1} \right) + \text{trace}(k v_z w_i v_z^T) \end{aligned} \qquad (15)$$

where $v_z = Z^{-1} v$, $\text{trace}(k v_z w_i v_z^T) = k w_i \sum_{i=1}^{m+3} \left(\frac{v_i}{z_{ii}} \right)^2 \equiv q > 0$, $k = 1 \left/ \left(1 + \sum_{i=1}^{m+3} \frac{v_i^2}{z_{ii}} \right) \right.$, then

$$GDOP_{n-1}^{i}{}^2 = GDOP_n^2 + q > GDOP_n^2 \qquad (16)$$

The above formula shows that GDOP decreases with the increase of the number of satellites. Therefore, in the process of positioning, if the satellite has no faults, all visible satellite should be selected as possible for positioning calculation. However, when the number of visible satellites is large, the positioning calculation burden will be heavy. And GDOP does not decrease indefinitely with the increase of the number of satellites. As the number of satellites increases to some extent, the decrease of the number of satellites is not obvious.

From formula (12), we can get the following conclusion: the less ΔG_i is, the smaller the effect of the ith satellite on $GDOP_n$ is, and the removal of this satellite has little effect on the original GDOP. Thus, we can choose the satellites whose ΔG_i is larger to get a better geometry distribution. By sorting ΔG_i, choosing some visible satellites to calculate GDOP can reduce the dimension of observation matrix and the complexity of matrix inversion calculation, and it can also improve computational efficiency. The statistical experiment shows that when the value of $\Delta G_i / GDOP_n > \lambda$ (λ is 0.15–0.20) and the ith satellite is removed,[13,14] the GDOP value of the combination varies greatly, so that we can add the terminal condition to eliminate the relative satellite through the loop.

Algorithm design

From the analysis above, it can be seen that selecting satellites based on geometry and GDOP contribution can not only increase the speed of satellite selection but also can select the combination with better geometry. The flow chart of satellite selection algorithm based on geometry and GDOP contribution is shown in Figure 1.

The general idea is as follows: firstly, select the satellites that meet the basic positioning requirements with a good geometric layout according to the elevation angles and azimuth angles, so that it can increase the initial satellite selection speed; secondly, based on GDOP contribution, the satellites whose GDOP contribution is smaller are eliminated step by step, but the positioning performance need to meet the setting criteria. Specific steps are as follows
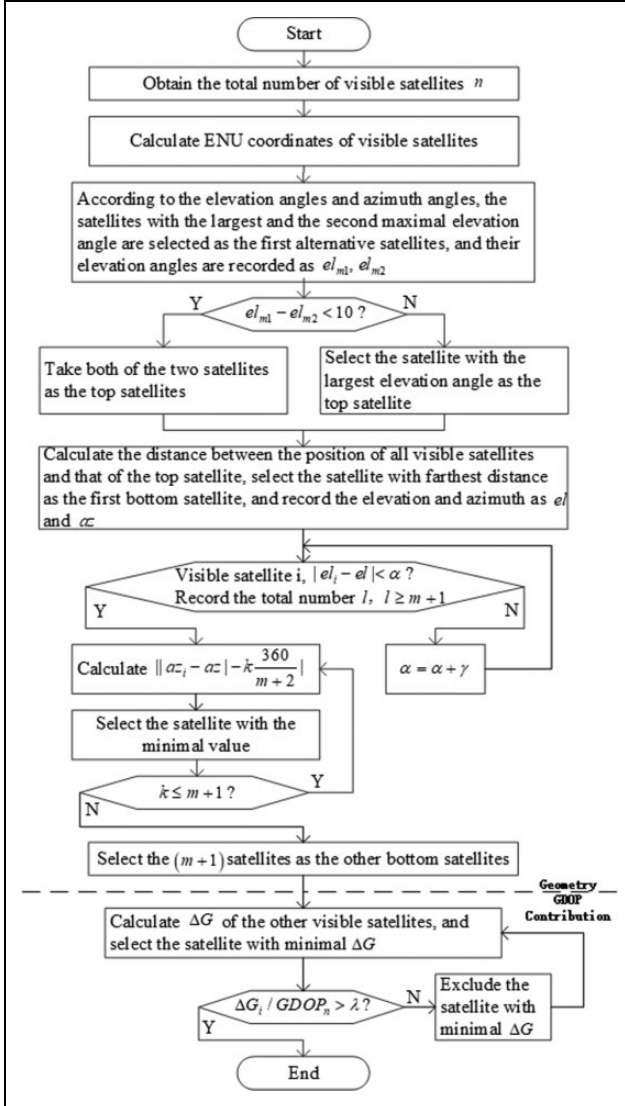
Figure 1. Flow chart of satellite selection. ENU: East-North-Up; GDOP: geometric dilution of precision.

