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Cosmological implications of exponential harmonic field with collisional matter

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Abstract. We have studied the evolution of cosmological parameters by considering exponential harmonic field with collisional matter. A comparison has been made with the behavior of these parameters in the presence of ordinary matter and the model ΛCDM. We have also compared the evolution of these parameters with the ones obtained in the modified gravity $f(R)$ and $f(R,T)$ theory case. The results are in line with those of the modified gravity so that the harmonic exponential field can be used to explain why the Universe has gone from the deceleration phase to the acceleration phase.

Keywords. Collisional matter—exponential field—dark energy.

1. Introduction

The recent observations have shown that the Universe is in a phase of accelerated expansion (Riess *et al.* 1998, 2004; Springel *et al.* 2006). This acceleration of the Universe is due to two phenomena: dark matter and dark energy, which represent 95% of the content of the Universe, 28% for the dark matter and 67% for the dark energy (Oikonomou *et al.* 2014; Bernal *et al.* 2017; Aubourg *et al.* 2015). It should be that this phase of accelerated expansion is characterized by a negative state equation $w \simeq -1$. The major challenge of the scientific work is to show why the Universe has gone from the deceleration phase to the recent acceleration phase. Hence several theories were developed to explain this fact: the modified gravity $f(R,T)$ and $f(R)$. Let us mention that the Universe is accelerated for low values of the redshift, and its values which separate the acceleration and deceleration phases are called redshift transition, noted z_t, and is based on observations $z_t = 0.46 \pm 0.13$ (Riess *et al.* 2004, 2007).

Several authors used scalar fields to explain the acceleration of the Universe. Nozari (2014), for example, used the tachyon field to study the cosmological evolution of late-time to obtain very interesting results. Harko *et al.* (2016) studied the early-time and late-time cosmology for one model of modified gravity with extended non-minimal derivative couplings.

This work focusses on the behavior of cosmological parameters for the exponential harmonic field.

Overwise, Eells and Lemaire (1978), based on the harmonic maps, introduced exponential harmonic maps which they defined as a regular extremum of the exponential energy. The main interest of the use of exponentially harmonic maps is the fact that Lagrangian of this field is a generalization of bosonic string Lagrangian (with only a dilatonic field) written in Einstein frame (Lidsey *et al.* 1998). Indeed, if $\lambda \approx 0$, this model tends to bosonic string Lagrangian.

Kanfon *et al.* (2002) introduced exponentially harmonic field in cosmology and showed that this field can reproduce the phenomena of quintessence. Ten years later, they showed that exponentially harmonic field can play inflation (Kanfon & Lambert 2012).

Here, we focus on the era after recombination, where, in addition to the ordinary matter in the Universe, there are exponentially harmonic field and another matter in self-interaction. The objective is to verify if the exponentially harmonic field can be a good choice to explain the late time cosmology. Let us note that the model of collisional matter has been studied and showed interesting results (Kleidis and Spyrou 2011; Freese & Lewis

2002; Gondolo & Freese 2002; Xu & Huang 2012). Also the evolution of cosmological parameters with self-interacting matter has already been done by Oikonomou & Karagiannakis (2014) by considering the modified gravity $f(R)$ and by Baffou *et al.* (2016) by considering the modified gravity $f(R, T)$.

This paper is organized as follows: In section 2, we present the model. The collisional matter is recalled in section 3. Section 4 is devoted to the study of time evolution of the cosmological parameters where the Universe is considered to be filled by the usual ordinary matter and the collisional matter in the presence of exponentially harmonic field. In section 5, we examine the evolution of the equation of state of dark energy in the case of exponentially harmonic field, where the matter content is assumed as a fluid which is composed of collisional matter and radiation. Three types of potentials have been used: quadratic potential, exponential potential and Higgs potential. We conclude in section 6.

2. Presentation of the model

Let us consider the following action of F-harmonic scalar minimally coupled (Kanfon *et al.* 2002):

$$\int_M \mathrm{d}^4x\sqrt{-g}\left(-\frac{R}{2k^2}+F(\phi, e(\phi))\right), \tag{1}$$

where $k^2 = 8\pi G$, $e(\phi) = (\frac{\dot{\phi}^2}{2})$ and F is a F-harmonic map. If we pose

$$F(\phi, e(\phi)) = \exp\left(\frac{\lambda\dot{\phi}^2}{2}\right) - 1 - V(\phi), \tag{2}$$

we obtain the specific F-harmonic action, precisely the exponential harmonic action

$$\int_\lambda \mathrm{d}^4x\sqrt{-g}\left(-\frac{R}{2k^2}+\exp\left(\frac{\lambda\dot{\phi}^2}{2}\right) - 1 - V(\phi)\right). \tag{3}$$

Note that the general form of the last action can be present in the following form:

$$\int_\lambda \mathrm{d}^4x\sqrt{-g} \times \left(-\frac{R}{2k^2}+\exp\left(\frac{\lambda}{2}\partial_\alpha\phi\partial^\alpha\phi\right) - 1 - V(\phi)\right). \tag{4}$$

Here, we used action of the field with the Lagrangian of matter. Let us consider that

$$\int_\lambda \mathrm{d}^4x\sqrt{-g} \times \left(-\frac{R}{2k^2}+\exp\left(\frac{\lambda}{2}\partial_\alpha\phi\partial^\alpha\phi\right) - 1 - V(\phi)\right) + S_m, \tag{5}$$

where g is the trace of the metric $g_{\mu\nu}$, λ is the normalization parameter and R is the Ricci curvature, $R = g_{\mu\nu}R^{\mu\nu}$.

If we vary equation (5) with respect to $g_{\mu\nu}$, we deduce the equation of Einstein as

$$G_{\mu\nu} = k^2 T_{\mu\nu}, \tag{6}$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy-momentum tensor, and we identify as follows:

$$\begin{aligned} R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R = k^2\Bigg[&\frac{\lambda}{2}\partial_\mu\phi\partial_\nu\phi\exp\left(\frac{\lambda}{2}g_{\mu\nu}\partial_\alpha\phi\partial_\alpha\phi\right) + \\ &-\frac{1}{2}g_{\mu\nu}\exp\left(\frac{\lambda}{2}g_{\mu\nu}\partial_\alpha\phi\partial_\alpha\phi\right) \\ &-\frac{1}{2} - V(\phi)\Bigg] \\ &+k^2\frac{1}{\sqrt{g}}\frac{\delta S_m}{\delta g^{\mu\nu}}. \end{aligned} \tag{7}$$

Additionally, we vary (5) with respect to ϕ and this provides the Klein–Golden equation:

$$\ddot{\phi}(1+\lambda\dot{\phi}^2) + 3H\dot{\phi} + \frac{V^{'}(\phi)}{\lambda}\mathrm{e}^{(-\frac{\lambda}{2}\dot{\phi}^2)} = 0 \tag{8}$$

We are interested in investigating the cosmological implications of the minimal coupling. Hence, we focus on a spatially-flat Friedmann–Robertson–Walker (FRW) background metric of the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 + a^2(t)\delta^{ij}\mathrm{d}xi\mathrm{d}xj, \tag{9}$$

where t is the cosmic time, x^i are the co-moving spatial co-ordinates and $a(t)$ is the scale factor.

