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The gravitational redshift of a optical vortex being different from that of an gravitational redshift plane of an electromagnetic wave

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Abstract. A hypothesis put forward in late 20th century and subsequently substantiated experimentally posited the existence of optical vortices (twisted light). An optical vortex is an electromagnetic wave that in addition to energy and momentum characteristic of flat waves also possesses angular momentum. In recent years optical vortices have found wide-ranging applications in a number of branches including cosmology. The main hypothesis behind this paper implies that the magnitude of gravitational redshift for an optical vortex will differ from the magnitude of gravitational redshift for flat light waves. To facilitate description of optical vortices, we have developed the mathematical device of gravitational interaction in seven-dimensional time-space that we apply to the theory of electromagnetism. The resulting equations are then used for a comparison of gravitational redshift in optical vortices with that of normal electromagnetic waves. We show that rotating bodies creating weak gravitational fields result in a magnitude of gravitational redshift in optical vortices that differs from the magnitude of gravitational redshift in flat light waves. We conclude our paper with a numerical analysis of the feasibility of detecting the discrepancy in gravitational redshift between optical vortices and flat waves in the gravitational fields of the Earth and the Sun.

Keywords. Redshift—optical vortex—twisted light—angular momentum—relativistic rotator—orbital angular momentum.

1. Introduction

High monochromaticity and directionality being two prominent features of laser emission, its properties can thus be described using the concept of a wavefront where the phase of optical vibrations is identical in all points. If the laser beam has no optical vortices (twisted light) for $n = 0$, it can be represented by a system of surfaces having the same phase in every point. The distance between such neighboring surfaces is equal to the wavelength λ, with photon energies proportional to their frequencies $E = h\nu$ where h is the Planck constant and the momentum equals $p = h/\lambda$. The situation becomes different if optical vortices $n \neq 0$ are present in the laser beam. In this case an orbital angular momentum $L = hn/(2\pi)$ is imparted to photons with n being the orbital topological charge (Torres & Torner 2011; Arita *et al.* 2013). A wavefront in such an optical vortex comprises a helicoid, i.e. a spiral winding toward the wave propagation direction, with a special central point where amplitude is zero and phase is indefinite Refs. (Allen *et al.* 1992; He *et al.* 1995; Mehmood *et al.* 2014; McMorran *et al.* 2011; Kazak & Tolstik 2010). For instance, at $n = \pm 1$, it takes one turn around the wave vector to for the phase of the wave to change by 2π, while at $n = \pm 2$ a half-turn has the same effect.

Optical vortices have found wide-ranging application in recent years: thanks to their unique properties, they can be used efficiently in optical communications systems, can be harnessed for optical tweezers, and are remarkably useful in gravitation and cosmology (Portnov 2015; Tamburini *et al.* 2011).

This paper examines the magnitude of redshift in optical vortices and weak gravitational fields.

2. The optical vortex equation

The standard theory of gravitation uses the point-mass model to describe the motion of test bodies despite of the fact that all astrophysical objects have dimensions and may rotate around their axes. To overcome this contradiction, relativism was generalized to extended rotating bodies, which allowed the application of models of relativistic extended bodies to elementary particles (the relativistic rotator of the relativistic rotator) (Minkevich 2011; Huang *et al.* 2014; Popawski 2010; Hehl 2007; Banerjee 2010; Babourova *et al.* 2016; Kuvschinova & Panov 2014; Capozziello & Vignolo 2010; Gitman 2009; Staruszkiewicz 2008; Sadurni 2009). This direction in fundamental theoretical physics describes the processes occurring in the space of the symmetry parameters of the Lie group. For example, to describe the motion of rigid bodies, we can consider the rotation group $SO(3)$, a topological space formed by a set of points, each of which is a rotation of g in the Euclidean space R_3. When parameterized by Euler angles: φ is the rotation angle, ψ is the precession angle, θ is the nutation angle, the rotation can be represented as the composition of three rotations $g(\varphi, \psi, \theta)$.

