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Fault isolation of thrusters under redundancy in frame-structure unmanned underwater vehicles

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Abstract

This article deals with fault isolation issue for the redundant thrusters of frame-structure unmanned underwater vehicles (UUVs). Consistency check is adopted to accomplish this task while solving the reformed control input equations that are produced after getting rid of some fault-free terms from the given equations. Specially selected column vectors from the given control matrix together with the corresponding hypothetical thrust output faults are taken as the known/unknown elements for these equations. Redundant relations among the thrusters support the vector selection, which are revealed by analyzing the maximally linearly independent vectors of the given control matrix. Simulation with faulty thrusters under redundancy in a frame-structure unmanned underwater vehicle illustrates the effectiveness of the proposed methodologies.

Keywords

Fault isolation, unmanned underwater vehicle, thruster, redundancy, consistency check

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Introduction

Unmanned underwater vehicle (UUV) has been part of the currently researched emphases, with which the human beings could explore and exploit resources under the sea more conveniently.[1] However, the watery undersurface environment obstructs UUVs to carry out tasks freely and might induce faults on the working actuators. Fault diagnosis (FD) is a rigorous way to get the faulty situations on a UUV and would provide early and exact fault information.[2] Thus effective fault-tolerant control strategies could be designed for the safety assurance.[3,4]

FD contains the processes of fault detection, isolation, and identification, where different levels of fault information are generated. Fault isolation (FI) is used to locate the most possible place of the fault that has occurred on the object. Many research studies have concerned about this problem.[5–7] In current research issues, FI is tied up with fault detection, where once the relative fault parameters are estimated, the faults are detected and isolated. Because the correspondence between the estimable fault parameter and the actual faulty object is exclusive. For example, Zhang et al. dealt with the UUV thruster FD while the UUV is not configured over-actuated, that is, there is no extra thruster, and the FI is straightly achieved based on unambiguous and

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exclusive mapping between the estimated parameters and the faulty thrusters.[8]

Actuators/sensors are usually deployed under redundancies, thus the correspondence between the estimable parameter that presents the fault and the actually faulty object might not be exclusive. Hardware and analytical redundancies are two choices to provide actuators/sensors and analytical models as redundancies for the researched objects. They could be used for FI by comparing readings between the provided redundancies and the researched objects.[9] The redundant relations among the researched objects and the hardware/analytical redundancies are set up in advance, but they only include the actively added ones used for FD. Series of observers are usually adopted in parallel for this issue to estimate and isolate the fault, and each one is against an estimable fault parameter corresponding to one object that might be a redundancy.[10–12] The problem is that the redundant relations among the objects have not been analyzed, thus the fault might be difficult to distinguish.

While the corresponding relationships among the estimated parameters and the faulty objects are not exclusive, Willersrud et al. builds a series of fault residuals of which each has or has not a specific relation with all the given faults of the operations, and thus a dependency table is generated to isolate the faults by comparing its signatures.[13] It tries to find ways of distinguishing the faults among the redundant elements; however, the redundant relations are not apparent and only one fault is assumed to act on the system at a given time instant.

Since actuators (especially the thrusters) of a UUV guide its movements, the faulty ones are necessary to be isolated. This article focuses on the FI issue against the redundant thrusters of the frame-structure UUVs, while some of the redundant thrusters may have got into faulty simultaneously and only the sailing velocity, position, and attitude outputs can be collected. A methodology based on solving reformed control input equations could effectively give the FI results.

As the given control matrix in the control input equations includes the deploying relations among all the thrusters, the analysis against this matrix is carried out to look for special thruster sets that have no redundant relation, however, may contain all the faulty thrusters. With special elimination, the given control input equations are reformed to contain only a set of chosen thrusters, and a redundant equation exists in each subset of control input equations corresponding to a destroyed redundant relation, which could produce an error equation. Consistency check can be carried out through checking all these errors from the error equations,[14,15] where the chosen set coincided with 0 error all the time contains all the faulty thrusters, and the corresponding reformed control equations will present these thrusters and fault magnitude.

The remainder of this article is organized as follows: the second section gives the preliminary support for the following researches; the third section displays the main results including the redundancy analysis and FI; the fourth section posts a simulation on a frame-structure UUV to illustrate the effectiveness of the given methodologies; and the fifth section concludes this article.

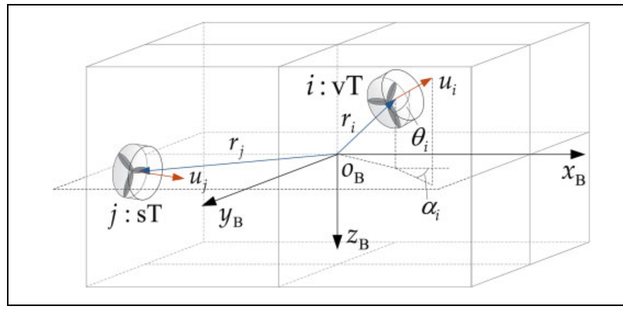

Figure 1. Frame of a frame-structure UUV. UUV: unmanned underwater vehicle.

Preliminaries and problem statement

The control input equation of the UUVs is usually expressed as[16]

$$\tau = B\boldsymbol{u} \tag{1}$$

where τ is the output vector of equation (1) and also the model control input vector of forces and moments for the dynamics system; B represents the control matrix that contains the deploying relations among all the thrusters; and $\boldsymbol{u}$ indicates the input vector of equation (1) and also the thrust output vector of the corresponding thrusters.

Figure 1 displays the frame of a frame-structure UUV configured with m thrusters among which the thruster vT is a vector and sT is a fixed one. The details of B and $\boldsymbol{u}$ in equation (1) are

$$B = \begin{bmatrix} \boldsymbol{e}_1 & \cdots & \boldsymbol{e}_i & \cdots & \boldsymbol{e}_m \\ \boldsymbol{r}_1 \times \boldsymbol{e}_1 & \cdots & \boldsymbol{r}_i \times \boldsymbol{e}_i & \cdots & \boldsymbol{r}_m \times \boldsymbol{e}_m \end{bmatrix} \tag{2}$$

$$\boldsymbol{e}_i = [\, \sin\theta_i \cos\alpha_i \quad \sin\theta_i \sin\alpha_i \quad \cos\theta_i \,]^\mathrm{T} \tag{3}$$

$$\boldsymbol{u} = [\, u_1 \quad \cdots \quad u_i \quad \cdots \quad u_m \,]^\mathrm{T} \tag{4}$$

where $\boldsymbol{e}_i$ represents the unit direction vector of the thrusts generated by the ith thruster; $\theta_i \in [0, \pi]$ is the angle between the ith thrust vector and the z_B coordinate axis; $\alpha_i \in (-\pi, \pi]$ is the angle between the projection of the ith thrust vector on $x_\mathrm{B} o_\mathrm{B} y_\mathrm{B}$ plane and the x_B coordinate axis; $\boldsymbol{r}_i$ indicates the position vector of the ith thruster from the buoyant centre o_B; and u_i is the corresponding thrust output. For the thruster vT labeled as i, angles θ_i and/or α_i are variables.