We deduce from equation (7), the Friedman equations

$$3H^2 = k^2\left[\frac{1}{2}\mathrm{e}^{\left(\frac{\lambda}{2}\dot{\phi}^2\right)}\left(-1+\lambda\dot{\phi}^2\right) + \frac{1}{2} + V(\phi) + T^m_{00}\right] \tag{10}$$

and

$$2\dot{H} + 3H^2 = k^2\left[-\frac{1}{2}\mathrm{e}^{\left(\frac{\lambda}{2}\dot{\phi}^2\right)} + \frac{1}{2} + V(\phi) + T^m_{11}\right]. \tag{11}$$

3. Recall of the collisional matter model

Note that the model of collisional matter was introduced for the first time by Fock (1959) and then by Kleidis and Spyrou (2011). This model is also studied in $f(R)$ gravity (Oikonomou & Karagiannakis 2014) and in $f(R, T)$ gravity (Baffou *et al.* 2016). The term ε_m is the total mass-energy density corresponding to matter and ρ_m is the energy density of matter. This total energy density ε_m is then given by the following expression:

$$\varepsilon_m = \rho_m + \rho_m \Pi, \tag{12}$$

where ρ_m refers to the part that does not change due to the usual matter content (Kleidis and Spyrou 2011; Fock 1959). When matter is collisional, the energy momentum tensor receives another contribution in terms of potential energy, that is Π, that includes all the extra interactions between the collisional matter. Then $\rho_m \Pi$ expresses the energy density part of the energy momentum tensor associated with thermodynamical content of the collisional matter (Baffou *et al.* 2016). This fluid is obviously not dust, but has a positive pressure and satisfies the following equation of state:

$$P_m = w\rho_m \tag{13}$$

with

$$0 < w < 1. \tag{14}$$

w denotes the parameter of equation of state of the collisional matter. $w = 0$ represents the non-collisional matter case. In this case, $\varepsilon_m = \rho_m$.

The potential energy density, expressing collisional matter can be written as

$$\Pi = \Pi_0 + 3w \ln\left(\frac{\rho_m}{\rho_{m0}}\right) \tag{15}$$

with ρ_{m0} and Π_0 being their current values. Then from equation (12) to (15), the total-energy density of the Universe can be written as

$$\varepsilon_m = \rho_m \left(1 + \Pi_0 + 3w \ln\left(\frac{\rho_m}{\rho_{m0}}\right)\right). \tag{16}$$

ε_m is the T_{00} component of the energy momentum tensor.

Throughout, the continuity equation and the motions of the volume elements in the interior of a continuous medium can be read

$$\nabla^{\mu} T_{\mu\nu} = 0. \tag{17}$$

Using the FRW line element (9), the conservation law of the equation (17) is

$$\dot{\varepsilon_m} + 3H(\varepsilon_m + p_m) = 0, \tag{18}$$

where $H = \frac{\dot{a}}{a}$ with a as the scalar factor. If one considers the scalar factor, one can give the following definition:

$$\rho_m = \rho_{m0}\left(\frac{a_0}{a}\right)^3 \tag{19}$$

with a_0 as the present value of the scale factor. The collisional matter is actually described by equations (16) and (19) and used in the rest of the paper. The value of Π_0 is equal to

$$\Pi_0 = \left(\frac{1}{\Omega_M} - 1\right). \tag{20}$$

4. Late-time cosmology

4.1 *Deceleration parameter*

Here we investigate several cosmological models in the framework of gravitational theories of the exponential scalar field focusing on the late-time evolution. For this, we use the Friedmann equations from the field equations (10) and (11):

$$3H^2 = k^2\left[\frac{1}{2}\mathrm{e}^{\left(\frac{\lambda}{2}\dot{\phi}^2\right)}\left(-1+\lambda\dot{\phi}^2\right) + \frac{1}{2} + V(\phi) + \varepsilon_m\right], \tag{21}$$

$$2\dot{H} + 3H^2 = -k^2(P_\phi + P_m), \tag{22}$$

where

$$\rho_\phi = \frac{1}{2}\mathrm{e}^{\left(\frac{\lambda}{2}\dot{\phi}^2\right)}\left(-1 + \lambda\dot{\phi}^2\right) + \frac{1}{2} + V(\phi), \tag{23}$$

$$P_\phi = \frac{1}{2}\mathrm{e}^{\left(\frac{\lambda}{2}\dot{\phi}^2\right)} - \frac{1}{2} - V(\phi). \tag{24}$$

ρ_ϕ and P_ϕ denote the energy density and pressure of the dark energy.

We introduce the deceleration parameter q, which is an indicator of the accelerated expansion, and is defined as follows (Harko *et al.* 2016):

$$q = \frac{\mathrm{d}}{\mathrm{d}t}\frac{1}{H} - 1. \tag{25}$$

The negatives values of q correspond to accelerating evolution. We can use the redshift z, defined as

$$1 + z = \frac{a_0}{a}. \tag{26}$$

In the following, we have posed $a_0 = 1$. The equations (19) and (16) lead to

$$\varepsilon_m = \rho_{mo} a^{-3}\left[1 + \Pi_o + 3wln(a)\right]. \tag{27}$$

Thus, time derivatives can be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}t} = -H(z)(1+z)\frac{\mathrm{d}}{\mathrm{d}z}. \tag{28}$$

If we combine equations (21) and (22), and use (8), we obtain the following system:

$$\begin{cases} 4\dot{H}+12H^2 = \mathrm{e}^{(-\frac{\lambda}{2}\dot{\phi}^2)}\left(-2+\lambda\dot{\phi}^2\right)+2+4V(\phi) \\ \qquad\qquad +2\varepsilon_m+2p_m, \\ \ddot{\phi}\left(1+\lambda\dot{\phi}^2\right)+3H\dot{\phi}+\frac{V'(\phi)}{\lambda}\mathrm{e}^{(-\frac{\lambda}{2}\dot{\phi}^2)}. \end{cases} \tag{29}$$

