

# Intelligent scheduling method for energy saving operation of multi-train based on genetic algorithm and regenerative kinetic energy

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**Abstract:** An intelligent scheduling method for energy saving operation of multi-train is proposed based on genetic algorithm and regenerative kinetic energy. Considering the morning and evening peak, departure time, and total vehicle number, the energy consumption optimisation model of multidimensional state vector subspace of train is established for the train departure interval with the lowest total energy consumption. Two intelligent control models are used to formulate the time interval scheduling plan depending on whether to consider the morning and evening peak. The train operation diagram is solved by Matlab, and the validity and feasibility of the proposed algorithm are verified. This study provides an intelligent and efficient algorithm and solution for multi-train intelligent scheduling problem.

## 1 Introduction

With the rapid development of urban rail transit in China, the energy consumption caused by urban rail transit is becoming more and more serious. For example, the electricity consumption of Beijing rail transit in 2015 is 1 billion 390 million degrees, ~1.2% of the total electricity consumption in Beijing [1]. It can be seen that the cost of energy consumption is huge in rail transit operation. Therefore, it is of great significance to study the energy saving problem of urban rail transit [2].

The research on train energy saving operation focuses on driving strategy, energy saving is achieved by optimising the working conditions of trains. Some scholars have solved the working point transformation by analytical method [3, 4], but the non-linear characteristics of train operation process, computer simulation has become a major research method [5, 6]. In addition, heuristic method is also widely used in the energy saving optimisation of train operation [7–10], Zhou and other scholars [6] analysed and summarised the optimal operation algorithm of an electric locomotive. In addition to the energy saving driving strategy, some scholars have studied the influence of line conditions, operating parameters, and energy saving on train operation [11, 12]. In terms of train operation organisation, Xun and other scholars [13, 14] studied from the angle of regenerative braking of train energy. Chevrier and other scholars have studied the energy saving operation control of trains by studying a single interval [15], the speed curves of multiple energy consumption and running time synergy optimisation can be obtained.

In summary, most of the studies on the optimisation of train energy saving operation are based on single interval, focusing on the control level of train driving. However, there are few researches on intelligent dispatching of trains. In this paper, the energy consumption optimisation model of multidimensional vector subspace of train is set up under the conditions of the morning and evening peak, the interval of departure time, and the total number of trains. The energy saving operation and intelligent scheduling method of multi-train-based are analysed and studied based on the genetic algorithm (GA) and the regenerative kinetic energy.

## 2 Build a model

### 2.1 Multidimensional state vector subspace model

**2.1.1 Force analysis:** According to the experience at home and abroad, the most energy-efficient and fastest operation strategy is usually adopted in the simulation of urban rail transit train operation, so taking into account energy conservation, we need to follow the following four principles and assumptions [16]:

- i. A single-point model is used for dynamic model analysis.
- ii. The weight of a train is 30 t.
- iii. The acceleration process is calculated according to the maximum traction force.
- iv. The maximum braking force is adopted in the parking brake.
- v. In addition to the parking brake, air brake is avoided as far as possible during operation.
- vi. In the middle of the train operation, the strategy of accelerating and idling alternately is adopted.

In the course of track operation, the train is subjected to gravity, the force of the track, train traction  $F$ , train braking force  $B$ , and train running resistance  $W$ .

**2.1.2 Calculation of train traction force:** The traction force of trains (kN) has different maximum values at different speeds, and the equation of train traction force is as follows:

$$F = \mu F_{\max} \mu$$

$$\left\{ \begin{array}{ll} 203 & 0 \leq v \leq 51.5 \text{ km/h} \\ -0.002032v^3 + 0.4928v^2 - 42.13v + 13430 & 51.5 < v \leq 80 \text{ km/h} \end{array} \right. \quad (1)$$

where  $u$  is the percentage of actual output acceleration and maximum acceleration,  $0 \leq u \leq 1$ .

$a_{\max} = 1$ , then  $u = a$ ;  $F_{\max}$  is the maximum traction force of train;  $v$  the train running speed.

