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On synchronisation of a class of complex chaotic systems with complex unknown parameters via integral sliding mode control

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Abstract. Chaotic systems demonstrate complex behaviour in their state variables and their parameters, which generate some challenges and consequences. This paper presents a new synchronisation scheme based on integral sliding mode control (ISMC) method on a class of complex chaotic systems with complex unknown parameters. Synchronisation between corresponding states of a class of complex chaotic systems and also convergence of the errors of the system parameters to zero point are studied. The designed feedback control vector and complex unknown parameter vector are analytically achieved based on the Lyapunov stability theory. Moreover, the effectiveness of the proposed methodology is verified by synchronisation of the Chen complex system and the Lorenz complex systems as the leader and the follower chaotic systems, respectively. In conclusion, some numerical simulations related to the synchronisation methodology is given to illustrate the effectiveness of the theoretical discussions.

Keywords. Chaos synchronisation; integral sliding mode control; adaptive control; complex systems.

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1. Introduction

Chaos control and synchronisation consist of designing a proper control law such that the follower chaotic system tracks the output of the leader chaotic system as time tends to infinity. Many control and synchronisation methods have been investigated in recent years to address it. Although the synchronisation methods investigated can be dynamically classified into two categories, integer-order methods and fractional-order methods, technically these can be classified as active break methods [1,2], adaptive methods [3–5], sliding mode [6–9], projective synchronisation [10–12], backstepping control methods [13], lag synchronisation [14], generalised method [15,16], modified function projective synchronisation (MFPS) [17–21] and many other synchronisation methods [22,23].

Unfortunately, in spite of a wide range of studies on synchronisation schemes, there are concerns regarding real systems with real parameters. However, in some real-world situations, for example, electromagnetic fields and laser systems, the variables and/or the parameters of the dynamical systems are complex. Furthermore, in some applied fields such as secure communications, where the quantity of variables and parameters specifies the capacity of transmitted information and the secure communication rate, synchronisation of complex chaotic systems can be considered to increase the capacity of transmitted information without fear of losing the quality.

Recently, Mahmoud *et al* [24] have investigated identical synchronisation problem of the complex Chen and Lu chaotic systems and have presented their chaotic attractors for a wide range of values of the corresponding system parameters; and in [25], they have proposed non-identical synchronisation between two non-identical chaotic Lu and Chen systems. After that, a wide variety of synchronisation schemes were developed by researchers [26–30] to control the behaviour of complex chaotic systems. It is worth mentioning that until now, no published paper is devoted to synchronisation problem of complex chaotic systems via sliding mode control, especially with the uncertainty in system parameters. Moreover, sliding mode control methodology provides a fast and reliable synchronisation strategy, in comparison with other methods such as projective method, lag method and so on. Therefore, in this paper, the synchronisation problem of a class of complex chaotic systems with unknown parameters are investigated by designing an ISMC method. An adaptive control law and parameter estimation law are developed based on the Lyapunov stability theory. In addition, the proposed ISMC method is verified by chaos synchronisation of two complex chaotic systems, the Chen complex system and the Lorenz complex system. Finally, some numerical simulations are presented to illustrate the effectiveness of theoretical discussions.

The rest of this paper is organised as follows: Section 2 gives chaos synchronisation between a class of complex chaotic systems via ISMC method. In §3, the problem of synchronisation between the Chen complex system and the Lorenz complex system is presented via ISMC method. In addition, some numerical simulations are presented to verify the theoretical analysis. Finally, some concluding remarks are given in §4.

2. Mathematical modelling and ISMC synchronisation

In this section, chaos synchronisation problem for a class of chaotic systems is presented. An appropriate adaptive control law and a parameter estimation strategy are derived based on Lyapunov stability function. Consider a class of complex chaotic systems as follows:

$$\dot{x} = Af(x) + F(x), \tag{1}$$

where $x = (x_1^r + jx_1^i, x_2^r + jx_2^i, \ldots, x_n^r + jx_n^i)^T \in \mathbf{C}^{n\times 1}$ is a complex vector of the leader system state variables. $f(x)$ in $n \times n$ and $F(x)$ in $n \times n$ are the complex matrices of linear and nonlinear functions, respectively. $A \in \mathbf{C}^{n\times 1}$ is the unknown parameter vector of the system. The symbol j denotes the imaginary notation of a complex number, $j = \sqrt{-1}$. The superscripts r and i indicate the real and imaginary parts of a complex number, respectively.