Now, let us introduce some dimensionless paramaters:

$$\tau = H_0 t, \quad H = H_0 h, \quad \phi = H_0\Phi,$$
$$\lambda = \frac{\mu}{H_0^2}, \quad V(\phi) = 3H_0 V(\Phi), \tag{30}$$

with H_0 a constant. From (30), we have

$$\dot{\phi} = H_0\frac{\mathrm{d}\Phi}{\mathrm{d}\tau},$$
$$\ddot{\phi} = H_0^2\frac{\mathrm{d}^2\Phi}{\mathrm{d}\tau^2},$$
$$\dot{H} = H_0^2\frac{\mathrm{d}h}{\mathrm{d}\tau}. \tag{31}$$

Using the new variables (30) and (31), system (29) becomes

$$\begin{cases} \frac{\mathrm{d}\Phi}{\mathrm{d}\tau} = \Xi, \\ \frac{\mathrm{d}h}{\mathrm{d}\tau} = -3h^2+\mathrm{e}^{(\frac{\mu}{2}\Xi^2)}(-2+\mu\Xi^2)+2+4V(\Phi) \\ \qquad +2\varepsilon_m+2p_m, \\ \frac{\mathrm{d}\Xi}{\mathrm{d}\tau} = -\frac{3h\Xi}{1+\mu\Xi^2}-\frac{V'(\Phi)}{\mu(1+\mu\Xi^2)}\mathrm{e}^{-(\frac{\mu}{2}\Xi^2)}. \end{cases}$$

In terms of the dimensionless variables, the deceleration parameter (25) becomes

$$q = \frac{\mathrm{d}}{\mathrm{d}\tau}\left(\frac{1}{h}\right)-1, \tag{32}$$
$$q = \frac{1}{2}\left(1+3w_{\mathrm{eff}}\right). \tag{33}$$

The dimensionless time-redshift relation (28) becomes

$$\frac{\mathrm{d}}{\mathrm{d}\tau} = -h(z)(1+z)\frac{\mathrm{d}}{\mathrm{d}z}. \tag{34}$$

System (32) is the main tool to be used for analysing the cosmological evolution taking into account the exponential harmonic field. In the following, we determine system (32) in the context of the redshift and we plot the decceleration parameters and state equations according to the different expressions of potential. Three potentials have been made: pawer-low potential, exponential potential and Higgs potential (Harko *et al.* 2016).

4.2 *Parameters behaviors*

4.2.1 *Power-law potential:* $V = b\Phi^m$. Using equations (16), (19) and (26), system (32) becomes

$$\begin{cases} \frac{\mathrm{d}\Phi}{\mathrm{d}z} = -\frac{\Xi}{(1+h)h}, \\ \frac{\mathrm{d}h}{\mathrm{d}z} = \frac{3h}{1+z}-\frac{\mathrm{e}^{(\frac{\mu}{2}\Xi^2)}(-2+\mu\Xi^2)}{(1+z)h} \\ \qquad -\frac{2+4b\Phi^m+2(1+w)\rho_{m0}(1+z)^3(1+\Pi_0+3wln(1+z))}{(1+z)h}, \\ \frac{\mathrm{d}\Xi}{\mathrm{d}z} = \frac{3\Xi}{1+\mu\Xi^2}+\frac{mb\Phi^{m-1}}{\mu(1+\mu\Xi^2)}\mathrm{e}^{-(\frac{\mu}{2}\Xi^2)}. \end{cases}$$

4.2.2 *Exponential potential:* $V = v_0\exp(-\mu_o\Phi)$. Equation (32) becomes

$$\begin{cases} \frac{\mathrm{d}\Phi}{\mathrm{d}z} = -\frac{\Xi}{(1+h)h}, \\ \frac{\mathrm{d}h}{\mathrm{d}z} = \frac{3h}{1+z}-\frac{\mathrm{e}^{(\frac{\mu}{2}\Xi^2)}(-2+\mu\Xi^2)}{(1+z)h} \\ \qquad -\frac{2+4v_o e^{(-\mu_o\Phi)}+2(1+w)\rho_{m0}(1+z)^3(1+\Pi_0+3wln(1+z))}{(1+z)h}, \\ \frac{\mathrm{d}\Xi}{\mathrm{d}z} = \frac{3\Xi}{1+\mu\Xi^2}-\frac{\mu_o v_o \mathrm{e}^{-\mu_o\Phi}}{\mu(1+\mu\Xi^2)}\mathrm{e}^{-(\frac{\mu}{2}\Xi^2)}. \end{cases} \tag{35}$$

4.2.3 *Higg's potential:* $V = v_o-\frac{1}{2}m^2\Phi^2+\frac{1}{4}\Lambda\Phi^4$. We assume that v_o, m and Λ are constants. According to Bezares-Roder Nils *et al.* (2007, 2010) where the Higgs potential is considered, we notice that $v_o = \frac{3}{4}\frac{\mu^4}{\alpha}$ with $\mu^2 > 0$ and $\alpha > 0$ as real-valued constants. Equation (32) becomes

$$\begin{cases} \frac{\mathrm{d}\Phi}{\mathrm{d}z} = -\frac{\Xi}{(1+h)h}, \\ \frac{\mathrm{d}h}{\mathrm{d}z} = \frac{3h}{1+z}-\frac{\mathrm{e}^{(\frac{\mu}{2}\Xi^2)}(-2+\mu\Xi^2)+2+4(v_o-\frac{1}{2}m^2\Phi^2+\frac{1}{4}\Lambda\Phi^4)}{(1+z)h} \\ \qquad -\frac{+2(1+w)\rho_{m0}(1+z)^3(1+\Pi_0+3wln(1+z))}{(1+z)h}, \\ \frac{\mathrm{d}\Xi}{\mathrm{d}z} = \frac{3\Xi}{1+\mu\Xi^2}+\frac{(-m\Phi+\Lambda\Phi^3)}{\mu(1+\mu\Xi^2)}\mathrm{e}^{-(\frac{\mu}{2}\Xi^2)}. \end{cases} \tag{36}$$

In Figures 1, 2 and 3, we plot deceleration parameters and equation of state for the power-law potential, exponential potential and Higgs potential.

In Fig. 1, the curve showing the evolution of the deceleration parameter in the presence of collisional matter is closer to the model ΛCDM than the non-collisional model. We also note that the transition from the deceleration phase to the acceleration phase is also carried out from high redshift to the low redshift values for collisional matter, ΛCDM and non-collisional matter. Futhermore, we see that this transition is realized for equation of state. These results are similar to that obtained by Oikonomou & Karagiannakis (2014) and Baffou *et al.* (2016).