On the group variety of a Lie group there exists the so-called Killing-Cartan metric, which for the rotation group has the form of a Euclidean metric, which implies that the variety of rotation groups is locally Euclidean (Cho 1975; Ne'eman & Regge 1978; Toller 1978; Toller & Vanzo 1978; Cognola *et al.* 1979). For convenience, in addition to the space-time coordinates, we will use the rotation coordinates determined by the Euler angles:

$$\begin{aligned} &x^0 = ct,\ x^1 = r,\ x^2 = \vartheta,\ x^3 = \phi, \\ &x^4 = r^4\varphi,\ x^5 = r^5\psi,\ x^6 = r^6\theta, \end{aligned} \tag{1}$$

where r^4, r^5, r^6 are coordinate coefficients:

$$r^4 = r^5 = r^6 = \sqrt{\frac{J}{m}}, \tag{2}$$

J is the moment of inertia of the test body relative to axes of rotation, precession and nutation, m is the mass of the body. The form of the metric in the coordinate space will be determined by the formula:

$$G_{AB} = g_{\alpha\beta} h^{\alpha}_{A} h^{\beta}_{B},$$

where h^{α}_{A} is the matrix of the tetrad coefficients, Greek symbols run from zero to 3, and the uppercase Latin characters are from zero to 6.

In this case, the calculations yield the following components for a non-perturbed metric:

$$\overset{0}{G}_{00} = 1,$$
$$\overset{0}{G}_{ab} = -1,$$
$$\overset{0}{G}_{45} = \overset{0}{G}_{54} = -\cos\theta$$

and the inverse matrix:

$$\overset{0}{G}{}^{00} = 1,$$
$$\overset{0}{G}{}^{11} = \overset{0}{G}{}^{22} = \overset{0}{G}{}^{33} = \overset{0}{G}{}^{66} = -1,$$
$$\overset{0}{G}{}^{44} = \overset{0}{G}{}^{55} = -(\sin\theta)^{-2},$$
$$\overset{0}{G}{}^{54} = \overset{0}{G}{}^{45} = \cos\theta(\sin\theta)^{-2},$$

where the numerical values over G refer to the order of the perturbation. To the resulting metric there corresponds a Riemannian Levi-Civita connection for which the affine-coupling coefficients are calculated by the formula:

$$\Gamma^{C}_{AB} = \frac{1}{2} G^{CD}(\partial_A G_{BD} + \partial_B G_{AD} - \partial_D G_{AB}).$$

As a result, we obtain unperturbed connectivity components:

$$\overset{0}{\Gamma}{}^{4}{}_{46} = \overset{0}{\Gamma}{}^{4}{}_{64} = \overset{0}{\Gamma}{}^{5}{}_{56} = \overset{0}{\Gamma}{}^{5}{}_{65} = \sqrt{\frac{m}{J}}\frac{\cot\theta}{2},$$
$$\overset{0}{\Gamma}{}^{4}{}_{56} = \overset{0}{\Gamma}{}^{4}{}_{65} = \overset{0}{\Gamma}{}^{5}{}_{46} = \overset{0}{\Gamma}{}^{5}{}_{64} = -\sqrt{\frac{m}{J}}\frac{1}{2\sin\theta},$$
$$\overset{0}{\Gamma}{}^{6}{}_{45} = \overset{0}{\Gamma}{}^{6}{}_{54} = \sqrt{\frac{m}{J}}\frac{\sin\theta}{2},$$

all other components of Γ^{C}_{AB} are equal to zero. These coupling coefficients correspond to the curvature tensor:

$$R^{A}_{BCD} = \partial_C \Gamma^{A}_{BD} - \partial_D \Gamma^{A}_{BC} + \Gamma^{A}_{FC}\Gamma^{F}_{BD} - \Gamma^{A}_{FD}\Gamma^{F}_{BC}.$$

Calculating the components of this tensor, we can establish the equalities:

$$\Lambda_{MN} = \overset{0}{R}_{MN},$$

$$\Lambda_{MN} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{m}{2J} & \frac{m\cos\theta}{2J} & 0 \\ 0 & 0 & 0 & 0 & \frac{m\cos\theta}{2J} & \frac{m}{2J} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{m}{2J} \end{pmatrix},$$

$$\overset{0}{R} = -\frac{3m}{2J},$$

which prove that the variety of the rotation group is a space of constant curvature. This allows us to write the gravitational field equations in the form:

$$R_{AB} = æ\left(T_{AB} - \frac{1}{2}G_{AB}T\right) + \Lambda_{AB},$$

where T_{AB} is the momentum energy tensor, and Λ_{AB} is the curvature tensor of the empty space.