Now, take the complex chaotic system presented in (1) as the leader system with unknown system parameter vector $A \in \mathbf{C}^{n\times 1}$. Then the follower complex system can be described as follows:

$$\dot{y} = Bg(y) + G(y) + u, \tag{2}$$

where $y = (y_1^r + jy_1^i, y_2^r + jy_2^i, \ldots, y_n^r + jy_n^i) \in \mathbf{C}^{n\times 1}$ stands for the state variable vector of the follower system and u declares control feedback vector, to be designed. $B \in \mathbf{C}^{n\times 1}$ stands for the complex parameter vector of the linear part of the system. $g(y)$ in $n \times n$ and $G(y)$ in $n \times n$ are the complex matrices of linear and nonlinear functions, respectively. The disparity between the leader and the follower state variables defined in (1) and (2) can be represented by $e = (e_1, e_2, \ldots, e_n)^T \in \mathbf{C}^{n\times 1}$ as follows:

$$e = e^r + je^i = y^r - x^r + j(y^i - x^i). \tag{3}$$

Then, the error dynamics between two chaotic systems can be obtained as follows:

$$\dot{e} = \dot{e}^r + j\dot{e}^i = \dot{y}^r - \dot{x}^r + j(\dot{y}^i - \dot{x}^i). \tag{4}$$

The ultimate goal of synchronisation is to design an appropriate feedback controller to force the motion trajectories of the follower chaotic system to track the leader one. To this end, an active ISMC is designed in this section that is capable of synchronising the leader and the follower attractors and forcing synchronisation error in (3) to converge to zero. Given the dynamic synchronisation errors in (4), then, $s = (s_1^r, \ldots, s_n^r; s_1^i, \ldots, s_n^i)^T$, the sliding surface of the ISMC method can be defined as follows:

$$\begin{aligned} s_k^r &= ({}_0D_t^1 + \lambda_{rk})({}_0D_t^{-1} e^r), \\ s_k^i &= ({}_0D_t^1 + \lambda_{ik})({}_0D_t^{-1} e_k^i), \quad k = 1, 2, \ldots, n, \end{aligned} \tag{5}$$

where ${}_0D_t^1$ and ${}_0D_t^{-1}$ represent the time derivative and the integral of the sliding surface, respectively. $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_n) \in \mathbf{C}^{1\times n}$, where $\lambda_k = \lambda_{rk} + j\lambda_{ik}$ is the arbitrary constant coefficient vector. Then, time derivative of the sliding surface can be given in vector form as follows:

$$\dot{s} = \dot{e} + \boldsymbol{\lambda} e. \tag{6}$$

In addition, according to the exponential reaching law described in [31], the derivative of the sliding surface can be given as follows:

$$\dot{s} = -\boldsymbol{\xi}\,\mathrm{sgn}(s) - \mathbf{k}s, \tag{7}$$

where $\boldsymbol{\xi} = (\xi_1, \xi_2, \ldots, \xi_n)$ and $\mathbf{k} = (k_1, k_2, \ldots, k_n)$ are the positive real constant vectors and sgn is the signum function. Then, considering the time derivative of the sliding surface in (6) and the exponential reaching law presented by eq. (7), one can obtain

$$\dot{e} + \boldsymbol{\lambda} e = -\boldsymbol{\xi}\,\mathrm{sgn}(s) - \mathbf{k}s. \tag{8}$$

This equation can be simplified by substituting the synchronisation error e and its dynamical representative $\dot{e}$ from eqs (3) and (4) as follows:

$$\dot{y} - \dot{x} + \boldsymbol{\lambda} y - \boldsymbol{\lambda} x = -\boldsymbol{\xi}\,\mathrm{sgn}(s) - \mathbf{k}s. \tag{9}$$

Then substituting the state variable vectors x and y and their dynamic representatives $\dot{x}$ and $\dot{y}$ from eqs (1) and (2) gives

$$\begin{aligned} &Bg(y) + F(y) + u - Af(x) - F(x) + \boldsymbol{\lambda} e \\ &\quad = -\boldsymbol{\xi}\,\mathrm{sgn}(s) - \mathbf{k}s. \end{aligned} \tag{10}$$