Usually, the input $\boldsymbol{u}$ in equation (1) is designed based on the demands of sailing velocity, position, and attitude and is used to produce the output vector τ. While any working thruster gets faulty, an undesired τ will be produced with the same designed $\boldsymbol{u}$. To make equation (1) work in a faulty case, we place some additive terms in equation (1) as the faults added into the system, that is

$$B_{\mathrm{f}}(\boldsymbol{u}_{\mathrm{d}} + \boldsymbol{f}_{\boldsymbol{u}}) = \tau_{\mathrm{d}} + \boldsymbol{f}_{\tau} \tag{5}$$

where $\boldsymbol{u}_{\mathrm{d}}$ and $\boldsymbol{f}_{\boldsymbol{u}}$ represent respectively the desired input and the additive input fault; τ_{d} and $\boldsymbol{f}_{\tau}$ are the desired output and the additive output fault, respectively; B_{f} is altered from B while the angles of vector thrusters contain additive faults.

If the frame-structure UUV is configured with redundant thrusters, the fault can hardly be diagnosed directly while $\boldsymbol{f}_{\tau}$ has been generated. Although equation (5) shows the strict correlation between $\boldsymbol{f}_{\boldsymbol{u}}$ and $\boldsymbol{f}_{\tau}$, the redundant relation increases the number of unknown elements in $\boldsymbol{f}_{\boldsymbol{u}}$, which is greater than the number of the usable equations in equation (5) and causes the solutions of equation (5) to be not unique. Thus, special solution will be generated in this article. A lemma is firstly introduced to justify whether there's any redundancy among the thrusters[17]:

Lemma 1. Consider control matrix $B \in \mathbb{R}^{n\times m}$ and the rank $r = \mathrm{rank}B$. If $r < m$, redundancy exists among the thrusters.

Lemma 1 indicates that the redundant relations among the thrusters are contained in control matrix B. A definition is given below for the subsequent analyses:

Definition 1. S_{maxLIV} is defined as a set of r maximally linearly independent column vectors from a given matrix B.

Definition 1 interprets that these r vectors are linearly independent; however, any $r + 1$ column vector from B are linearly dependent. Thus, any column vector of B could be represented by linearly combing the r vectors of S_{maxLIV}. The linear combination can be regarded as a redundancy of the former one and could replace it in some applications. With this analysis, the redundant relations in control matrix B might be got by finding the corresponding S_{maxLIV}.

Suppose that $B = [\boldsymbol{b}_1, \cdots, \boldsymbol{b}_j, \cdots, \boldsymbol{b}_m] \in \mathbb{R}^{n\times m}$ and $r = \mathrm{rank}B$, where $\boldsymbol{b}_j \in \mathbb{R}^{n\times 1}$ is the jth $(j = 1, 2, \cdots, m)$ column vector corresponding to the jth element of $\boldsymbol{u}$ in equation (1). Algorithm 1 given below is used to generate the S_{maxLIV} of B.[18]

With S_{maxLIV}, the redundancy relations and the probable combinations of faulty thrusters are hopeful to be discovered, which could help to reveal the FI issue through solving some equations.

Main results

This section will give the main methodology for isolating the faulty thrusters. Since redundant thrusters are usually mounted on a frame-structure UUV to make it over-actuated, it is necessary to clarify the redundant relations among the working thrusters for FI.

Algorithm 1 Solution of S_{maxLIV}

(1) Apply the elementary row transformation for B to get an echelon matrix

$$B_{\mathrm{e}} = \begin{bmatrix} 0 & \cdots & 0 & \tilde{b}_{1j_1} & * & \cdots & * & \tilde{b}_{1j_2} & * & \cdots & \tilde{b}_{1j_r} & * & \cdots & * \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \tilde{b}_{2j_2} & * & \cdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \tilde{b}_{rj_r} & * & \cdots & * \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix} \tag{6}$$

where $\tilde{b}_{ij_i} \neq 0$ $(i = 1, 2, \cdots, r)$ is the nonzero primary data of the ith line; $j_i = 1, 2, \cdots, m$ represents the unchanged column number; and $*$ indicates the matrix element.

(2) Pick $\forall i_1 \in \{j_1, \cdots, j_2 - 1\}$, $\forall i_2 \in \{j_2, \cdots, j_3 - 1\}, \ldots,$ $\forall i_r \in \{j_r, \cdots, m\}$. If

$$\mathrm{rank}\begin{bmatrix} \tilde{b}_{1i_1} & \tilde{b}_{1i_2} & \cdots & \tilde{b}_{1i_r} \\ 0 & \tilde{b}_{2i_2} & \cdots & \tilde{b}_{2i_r} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \tilde{b}_{ri_r} \end{bmatrix} = r \tag{7}$$

then $\{\boldsymbol{b}_{i_1}, \boldsymbol{b}_{i_2}, \cdots, \boldsymbol{b}_{i_r}\}$ is an S_{maxLIV} of B.

Redundancy analysis

Equation (1) transforms $\boldsymbol{u}$, the vector of all thruster outputs, into a vector of forces and moments through matrix B, where each column of B corresponds to an exclusive element of $\boldsymbol{u}$. It is clear that the deploying relations among all the thrusters are contained in B. Thus, analyzing the relations of all columns of B will help to obtain the thruster relations including the redundant ones.

