Leader-following consensus of general fractional-order linear multi-agent systems via event-triggered control

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Abstract: The leader-following consensus problem of the general fractional-order linear multi-agent systems via event-triggered control is considered. An effective event-trigger controller is designed, and then the leader-following consensus problem of the controlled multi-agent systems is studied by using the Lyapunov theory of fractional-order systems and linear matrix inequality method. The consensus condition and the convergence rate of the system are obtained based on the Mittag–Leffler stability of fractional-order systems. Simulation indicates the effectiveness of the theoretical results.

1 Introduction

The distributed coordination control in multi-agent systems is widely applied in multidisciplines engineering such as the formation control of unmanned vehicles, the cooperation behaviour of intelligent robots etc. [1–4]. The common advantage is that the global collative behaviour of the system can be realised through local information exchange between each individual agent. This is the so-called consensus problem in multi-agent systems with distributed coordination control. A special case is that the consensus of a group relies on a leader agent, whose state is independent of all the other follower agents. It is called the leader-following consensus. For more information about the recently developed results in this area, see [5, 6].

However, a series of research achievements show that many physical systems are more suitable to be described by fractional-order dynamic model, for example, the synchronisation behaviour of agents in complex circumstances such as the macromolecule fluids or porous media [7], vehicles moving on road surface of viscoelastic materials and so on [8]. Also, many phenomena can be explained naturally by a coordinated behaviour of agents with fractional-order dynamics [9]. Hence, it is very meaningful to study the consensus problems of fractional-order systems. To the best of our knowledge, the consensus of fractional-order systems was first investigated in [10], the convergence analysis of consensus to fractional-order systems was further discussed in [11]. Recently, the leader-following consensus problem of fractional-order systems also considered by some authors [12].

It is noted that the work mentioned above all assumes that the information received by agents at each time will be transmitted to controllers, i.e. the consensus problem can be viewed as time-driven consensus. Compared to the time-driven consensus, the event-triggered consensus is more realistic. In this case, the information transmission and controller update can occur only when the events are triggered. Hence, to use the limited communication network resources efficiently, such novel controller has been widely introduced in the field of networked control systems. In [13, 14], centralised and distributed event-triggered control strategies have been employed to study the first-order or second-order consensus problem of multi-agent systems. Also, the leader-following asymptotic or exponential consensus problem based on event-triggered control is considered in [15, 16]. Motivated by these researches, this paper deals with event-triggered control design problem of general fractional-order linear systems and aims to obtain the condition of achieving the leader-following consensus.

The main contribution of this paper is that, first, we will extend the exponential consensus conclusion of integer-order model to the fractional-order case, based on the Mittag–Leffler stability of the fractional-order system. Second, the event-triggered control strategy is proposed to the fractional-order system, there are few reports in this field.

The rest of this paper is organised as follows. In Section 2, we will give preliminary knowledge about graph theory and fractional calculus, and then the leader-following consensus of fractional-order systems based on the Mittag–Leffler stability condition is introduced briefly in Section 3. In Section 4, we propose a class of event-triggered controller to achieve the consensus and the convergence rate of the controlled multi-agent systems is estimated. The numerical simulation illustrates the effectiveness of the theoretical results in Section 5. Finally, in Section 6 some concluding remarks are drawn from the investigation.

2 Preliminaries

2.1 Graph theory

A digraph noted as $\mathcal{G} = (\mathcal{V},\ \mathcal{E},\ \mathcal{A})$, in which $\mathcal{V} = (v_1,\ v_2,\ \dots,\ v_N)$ is the set of agents, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges and $\mathcal{A} = (a_{ij})_{N\times N}$ is the weighted adjacency matrix of $\mathcal{G}$. If the directed edge $(j,\ i) \in \mathcal{E}$, agent j is called a neighbour of the agent i with $a_{ij} > 0$, and agent i can receive information from the agent j; otherwise, $a_{ij} = 0$. The degree matrix is $\mathcal{D} = \mathrm{diag}\{d_1,\ d_2,\ \dots,\ d_N\}$ and $d_i = \sum_{j=1, j\neq i}^{N} a_{ij}$, the Laplacian matrix $\boldsymbol{L}$ of the weighted digraph $\mathcal{G}$ is defined as $\boldsymbol{L} = (l_{ij}) \in R^{N\times N} = \mathcal{D} - \mathcal{A}$. Let $D = \mathrm{diag}\{\hat{d}_1,\ \hat{d}_2,\ \dots,\ \hat{d}_N\}$ be the leader adjacency matrix of the union graph $\bar{\mathcal{G}} = \mathcal{G} \cup 0$, and we can also define $\bar{L} = \boldsymbol{L} + D = (\bar{l}_{ij})_{N\times N}$.