One of the postulates of fundamental physics is the variational principle of the extremal action, according to which the trajectory of the motion of the physical system realizes the extremum of some functional composed of the dynamic variables of the given system. In the theory of gravitation, such an extremum is the motion of the body along the geodesic, that is, obeying the equation of motion:

$$u^A\left(\partial_A u^B + \Gamma^B_{CA}u^C\right) = 0.$$

The above equations describing the motion of test solids in gravitational fields in extended space-time with rotation allow one to describe optical vortices as extended rotating objects.

Components of the metric of unperturbed spherical space-time with rotation appear as follows at zero-order decomposition:

$$\overset{0}{G}_{00} = 1, \tag{3}$$
$$\overset{0}{G}_{22} = -r^2, \tag{4}$$
$$\overset{0}{G}_{11} = \overset{0}{G}_{44} = \overset{0}{G}_{55} = \overset{0}{G}_{66} = -1, \tag{5}$$
$$\overset{0}{G}_{33} = -r^2(\sin\vartheta)^2, \tag{6}$$
$$\overset{0}{G}_{45} = \overset{0}{G}_{54} = -\cos\theta, \tag{7}$$

where r, ϑ, ϕ are the radial coordinate, the zenith and azimuth angles of the spherical space, respectively. The components of the metric of the perturbed space, of the second and third order of the expansion can be obtained from gravitational equations in the form of equalities:

$$\overset{2}{G}_{00} = -\frac{æ}{4\pi}\int\frac{\overset{0}{T}{}^{00}(x)}{|x^1 - x|}d^3x, \tag{8}$$
$$\overset{2}{G}_{ab} = -\overset{0}{G}_{ab}\overset{2}{G}_{00}, \tag{9}$$
$$\overset{3}{G}_{a0} = -\overset{0}{G}_{ab}\frac{æ}{2\pi}\int\frac{\overset{1}{T}{}^{b0}(x)}{|x^1 - x|}d^3x, \tag{10}$$

Here lowercase Latin letters iterate from 1 to 6. We find that the components of the metric will be composed of unperturbed and perturbed parts:

$$G_{00} = \overset{0}{G}_{00} + \overset{2}{G}_{00},$$
$$G_{ab} = \overset{0}{G}_{ab} + \overset{2}{G}_{ab},$$
$$G_{a0} = \overset{1}{G}_{a0} + \overset{3}{G}_{a0}.$$

Interactions between charged particles will be described using a force field whose properties in contrast to the classical theory - will be characterized by a 7-dimensional vector of electric E_i and magnetic H_i field strength with functions of coordinates, time and orientation angle comprising vector components. As shown in Ref. (Portnov 2016), Maxwell's equations for an empty 7-dimensional time-space have the form:the form:

$$G^{ks}\partial_k H_s = 0, \tag{11}$$
$$\varepsilon_{sdh}G^{dm}G^{hf}\partial_m E_f = -\frac{1}{c}\frac{\partial}{\partial t}H_s, \tag{12}$$
$$G^{ks}\partial_k E_s = 4\pi\rho, \tag{13}$$
$$\varepsilon_{klm}G^{lh}G^{mf}\partial_h H_f = \frac{1}{c}\frac{\partial E_k}{\partial t} + \frac{4\pi}{c}j_k, \tag{14}$$

where ε_{kni} are symbols similar to the Levi-Civita notation; ρ is bulk density of electric charge; j_k are current density. Considering that the space-time at hand does not permit for multiplication of spatial and angular coordinates, the structure of ε_{kni} symbols is as follows:

$$\varepsilon_{ikl} = \begin{Bmatrix} -1 & (1,2,3);\ (2,3,1);\ (3,1,2); \\ & (4,5,6);\ (5,6,4);\ (6,4,5) \\ +1 & (3,2,1);\ (1,3,2);\ (2,1,3); \\ & (6,5,4);\ (4,6,5);\ (5,4,6) \\ 0 & \end{Bmatrix}$$