Definition 1 manifests that if a column vector $\boldsymbol{b}_j$ from B does not lie in some

$$S_{\mathrm{maxLIV}k} \triangleq \{\boldsymbol{b}_{k_1}, \boldsymbol{b}_{k_2}, \cdots, \boldsymbol{b}_{k_r}\} \tag{8}$$

where k is a serial number and k_1, $k_2, \ldots,$ and k_r are column numbers, then $\boldsymbol{b}_j$ can be linearly expressed by the vectors of $S_{\mathrm{maxLIV}k}$, that is

$$u_j \boldsymbol{b}_j = \sum_{i=1}^{r} u_{k_i} \boldsymbol{b}_{k_i} \tag{9}$$

where coefficients u_j and u_{k_i} are corresponding elements of $\boldsymbol{u}$. Apparently, the combination of the r thrusters with input u_{k_i} is a redundancy of the jth one with input u_j. Choosing $\forall p \in \{1, 2, \cdots, r\}$, it is apparent that

$$u_{k_p} \boldsymbol{b}_{k_p} = u_j \boldsymbol{b}_j - \sum_{i=1, i\neq p}^{r} u_{k_i} \boldsymbol{b}_{k_i} \tag{10}$$

So $\{\boldsymbol{b}_{k_1}, \boldsymbol{b}_{k_2}, \cdots, \boldsymbol{b}_{k_{p-1}}, \boldsymbol{b}_j, \boldsymbol{b}_{k_{p+1}}, \cdots, \boldsymbol{b}_{k_r}\}$ is an S_{maxLIV}. Equation (10) indicates that any thruster could have a redundancy; however, some coefficients might work beyond the feasible regions, for example, being negative or zero.

A theorem is necessary to be displayed for analyzing the redundant relation between any two groups of thrusters on the basis of the aforementioned inference. Take any two groups of thrusters that are labeled as v and w with respective m^{v} and m^{w} thrusters. The output forces and moments provided by these thrusters are assumed to be $B^{\text{v}}\boldsymbol{u}^{\text{v}}$ and $B^{\text{w}}\boldsymbol{u}^{\text{w}}$, respectively, where $B^{\text{v}} \in \mathbb{R}^{n\times m^{\text{v}}}$ and $B^{\text{w}} \in \mathbb{R}^{n\times m^{\text{w}}}$. Defining $B = [B^{\text{v}}, B^{\text{w}}]$ and $r = \text{rank}B$, the theorem is shown below.

Theorem 1. If there's an $S_{\text{maxLIV}k}$ of B that satisfies

$$S_{\text{maxLIV}k} = \{\boldsymbol{b}^{\text{w}}_{k_1}, \boldsymbol{b}^{\text{w}}_{k_2}, \cdots, \boldsymbol{b}^{\text{w}}_{k_r}\} \tag{11}$$

where $k_i \in \{1, \cdots, m^{\text{w}}\}(i = 1, \cdots, r)$, then the thrusters corresponding to the column vectors of $S_{\text{maxLIV}k}$ in group w constitute a redundant relation with the ones in group v.

Proof. Suppose that $S_{\text{maxLIV}k}$ in equation (11) has been found by using Algorithm 1. Being similar to equation (9), it is clear that

$$u^{\text{v}}_j \boldsymbol{b}^{\text{v}}_j = \sum_{i=1}^{r} u^{\text{w}}_{jk_i} \boldsymbol{b}^{\text{w}}_{k_i} \tag{12}$$

where $\boldsymbol{b}^{\text{v}}_j$ is the jth ($j = 1, \cdots, m^{\text{v}}$) column vector of B^{v}; u^{v}_j and $u^{\text{w}}_{jk_i}$ are coefficients corresponding to vectors $\boldsymbol{b}^{\text{v}}_j$ and $\boldsymbol{b}^{\text{w}}_{k_i}$, respectively.

Adding up equation (12) for all j generates

$$\sum_{j=1}^{m^{\text{v}}} u^{\text{v}}_j \boldsymbol{b}^{\text{v}}_j = \sum_{i=1}^{r} u^{\text{w}}_{k_i} \boldsymbol{b}^{\text{w}}_{k_i} \tag{13}$$

where

$$u^{\text{w}}_{k_i} = \sum_{j=1}^{m^{\text{v}}} u^{\text{w}}_{jk_i} \tag{14}$$

Thus,

$$B^{\text{v}}\boldsymbol{u}^{\text{v}} = \sum_{i=1}^{r} u^{\text{w}}_{k_i} \boldsymbol{b}^{\text{w}}_{k_i} \tag{15}$$

where $\boldsymbol{u}^{\text{v}} \triangleq [u^{\text{v}}_1, u^{\text{v}}_2, \cdots, u^{\text{v}}_{m^{\text{v}}}]^{\text{T}}$. This completes the proof.

Using Theorem 1, the redundant relations of all the thrusters deployed on a frame-structure UUV can be clarified.

Algorithm 1 helps to find all S_{maxLIV} of B. However, some *local* S_{maxLIV} might exist, where the column vectors of different local S_{maxLIV} are linearly independent, that is, they have no redundant relation with each other. For example, let $B \triangleq [B_1, B_2, \boldsymbol{b}_1, \boldsymbol{b}_2] \in \mathbb{R}^{n\times m}$, where B_1 and B_2 contain vectors of redundant relations for $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$, respectively, but have no vector of redundant relation with each other. Then, the vectors of B_1 and B_2 constitute separate local S_{maxLIV}. By utilizing Theorem 1, the following Algorithm 2 is able to find out all the local S_{maxLIV}.

Algorithm 2 Solution of local S_{maxLIV}

(1) Give any S_{maxLIV} of B. Let matrices $S \in \mathbb{R}^{n\times r}$ and $R \in \mathbb{R}^{n\times(m-r)}$ contain the column vectors of S_{maxLIV} and the rest column vectors of B, respectively.
(2) Let $j = 1, 2, \cdots, m - r$. Pick column vector $\boldsymbol{b}^R_j$ of R.
(3) Let $i = 1, 2, \cdots, r$. Pick i column vectors from S. Use Theorem 1 to check whether they constitute a redundancy of $\boldsymbol{b}^R_j$ without any zero coefficient. If not, redo step (3). If yes, put $\boldsymbol{b}^R_j$ and the i column vectors in a set, namely redundant set; and restart step (2) with another j if at least one column vector of R has not been visited, otherwise goto step (4).
(4) Compare column vectors among all the redundant sets, and merge the sets that have at least one same column vector. The last left sets contain the local redundant relations. Changing these sets to corresponding matrices and using Algorithm 1 will finally generate all local S_{maxLIV} of B.

Proof. To find local S_{maxLIV}, the local redundant relations are necessary to be confirmed as the given S_{maxLIV} cannot make a redundant relation alone. Thus, matrices S and R are firstly set up to contribute column vectors jointly for constructing redundant relation.