Lemma 1: A non-singular matrix $\boldsymbol{A} = (a_{ij})_{n\times n}$ with $a_{ij} \le 0,\ (i \ne j)$ is called an M-matrix, the following statements are equivalent [17]:

(i) All eigenvalues of $\boldsymbol{A}$ have positive real parts. That is $\mathrm{Re}(\lambda_i(\boldsymbol{A})) > 0 \quad i = 1,\ 2,\ \ldots,\ n$.
(ii) There exists a positive definite diagonal matrix $\boldsymbol{\Xi}$ such that $\boldsymbol{\Xi A} + A^{\mathrm{T}}\boldsymbol{\Xi}$ is positive definite.

2.2 Fractional calculus

The definition of Riemann–Liouville fractional integral [18] is

$$ {}_0I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}}\,\mathrm{d}\tau $$

and the Caputo fractional-order derivative [18] is defined as

$$ {}_0^C D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}}\,\mathrm{d}\tau $$

where n is the integer satisfying $n-1 < \alpha \le n$ and $\Gamma(z)$ is the Gamma function satisfying $\Gamma(z+1) = z\Gamma(z)$ for $z > 0$. In this paper, we consider the case of $0 < \alpha \le 1$. According to the fractional calculus theory, the following formula ${}_0^C D_t^{\alpha}({}_0I_t^{\alpha} f(t)) = f(t)$ holds.

Lemma 2: Let $V(t)$ be a continuous function on $[0,\ +\infty)$ satisfying ${}_0^C D_t^{\alpha} V(t) \le \theta V(t)$, where θ is a constant. Then

$$ V(t) \le V(0) E_{\alpha}(\theta t^{\alpha}) \quad t > 0. $$

The Mittag–Leffler function is $E_{\alpha}(z) = \sum_{k=0}^{\infty} (z^k/\Gamma(k\alpha+1))$. As $\alpha = 1$, the Mittag–Leffler function [19] evolves into the exponential function.

Lemma 3: Let $\boldsymbol{x}(t) \in \mathbb{R}^n$ be a vector of a differentiable function. Then, for any time instant $t \ge t_0$, the following relationship holds:

$$ \frac{1}{2}{}_0^C D_t^{\alpha}(x^{\mathrm{T}}(t)P\boldsymbol{x}(t)) \le x^{\mathrm{T}}(t)P{}_0^C D_t^{\alpha}\boldsymbol{x}(t) $$

where $P \in \mathbb{R}^{n\times n}$ is a symmetric and positive definite or semi-definite matrix [19].

3 Problem formation

Consider a group of N identical follower agents and a leader with general continuous time linear dynamics over directed network topology. The dynamics of the ith agent is described by

$$ {}_0^C D_t^{\alpha} x_i(t) = \boldsymbol{A}x_i(t) + \boldsymbol{B}u_i(t) \quad i = 1,\ 2,\ \ldots,\ N. \tag{1} $$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ denote the state and control input of agent i, respectively, $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ and $\boldsymbol{B} \in \mathbb{R}^{n\times m}$ are constant matrices. The leader indexed by 0 has dynamics given by

$$ {}_0^C D_t^{\alpha} x_0(t) = \boldsymbol{A}x_0(t). \tag{2} $$

Definition 1: The leader-following consensus is said to be reached under the Mittag–Leffler stability, if there exist positive constants $k > 0$, $\lambda > 0$, $0 < \beta \le 1$ and $T > 0$, such that

$$ \|x_i(t) - x_0(t)\| \le k(E_{\alpha}(-\lambda t^{\alpha}))^{\beta}, \tag{3} $$

for all $t > T$, λ is called the convergence rate.

Remarks 1: If the system is Mittag–Leffler stable, which implies the asymptotic stability. As $\alpha = 1$, it converts to the traditional exponential stability of the integer-order system. Hence, under Definition 1, the state of the following agents will converge to the leader agent with a general exponential convergence rate as time t is larger enough.