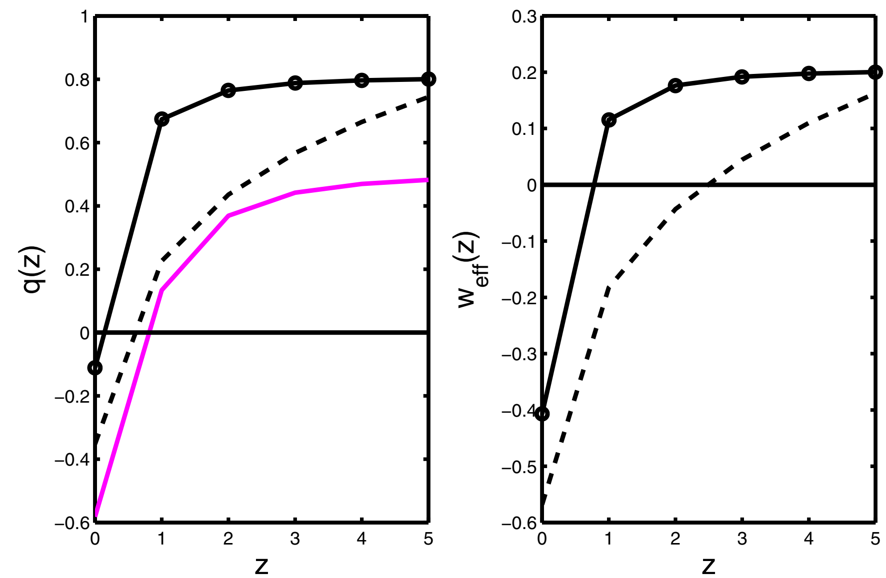

Figure 1. The graphs show evolution of the deceleration parameter (*left*), and effective equation of state parameter (*right*), as a function of the redshift, for exponential harmonic field in the case of the power-law potential. The dot, dashed and magenta refer to non-collisional matter, collisional matter and ΛCDM model, respectively, with $\lambda = 0.1$ and $m = 5$.

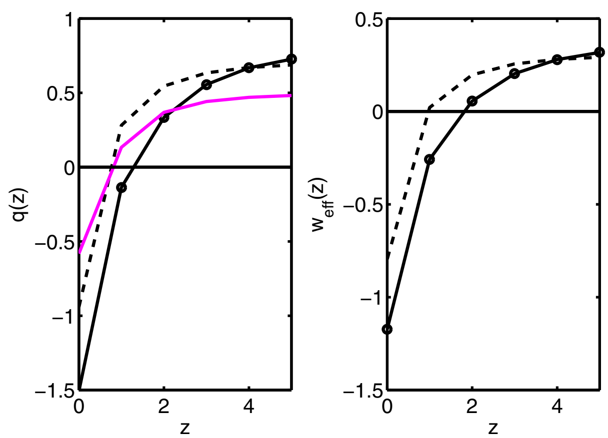

Figure 2. The graphs show evolution of the deceleration parameter (*left*), and effective equation of state parameter (*right*) as a function of the redshift, for exponential harmonic field in the case of the exponential potential. The dot, dashed and the magenta refer to non-collisional matter, collisional matter and ΛCDM model, respectively, with $\lambda = -0.01$, $m = -5$ and $v = -0.1$.

As seen in Fig. 2, for the curve representing the deceleration parameter evolution from the high redshift to the low redshift, we note that the transition from the deceleration phase to the acceleration phase is carried out in the presence of the collisional matter, ΛCDM model and non-collisional matter. Moreover, in the presence of the collisional matter, the curve is closer to the ΛCDM model than that of the non-collisional matter in the acceleration phase. But the curves showing the evolution of the deceleration parameter in the presence of the collisional matter and the non-collisional matter, coincides at high values of the redshift in the deceleration phase. This effect leads to the assertion that collisional matter does not exist at early times but it arises from ordinary matter at the low redshift and increases to ΛCDM, the low redshift as they see in modified gravity (Baffou *et al.* 2016).

Figure 3 shows the evolution of the deceleration parameter and the equation of state for an ordinary matter, a matter in self-interaction and the ΛCDM model for the Higgs potential of the exponential harmonic field. As for the previous potentials, the parameter behaviours (deceleration parameter and equation of state parameter) observed in Fig. 3 are similar to the existing results.

4.3 *Cardassian matter and exponential harmonic field*

We use the exponential harmonic field model and the Cardassian model. Note that Freese & Lewis (2002), Gondolo & Freese (2002) and Xu & Huang (2012) were the first who developped the Cardassian model. Other authors too have shown interest in this model (Baffou *et al.* 2016; Oikonomou & Karagiannakis 2014), in $f(R, T)$ and $f(R)$ modified gravity for study of the late-time cosmology. The Cardassian model of matter is characterized by negative pressure and there is no vacuum energy whatsoever. According to the so-called Cardassian model of matter, the total energy density ε_m of matter is given by

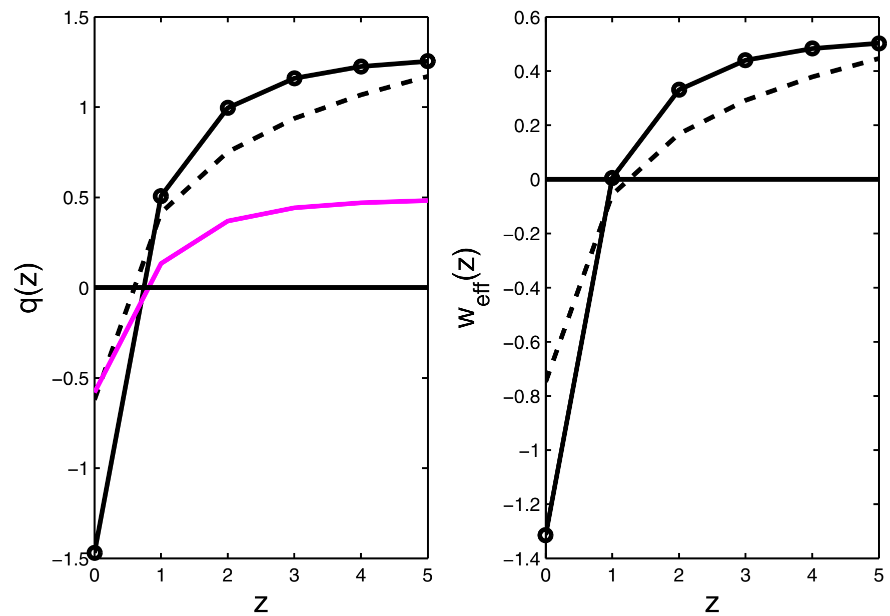

Figure 3. The graphs show evolution of the deceleration parameter (*left*), and effective equation of state parameter (*right*) as a function of the redshift, for exponential harmonic field in the case of the Higgs potential. The dot, dashed and magenta refer to non-collisional matter, collisional matter and ΛCDM model, respectively, with $v = 0.1$, $m = 5$ and $\Lambda = 2$.

$$\epsilon_m = \rho + \rho K(\rho), \tag{37}$$

where ρ stands for the ordinary matter-energy density and $K(\rho)$ is the term describing the new interacting matter and is, in general, a function of the ordinary mass-energy density.