Steps (1), (2), and (3) produce all the redundant relations; and step (4) combines the relations that have common column vectors. As the former three steps are apparent, only step (4) is necessary to be proved. Suppose there are two different redundant relation sets $S^{\text{v}} = \{\boldsymbol{b}^{\text{v}}_1, \boldsymbol{b}^{\text{v}}_2, \cdots, \boldsymbol{b}^{\text{v}}_{m^{\text{v}}-1}, \boldsymbol{b}^{\text{c}}\}$ and $S^{\text{w}} = \{\boldsymbol{b}^{\text{w}}_1, \boldsymbol{b}^{\text{w}}_2, \cdots, \boldsymbol{b}^{\text{w}}_{m^{\text{w}}-1}, \boldsymbol{b}^{\text{c}}\}$ which have only one common column vector $\boldsymbol{b}^{\text{c}}$, where m^{v} and m^{w} are column vector numbers of S^{v} and S^{w}, respectively.

Choose S_{maxLIV} of S^{v} as $\{\boldsymbol{b}^{\text{v}}_{k_1}, \boldsymbol{b}^{\text{v}}_{k_2}, \cdots, \boldsymbol{b}^{\text{v}}_{k_{r^{\text{v}}-1}}, \boldsymbol{b}^{\text{c}}\}$ and of S^{w} as $\{\boldsymbol{b}^{\text{w}}_{j_1}, \boldsymbol{b}^{\text{w}}_{j_2}, \cdots, \boldsymbol{b}^{\text{w}}_{j_{r^{\text{w}}-1}}, \boldsymbol{b}^{\text{c}}\}$, where r^{v} and r^{w} indicate ranks of the matrices corresponding to S^{v} and S^{w}. Any other vector $\boldsymbol{b}^{\text{v}}_{i^{\text{v}}}(i^{\text{v}} \in \{1, 2, \cdots, m^{\text{v}} - 1\}, i^{\text{v}} \neq k_1, k_2, \cdots, k_{r^{\text{v}}-1})$ from S^{v} and $\boldsymbol{b}^{\text{w}}_{i^{\text{w}}}(i^{\text{w}} \in \{1, 2, \cdots, m^{\text{w}} - 1\}, i^{\text{w}} \neq j_1, j_2, \cdots, j_{r^{\text{w}}-1})$ from S^{w} could be expressed by the vectors of these two S_{maxLIV}, respectively, that is

$$\boldsymbol{b}^{\text{v}}_{i^{\text{v}}} = u^{\text{v}}\boldsymbol{b}^{\text{c}} + \sum_{i=1}^{r^{\text{v}}-1} u^{\text{v}}_{k_i} \boldsymbol{b}^{\text{v}}_{k_i} \tag{16}$$

$$\boldsymbol{b}^{\text{w}}_{i^{\text{w}}} = u^{\text{w}}\boldsymbol{b}^{\text{c}} + \sum_{i=1}^{r^{\text{w}}-1} u^{\text{w}}_{j_i} \boldsymbol{b}^{\text{w}}_{j_i} \tag{17}$$

where u^{v}, u^{w}, $u^{\text{v}}_{k_i}$, and $u^{\text{w}}_{j_i}$ are coefficients and the former two are supposed to be nonzero. By combining equations (16) and (17) and eliminating $\boldsymbol{b}^{\text{c}}$, the following equation is achieved.

$$\boldsymbol{b}^{\text{v}}_{i^{\text{v}}} = \frac{u^{\text{v}}}{u^{\text{w}}}\boldsymbol{b}^{\text{w}}_{i^{\text{w}}} + \sum_{i=1}^{r^{\text{v}}-1} u^{\text{v}}_{k_i} \boldsymbol{b}^{\text{v}}_{k_i} - \frac{u^{\text{v}}}{u^{\text{w}}} \sum_{i=1}^{r^{\text{w}}-1} u^{\text{w}}_{j_i} \boldsymbol{b}^{\text{w}}_{j_i} \tag{18}$$

Equation (18) indicates that the randomly picked vector $\boldsymbol{b}_{i^{\mathrm{v}}}^{\mathrm{v}}$ from S^{v} can be represented by linearly combining the vectors of set $\{\boldsymbol{b}_{i^{\mathrm{w}}}^{\mathrm{w}}, \boldsymbol{b}_{k_1}^{\mathrm{v}}, \boldsymbol{b}_{k_2}^{\mathrm{v}}, \cdots, \boldsymbol{b}_{k_{r^{\mathrm{v}}-1}}^{\mathrm{v}}, \boldsymbol{b}_{j_1}^{\mathrm{w}}, \boldsymbol{b}_{j_2}^{\mathrm{w}}, \cdots, \boldsymbol{b}_{j_{r^{\mathrm{w}}-1}}^{\mathrm{w}}\}$, which behaves as an S_{maxLIV} of the merged set, similarly to $\boldsymbol{b}_{i^{\mathrm{w}}}^{\mathrm{w}}$.

Because the deduced redundant relation is related to the vectors of both former sets, it is necessary to merge the sets that have common column vectors. If the redundant relation sets have more than one common column vectors, similar processes could be deduced to get the same conclusion. Apparently, while any two local redundant relation sets have no common column vector, they cannot be merged.

The proof is completed.

The rest vectors of S do not constitute any local redundant relation with any vector of R, which are all independent and called *individual* vectors. With local S_{maxLIV} and individual vectors, the following subsection will display the deduction of FI methodology.

Fault isolation

To diagnose thruster faults, fault detection is usually used firstly to generate estimates of predefined fault parameters, after which FI is utilized to produce more accurate fault information for the fault-tolerant control. This subsection will figure out the FI issue on the basis of the stage that the additive output fault $\boldsymbol{f}_\tau$ in equation (5) has already been estimated through some published method.[19]

As the mounted thrusters are not far away from each other restricted to UUV size, and the ones contained in the same redundant relation could all have fault suspicion, it could image that each thruster in the same redundant relation bears the similar fault risk. Then, some assumptions are prepared.

Assumption 1. The disturbance is negligible.

The main job of this article is to reveal the information that hides in equation (1); however, the parameter estimation and fault-tolerant control are not included, so the disturbance related to those two steps is not considered temporarily.

Assumption 2. The redundant relation of two thrusters is nonexistent, that is $\boldsymbol{b}^{\mathrm{v}} \neq k\boldsymbol{b}^{\mathrm{w}}$.