In this paper, we will study the leader–follower consensus problems of (1) and (2) with the event-triggered control strategy. The event-triggered controller designed as

$$ \begin{aligned} u_i(t) &= -K\sum_{j=1}^{N} a_{ij}(x_i(t_k^i) - x_j(t_k^i)) - K\hat{d}_i(x_i(t_k^i) - x_0(t_k^i)) \\ &\quad t \in [t_k^i,\ t_{k+1}^i), \end{aligned} \tag{4} $$

where $K \in \boldsymbol{R}^{m\times n}$ is the control gain to be designed later, t_k^i is the triggering times to be determined and $\hat{d}_i = 1$ if the agent i is connected to the leader, $\hat{d}_i = 0$ otherwise. The measurement error between the agent i and the leader is defined as $\varepsilon_i(t) = x_i(t) - x_0(t)$. By introducing the Laplacian matrix $\boldsymbol{L} = (l_{ij})_{N\times N}$, the multi-agent system (1) and (2) controlled by (4) can be rewritten as

$$ \begin{aligned} {}_0^C D_t^{\alpha}\varepsilon_i(t) &= \boldsymbol{A}\varepsilon_i(t) - \boldsymbol{B}K\sum_{j=1}^{N} l_{ij}\varepsilon_j(t_k^i) - \boldsymbol{B}K\hat{d}_i\varepsilon_i(t_k^i) \\ &\quad t \in [t_k^i,\ t_{k+1}^i). \end{aligned} \tag{5} $$

4 Event-trigger design and analysis

4.1 Event-triggered controller design

let $q_i(t) = \sum_{j=1}^{N} \bar{l}_{ij}\varepsilon_j(t)$ and $f_i(t) = q_i(t_k^i) - q_i(t)$, then $q_i(t_k^i) = f_i(t) + q_i(t)$. Designing the control gain $K = B^{\mathrm{T}}P$, where P is a positive definite matrix, the vector form of (5) can be expressed as

$$ {}_0^C D_t^{\alpha}\varepsilon(t) = (I_N \otimes \boldsymbol{A})\varepsilon(t) - (I_N \otimes BB^{\mathrm{T}}P)(f(t) + q(t)). \tag{6} $$

where $\varepsilon(t) = (\varepsilon_1^{\mathrm{T}}(t),\ \ldots,\ \varepsilon_N^{\mathrm{T}}(t))$, $f(t) = (f_1^{\mathrm{T}}(t),\ \ldots,\ f_N^{\mathrm{T}}(t))$, and $q(t) = (q_1^{\mathrm{T}}(t),\ \ldots,\ q_N^{\mathrm{T}}(t))$.

Consider a Lyapunov function candidate V defined as $V(t) = \varepsilon^{\mathrm{T}}(t)(\boldsymbol{\Xi} \otimes P)\varepsilon(t)$. Lemma 3 can be used to calculate the upper bound of ${}_0^C D_t^{\alpha} V(t)$ along the trajectory of system (6)

$$ \begin{aligned} {}_0^C D_t^{\alpha} V(t) \le{}& \varepsilon^{\mathrm{T}}(t)\big(\boldsymbol{\Xi} \otimes (A^{\mathrm{T}}P + P\boldsymbol{A})\big)\varepsilon(t) - 2\varepsilon^{\mathrm{T}}(t) \\ &\times (\boldsymbol{\Xi} \otimes PBB^{\mathrm{T}}P)q(t) - 2\varepsilon^{\mathrm{T}}(t)(\boldsymbol{\Xi} \otimes PBB^{\mathrm{T}}P)f(t). \end{aligned} \tag{7} $$

Since $q(t) = (\bar{L} \otimes I_N)\varepsilon(t)$, i.e. $\varepsilon(t) = (\bar{L}^{-1} \otimes I_N)q(t)$, inequality (7) can be rewritten as

$$ \begin{aligned} {}_0^C D_t^{\alpha} V(t) \le{}& q^{\mathrm{T}}(t)\Big(\big(\bar{L}^{-1}\big)^{\mathrm{T}}\boldsymbol{\Xi}\bar{L}^{-1} \otimes (A^{\mathrm{T}}P + P\boldsymbol{A})\Big)q(t) \\ &- q^{\mathrm{T}}(t)\Big([(\bar{L}^{-1})^{\mathrm{T}}\boldsymbol{\Xi} + \boldsymbol{\Xi}\bar{L}^{-1}] \otimes PBB^{\mathrm{T}}P\Big)q(t) \\ &- q^{\mathrm{T}}(t)\Big([(\bar{L}^{-1})^{\mathrm{T}}\boldsymbol{\Xi} + \boldsymbol{\Xi}\bar{L}^{-1}] \otimes PBB^{\mathrm{T}}P\Big)f(t) \\ ={}& q^{\mathrm{T}}(t)\big(\Phi_1 \otimes (A^{\mathrm{T}}P + P\boldsymbol{A})\big)q(t) - q^{\mathrm{T}}(t) \\ &\times (\Phi_2 \otimes PBB^{\mathrm{T}}P)q(t) - q^{\mathrm{T}}(t)(\Phi_2 \otimes PBB^{\mathrm{T}}P)f(t). \end{aligned} \tag{8} $$