**2.1.3 Total resistance of train operation:** The total resistance of trains can be divided into basic resistance and additional resistance according to their causes.

i. Calculation equation of train total resistance

$$W = (w_0 + w_i + w_c) \times g \times M / 1000 \quad (2)$$

where  $w_0$  is the basic resistance (N/kN);  $w_i$  the resistance coefficient of unit ramp (N/kN);  $w_c$  the resistance coefficient of unit curve (N/kN);  $M$  the train weight (kg).

ii. Calculation equation of train total resistance after considering the calculation equation of basic resistance and additional resistance:

$$W = \left( A + Bv + Cv^2 + i + \frac{C}{R} \right) \times g \times M / 1000 \quad (3)$$

where  $v$  is the train speed (km/h);  $A$ ,  $B$ , and  $C$  are the train resistance coefficient, separately values: 2.031, 0.0622, 0.001807;  $i$  is the line slope (%). If  $i$  is a positive value, indicating the upslope, and if  $i$  is a negative value, representing a downhill;  $R$  is the radius of curvature (m);  $C$  an empirical constant reflecting many factors affecting the resistance of curves. China's rail transit is generally 600.

The calculation equation of train resistance after substituting for each parameter is

$$W = 1942.95 \left( 2.031 + 0.0622v + 0.001807v^2 + i + \frac{0.6}{R} \right) \quad (4)$$

**2.1.4 Calculation of train braking force:** Braking force  $B$  (kN) has different maximum values at different speeds, and the braking force calculation equation is

$$B = \mu B_{\max} = \mu \begin{cases} 166 & 0 \leq v \leq 77 \text{ km/h} \\ 0.1343v^2 - 25.07v + 1300 & 77 < v \leq 80 \text{ km/h} \end{cases} \quad (5)$$

**2.1.5 Train mechanics model under different working conditions:**

i. Traction condition: the force acting on the train has traction  $F$  and running resistance  $W$ , its resultant force is:

$$C = F - W \quad (6)$$

ii. Cruise condition: the train is running at a constant speed and the resultant force is 0. Whether the train needs traction or braking depends on the total resistance of the train at that time

$$C = 0 \quad (7)$$

iii. Idle running condition: only running resistance on the train  $W$ , its resultant force is

$$C = -W \quad (8)$$

iv. Braking condition: the force acting on the train is braking force and running resistance, its resultant force is

$$C = -(B + W) \quad (9)$$

**2.1.6 Establishment of numerical integration calculation model for train running state:** Train traction force, braking force, and train road condition are non-linear piecewise functions, it is difficult to solve the train running condition by traditional analytical calculation method. The numerical integration model of train operation is established based on the idea of numerical integration in order to solve the state values of acceleration, speed, and distance during train operation.

$S_0$ ,  $v_0$ , and  $a_0$  indicate the initial state parameters of the train, the calculation formula is as follows:

$$\begin{cases} S_0 = 0 \\ v_0 = 0 \\ a_0 = \frac{\mu F_{\max}(v) - (w_0(0) + w_i + w_c) \times g \times M / 1000}{M} \end{cases} \quad (10)$$

where when  $v = v_0$ , actual output traction of the train is  $F_{\max}(v)$ ; when  $v = v_0$ , basic resistance of the train is  $w_0(0)$ .

$S_{i+1}$ ,  $v_{i+1}$ , and  $a_{i+1}$  express train state parameters at  $i+1$  time, the calculation formula is as follows:

$$\begin{cases} S_{i+1} = S_i + v_i \Delta t + \frac{1}{2} a_i \Delta t^2 \\ v_{i+1} = v_i + a_i \Delta t \\ a_{i+1} = \frac{\mu F_{\max}(v_i) - (w_0(v_i) + w_i + w_c) \times g \times M / 1000}{M} \end{cases} \quad (11)$$

where when  $v = v_i$ , actual output traction of the train is  $F_{\max}(v_i)$ ; when  $v = v_i$ , the basic resistance of the train is  $w_0(v_i)$ .

When the time interval is very small, the traction force and resistance of the train are almost constant. According to the state parameters of the train at a certain time, the acceleration equation can be used to calculate the train running state parameters after the train time interval  $\Delta t$ . Therefore, by setting the appropriate time interval  $\Delta t$ , the state of each train can be approximately solved.

**2.1.7 Establishment of energy consumption model for single train:** The calculation method of energy consumption model for single train is as follows [4]:

$$E_{i+1} = E_i + F_i \cdot S_i \quad (12)$$

where  $i$  is the express station site;  $F_i$  the train traction at  $i$  station;  $S_i$  the train running distance at  $i$  site;  $E_i$  the total energy consumption of train at  $i$  site;  $E_{i+1}$  the total energy consumption of trains at  $i+1$  sites.

**2.1.8 Calculation of state vector in train operation:** In the course of train  $i$  running in the line interval, its state vectors  $[S(t), v(t), a(t), E_{\text{traction}}(t), E_{\text{cruise}}(t), E_{\text{Transformation of the brake}}(t)]_i^T$  are all state sub-vectors on the train running state space set during the whole departure time.