If the redundant relation is supposed to exist, k is ± 1 based on equation (2). Thus, these two thrusters are placed in line with the same thrust direction or with the opposite thrust direction but the line passes through the buoyant centre. These two thruster deployments are a little inappropriate.

Assumption 3. The thruster fault is considered to behave on thrust magnitude and/or thrust direction only if the thruster is a vector.

To simplify and concentrate on the present preliminary issue, more complicated forms of faults are not included.

With these assumptions, a proposition is produced.

Proposition 1. The thrusters corresponding to the column vectors of an S_{maxLIV} will not get out of order simultaneously.

Proof. Restricting to the actual UUV size, the thrusters are mounted not far away from each other. The ones in the same redundant relation face common tasks and similar environments, thus the fault probabilities are similar. The probability of getting into faulty *simultaneously* for all the thrusters corresponding to an S_{maxLIV} is the product of respective fault probabilities of these thrusters, which is much smaller than the one of any thruster in the redundant relation. Thus, the faults in a redundant relation could occur *simultaneously* on some thrusters but not all the ones in an S_{maxLIV}. This completes the proof.

Remark 1. Proposition 1 emphasizes on the occurrence simultaneity of faults; however, the thrusters corresponding to an S_{maxLIV} might get out of order one by one. The FI mechanism given later will not wait to isolate the faulty thrusters until that all the ones corresponding to an S_{maxLIV} have got into faulty.

A theorem is given to search for the special set that contains the column vectors corresponding to all the faulty thrusters in a redundant relation.

Theorem 2. The column vectors corresponding to the faulty thrusters that belong to the same redundant relation could be contained in one S_{maxLIV}.

Proof. Suppose there are k thrusters that belong to the same redundant relation and are faulty. An S_{maxLIV} with r column vectors of this redundant relation is chosen randomly, where some k^S vectors correspond to faulty thrusters.

If $k^S < k < r$, any column vector $\boldsymbol{b}^R$ of the rest $k - k^S$ ones corresponding to the faulty thrusters can be represented by the vectors $\boldsymbol{b}_j^S$ from S_{maxLIV}, that is,

$$u^R \boldsymbol{b}^R = \sum_{j=1}^{r} u_j^S \boldsymbol{b}_j^S \tag{19}$$

where u^R and u_j^S are coefficients. Similarly to the change from equation (9) to equation (10), $u^R \boldsymbol{b}^R$ could be exchanged with some fault-free term on the right side of equal sign, such as

$$u_i^S \boldsymbol{b}_i^S = u^R \boldsymbol{b}^R - \sum_{j=1, j\neq i}^{r} u_j^S \boldsymbol{b}_j^S \tag{20}$$

where $u_i^S \boldsymbol{b}_i^S$ represents the fault-free term. Thus, the set $\{\boldsymbol{b}_1^S, \cdots, \boldsymbol{b}_{i-1}^S, \boldsymbol{b}^R, \boldsymbol{b}_{i+1}^S, \cdots \boldsymbol{b}_r^S\}$ could be taken as a substituted S_{maxLIV}. By taking the above steps repeatedly, all the $k - k^S$ vectors could be exchanged into an S_{maxLIV}.

While $k \geq r$, any r column vectors corresponding to faulty thrusters can be gradually exchanged into an S_{maxLIV} based on the above process. However, no r

thrusters would get into faulty simultaneously on the basis of Proposition 1. If faults occur on thrusters one by one in a period of time, the following FI methodology will not wait to work after r thrusters in a redundant relation have got out of order. Thus, the case of $k \geq r$ is not an issue in current research.

The proof is completed.

To reach the final goal of FI, choose $B \in \mathbb{R}^{n\times m}$ as the control matrix for a given frame-structure UUV with $r \triangleq \text{rank}B$. Suppose $B = [\boldsymbol{b}_1, \boldsymbol{b}_2, \cdots, \boldsymbol{b}_m]$ and thrust input $\boldsymbol{u}$ has the form from equation (4). While all thrusters are working, it is apparent that

$$\boldsymbol{b}_1 u_1 + \boldsymbol{b}_2 u_2 + \cdots + \boldsymbol{b}_m u_m = \tau_{\mathrm{d}} + \boldsymbol{f}_\tau \tag{21}$$

Toward the i th of all fixed thrusters, its output effort is

$$\boldsymbol{b}_i^{\mathrm{s}}(u_{\mathrm{d}i}^{\mathrm{s}} + \boldsymbol{f}_{ui}^{\mathrm{s}}) = \tau_{\mathrm{d}i}^{\mathrm{s}} + \boldsymbol{f}_{\tau i}^{\mathrm{s}} \tag{22}$$

where the superscript s is used to mark parameters of the fixed thruster; $\boldsymbol{b}_i^{\mathrm{s}}$ is a column vector from B; $u_{\mathrm{d}i}^{\mathrm{s}}$ and $\boldsymbol{f}_{ui}^{\mathrm{s}}$ represent respectively the desired input and the additive input fault of the i th fixed thruster; $\tau_{\mathrm{d}i}^{\mathrm{s}}$ and $\boldsymbol{f}_{\tau i}^{\mathrm{s}}$ are parts of τ_{d} and $\boldsymbol{f}_\tau$, respectively, and are all contributed by the i th fixed thruster. Since

$$\boldsymbol{b}_i^{\mathrm{s}} u_{\mathrm{d}i}^{\mathrm{s}} = \tau_{\mathrm{d}i}^{\mathrm{s}} \tag{23}$$

for that desired thrust output $u_{\mathrm{d}i}^{\mathrm{s}}$ produces desired model input $\tau_{\mathrm{d}i}^{\mathrm{s}}$, it could get that

$$\boldsymbol{b}_i^{\mathrm{s}} \boldsymbol{f}_{ui}^{\mathrm{s}} = \boldsymbol{f}_{\tau i}^{\mathrm{s}} \tag{24}$$

Toward the j th of all vector thrusters, its output effort is

$$\boldsymbol{b}_j^{\mathrm{v}}(u_{\mathrm{d}j}^{\mathrm{v}} + \boldsymbol{f}_{uj}^{\mathrm{v}}) \triangleq \boldsymbol{b}_j^{\mathrm{v}} u_j^{\mathrm{v}} = \tau_{\mathrm{d}j}^{\mathrm{v}} + \boldsymbol{f}_{\tau j}^{\mathrm{v}} \tag{25}$$

where the definitions of these parameters are similar to the fixed one's and are omitted. Because vector $\boldsymbol{b}_j^{\mathrm{v}}$ contains angle variables which might be faulty (see equation (3)), the terms on the left side of the second equal sign is nonlinear and cannot be simplified as the fixed thruster.