Derive a 7-dimensional curl from the Eq. (12):

$$\varepsilon_{eru}G^{rb}G^{us}\partial_b\varepsilon_{sdh}G^{dm}G^{hf}\partial_m E_f = -\frac{1}{c}\frac{\partial}{\partial t}\varepsilon_{eru}G^{rb}G^{us}\partial_b H_s.$$

Proceeding from the Eq. (14) in free space $\rho = 0$, $j_k = 0$, a wave equation of the following form can be obtained:

$$\varepsilon_{eru}\varepsilon_{sdh}G^{rb}G^{us}G^{dm}G^{hf}\partial_b\partial_m E_f = -\frac{1}{c^2}\frac{\partial^2 E_e}{\partial t^2}.$$

The resulting wave equation can be transformed as below with indices in Levi-Civita symbols raised:

$$\varepsilon_{eru}\varepsilon^{mfu}G^{rb}\partial_b\partial_m E_f = -\frac{1}{c^2}\frac{\partial^2 E_e}{\partial t^2}.$$

The product $\varepsilon_{eru}\varepsilon^{mfu}$ is a true tensor of the 6th rank that can be expressed as a combination of products of unit tensor components $\delta^i_k = G_{kn}G^{ni}$ using the following formula:

$$\varepsilon_{eru}\varepsilon^{mfu} = \delta^f_e\delta^m_r - \delta^m_e\delta^f_r.$$

This transform enables us to cast the wave equation into the form:

$$(\delta_e^f \delta_r^m - \delta_e^m \delta_r^f) G^{rb} \partial_b \partial_m E_f = -\frac{1}{c^2} \frac{\partial^2 E_e}{\partial t^2}.$$

which can be simplified to result in:

$$G^{mb} \partial_m \partial_b E_e - \partial_e G^{mb} \partial_b E_m = -\frac{1}{c^2} \frac{\partial^2 E_e}{\partial t^2}.$$

The other term in the left-hand part of the equation is zero according to the Eq. (13), thus the final wave equation for the electric component will appear as follows:

$$G^{mb} \partial_m \partial_b E_e + \frac{1}{c^2} \frac{\partial^2 E_e}{\partial t^2} = 0. \tag{15}$$

A similar transform applied to the Eq. (11) will result in the wave equation for the magnetic component.

We will assume the components of the electrical induction vector to be a function of temporal, spatial and rotational coordinates $\widetilde{E} = E_2(t, x^1, x^4)$, where $x^1 = r, x^4 = r^4 \varphi$ see (1), (2).

For a space with a massive rotating body, in view of Eqs. (3)-(7) and Eqs. (8)-(10), the Eq. (15) will appear as follows:

$$-\left(\frac{1}{1 + \dfrac{2GM}{c^2 r}}\right) \left(\partial_r^2 + w \partial_\varphi^2\right) \widetilde{E} + \frac{1}{c^2} \frac{\partial^2 \widetilde{E}}{\partial t^2} = 0,$$

where

$$w = \frac{c^6 r^2 - 4G^2 M^2 c^2 + 16 M \Omega^2 G^2 J_\Omega (\cos\theta)^2}{(c^6 r^2 - 4G^2 M^2 c^2 + 16 G^2 J_\Omega M \Omega^2) R_i^2 (\sin\theta)^2},$$

M is mass, J_Ω is the moment of inertia, Ω is the angular velocity of the body creating the gravitational field, G is Newtons gravitational constant, and $R_i = \sqrt{J/m}$ is a certain constant of the electromagnetic wave which can be provisionally named inertial radius. Assuming the nutation angle θ of the electromagnetic wave to be equal to $\pi/2$, the equation can be simplified to the form:

$$\left(\frac{1}{1 + \dfrac{2GM}{c^2 r}}\right) \left(\partial_r^2 + \frac{1}{f} \partial_\varphi^2\right) \widetilde{E} - \frac{1}{c^2} \frac{\partial^2 \widetilde{E}}{\partial t^2} = 0, \tag{16}$$

where

$$f = \left(1 + \frac{16 G^2 J_\Omega M \Omega^2}{c^6 r^2 - 4 G^2 M^2 c^2}\right) R_i^2.$$