In the original Cardassian model, the function $\rho K(\rho)$ takes the following form:

$$\rho K(\rho) = B\rho^l, \tag{38}$$

where B is a known real number, $l < \frac{2}{3}$ in order to guarantee the acceleration. From (38) in (37), we have the total density-energy as

$$\varepsilon_m = \rho + B\rho^l. \tag{39}$$

If we take into account (19) and cast it in the form $\rho = \rho_{m0}(1+z)^3$, then $a_0 = 1$ and $\frac{1}{a} = (1+z)$. The expression (39) may be expressed in terms of the redshift (z) as

$$\varepsilon_m = \rho_{m0}(1+z)^3[1 + B\rho_{m0}^{(l-1)}(1+z)^{3(l-1)}]. \tag{40}$$

Following Oikonomou & Karagiannakis (2014) and Baffou *et al.* (2016), we assume that the late-time evolution is governed by the geometric dark fluid and gravitating fluid with respect to negative pressure and positive pressure, satisfying the equation of state

$$p = w_k \rho. \tag{41}$$

Using (30) and (31), system (32) becomes

$$\begin{cases} \frac{\mathrm{d}\Phi}{\mathrm{d}\tau} = \Xi, \\ \frac{\mathrm{d}h}{\mathrm{d}\tau} = -3h^2 + \mathrm{e}^{(\frac{\mu}{2}\Xi^2)}(-2 + \mu\Xi^2) + 2 + 4V(\Phi) \\ +2\rho_{mo}(1+z)^3[w + 1 + B\rho^{l-1}(1+z)^{3(l-1)}], \\ \frac{\mathrm{d}\Xi}{\mathrm{d}\tau} = -\frac{3h\Xi}{1+\mu\Xi^2} - \frac{V'(\Phi)}{\mu(1+\mu\Xi^2)}\mathrm{e}^{-(\frac{\mu}{2}\Xi^2)}. \end{cases} \tag{42}$$

In the following, we will determine system (42) in the context of the redshift and we will plot the deceleration parameters and equation of state by exploring the different expressions of the potential. Three potentials have been considered: power-law potential, exponential potential and Higgs potential.

4.3.1 *Power-law potential:* $V = b\Phi^m$. In Fig. 4, the deceleration parameter and state of equation for the power-law potential have been plotted.

$$\begin{cases} \frac{\mathrm{d}\Phi}{\mathrm{d}z} = -\frac{\Xi}{(1+h)h}, \\ \frac{\mathrm{d}h}{\mathrm{d}z} = \frac{3h}{1+z} - \frac{\mathrm{e}^{(\frac{\mu}{2}\Xi^2)}(-2+\mu\Xi^2)}{(1+z)h} \\ \qquad - \frac{2+4b\Phi^m+2\rho_{m0}(1+z)^3\left[w+1+B\rho_{mo}^{l-1}(1+z)^{3(l-1)}\right]}{(1+z)h}, \\ \frac{\mathrm{d}\Xi}{\mathrm{d}z} = \frac{3\Xi}{1+\mu\Xi^2} + \frac{mb\Phi^{m-1}}{\mu(1+\mu\Xi^2)}\mathrm{e}^{-(\frac{\mu}{2}\Xi^2)}. \end{cases} \tag{43}$$

4.3.2 *Exponential potential:* $V = v_0 \exp(-\mu_o\Phi)$. In Fig. 5, the deceleration parameter and state of equation for the exponential potential have been plotted.

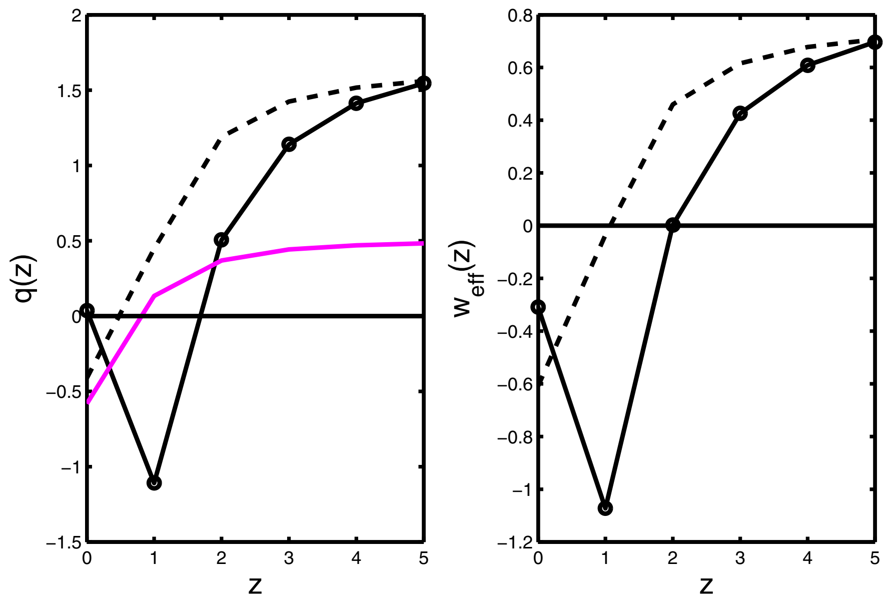

Figure 4. Evolution of deceleration parameter (*left*) and of effective equation of state parameter (*right*) as a function of the redshift, for exponential harmonic field in the case of the power-law potential. The dot, dashed and the magenta refer to non-collisional matter, collisional matter and ΛCDM model, respectively, with $\lambda = -0.01$ and $m = 0.5$.