Remark 2. The usage of eliminating desired inputs from control input equations for fixed thrusters is to reduce the unnecessary effects. However, the desired inputs of vector thrusters cannot be eliminated because of the nonlinearity.

Suppose there are m^{s} fixed and m^{v} vector thrusters. Based on the above simplifying derivations, equation (21) is reformed as

$$\sum_{i=1}^{m^{\mathrm{s}}} \boldsymbol{b}_i^{\mathrm{s}} \boldsymbol{f}_{ui}^{\mathrm{s}} + \sum_{j=1}^{m^{\mathrm{v}}} \boldsymbol{b}_j^{\mathrm{v}} u_j^{\mathrm{v}} = \sum_{j=1}^{m^{\mathrm{v}}} \tau_{\mathrm{d}j}^{\mathrm{v}} + \boldsymbol{f}_\tau \tag{26}$$

which is not solvable if some thrusters are redundant for that $r < m$ indicates that the equations for the unknowns are lacking.

Suppose there are m^{I} *individual* column vectors and m^{L} independent sets of *local* redundant relations. Define r_i as the column vector number of the local S_{maxLIV} from the i th local redundant relation.

With Theorem 2, the vectors corresponding to all faulty thrusters in a local redundant relation could be contained by a local S_{maxLIV}. While constructing equation (26), if all the thrusters corresponding to the vectors from one local S_{maxLIV} of each independent local redundant relation, together with the individual vectors, are kept and the other fault-free (assumed) ones are eliminated, the following group of equations will be got with r unknown parameters.

$$\sum_{i=1}^{m^{\mathrm{L}}} \sum_{j=1}^{r_i} \boldsymbol{b}_{i_j} u_{i_j} + \sum_{k=1}^{m^{\mathrm{I}}} \boldsymbol{b}_k^{\mathrm{I}} u_k^{\mathrm{I}} - \tau_{\mathrm{d}}^{\mathrm{v}} = \boldsymbol{f}_\tau \tag{27}$$

where $\boldsymbol{b}_{i_j}$ indicates the j th vector of the chosen S_{maxLIV} in the i th redundant relation; $\boldsymbol{b}_k^{\mathrm{I}}$ is the k th vector of all the individual vectors; u_{i_j} and u_k^{I} are unknown parameters corresponding to $\boldsymbol{b}_{i_j}$ and $\boldsymbol{b}_k^{\mathrm{I}}$, respectively, and equal to $\boldsymbol{f}_{ui}^{\mathrm{s}}$ from equation (24) or u_j^{v} from equation (25) based on the thruster type (fixed or vector); $\tau_{\mathrm{d}}^{\mathrm{v}}$ is the desired model control input vector corresponding to the chosen vector thrusters.

For some thruster combinations in equation (27), all the faulty thrusters or the redundancies of some faulty ones are contained. Thus, with the help of trigonometric functions ($\sin^2 * + \cos^2 * = 1$) corresponding to the angle variables of vector thrusters, they (equation (27)) are solvable. However, the actual faults are confused with the redundant results among all the solutions, which cannot be isolated by comparing these solutions with some predefined thresholds.

Remark 3. The terms of fault-free (assumed) thrusters are eliminated from equation (26) to produce equation (27) for the reason that the corresponding terms on both sides of the equal sign are always equivalent based on analyzing equations (24) and (25).

Against the above issue, a further simplification process is necessary. As Proposition 1 says, not all thrusters corresponding to an S_{maxLIV} will get into faulty simultaneously. Thus, combining any $r_i - 1 (i = 1, 2, \cdots, m^{\mathrm{L}})$ column vectors from one S_{maxLIV} of each local redundant relation with all individual vectors ($\sum_{i=1}^{m^{\mathrm{L}}}(r_i - 1) + m^{\mathrm{I}}$ vectors in total) will obtain the vectors corresponding to all faulty thrusters.

Picking all individual vectors and any $r_i - 1$ column vectors from a local S_{maxLIV} of each (i th) redundant relation could constitute a matrix

$$B^\dagger \in \mathbb{R}^{n\times\left[m^{\mathrm{I}} + \sum_{i=1}^{m^{\mathrm{L}}}(r_i - 1)\right]} \tag{28}$$

Define an unknown vector $u^\dagger$ corresponding to the column vectors of $B^\dagger$, where the element (u^{s}) corresponding to a fixed thruster indicates the additive thrust fault ($\boldsymbol{f}_u^{\mathrm{s}}$) merely and the element (u^{v}) corresponding to a vector thruster is the faulty thrust input $(u_{\mathrm{d}}^{\mathrm{v}} + \boldsymbol{f}_u^{\mathrm{v}})$. Let the desired output vector of all the vector thrusters contained in $B^{\dagger}$ be $\tau_{\mathrm{d}}^{\mathrm{v}\dagger}$. Referring to equation (5), $\boldsymbol{f}_u^{\mathrm{s}}$ and $\boldsymbol{f}_u^{\mathrm{v}}$ are elements of $\boldsymbol{f}_{\mathbf{u}}$, $u_{\mathrm{d}}^{\mathrm{v}}$ is an element of $\boldsymbol{u}_{\mathrm{d}}$, and $\tau_{\mathrm{d}}^{\mathrm{v}\dagger}$ is part of τ_{d}.

Reconsidering about constructing equation (27) with only the aforementioned $\sum_{i=1}^{m^{\mathrm{L}}}(r_i - 1) + m^{\mathrm{I}}$ vectors directly gives

$$B^{\dagger}\boldsymbol{u}^{\dagger} - \tau_{\mathrm{d}}^{\mathrm{v}\dagger} = \boldsymbol{f}_{\tau} \tag{29}$$

where $B^{\dagger}$, $\boldsymbol{u}^{\dagger}$, and $\tau_{\mathrm{d}}^{\mathrm{v}\dagger}$ are defined as before; and $\boldsymbol{f}_{\tau}$ could be estimated during the fault detection.

On the basis of the preceding preparation, the following FI methodology could finally be produced.

Theorem 3. For each $B^{\dagger}$ with the corresponding equation (29), if the consistency check is not conflicting, and the number of solved nonzero additive fault parameters ($u^{\mathrm{s}} \neq 0$ and $u^{\mathrm{v}} - u_{\mathrm{d}}^{\mathrm{v}} \neq 0$) from different equation (29) is the least, then the thrusters corresponding to nonzero parameters are faulty.