Then the following feedback controller can be obtained:

$$u = \hat{A}f(x) - \hat{B}g(y) - F(y) + F(x) - \boldsymbol{\lambda}e - \boldsymbol{\xi}\,\mathrm{sgn}(s) - \mathbf{k}s, \tag{11}$$

where $\hat{A}$ and $\hat{B}$ declare the estimation of A and B vectors, respectively; which can be estimated as

$$\dot{\hat{A}} = sf(x),$$
$$\dot{\hat{B}} = -sf(y). \tag{12}$$

In the following theorem, synchronisation between two classes of complex chaotic systems using ISMC method is proved to achieve synchronisation between a class of complex chaotic systems presented in (1) and (2).

Theorem 1. *The motion trajectories of the follower chaotic system state variables in eq.* (2) *with the initial state values* $y(0) \in \mathbf{R}^3$, *using the feedback controller presented in* (11) *with coefficient vectors* $\boldsymbol{\lambda}, \boldsymbol{\xi}, \mathbf{k} > 0$ *and system parameter estimation in* (12), *will track the trajectories of the leader attractors in eq.* (1). *Furthermore, the synchronisation error vector e in eq.* (5) *asymptotically converges to zero.*

Proof. Let the Lyapunov candidate function be as follows:

$$V = \frac{1}{2}(s^2 + \bar{A}\bar{A}^T + \bar{B}\bar{B}^T), \tag{13}$$

where $\bar{A} = \hat{A} - A$ and $\bar{A} = \hat{B} - B$. Clearly, V is positive definite. The derivative of the Lyapunov function V with respect to time is:

$$\dot{V} = s\dot{s} + \bar{A}\dot{\hat{A}} + \bar{B}\dot{\hat{B}}$$
$$= s[Bg(y) + F(y) + u - Af(x)F(x) + \boldsymbol{\lambda}e]$$
$$+ \bar{A}\dot{\hat{A}} + \bar{B}\,\dot{\hat{B}}. \tag{14}$$

By substituting the designed feedback controller u in (11) and the parameter estimation in (12), the derivative of the Lyapanov function in (14) can be simplified as

$$\dot{V} = -\boldsymbol{\xi}s\,\mathrm{sgn}(s) - \mathbf{k}s^2. \tag{15}$$

Hence $\dot{V}$ is negative definite when the coefficient vectors $\boldsymbol{\xi}$ and $\mathbf{k}$ are positive. Consequently, according to the Lyapunov stability theorem, the leader complex chaotic system in (1) and the follower complex chaotic system in (2) will be asymptotically synchronised with the control input vector in eq. (11) and parameter estimation in eq. (12). So the proof is complete. □

3. Simulation results

In this section, chaos synchronisation between two non-identical complex chaotic systems, the Chen complex system and the Lorenz complex system, is addressed. Take the Chen complex chaotic system first introduced in [32] as follows:

$$\dot{x}_1 = -\alpha_1 x_1 + \alpha_1 x_2,$$
$$\dot{x}_2 = -\alpha_1 x_1 + \alpha_2 x_1 + \alpha_2 x_2 - x_1 x_3,$$
$$\dot{x}_3 = \frac{1}{2}(\bar{x}_1 x_2 + x_1 \bar{x}_2) - \alpha_3 x_3, \tag{16}$$

where $x_1 = x_1^r + jx_1^i$ and $x_2 = x_2^r + jx_2^i$ are complex state vectors and $x_3 = x_3^r$ is a real state variable. $\alpha = (\alpha_1^r + j\alpha_1^i, \alpha_2^r + j\alpha_2^i, \alpha_3)$ is the complex parameter vector of the system. Hence, the Chen complex chaotic system can be represented with a five-dimensional (5D) dynamical real system as follows:

$$\dot{x}_1^r = -\alpha_1^r x_1^r + \alpha_1^i x_1^i + \alpha_1^r x_2^r - \alpha_1^i x_2^i$$
$$\dot{x}_1^i = -\alpha_1^i x_1^r - \alpha_1^r x_1^i + \alpha_1^i x_2^r + \alpha_1^r x_2^i$$
$$\dot{x}_2^r = -\alpha_1^r x_1^r + \alpha_1^i x_1^i + \alpha_2^r x_1^r - \alpha_2^i x_1^i + \alpha_2^r x_2^r - \alpha_2^i x_2^i - x_1^r x_3$$
$$\dot{x}_2^i = -\alpha_1^i x_1^r - \alpha_1^r x_1^i + \alpha_2^i x_1^r + \alpha_2^r x_1^i + \alpha_2^i x_2^r + \alpha_2^r x_2^i - x_1^i x_3$$
$$\dot{x}_3 = \frac{1}{2}(x_1^r x_2^r + x_1^i x_2^i) - \alpha_3 x_3. \tag{17}$$

The chaotic behaviour of the Chen complex system is shown in figure 1, with system parameters, $\alpha_1 = 2 + 2i, \alpha_2 = 3 + i, \alpha_3 = 4$.

Consider the Chen complex system in (17) as the leader system. Then, the corresponding follower complex system can be given by the Lorenz chaotic system [33]. The structure of the Lorenz chaotic system is

$$\dot{y}_1 = -\alpha_1 y_1 + \alpha_1 y_2$$
$$\dot{y}_2 = \alpha_2 y_1 - y_2 - y_1 y_3$$
$$\dot{y}_3 = y_1 y_2 - \alpha_3 y_3 \tag{18}$$

which can be presented as the follower system in the complex manner as follows:

$$\dot{y}_1 = -\hat{\alpha}_1 y_1 + \hat{\alpha}_1 y_2 + u_1,$$
$$\dot{y}_2 = \hat{\alpha}_2 y_1 - y_2 - y_1 y_3 + u_2,$$
$$\dot{y}_3 = \frac{1}{2}(\bar{y}_1 y_2 + y_1 \bar{y}_2) - \hat{\alpha}_3 y_3 + u_3, \tag{19}$$

where $y = (y_1^r + jy_1^i, y_2^r + jy_2^i, y_3^r)$ is the complex state variable vector. $\hat{\alpha} = (\alpha_1^r + j\hat{\alpha}_1^i, \hat{\alpha}_2^r + j\hat{\alpha}_2^i, \hat{\alpha}_3^r)$ indicates

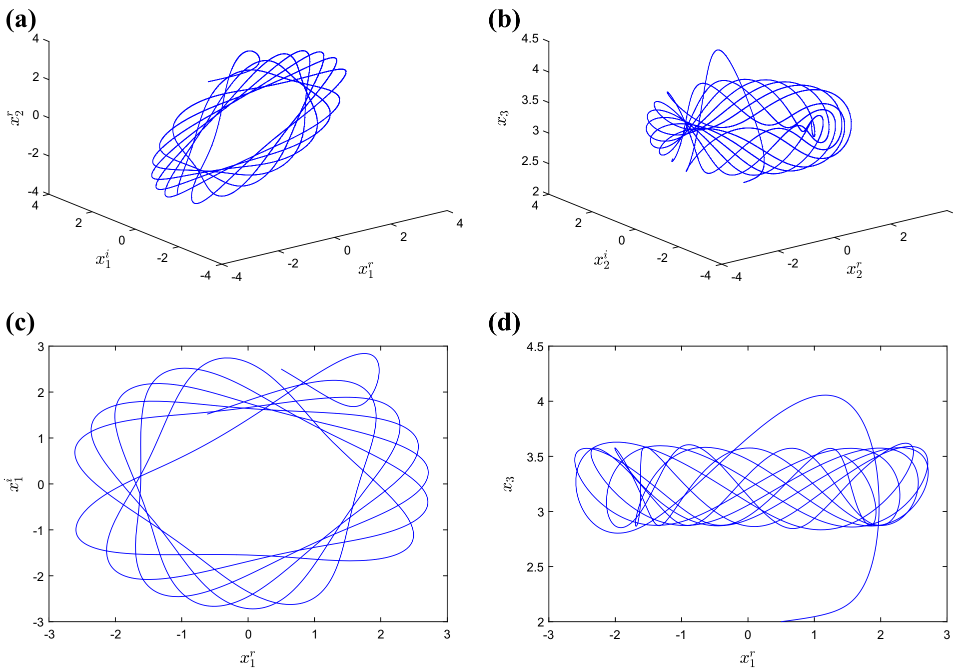

Figure 1. Some chaotic attractors of the Chen complex chaotic system (17).