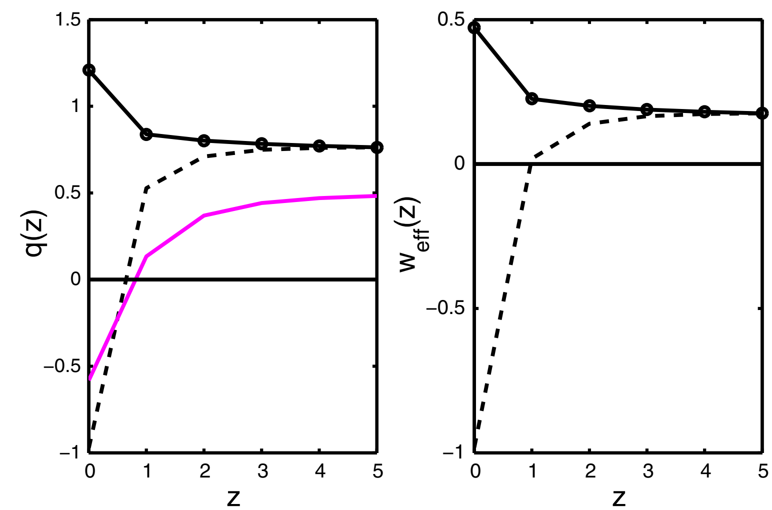

Figure 5. Evolution of deceleration parameter (*left*), and of effective equation of state parameter (*right*) as a function of the redshift, for exponential harmonic field in the case of the exponential potential. The dot, dashed and the magenta refer to non-collisional matter, collisional matter and ΛCDM model, respectively, with $\lambda = -0.01$, $l = 0.5$, $B = 0.2$ and $v = -0.1$.

4.3.3 *Higg's potential:* $V = v_o - \frac{1}{2}m^2\Phi^2 + \frac{1}{4}\Lambda\Phi^4$. In Fig. 6, the deceleration parameter and state of equation for the Higgs potential have been plotted.

In Fig. 4, for the deceleration parameter, we found that the phase of the deceleration to the acceleration phase is carried out from high to low values of the redshift for the Cardassian matter, ordinary matter and the ΛCDM model. The curve of the deceleration parameter representing the Cardassian matter and the ordinary matter is confused in the deceleration phase. While in the acceleration phase, the curve reflecting the behavior of the deceleration parameter in the presence of the Cardassian material moves away from that in the presence of the ordinary matter and approaches the curve of the ΛCDM model, which in the equation of state, we note that the transition from the decelerated phase to the accelerated phase is realized from high

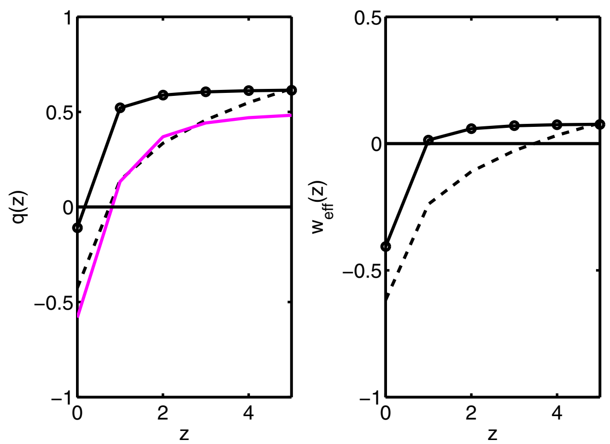

Figure 6. Evolution of deceleration parameter (*left*), and of effective equation of state parameter (*right*), as a function of the redshift, for exponential harmonic field in the case of the Higgs potential. The dot, dashed and the magenta refer to non-collisional matter, collisional matter and ΛCDM model, respectively, with $\upsilon = -0.1$, $l = 0.1$, $B = 0.45$ and $\Lambda = 0.2$.

to low values of the redshift in the same order of the content of the Universe. These results obtained for the harmonic exponential field are similar to those obtained in modified gravitation $f(R, T)$ and $f(R)$.

Figure 5 also shows the evolution of the deceleration parameter and the state equation parameter in the presence of the Cardassian material, of the ordinary matter and the ΛCDM model for the exponential potential. It has been found that the transition of the deceleration phase at the acceleration phase is performed for the deceleration parameter and the equation of state parameter. We found that in the deceleration phase, the curve representing the deceleration parameter and the equation of state parameter in the presence of the Cardassian matter and the ordinary matter are confounded. The deceleration parameter in the presence of the cardassian matter approximates the curve of the ΛCDM model. It should be noted that the curves of the exponential potential curves are also those obtained in the modified gravity $f(R, T)$ but are not the same for the gravity $f(R)$ for the exponential form of their respective models.

In Fig. 6, we can see the transition from the deceleration phase to the acceleration phase is carried out from high to low values of the redschift for the Cardassian matter, ordinary matter and for the ΛCDM model.

In the presence of Cardassian matter, we can see the same behaviour for deceleration parameter $q(z)$ and equation of state parameter in the case of quadratic potential as some authors have found in modified gravity

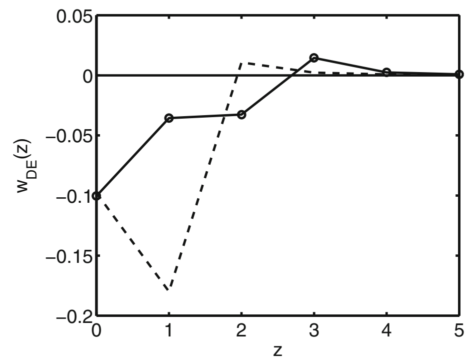

Figure 7. The graphs show w_{DE} as a function of the redshift z of the quadratic potential of collisional matter for $w = 0.6$ (dashed) and the non-collisional matter (dot) with $m = 0.25$ and $\lambda = 0.009$.

$f(R, T) = \lambda_o(\lambda + R)^n + T^\alpha$ and $f(R) = \lambda_o(\lambda + R)^n$. At low redshifts, in the exponential potential case, the cosmological parameters show the same behaviour as modified gravity $f(R, T) = R_o e^{\beta R} + T^\alpha$. When the redshift z increases, the trajectories of the previous models with cardassian matter show similar behaviors with exponential potential for harmonic exponential field.

5. Equation of state oscillations for exponential harmonic field with collisional matter

In section 4, we investigated the effect of collisional matter in the presence of harmonic exponential field on the late-time evolution of the Universe. In the present work, we shall consider the matter domination eras and also study the oscillatory behavior of exponential harmonic field as dark energy. We performed cosmology evolution in the context of exponential harmonic field with the matter composed by collisional matter and radiation (Baffou *et al.* 2016; Oikonomou *et al.* 2014).