Figure 2. Comparison of vertical positioning error before and after satellite selection based on GPS/BDS. BDS: Beidou Satellite Navigation System.

Step 1: Obtain the total number of visible satellites and calculate the position coordinates in the Earth-Centered-Earth-Fixed (ECEF) coordinate system of each satellite;

Step 2: Convert the coordinates of the visible satellites from the ECEF coordinate system to the ENU coordinate system;

Step 3: Calculate the elevation and azimuth of all visible satellites and select the satellites with largest and the second maximal elevation angle as the top satellite to be selected. Check the difference between that of the two satellites, if it is less than $10°$, select them as top satellites, if not, select only the satellite with the largest elevation angle as the top satellite[15];

Step 4: Calculate the distance between the position of all visible satellites and that of the top satellite with the largest elevation angle and select the satellite with furthest from the distance as the first bottom satellite;
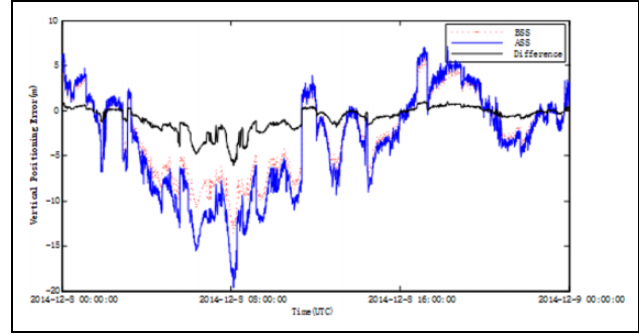
Step 5: With the exception of the selected satellites, calculate the difference of other satellites' elevation angles and that of the first bottom satellite, and the satellites whose absolute value of the difference less than α are selected. To make the geometry better, the elevation angles of the bottom satellites should be in the same range, that is, the smaller α, the better the result. Generally, α is less than $30°$. The initial value of α is $5°$, the step γ is $5°$, the total number l is recorded for each selection, if $l \geq m + 1$ (m is the number of GNSSs participating in the calculation), go to step 6, otherwise, continue to step 5 after the step α is increased;

Step 6: Calculate the difference between the azimuth angles of the visible satellites obtained in step 5 and that of the first bottom satellite and then calculate the difference between their absolute values and $k\frac{360}{m+2}$ ($k = 1 : (m + 1)$), the lowest difference satellite is selected in turn to obtain $(m + 1)$ bottom satellites. Thus, satellite selection based on geometry is to the end, the total number of selected satellites is $(m + 3)$ or $(m + 4)$, it can meet the basic requirements of multi-GNSS positioning;

Step 7: Calculate ΔG of the remaining visible satellites and then sort them according to the size of ΔG, select the satellite with the smallest value of ΔG;

Step 8: According to the determined conditions, if $\Delta G_i/\text{GDOP}_n > \lambda$ (λ is taken as 0.2 in this article), satellite selection is to the end, otherwise, exclude the visible satellite with the smallest value of ΔG, then repeat steps 7 and 8.