The interval between the train  $i$  and the given departure time is  $H = \{h_1, \wedge h_{99}\}$ , the formula for the state vector  $[S(t), v(t), a(t), E_{\text{traction}}(t), E_{\text{cruise}}(t), E_{\text{Transformation of the brake}}(t)]_i^T$  at  $i$  time is as follows:

$$S(t) = \begin{cases} 0 & \left( t \leq \sum_{j=1}^{i-1} h_j \right) \\ S(t - \Delta t) + v(t - \Delta t) \cdot \Delta t + \frac{1}{2} a(t - \Delta t) \Delta t^2 & \left( t > \sum_{j=1}^{i-1} h_j \right) \end{cases} \quad (13)$$

$$v(t) = \begin{cases} 0 & \left( t \leq \sum_{j=1}^{i-1} h_j \right) \\ v(t - \Delta t) + a(t - \Delta t) \Delta t & \left( t > \sum_{j=1}^{i-1} h_j \right) \end{cases} \quad (14)$$

$$a(t) = \begin{cases} 0 & \left( t \leq \sum_{j=1}^{i-1} h_j \right) \\ \frac{F_{\text{traction}}(t) - W(t)}{M} & \left( t > \sum_{j=1}^{i-1} h_j \right) \end{cases} \quad (15)$$

$$E_{\text{traction}}(t) = \begin{cases} 0 & \left( t \leq \sum_{j=1}^{i-1} h_j \right) \\ F_{\text{traction}}(t) \left[ v(t - \Delta t) \cdot \Delta t + \frac{1}{2} a(t - \Delta t) \Delta t^2 \right] & \left( t > \sum_{j=1}^{i-1} h_j \right) \end{cases} \quad (16)$$

$$E_{\text{cruise}}(t) = \begin{cases} 0 & \left( t \leq \sum_{j=1}^{i-1} h_j \right) \\ W(t) \left[ v(t - \Delta t) \cdot \Delta t + \frac{1}{2} a(t - \Delta t) \Delta t^2 \right] & \left( t > \sum_{j=1}^{i-1} h_j \right) \end{cases} \quad (17)$$

$$E_{\text{transformation of the brake}}(t) = \begin{cases} 0 & \left( t \leq \sum_{j=1}^{i-1} h_j \right) \\ \frac{1}{2} M [v(t)^2 - v(\Delta t)^2] & \left( t > \sum_{j=1}^{i-1} h_j \right) \end{cases} \quad (18)$$

where  $S(t)$  is the train operation displacement at  $t$  time;  $V(t)$  the train running speed at  $t$  time;  $a(t)$  the acceleration of train running at  $t$  time;  $S(t - \Delta t)$  the train displacement at  $\Delta t$  moment before  $t$  time;  $V(t - \Delta t)$  the train speed at  $\Delta t$  moment before  $t$  time;  $a(t - \Delta t)$  the train acceleration at  $\Delta t$  moment before  $t$  time;  $F_{\text{traction}}(t)$  the train traction at  $t$  moment, is the amount related to the condition of the road and the control strategy;  $W$  the total resistance of train operation.

So, the energy consumed by all trains can be expressed by the equation at  $t$  time

$$E(t) = \sum_{i=1}^n E_{\text{traction}^i}(t) + \sum_{i=1}^n E_{\text{traction}^i}(t) - \sum_{i=1}^n E_{\text{transformation of the brake}^i}(t) \quad (19)$$

As when the train stops on the platform, the traction, resistance, and braking force do not do work,  $\int_0^{T_0} E(t) dt$  is the total energy consumption of  $E(t)$  at the time interval of  $H = \{h_1, \wedge h_n\}$ .

## 2.2 Genetic algorithm

The numerical integration model, GA, and the principle of utilisation of train regeneration energy are applied to the multidimensional state vector subspace model established in the process of train running energy consumption.

Among them, the GA abandons the traditional search method, simulates the process of natural biological evolution, and uses artificial evolution to randomise the search of the target space. Fig. 1 is a typical GA flow.