Proof. It is apparent that $m^{\mathrm{I}} + \sum_{i=1}^{m^{\mathrm{L}}} r_i$ independent vectors are contained in each equation (27). Thus, there are $m^{\mathrm{I}} + \sum_{i=1}^{m^{\mathrm{L}}} r_i$ independent fault control equations in equation (27), with which those unknown parameters can be calculated. For the ith redundant relation, there are r_i independent equations.

Suppose the equations corresponding to a local S_{maxLIV} of the ith group in equation (27) has been found as

$$\begin{bmatrix} b_{i_1 i_1} & b_{i_1 i_2} & \cdots & b_{i_1 i_{r_i}} \\ b_{i_2 i_1} & b_{i_2 i_2} & \cdots & b_{i_2 i_{r_i}} \\ \vdots & \vdots & \ddots & \vdots \\ b_{i_{r_i} i_1} & b_{i_{r_i} i_2} & \cdots & b_{i_{r_i} i_{r_i}} \end{bmatrix} \begin{bmatrix} u_{i_1} \\ u_{i_2} \\ \vdots \\ u_{i_{r_i}} \end{bmatrix} - \tau_{\mathrm{d}_i}^{\mathrm{v}} = f_{\tau_i} \tag{30}$$

where $i_j\ (j = 1, 2, \cdots, r_i)$ represents the i_jth number, where $1 \leq i_j \leq n$; the meanings of other parameters are easily known and are omitted.

While generating equation (29), one of the r_i thrusters is supposed to be fault-free. Without loss of generality, suppose that thruster i_{r_i} is fault-free. Thus, $u_{i_{r_i}} = 0$ (for a fixed or a vector thruster) or $\boldsymbol{b}_{i_{r_i}} u_{i_{r_i}} - \tau_{\mathrm{d}_i i_{r_i}}^{\mathrm{v}} = 0$ (for a vector thruster only, where the undefined parameters correspond to the i_{r_i}th vector thruster), and equation (30) is changed to be

$$\begin{bmatrix} b_{i_1 i_1} & b_{i_1 i_2} & \cdots & b_{i_1 i_{r_i-1}} \\ b_{i_2 i_1} & b_{i_2 i_2} & \cdots & b_{i_2 i_{r_i-1}} \\ \vdots & \vdots & \ddots & \vdots \\ b_{i_{r_i} i_1} & b_{i_{r_i} i_2} & \cdots & b_{i_{r_i} i_{r_i-1}} \end{bmatrix} \begin{bmatrix} u_{i_1} \\ u_{i_2} \\ \vdots \\ u_{i_{r_i-1}} \end{bmatrix} - \tilde{\tau}_{\mathrm{d}_i}^{\mathrm{v}} = \boldsymbol{f}_{\tau_i} \tag{31}$$

where $\tilde{\tau}_{\mathrm{d}_i}^{\mathrm{v}} \triangleq \tau_{\mathrm{d}_i}^{\mathrm{v}} - \tau_{\mathrm{d}_i i_{r_i}}^{\mathrm{v}}$ replaces $\tau_{\mathrm{d}_i}^{\mathrm{v}}$ only if thruster i_{r_i} is a vector.

Define

$$B_i = \begin{bmatrix} b_{i_1 i_1} & b_{i_1 i_2} & \cdots & b_{i_1 i_{r_i-1}} \\ b_{i_2 i_1} & b_{i_2 i_2} & \cdots & b_{i_2 i_{r_i-1}} \\ \vdots & \vdots & \ddots & \vdots \\ b_{i_{r_i-1} i_1} & b_{i_{r_i-1} i_2} & \cdots & b_{i_{r_i-1} i_{r_i-1}} \end{bmatrix} \tag{32}$$

and

$$\bar{\boldsymbol{b}}_i = [\, b_{i_{r_i} i_1} \quad b_{i_{r_i} i_2} \quad \cdots \quad b_{i_{r_i} i_{r_i-1}} \,] \tag{33}$$

If there is no vector thruster in the ith group, then,

$$\begin{bmatrix} u_{i_1} \\ u_{i_2} \\ \vdots \\ u_{i_{r_i-1}} \end{bmatrix} = [B_i]^{-1}[\boldsymbol{f}_{\tau_i}(i_1 : i_{r_i-1}) + \tilde{\tau}_{\mathrm{d}_i}^{\mathrm{v}}(i_1 : i_{r_i-1})] \tag{34}$$

and

$$\begin{aligned} &\bar{\boldsymbol{b}}_i[B_i]^{-1}\,[\boldsymbol{f}_{\tau_i}(i_1 : i_{r_i-1}) + \tilde{\tau}_{\mathrm{d}_i}^{\mathrm{v}}(i_1 : i_{r_i-1})] \\ &\quad = \boldsymbol{f}_{\tau_i}(i_{r_i}) + \tilde{\tau}_{\mathrm{d}_i}^{\mathrm{v}}(i_{r_i}) \end{aligned} \tag{35}$$

where $*(i_1 : i_{r_i-1})$ and $*(i_{r_i})$ indicate that the elements of the column vector '*' are chosen from i_1 to i_{r_i-1} and i_{r_i}, respectively. Equation (34) gives the solution of additive faults where the nonzero ones are faults; and Equation (35) is a redundant equation to check the consistency. If the results from equation (34) cannot support equation (35), the consistency check fails and there must be some wrongly chosen thrusters, that is, the chosen $B^{\dagger}$ does not contain all the vectors corresponding to faulty thrusters.

Toward the case that some vector thrusters are included in the chosen ith group, equation (34) is not suitable because there might be unknown angle faults in B_i. For this problem, using equation (31) together with correlated trigonometric functions ($\sin^2 * + \cos^2 * = 1$) could get the unknown parameters and also the redundant equation for consistency check.

Against each group of $r_i - 1$ vectors, the aforementioned strategies are suitable. In addition, the issues for individual vectors are simple where the equation that contains any individual vector in equation (29) can be directly solved since it has no correlation with other unknowns. The deduced result directly shows the fault degree of the corresponding individual thruster.

Because some raw redundant relations were merged to be local ones in Algorithm 2, there might be small redundant relations which are parts of some local S_{maxLIV}. For this case, if all the redundancies of a thruster are included by some chosen $r_i - 1$ thrusters, the deduced result based on the above process will not violate the consistency check; however it could confuse the judgement of faulty thrusters. To solve this problem, a simple way is comparing the number of abruptly emerged faulty thrusters among which the least group contains the right answer. The explanation is referred to Proposition 1.