Simulation analysis

The observation data of JFNG tracking station of the MGEX network is selected to analyze. The data in the 342nd day of 2014 is selected. Two programs are respectively tested, GPS/BDS and GPS/BDS/GLONASS. The comparison of the positioning error and the elapsed time before and after satellite selection is made using the
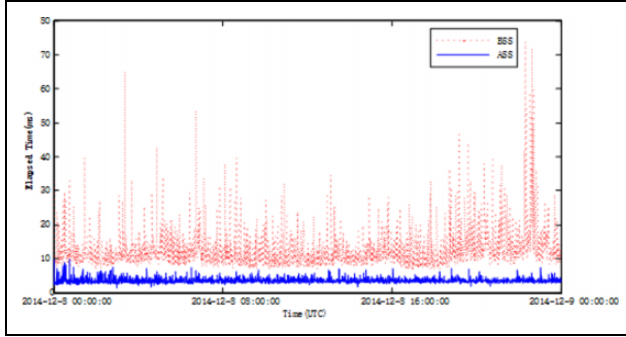
Figure 3. Comparison of elapsed time before and after satellite selection based on GPS/BDS. BDS: Beidou Satellite Navigation System.
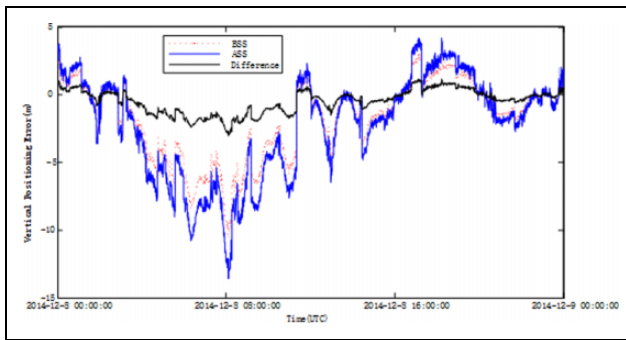

Figure 4. Comparison of vertical positioning error before and after satellite selection based on GPS/BDS/GLONASS. BDS: Beidou Satellite Navigation System.
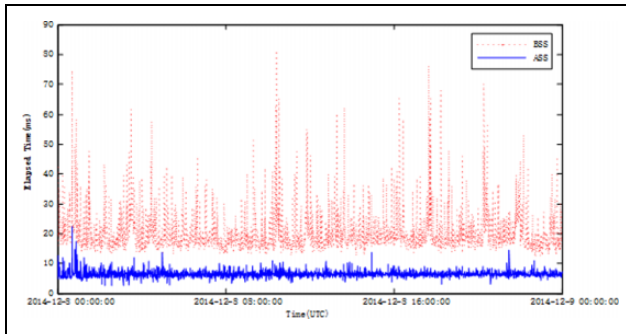

Figure 5. Comparison of elapsed time before and after satellite selection based on GPS/BDS/GLONASS. BDS: Beidou Satellite Navigation System.

observation data of JFNG tracking station for 24 h statistically. The results are shown in Figures 2 to 5, where vertical error is taken as an example for the positioning error.

In the figures and table (Table 1) above, BSS stands for Before Satellite Selection, ASS stands for After Satellite Selection. From the results above, it can be seen that for the dual constellations combination of GPS and Beidou regional service system, with the satellite selection method, the mean and root mean square (RMS) values of positioning error in the vertical direction are both slightly

Table 1. Performance comparison of different programs.

	GPS/BDS		GPS/BDS/GLONASS	
	BSS	ASS	BSS	ASS
Mean (m)	4.827	5.283	2.983	3.341
RMS (m)	5.292	5.651	3.039	3.517
Mean elapsed time (ms)	15.336	3.591	25.126	6.523

BDS: Beidou Satellite Navigation System; BSS: before satellite selection; ASS: after satellite selection.

increased, but the average time consumption is greatly reduced. This is also true for the combination of GPS/BDS/GLONASS, the purpose of this method is to select as few as visible satellites with less calculation fast, while having less impact on positioning accuracy.