From equations (10) and (11), we have

$$3H^2 = k^2\left[\frac{1}{2}e^{(\frac{\lambda}{2}\dot{\phi}^2)}(-1+\lambda\dot{\phi}^2) + \frac{1}{2} + V(\phi) + \rho_{\mathrm{matt}}\right], \tag{44}$$

$$2\dot{H} + 3H^2 = k^2\left[-\frac{1}{2}e^{(\frac{\lambda}{2}\dot{\phi}^2)} + \frac{1}{2} + V(\phi) + P_{\mathrm{matt}}\right], \tag{45}$$

where ρ_{matt} and P_{matt} denote the energy density and pressure of all perfect fluids of generic matter, respectively. We assume that the Universe is filled with collisional matter (self-interacting matter), the relativistic matter (radiation) and exponential harmonic field as dark energy. The matter energy density ρ_{matt} is given by

$$\rho_{\mathrm{matt}} = \varepsilon_m + \rho_{ro}a^{-4}, \tag{46}$$

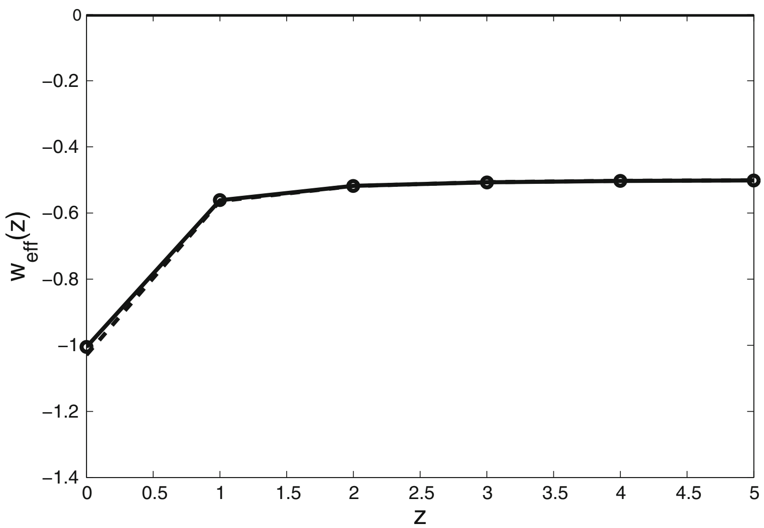

Figure 8. The graphs show w_{eff} as a function of the redshift z of the quadratic potential of collisional matter for $w = 0.6$ (dashed) and the non-collisional matter (dot) with $m = 0.5$ and $\lambda = -0.1$

where ρ_{ro} is the current energy density of radiation. The pressure of all perfect fluids of matter is given by

$$P_{\mathrm{matt}} = p_m + p_r. \tag{47}$$

From (27), we have

$$\rho_{\mathrm{matt}} = \rho_{mo}a^{-3}\left[1 + \Pi_o + 3wln(a)\right] + \rho_{ro}a^{-4}. \tag{48}$$

If $g(a) = a^{-3}\left[1 + \Pi_o + 3wln(a)\right]$ and $\chi = \frac{\rho_{ro}}{\rho_{mo}}$, we have

$$\rho_{\mathrm{matt}} = \rho_{mo}(g(a) + \chi a^{-4}), \tag{49}$$

where $\chi = (3.1).10^{-4}$ and the parameter $g(a)$ describes the nature of the collisional matter (viewed as perfect fluid). Combining (44) and (45) and using (8), we have

$$\begin{cases} 4\dot{H} + 12H^2 = \mathrm{e}^{(-\frac{\lambda}{2}\dot{\phi}^2)}(-2+\lambda) + 2 + 4V(\phi) \\ \qquad\qquad + 2\rho_{\mathrm{matt}} + 2P_{\mathrm{matt}} \\ \ddot{\phi}(1+\lambda\dot{\phi}^2) + 3H\dot{\phi} + \frac{V'(\phi)}{\lambda}\mathrm{e}^{(-\frac{\lambda}{2}\dot{\phi}^2)}. \end{cases} \tag{50}$$

Using the new variables (30) and (31), the system (50) becomes

$$\begin{cases} \frac{\mathrm{d}\Phi}{\mathrm{d}\tau} = \Xi, \\ \frac{\mathrm{d}h}{\mathrm{d}\tau} = -3h^2 + \mathrm{e}^{(\frac{\mu}{2}\Xi^2)}(-2+\mu\Xi^2) + 2 + 4V(\Phi) \\ \qquad +2(1+w)\rho_{\mathrm{matt}}, \\ \frac{\mathrm{d}\Xi}{\mathrm{d}\tau} = -\frac{3h\Xi}{1+\mu\Xi^2} - \frac{V'(\Phi)}{\mu(1+\mu\Xi^2)}\mathrm{e}^{-(\frac{\mu}{2}\Xi^2)}. \end{cases} \tag{51}$$

The energy density can take the following expression:

$$\rho_{\mathrm{matt}} = \rho_{mo}a^{-3}[1 + \Pi_o - 3wln(a^{-1}) + \chi(a^{-1})]. \tag{52}$$

In the context of the redshift, and (52), system (51) becomes

$$\begin{cases} \frac{\mathrm{d}\Phi}{\mathrm{d}z} = -\frac{\Xi}{(1+h)h}, \\ \frac{\mathrm{d}h}{\mathrm{d}z} = \frac{3h}{1+z} - \frac{\mathrm{e}^{(\frac{\mu}{2}\Xi^2)}(-2+\mu\Xi^2)}{(1+z)h} \\ -\frac{2+4V(\Phi)+2\rho_{m0}(1+w)(1+z)^3[1+\Pi_o-3wln(1+z)+\chi(1+z)]}{(1+z)h}, \\ \frac{\mathrm{d}\Xi}{\mathrm{d}z} = \frac{3\Xi}{1+\mu\Xi^2} + \frac{V'(\Phi)}{\mu(1+\mu\Xi^2)}\mathrm{e}^{-(\frac{\mu}{2}\Xi^2)}. \end{cases} \tag{53}$$

Equation (44) can then take the following form:

$$3H^2 = \rho_{\mathrm{DE}} + \rho_{\mathrm{matt}}. \tag{54}$$

We know that

$$W_{\mathrm{DE}} = \frac{P_{\mathrm{DE}}}{\rho_{\mathrm{DE}}} \Longrightarrow \rho_{\mathrm{DE}} = \frac{P_{\mathrm{DE}}}{W_{\mathrm{DE}}}. \tag{55}$$

Equation (55) in (54) give

$$3H^2 = \frac{P_{\mathrm{DE}}}{W_{\mathrm{DE}}} + \rho_{\mathrm{matt}}. \tag{56}$$

From (56), we have

$$\frac{P_{\mathrm{DE}}}{W_{\mathrm{DE}}} = 3H^2 - \rho_{\mathrm{matt}}. \tag{57}$$

Finally, by using ρ_{matt} of (48), we have

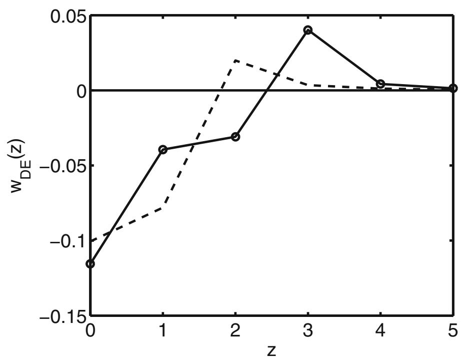

Figure 9. The graphs show the evolution cosmological of w_{DE} as a function of the redshift z of the exponential potential of collisional matter for $w = 0.6$ (blue) and non-collisional matter (red), with $\mu_o = 0.1$, $\upsilon = -0.1$ and $\nu = 0.1$.