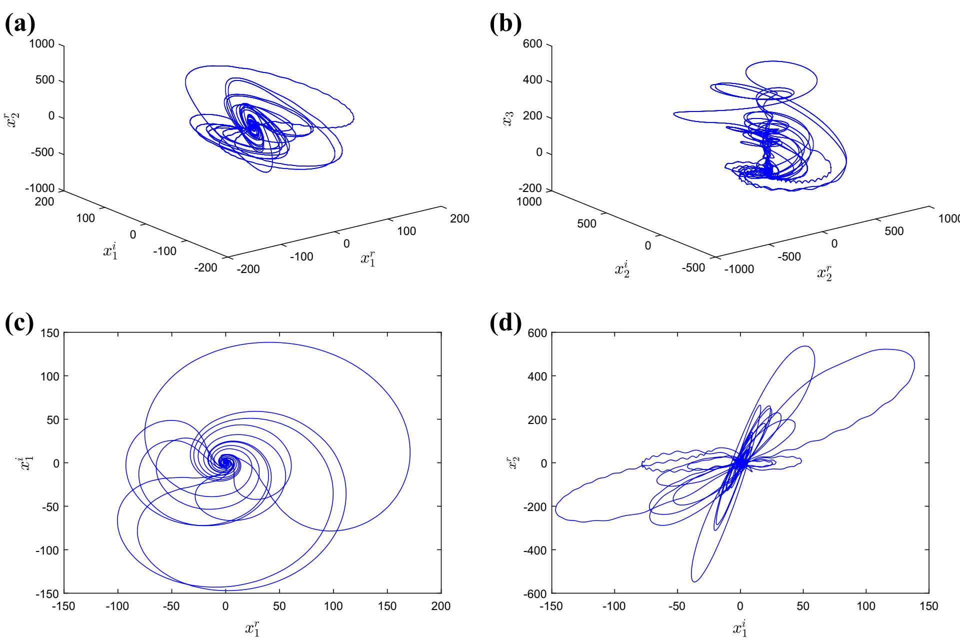

Figure 2. Some chaotic attractors of the Lorenz complex chaotic system (20).

the estimation of the complex parameter vector α, of the leader complex system (17). $u_1 = u_1^r + ju_1^i$ and $u_2 = u_2^r + ju_2^i$ are the complex feedback controllers and $u_3 = u_3^r$ is the real feedback control, to be designed. So the complex Lorenz follower system can be represented by 5D real chaotic system as

$$\dot{y}_1^r = -\hat{\alpha}_1^r y_1^r + \hat{\alpha}_1^i y_1^i + \hat{\alpha}_1^r y_2^r - \hat{\alpha}_1^i y_2^i + u_1^r$$

$$\dot{y}_1^i = -\hat{\alpha}_1^i y_1^r - \hat{\alpha}_1^r y_1^i + \hat{\alpha}_1^i y_2^r + \hat{\alpha}_1^r y_2^i + u_1^i$$

$$\dot{y}_2^r = -\hat{\alpha}_1^r y_1^r + \hat{\alpha}_1^i y_1^i + \hat{\alpha}_2^r y_1^r - \hat{\alpha}_2^i y_1^i + \hat{\alpha}_2^r y_2^r - \hat{\alpha}_2^i y_2^i - x_1^r y_3 + u_2^r$$

$$\dot{y}_2^i = -\hat{\alpha}_1^i y_1^r - \hat{\alpha}_1^r y_1^i + \hat{\alpha}_2^i y_1^r + \hat{\alpha}_2^r y_1^i + \hat{\alpha}_2^i y_2^r + \hat{\alpha}_2^r y_2^i - y_1^i y_3 + u_2^i$$