we note $\Phi_1 = (\bar{L}^{-1})^{\mathrm{T}}\boldsymbol{\Xi}\bar{L}^{-1}$, $\Phi_2 = (\bar{L}^{-1})^{\mathrm{T}}\boldsymbol{\Xi} + \boldsymbol{\Xi}\bar{L}^{-1}$ for simplicity, since $\bar{L}$ is an M-matrix, from Lemma 1, we have $\Phi_2 > 0$. By using the inequality $x^{\mathrm{T}}y \le (a/2)x^{\mathrm{T}}x + (1/2a)y^{\mathrm{T}}y$ for all $a > 0$, it has

$$ \begin{aligned} q^{\mathrm{T}}(t)(\Phi_2 \otimes PBB^{\mathrm{T}}P)f(t) \le{}& \frac{a}{2}\lambda_{\max}^2(\Phi_2)q^{\mathrm{T}}(t)q(t) \\ &+ \frac{1}{2a}\lambda_{\max}^2(PBB^{\mathrm{T}}P)f^{\mathrm{T}}(t)f(t). \end{aligned} \tag{9} $$

Suppose P be the solution to the following Riccati inequality:

$$A^{\mathrm{T}}P + P\boldsymbol{A} - \frac{\lambda_{\min}(\Phi_2)}{\lambda_{\max}(\Phi_1)} PBB^{\mathrm{T}}P + \frac{a\lambda_{\max}^2(\Phi_2)}{2\lambda_{\max}(\Phi_1)} I_N + \frac{\theta}{\lambda_{\max}(\Phi_1)} I_N < 0,$$

then it holds

$$\begin{aligned} {}_0^C D_t^{\alpha} V(t) \le & \lambda_{\max}(\Phi_1)\Bigg(A^{\mathrm{T}}P + P\boldsymbol{A} - \frac{\lambda_{\min}(\Phi_2)}{\lambda_{\max}(\Phi_1)} PBB^{\mathrm{T}}P \\ & + \frac{a\lambda_{\max}^2(\Phi_2)}{2\lambda_{\max}(\Phi_1)}\Bigg) q^{\mathrm{T}}(t)q(t) + \frac{1}{2a}\lambda_{\max}^2(PBB^{\mathrm{T}}P) f^{\mathrm{T}}(t)f(t) \\ \le & -\theta\sum_{i=1}^{N} q_i^{\mathrm{T}}(t)q_i(t) + \frac{1}{2a}\lambda_{\max}^2(PBB^{\mathrm{T}}P)\sum_{i=1}^{N} f_i^{\mathrm{T}}(t)f_i(t). \end{aligned} \tag{10}$$

We can see that $V(t) = q^{\mathrm{T}}(t)(\Phi_1 \otimes I_N)q(t)$, and a sufficient condition for ${}_0^C D_t^{\alpha} V(t) \le -\gamma V(t)$ is

$$f_i^{\mathrm{T}}(t)f_i(t) \le 2a\frac{(\theta - \gamma\lambda_{\max}(\Phi_1))}{\lambda_{\max}^2(PBB^{\mathrm{T}}P)} q_i^{\mathrm{T}}(t)q_i(t),$$

with $0 < \gamma < (\theta/\lambda_{\max}(\Phi_1))$. It implies that $\|f_i(t)\| \le \eta\|q_i(t)\|$, where $\eta = \sqrt{2a(\theta - \gamma\lambda_{\max}(\Phi_1))}/\lambda_{\max}(PBB^{\mathrm{T}}P)$. Hence, we define the triggering function and the triggering time instants satisfying

$$e_i(t) = \|f_i(t)\| - \eta\|q_i(t)\| \quad t \in [t_k^i,\ t_{k+1}^i), \tag{11}$$

$$t_{k+1}^i = \inf\{t > t_k^i : e_i(t) > 0\}. \tag{12}$$

for an agent i, the events will be triggered at $t = t_k^i$ when $e_i(t) = 0$, and then $f_i(t)$ is reset to 0 automatically. Now, we are in the position to realise the leader-following consensus with the event-triggered control strategy.