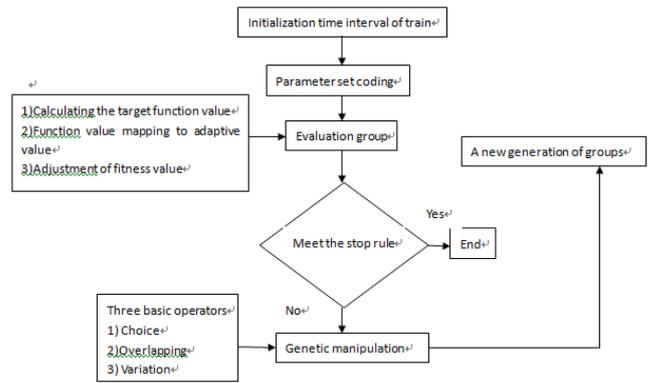


Fig. 1 Typical GA flowchart

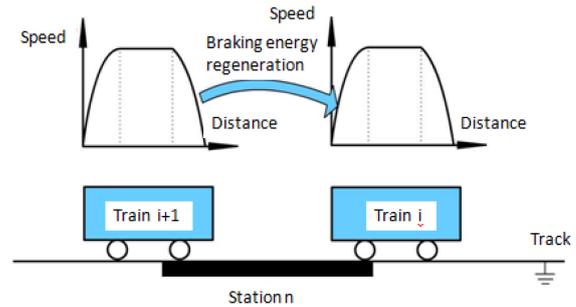


Fig. 2 Principle of regenerating energy for adjacent trains

## 2.3 Train regenerated energy

Train regeneration energy utilisation is a commonly used regenerative braking technology for urban rail transit trains. If the regenerative braking energy can be used by the adjacent trains in time, the energy loss can be reduced. The principle of regenerative energy utilisation between adjacent trains is shown in Fig. 2.

Trains produce energy  $E_{\text{reg}}$  during braking, the generated regenerated energy is  $E_{\text{reg}}$

$$E_{\text{reg}} = (E_{\text{mech}} - E_f) \cdot 95\% \quad (20)$$

where  $E_{\text{mech}}$  is the changes in the mechanical energy of the train during the braking process;  $E_f$  in order to overcome the basic resistance and additional resistance, the work is done during the braking process.

The regenerated energy used can be calculated according to the below formula

$$E_{\text{used}} = E_{\text{reg}} \cdot t_{\text{overlap}} / t_{\text{brake}} \quad (21)$$

where  $t_{\text{overlap}}$  is the overlap time between train  $i + 1$  braking time and train  $i$  acceleration time;  $t_{\text{brake}}$  the train  $i + 1$  braking time.

## 2.4 Principle of moving block

According to principle of moving block in train system, the speed limit  $V_{\text{limit}}$  is calculated as follows:

$$V_{\text{limit}} = \min(V_{\text{line}}, \sqrt{2LB_e}) \quad (22)$$

Among them,  $V_{\text{line}}$  is the line speed limit (km/h) of the current position of the train.  $L$  is the distance between the front and rear vehicles at present time (m).  $B_e$  is the maximum deceleration ( $\text{m/s}^2$ ) of train braking.