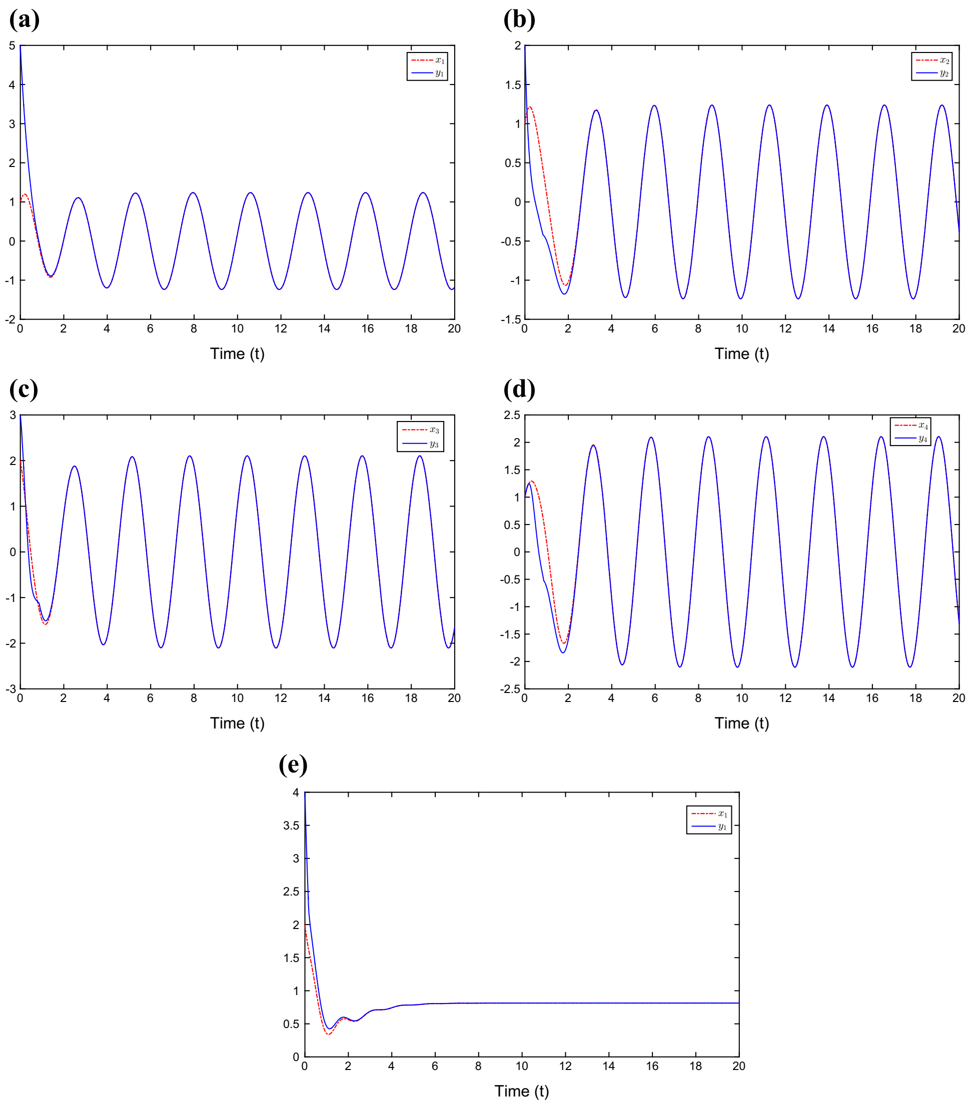

Figure 3. Synchronisation of the leader–follower complex chaotic systems (17) and (20) with complex parameters, using the ISMC method.

$$\dot{y}_3 = \frac{1}{2}(y_1^r y_2^r + y_1^i y_2^i) - \hat{\alpha}_3 y_3 + u_3. \tag{20}$$

The chaotic behaviour of the Lorenz complex system (20) is shown in figure 2, with system parameters considered constant as: $\alpha_1 = 1.5 + i, \alpha_2 = 2 + 3i$ and $\alpha_3 = -2$.