4.2 Analysis of leader-following consensus

Theorem 1: Suppose Assumptions 1 and 2 hold, then the event-triggered controller designed in (4) with triggering function (11) and the control gain $K = B^{\mathrm{T}}P$ ensures that the leader-following consensus of the system (1) and (2) can be reached asymptotically with convergence rate γ under the Mittag–Leffler stability condition. Furthermore, the Zeno behaviour can be excluded.

Proof: For the triggering function (11), when $\|f_i(t)\| \le \eta\|q_i(t)\|$, we can derive that ${}_0^C D_t^{\alpha} V(t) \le -\gamma V(t)$. Thus, for some $t_1 \ge 0$, $V(t) \le V(t_1)E_{\alpha}\big(-\gamma(t - t_1)^{\alpha}\big)$ holds. On the other hand, since $V(t) = \varepsilon^{\mathrm{T}}(t)(\boldsymbol{\Xi} \otimes I_N)\varepsilon(t)$, it is obvious that $V(t) \ge \lambda_{\min}(\boldsymbol{\Xi})\varepsilon_i^{\mathrm{T}}(t)\varepsilon_i(t)$ for all i. Hence

$$\|\varepsilon_i(t)\| \le \sqrt{\frac{V(t_k^i)}{\lambda_{\min}(\boldsymbol{\Xi})}}\big(E_{\alpha}(-\gamma(t - t_k^i)^{\alpha})\big)^{1/2}.$$

Which means that the leader-following consensus is reached asymptotically with convergence rate γ.

Next, we will show the Zeno behaviour can be excluded. For $t \in [t_k^i,\ t_{k+1}^i)$, we have ${}_{t_k^i}^C D_t^{\alpha} f_i(t) = -{}_{t_k^i}^C D_t^{\alpha} q_i(t)$, thus

$$\begin{aligned} {}_{t_k^i}^C D_t^{\alpha}\|f_i(t)\| &\le \|\boldsymbol{A}\| \cdot \|\varepsilon_i(t)\| + \|BB^{\mathrm{T}}P\| \cdot \|q_i(t_k^i)\| \\ &\le \big(\|\boldsymbol{A}\| + \|BB^{\mathrm{T}}P\|\big)\sqrt{\frac{V(t_k^i)}{\lambda_{\min}(\boldsymbol{\Xi})}}, \end{aligned}$$

straight calculation shows that

$$\|f_i(t)\| \le \frac{\|\boldsymbol{A}\|\sqrt{V(t_k^i)/\lambda_{\min}(\boldsymbol{\Xi})} + \|BB^{\mathrm{T}}P\|\|q_i(t_k^i)\|}{\Gamma(1+\alpha)}(t - t_k^i)^{\alpha}.$$

When the event is triggered, we have $\|f_i(t_{k+1}^i)\| \ge (\eta/(\eta+1))\|q_i(t_k^i)\|$.

Thus, a low bound of the inter-event time is determined by

$$t_{k+1}^i - t_k^i \ge \left(\frac{\Gamma(1+\alpha)(\eta/(\eta+1))\|q_i(t_k^i)\|}{(\|\boldsymbol{A}\| + \|BB^{\mathrm{T}}P\|)\sqrt{V(t_k^i)/\lambda_{\min}(\boldsymbol{\Xi})}}\right)^{1/\alpha} > 0,$$

which means that Zeno behaviour has been excluded. □

5 Numerical examples

In this section, a numerical example is illustrated to verify the effectiveness of the theoretical results for the leader-following consensus of the fractional-order linear multi-agent systems.

Example 1: Consider a multi-agent system contains a leader and four agents under directed topology described as in Fig. 1. The system matrices are given as follows:

$$\boldsymbol{A} = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -0.3 & 0.4 \\ 0 & -0.4 & -0.3 \end{pmatrix} \quad \boldsymbol{B} = \begin{pmatrix} 0.1 \\ 0 \\ -0.1 \end{pmatrix}.$$