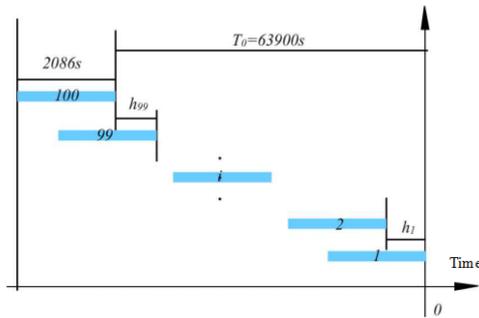


Fig. 3 Relationship between train running time series and total departure time

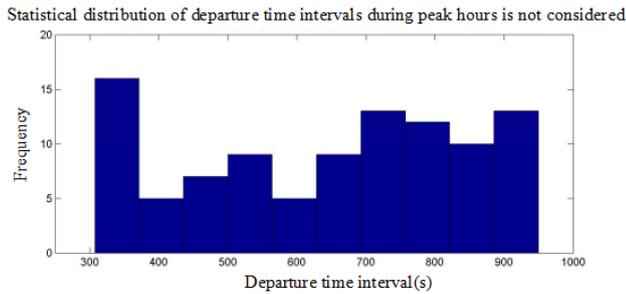


Fig. 4 Statistical distribution of departure time interval

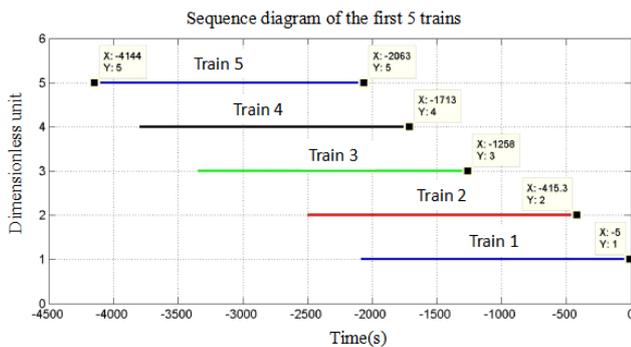


Fig. 5 Sequence diagram of the first five columns

### 3 Application research

#### 3.1 Application 1 – do not consider the peak of the morning and evening peak

The application of the research object is the Chongqing light rail No.3 line, from Si gongli to Long tousi, a total of 14 sites, contains: Si gongli (A0)–Nan ping (A1)–Gong mao (A2)–Tong yuanju (A3)–Liang lukou (A4)–Niu jiaotuo (A5)–Hua xinjie (A6)–Guan yinqiao (A7)–Hong qihegou (A8)–Jia zhoulu (A9)–Zheng Jiayuanzi (A10)–Tang jiayuanzi (A11)–Shi ziping (A12)–Chongqing North Railway Station South Square (A13)–Long tousi (A14). The departure time between starting trains and receiving trains per day is  $T_0 = 63,900$  s. Also, the total running time from the Si gongli station to the Long tousi Station is 2086 s.

**3.1.1 Establishment of multidimensional state vector subspace model:** In the study, 100 trains were tracked and the departure interval was not less than 5 minutes. The objective of the solution is to minimise the energy consumption of the train in the whole running time. So, the objective function is the integral value of the energy consumption in the running time. Since there are many constraints involved in solving the problem, in order to simplify the solution process, the external penalty function method is used to transform it into an unconstrained control problem.

Objective function:

$$\min E = \int_0^{T_0} E(t)dt \quad (23)$$

Constraints are shown in the following forms

$$\begin{cases} V(t) \leq V_{\text{limit}} & t \leq 63,900 \\ \sum_{i=1}^{99} h_i = T_0 \\ D_{\text{min}} \leq D_i(t) \leq D_{\text{max}} & t \leq 63,900 \\ t_i \leq 2086 & i \leq 100 \end{cases} \quad (24)$$

In the forms:  $D_i(t)$  is the stop time of the train;  $D(t)$  the interval time of train departure;  $D_{\text{min}}$  the shortest parking time of the train;  $D_{\text{max}}$  the longest parking time of the train;  $t_i$  the running time of the train  $i$ .

Among them,  $h_i$  represents the departure time of trains, and  $T_0$  is the total departure time of the 100 trains. Fig. 3 is a diagram of the train running time series and the total departure time

$$\sum_{i=1}^{99} h_i = T_0 \quad (25)$$

*Note:* the blue rectangular pattern represents the total running time from A1 station to A14 station of a single train, both of which are 2086s.

**3.1.2 Using MATLAB to solve application 1:** In MATLAB, the program is compiled to solve the statistical map and the interval time chart of departure time of the 100 trains. The statistical distribution of the departure time interval is shown in Fig. 4.

Under the factor that do not take the peak of the morning and evening into account, interval is  $>5$  min, so as to meet the application requirements.

In order to display the solution results more intuitively, the first five rows of departure time are extracted, and the first five sequence diagrams are shown below.

From Fig. 5, we can see that the departure time between the first column and the second column is 410.3 s. The departure time of trains at the rear is 842.7,455 and 350 s, respectively; it meets the requirements.

#### 3.2 Application 2 – taking the peak of the morning and evening into account

The basic situation of application 2 is the same as that of application 1, due to the fact that in real life, the departure time of the urban track during the operation is affected by the early peak (7200–12,600 s) and the late peak (43,200–50,400 s). Therefore, the departure interval at this stage should be adjusted to no more than 2.5 min and not  $<2$  min, at the same time, in order to better observe the application results, the train number is increased to 240 for simulation analysis.

**3.2.1 Establishment of multidimensional state vector subspace model:** The objective function is similar to formula (23) in application 1

$$\min E = \int_0^{T_0} E(t)dt \quad (26)$$

Constraints under the influence of peak time

$$\left\{ \begin{array}{ll} V(t) \leq V_{\text{limit}} & (t \leq 63,900) \\ \sum_{i=1}^{239} h_i = T_0 \\ V(t) \leq 2086 & (t \leq 63,900) \\ 5 \leq D_i(t) & 0 \leq t < 7200 \\ 2 \leq D(t) \leq 2.5 & 7200 \leq t < 12,600 \\ 5 \leq D_i(t) & 12,600 \leq t < 43,200 \\ 2 \leq D(t) \leq 2.5 & 43,200 \leq t < 50,400 \\ 5 \leq D_i(t) & 50,400 \leq t \leq 63,900 \end{array} \right. \quad (27)$$

where  $D_i(t)$  is the stopping time of train  $i$ ;  $D(t)$  the departure time of trains. The calculation of  $V_{\text{limit}}$  is the same as formula (22).