Then, according to the dynamics system error in (4), we have

$$\begin{aligned}
\dot{e}_1 &= \dot{e}_1^r + j\dot{e}_1^i = \dot{y}_1^r - \dot{x}_1^r + j(\dot{y}_1^i - \dot{x}_1^i)\\
\dot{e}_2 &= \dot{e}_2^r + j\dot{e}_2^i = \dot{y}_2^r - \dot{x}_2^r + j(\dot{y}_2^i - \dot{x}_2^i)\\
\dot{e}_3 &= \dot{e}_3^r = \dot{y}_3^r - \dot{x}_3^r.
\end{aligned} \tag{21}$$

In the following, an adaptive controller is designed for synchronisation of the considered complex chaotic systems, by considering the complex form of the leader dynamic system in (17) and the follower dynamic system in (20), which can be given as follows:

$$\begin{aligned}
u_1^r &= -\hat{\alpha}_1^r y_1^r + \hat{\alpha}_1^i y_1^i + \hat{\alpha}_1^r y_2^r - \hat{\alpha}_1^i y_2^i - \hat{\alpha}_1^r x_1^r\\
&\quad + \hat{\alpha}_1^i x_1^i + \hat{\alpha}_1^r x_2^r - \hat{\alpha}_1^i x_2^i - \lambda_1 e_1^r - \xi_1 \operatorname{sgn}(s_1^r) - k_1 s_1^r\\
u_1^i &= -\hat{\alpha}_1^i y_1^r - \hat{\alpha}_1^r y_1^i + \hat{\alpha}_1^i y_2^r + \hat{\alpha}_1^r y_2^i - \hat{\alpha}_1^i x_1^r\\
&\quad - \hat{\alpha}_1^r x_1^i + \hat{\alpha}_1^i x_2^r + \hat{\alpha}_1^r x_2^i - \lambda_2 e_1^i - \xi_2 \operatorname{sgn}(s_1^i) - k_2 s_1^i\\
u_2^r &= -\hat{\alpha}_2^r y_1^r + \hat{\alpha}_2^i y_1^i + y_2^r + y_1^r y_3 - \hat{\alpha}_1^r x_1^r + \hat{\alpha}_1^i x_1^i + \hat{\alpha}_2^r x_1^r\\
&\quad - \hat{\alpha}_2^i x_1^i + \hat{\alpha}_2^r x_2^r - \hat{\alpha}_2^i x_2^i - x_1^r x_3 - \lambda_3 e_2^r - \xi_3 \operatorname{sgn}(s_2^r)\\
&\quad - k_3 s_2^r\\
u_2^i &= -\hat{\alpha}_2^i y_1^i - \hat{\alpha}_2^i y_1^r + y_2^i + y_1^i y_3 - \hat{\alpha}_1^r x_1^i - \hat{\alpha}_1^i x_1^r + \hat{\alpha}_2^r x_1^i\\
&\quad + \hat{\alpha}_2^i x_1^r + \hat{\alpha}_2^r x_2^i + \hat{\alpha}_2^i x_2^r - x_1^i x_3 - \lambda_4 e_2^i - \xi_4 \operatorname{sgn}(s_2^i)\\
&\quad - k_4 s_2^i
\end{aligned}$$