Then the solution of this equation for the case of an electromagnetic wave traveling along the axis r and rotating around the same with phase turning at angle φ will appear as follows:

$$\widetilde{E}(t, r, \varphi) = A \cdot \exp\left(-i\left(\omega t - kr - n\varphi + \sigma\right)\right), \tag{17}$$

where $\omega = 2\pi\nu$ is the cyclical vibration frequency, $k = 2\pi/\lambda$ is the wave number, n is the orbital topological charge, numerically equal to the number of wave phase changes corresponding to a change of the angle φ by 2π radians. Substitution of Eq. (17) into Eq. (16) yields, after simplification:

$$\left(\frac{1}{1 + \dfrac{2GM}{c^2 r}}\right) \left(k^2 + \frac{n^2}{f}\right) - \frac{\omega^2}{c^2} = 0.$$

If the equation were modified to express wave frequency, the resulting equation would show the wave frequency in a gravity field:

$$\nu = \frac{c}{\lambda} \sqrt{\left(\frac{1}{1 + \dfrac{2GM}{c^2 r}}\right) \left(1 + \frac{n^2 \lambda^2}{4\pi^2 f}\right)},$$

or, decomposed in Taylor series:

$$\nu \approx \frac{1}{\sqrt{1 + \dfrac{2GM}{c^2 r}}} \left(\frac{c}{\lambda} + \frac{n^2 c \lambda}{8\pi^2 f}\right). \tag{18}$$

The laboratory wave frequency in an optical vortex, away from massive bodies:

$$\nu_0 \approx \frac{c}{\lambda} + \frac{n^2 c \lambda}{8\pi^2 R_i^2}. \tag{19}$$

It should be noted that the classical relation between wave frequency and wavelength breaks down when a topological charge $n \neq 0$ is present. The second term in Eq. (19) could be named the first topological adjustment for frequency.

Consider a dimensionless gravitational redshift quantity:

$$z = \frac{\nu_0 - \nu}{\nu},$$

where ν is the measured wavelength and ν_0 is the laboratory wavelength. With formulas (18) and (19) substituted, at the Newtonian limit with $2GM/(c^2 r) << 1$, the resulting redshift value will be a function of wavelength and topological charge:

$$z \approx \frac{\left(1+\frac{n^2\lambda^2}{8\pi^2 R_i^2}\right)\left(1+\frac{GM}{c^2 r}\right)}{1+\frac{n^2\lambda^2}{\left(1+\frac{16G^2 J_\Omega M\Omega^2}{c^6r^2-4G^2M^2c^2}\right)8\pi^2 R_i^2}} - 1. \quad (20)$$

3. Discussion

As is evident from Eq. (20), for a non-rotating gravitating body $\Omega = 0$ or for an electromagnetic wave having no topological charge $n = 0$, redshift value can be approximated by reduction to the classical formula:

$$z \approx \frac{GM}{c^2 r}.$$

When the electromagnetic wave possesses an orbital angular momentum $n \neq 0$ and the gravitating body is rotating $\Omega \neq 0$, gravitational redshift will differ from the classical one particularly by being wavelength-dependent and orbital topological charge.

For Earth the magnitude of corrections in the Eq. (20) is:

$$\frac{16G^2 J_\Omega M\Omega^2}{c^6r^2-4G^2M^2c^2} \sim 10^{-30},$$

while for the Sun it is:

$$\frac{16G^2 J_\Omega M\Omega^2}{c^6r^2-4G^2M^2c^2} \sim 10^{-21}.$$

Both are far below the margin of error afforded by contemporary measurement techniques. Thus, if corrections so minuscule were applied to the Eq. (20), they would be of no help for detecting the deviation of the standard redshift of a flat wave or the redshift of an optical vortex using state-of-the art measurement techniques on Earth and in the circumsolar range.

However, when the optical vortex passes near a rotating neutron star, the order of correction in Eq. (20) becomes significant, so that the dependence of the gravitational redshift on the wavelength λ and the topological charge n can be observed. In addition, if such a ray is recorded on the Earth from the gravitational redshift it will be possible to determine such characteristics of a neutron star as its mass M and the angular velocity of rotation Ω.

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