**3.2.2 Using MATLAB to solve application 2:** After modifying the model constraint conditions, the time interval between the 240 trains is solved, and the departure time curve is obtained, as shown in Fig. 6.

In consideration of the early and evening peaks, take 240 trains in section Si gongli–Long tousi of No.3 line of Chongqing as an example. The distribution chart of departure time intervals of each train is shown in the figure above, showing that the departure time between trains is 120 and 150 s during the morning and evening peaks. It meets the requirements.

At the same time, the statistical analysis of the departure time is made, and the statistical distribution curve considering the departure time interval at peak time is obtained, as shown in Fig. 6.

From Fig. 7, we can see more intuitively that there are 95 vehicles in the morning and evening peak, departure time interval is between 120 and 150 s. In the rest of the stage, the interval time of departure is >300 s. It also can be seen that the departure time is larger at the end of the last class, which conforms to the actual situation. So, it can be seen that the scheduling is reasonable.

### 3.3 Analysis and discussion

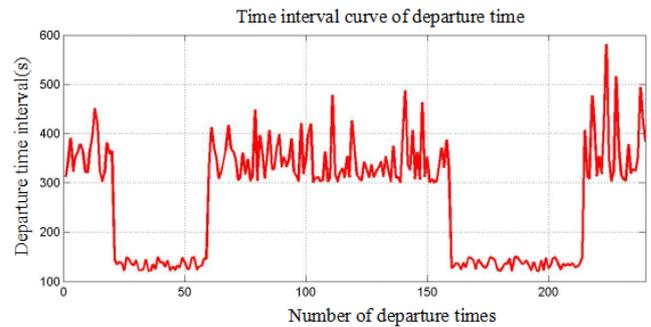
Based on the above model, on the basis of the determining state vector and time constraint in the process of train operation, between section Si gongli–Long tousi of No. 3 line of Chongqing light rail line and considering the two different constraints, one is the 100 trains are simulated without considering the peak hours, another is the 240 trains running in the morning and evening peak hours, the method of train intelligent scheduling is designed. Solving by MATLAB, the results meet the actual requirements. The starting time of each train under two different constraints is solvable. The all trains scheduling statistics are shown in Figs. 2–6. Example application simulation shows that the model is feasible, and the algorithm is effective.

## 4 Summary

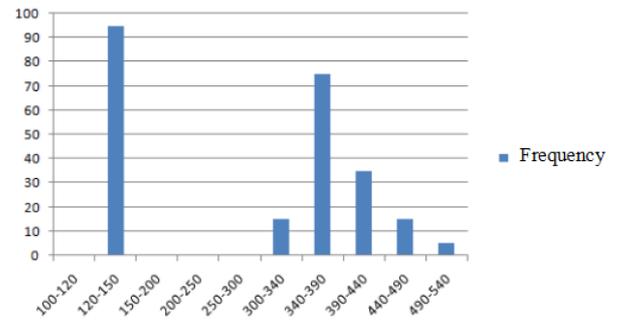
In order to further improve the level of intelligent and energy saving of train operation, based on the characteristics of urban rail train operation in China, a multidimensional state vector subspace model for train operation is established. In the research, the mobile block principle is used to propose an intelligent scheduling method for multi-train energy saving operation based on GA and regenerative kinetic energy. Also, through the example simulation application, the reasonableness of the scheduling is proved. It provides an effective and efficient algorithm and solution for multi-train intelligent and energy saving scheduling problem, which can effectively reduce the work intensity of the dispatcher and lay the foundation for realising the energy saving and intelligent of railway operation and dispatching.

## 5 Acknowledgments

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**Fig. 6** Curve of the time interval between the 240 trains during the rush hour period



**Fig. 7** Consider the statistical distribution of the departure time of 240 trains during rush hour

Chongqing city (cstc2016jcyjA0385, cstc2017jcyjAX0343); the Humanities and Social Science research project of the Chongqing Municipal Education Commission (15SKG133); Science and technology research project of the Chongqing Municipal Education Committee, Research on neural network expert system for production line switching decisions (KJ1503006).

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