Conclusion

In this article, to analyze the geometric characteristics of the best visible satellites combination, the layout of the elevation angles and azimuth angles of that is summarized. The satellite selection algorithm based on the best geometry is optimal, but it has a large amount of calculation and consumes long time. Therefore, a fast satellite selection algorithm based on both of geometry and GDOP contribution is proposed in this article. Firstly, according to elevation angles and azimuth angles, satellites with better geometry are selected to meet the basic positioning requirements and thus the initial satellite selection speed is improved. In addition, according to the GDOP contribution, the satellite with less contribution is gradually eliminated, so that the positioning performance meets the established criteria. Finally, this article selects the observation data of JFNG tracking station to simulate. On the premise of sacrificing a small amount of positioning accuracy, the simulation results show that the algorithm effectively solves the problem of matrix multiplication and matrix inversion in the best geometry algorithm and reduces the amount of computation and it can meet the real-time requirement of high dynamic users. The algorithm can be extended to satellite selection in any multiple constellation combination.

ORCID iD

Pengfei Zhang https://orcid.org/0000-0002-4249-3785

References

1. Zhang PF, Xu CD, Hu CS, et al. Time scales and time trans-formations among satellite navigation systems. In: *The 3 rd China Satellite Navigation Conference (CSNC2012)*, Guangzhou, China, 15–17 May 2012, pp. 491–502. Berlin: Springer.
2. Zhang GM, Huang GJ, and Zhang YL. A new satellite selec-tion algorithm for GPS Navigation. *J UEST China* 2000; 29(3): 221–224.
3. Huo GP, Miao LJ, and Zhao HS. GDOP Estimation method in tightly coupled integrated navigation system. *Trans Beijing Instit Techno* 2014; 34(8): 831–835.
4. Wu C and Ho Y. Genetic programming for the approximation of GPS GDOP. In: *Proceedings of the ninth international conference on machine learning and cybernetic*, Qingdao, China, 24–26 February 2010, pp. 2944–2949. New York, NY, USA: IEEE.
5. Wu C and Su W. A comparative study on regression models of GPS GDOP using soft-computing technique. In: *Proceed-ings of the 2009 IEEE international conference on fuzzy sys-tems (Fuzz-IEEE 2009)*, Jeju Island, South Korea, 20–24 August 2009, pp. 1513–1516. New York, NY, USA: IEEE.
6. Song JC, Xue GX, and Kang YN. A novel method for opti-mum global positioning system satellite selection based on a modified genetic algorithm. *PLOS One* 2016; 11(3): 1–14.
7. Blanco-Delgado N and Nunes F. Satellite selection method for multi-constellation GNSS using convex geometry. *IEEE Trans Veh Technol* 2010; 59(9): 4289–4297.
8. Liu S, Zhao GR, and Gao C. A fast satellite selection algo-rithm for GPS/BD Integrated Navigation System. *Elect Optics Control* 2017; 24(3): 32–35.
9. Nie Z, Gao Y, Wang Z, et al. A new method for satellite selection with controllable weighted PDOP threshold. *Surv Rev* 2017; 49(355): 285–293.
10. Chen CH and Zhang XL. A fast satellite selection approach for satellite navigation system. *Chinese J Elect* 2010; 38(12): 2887–2891.
11. Deng G. GPS satellite selection algorithm based on elevation angle and GDOP. *Digital Commun* 2010; (4): 47–50.
12. Wu RX and Cai TJ. An satellites selection algorithm based on elevation and azimuth. *Ship Elect Eng* 2009; 29(11): 73–75.
13. Li GY, Xu CD, Zhang PF, et al. A modified satellite selection algorithm based on satellite contribution for GDOP in GNSS. In: *International conference on mechanical and electronic engineering (ICMEE 2012)*, Hefei, China, 23–24 June 2012, pp. 415–421. Berlin: Springer.
14. Cong L and Tan ZZ. Satellite selection algorithm to improve precision and real-time performance of GPS positioning. *Syst Eng Electron* 2008; 30(10): 1914–1917.
15. Zhang PF, Xu CD, Hu CS, et al. Research on fast satellite selection algorithm based on geometry. In: *The 4th China satellite navigation conference (CSNC 2013)*, Lecture Notes in Electrical Engineering, 2013, *243*, pp. 545–554.