A simple checking shows that $(A,\ B)$ is stable. According to the graph theory, the Laplacian $\boldsymbol{L}$ and the matrix $\boldsymbol{D}$ written as

$$\boldsymbol{L} = \begin{pmatrix} 2 & -1 & 0 & -1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix} \quad \boldsymbol{D} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Obviously, $\bar{L} = \boldsymbol{L} + \boldsymbol{D} > 0$ is M-matrix, we can choose a positive definite diagonal matrix $\boldsymbol{\Xi}$ as

$$\boldsymbol{\Xi} = \begin{pmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0.792 & 0 & 0 \\ 0 & 0 & 0.812 & 0 \\ 0 & 0 & 0 & 0.798 \end{pmatrix}$$

such that $\Phi_1 = (\bar{L}^{-1})^{\mathrm{T}}\boldsymbol{\Xi}\bar{L}^{-1} > 0$, $\Phi_2 = (\bar{L}^{-1})^{\mathrm{T}}\boldsymbol{\Xi} + \boldsymbol{\Xi}\bar{L}^{-1} > 0$. Given $a = 1.6$, $\theta = 12$, the Riccati inequality

$$A^{\mathrm{T}}P + P\boldsymbol{A} - \frac{\lambda_{\min}(\Phi_2)}{\lambda_{\max}(\Phi_1)} PBB^{\mathrm{T}}P + \frac{a\lambda_{\max}^2(\Phi_2)}{2\lambda_{\max}(\Phi_1)} I_N + \frac{\theta}{\lambda_{\max}(\Phi_1)} I_N < 0$$

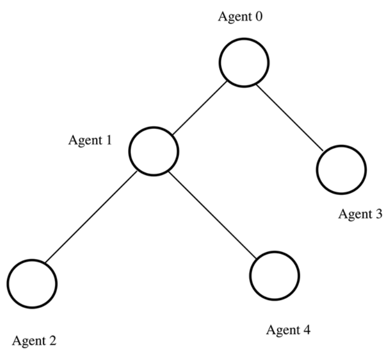

Fig. 1 *Topology of multi-agent systems*

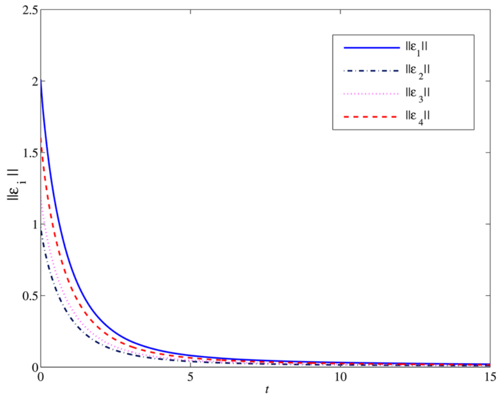

Fig. 2 *Trajectory of state error* $\|\varepsilon_i\|$, $i = 1, 2, 3, 4$

can be solved by using MATLAB. we can obtain a set of solutions $P > 0$ such as

$$P = \begin{pmatrix} 1.8416 & 1.2743 & 0.3952 \\ 1.2743 & 8.6112 & 1.2648 \\ 0.3952 & 1.2648 & 7.7020 \end{pmatrix} \quad K = B^{\mathrm{T}}P = \begin{pmatrix} 0.1419 \\ 0.0010 \\ -0.7307 \end{pmatrix}^T$$

Since $0 < \gamma < (\theta/\lambda_{\max}(\Phi_1)) = 1.0823$, chosen $\gamma = 0.9$, then $\eta = \sqrt{2a(\theta - \gamma\lambda_{\max}(\Phi_1))}/\lambda_{\max}(PBB^{\mathrm{T}}P) = 4.5906$. Under the control law (4) and the triggering function (11), the state error trajectories of $\|\varepsilon_i\| \quad i = 1, 2, 3, 4$ are converges to 0 very fast as shown in Fig. 2. Hence, the leader-following consensus is achieved. Note that: (i) the convergence speed of the consensus is based on Definition 1, which has a fast convergence rate similar to the exponential form. (ii) Since the event-time interval can be adjusted flexibly by changing the value of α due to value of $t^i_{k+1} - t^i_k$ is dependent on α. Hence, the event-triggered controller proposed in this paper is effective.

6 Conclusion

The proposed event-triggered controller combined with the triggering function is effective to guarantee the leader-following consensus of the controlled fractional-order multi-agent systems. The consensus condition of the system and the convergence rate estimate are based on the Mittag–Leffler stability of fractional-order systems, which have a faster convergence speed and it implies the asymptotic stability. In the future work, we will consider the leader-following consensus of event-trigger controlled multi-agent systems with large-scale network, but in this case, we should consider many actual factors such as non-linear factor and communication time delays.

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