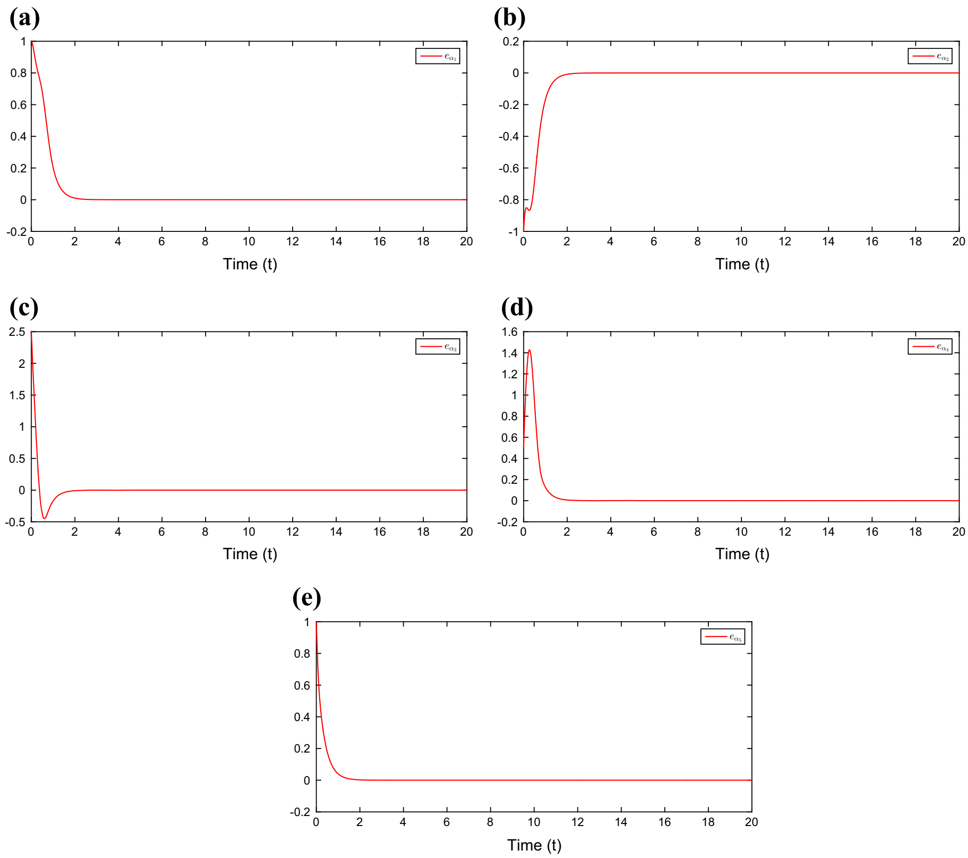

Figure 4. Errors of system parameters obtained from synchronisation of the leader–follower complex systems (17) and (20), using the ISMC method.

$$u_3 = -\frac{1}{2}(y_1^r y_2^r + y_1^i y_2^i) + \hat{\alpha}_3 y_3 + \frac{1}{2}(x_1^r x_2^r + x_1^i x_2^i) - \hat{\alpha}_3 x_3 - \lambda_5 e_3 - \xi_5 \operatorname{sgn}(s_3) - k_5 s_3. \tag{22}$$

In addition, complex system parameters $\alpha_1^r, \alpha_1^i, \alpha_2^r, \alpha_2^i$ and α_3 can be dynamically estimated as follows:

$$\begin{aligned}
\dot{\hat{\alpha}}_1 &= \dot{\hat{\alpha}}_1^r + j\dot{\hat{\alpha}}_1^i = -s_1(x_2 - x_1) + s_2 x_1 \\
&= -s_1^r(x_2^r - x_1^r) + s_1^i(x_2^i - x_1^i) + s_2^r x_1^r - s_2^i x_1^i \\
&\quad - j\left(s_1^i(x_2^r - x_1^r) - s_1^r(x_2^i - x_1^i) + s_2^r x_1^i + s_2^i x_1^r\right) \\
\dot{\hat{\alpha}}_2 &= \dot{\hat{\alpha}}_2^r + j\dot{\hat{\alpha}}_2^i = -s_2(x_1 + x_2) \\
&= -s_2^r(x_1^r + x_2^r) + s_2^i(x_1^i + x_2^i) \\
&\quad - j\left(s_2^i(x_1^r + x_2^r) - s_2^r(x_1^i + x_2^i)\right) \\
\dot{\hat{\alpha}}_3 &= \dot{\hat{\alpha}}_3^r = s_3 x_3.
\end{aligned} \tag{23}$$

Lyapunov's stability function can be utilised to prove that the equilibrium point $(e_1, e_2, e_3) = (0, 0, 0)$ of the leader system (17) is asymptotically stable. To verify this, some numerical results of the leader system (17) and the follower system (20) with the obtained feedback controller in (22) and the estimated system parameters in (23) are given in figures 3 and 4.

4. Conclusion

In this paper, ISMC method for the synchronisation of a class of complex chaotic systems is presented. An appropriate feedback controller and a system parameter strategy were designed based on the Lyapunov stability theorem. The performance evaluation of the proposed scheme was done by synchronisation of the Chen complex chaotic system and the Lorenz complex chaotic system. Finally, numerical analyses were performed to verify the effectiveness of the proposed ISMC method for the synchronisation of the complex chaotic systems. The results show that the synchronisation scheme provides the expected results from the point of view of both accuracy and speed.

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