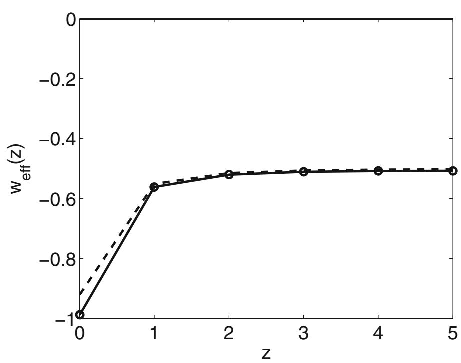

Figure 10. The graphs show $w_{\rm eff}$ as a function of the redshift z of the exponential potential of collisional matter for $w = 0.6$ (dashed) and the non-collisional matter (dot), with $\mu_o = 0.1$, $\upsilon = -2.9$ and $\nu = 0.1$

$$W_{\rm DE} = \frac{P_{\rm DE}}{3H^2 - \rho_{mo}a^{-3}[1 + \Pi_o + 3wln(a)] + \rho_{ro}a^{-4}}, \quad (58)$$

and using new variables (30) and (31) in the context of the redshift, gives

$$W_{\rm DE} = \frac{e^{(\frac{\mu_o}{2}\Xi^2)} - 1 - 2V(\phi)}{6h^2 - 2\rho_{mo}(1+z)^3[1 + \Pi_o - 3wln(1+z) + \chi(1+z)]}, \quad (59)$$

where $\chi = \frac{\rho_{ro}}{\rho_{mo}}$.

Equation (59) characterizes the equation to be used for describing the cosmological evolution of the dark energy in the Universe filled with collisional matter and radiation of the exponential harmonic field.

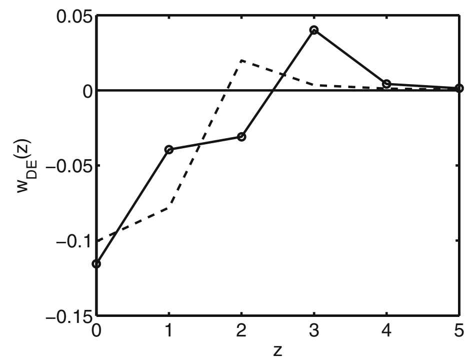

Figure 11. The graphs show $w_{\rm DE}$ as a function of the redshift z of the Higgs potential of collisional matter for $w = 0.6$ (dashed) and non-collisional matter (dot), with $m = 0.25$, $\upsilon = -0.1$ and $\Lambda = 1$.

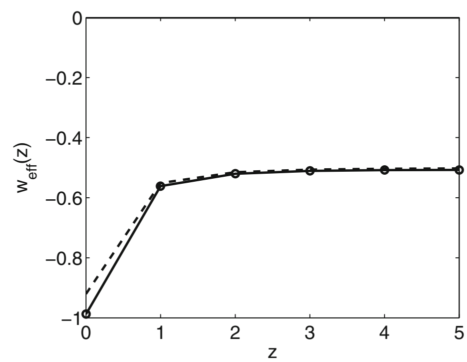

Figure 12. The graphs show $w_{\rm eff}$ as a function of the redshift z of the Higgs potential of collisional matter for $w = 0.6$ (dashed) the non-collisional matter (dot), with $m = 0.5$, $\upsilon = -0.1$ and $\Lambda = -0.5$

Now, we use three potentials to find the behavior of the two equations of state.

5.1 *Power-law potential:* $V = b\Phi^m$

Figures 7 and 8 show $w_{\rm DE}$ and $w_{\rm eff}$ as functions of the redshift z for power-law potential in collisional matter and non-collisional matter cases.

5.2 *Exponential potential:* $V = \upsilon_0 \exp(-\mu_o\Phi)$

Figures 9 and 10 show the cosmological behaviours of $w_{\rm DE}$ and $w_{\rm eff}$, respectively as functions of the redshift z for exponential potential in collisional matter and non-collisional matter cases.

5.3 *Higg's potential:* $V = v_o - \frac{1}{2}m^2\Phi^2 + \frac{1}{4}\Lambda\Phi^4$

Figures 11 and 12 show for w_{DE} and w_{eff} as functions for the redshift z for Higgs potential in collisional matter and non-collisional matter cases.

Figures 7, 8, 9, 10, 11 and 12 present the behavior of dark energy equation and the effective equation of state in the presence of collisional matter and radiation. We found that the effective equation of state parameter remains negative. Dark energy parameter have an oscillatory movement and goes to zero. These features prove that the co-existence of the collisional matter and radiation in the presence of exponential harmonic field do not change the well-known behaviors of the different parameters.

6. Conclusion

The purpose of this article is to investigate the effect of collisional matter on the late-time cosmological evolution in the case of the exponential harmonic field. Three potentials have been used: power-law potential, exponential potential and the Higgs potential.

We have assumed that in addition to the ordinary matter, there exists another matter in self-interaction with a positive pressure and exponential harmonic field in the Universe. We determined the behavior of the deceleration parameter and the equation of state parameter. We have found that the transition from the deceleration phase to the aceleration phase is performed. Basically we have noted that the curves that induce different parameters in the presence of the matter in self-interaction and exponential harmonic field are closer to those of the model ΛCDM than those in the presence of the ordinary matter, which confirms the necessity of the contribution of another matter in the Universe. We also studied the oxillatory behavior of the dark energy state parameter in the presence of the collision matter and radiation. The curves of the parameters in the presence of matter in self-interaction with the exponential harmonic field confirm those obtained in the modified gravities $f(R)$ and $f(R, T)$. Consequently, it can be affirmed that the exponential harmonic field is a serious candidate for explaining the transition from the aceleration phase to the deceleration phase of the Universe in the presence of the collisional matter.

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