The proof is completed.

Remark 4. The core of the above methodology is to destroy the redundancy of each thruster, thus the deduced results will exclusively contain all the faults.

Remark 5. Define an error variable as the difference of the terms on both sides of the equal sign of equation (35). If the error variable always tends to zero, the corresponding consistency check succeeds.

Remark 6. This article focuses on solving FI issue for the frame-structure UUVs. The algorithms and theorems are derived for most of the cases however the ones excluded in the assumptions. Thus, the robustness has been partly included in the theoretical derivations and proofs.

Remark 7. Since there are equations to be solved during the FI process, the computational issue is necessary to be claimed. Before solving equation (29), the consistency is firstly checked through observing those error variables. For the i th redundant relation, define the total number of thrusters to be T_i. Thus, the computational times of consistency check are

$$\sum_{i=1}^{m^{\mathrm{L}}} \binom{T_i}{r_i - 1} \tag{36}$$

That is, the total times of selecting $r_i - 1$ thrusters from T_i ones among each redundant relation, because each selection corresponds to an error variable. So the total computational times are

$$\sum_{i=1}^{m^{\mathrm{L}}} \left(\binom{T_i}{r_i - 1} + r_i - 1 \right) + m^{\mathrm{I}} \tag{37}$$

where the later $r_i - 1$ and m^{I} indicate that $r_i - 1$ and m^{I} equations should be solved for each redundant relation and all individual thrusters, respectively.

Based on the aforementioned methodologies, the faulty thrusters of frame-structure UUVs can be isolated, even if there are redundancies. The following section will illustrate the effectiveness of the methodologies with a simulation.

Simulation

This section displays a numerical example. The thrusters of the frame-structure UUV are deployed as shown in Figure 2, where four fixed thrusters (sT1, sT2, sT3, and sT4) are mounted in the same $x_{\mathrm{B}} o_{\mathrm{B}} y_{\mathrm{B}}$ plane but on the four corners of the UUV, another fixed thruster sT5 is vertically deployed through the middle of the top plane, and a vector thruster vT6 is mounted in the middle of the tail plane. The thrust directions of these five fixed thrusters are fastened with confirmed angles as shown in Figure 2; while the

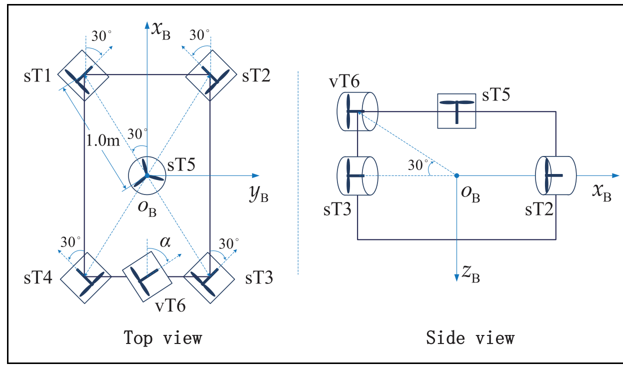

Figure 2. Thruster deployments of the frame-structure UUV. UUV: unmanned underwater vehicle.

direction angle α of vT6 can be altered as $\alpha \in [-90°, 90°]$ in the top plane that is parallel to the plane $x_{\mathrm{B}} o_{\mathrm{B}} y_{\mathrm{B}}$.

The UUV dynamics model is displayed as[16]

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + \boldsymbol{g}(\eta) = \tau \tag{38}$$

$$\dot{\eta} = J(\eta)\nu \tag{39}$$

where ν and η are vectors of velocities and position/Euler angles, respectively; τ is a vector of model control input; the matrices M, $C(\nu)$, and $D(\nu)$ denote inertia, Coriolis, and damping, respectively; $\boldsymbol{g}(\eta)$ is a vector of generalized gravitational and buoyancy forces.

The thrust outputs of thrusters sT1, $\cdots$, sT5 and vT6 could be represented by a vector $\boldsymbol{u} = [u_1, u_2, \cdots, u_6]^{\mathrm{T}}$ while the corresponding control matrix is derived as

$$B = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 & \cos\alpha \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 & \sin\alpha \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\sin\alpha \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}\cos\alpha \\ \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & 0 & -\frac{\sqrt{3}}{2}\sin\alpha \end{bmatrix} \tag{40}$$

$$\triangleq [\boldsymbol{b}_1, \boldsymbol{b}_2, \cdots, \boldsymbol{b}_6] \tag{41}$$

The rank of B is easily calculated as $r = 5$. Since $B \in \mathbb{R}^{6\times 6}$, there is at least one redundant thruster based on Lemma 1. By using Algorithm 2, $\boldsymbol{b}_5$ and $\boldsymbol{b}_6$ are found to be individual vectors, and only one local redundant relation is found among the thrusters sT1, sT2, sT3, and sT4 of which the local S_{maxLIV} are

$$\begin{cases} S_{\mathrm{maxLIV1}} = \{\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3\} \\ S_{\mathrm{maxLIV2}} = \{\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_4\} \\ S_{\mathrm{maxLIV3}} = \{\boldsymbol{b}_1, \boldsymbol{b}_3, \boldsymbol{b}_4\} \\ S_{\mathrm{maxLIV4}} = \{\boldsymbol{b}_2, \boldsymbol{b}_3, \boldsymbol{b}_4\} \end{cases} \tag{42}$$

Then, all $B^\dagger$ could be generated, for example, one $B^\dagger$ corresponding to S_{maxLIV1} is $B^\dagger = [\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_5, \boldsymbol{b}_6]$.

To clearly illustrate current work, strange faults are set up for some of these thrusters. The fault set up for sT1 is

$$f_1 = -2\,\mathrm{N}, (t \geq 30\ \mathrm{s}) \tag{43}$$

where t denotes the time instant from the beginning of the simulation, similarly hereinafter. The fault for sT2 is

$$f_2 = 3 - 3\,\mathrm{e}^{3.75 - t/16}\,\mathrm{N},\ (t \geq 60\ \mathrm{s}) \tag{44}$$

and for vT6 is

$$f_6 = \begin{cases} 4 - 4\,\mathrm{e}^{9 - t/10}\,\mathrm{N}, (90\,\mathrm{s} \leq t < 120\ \mathrm{s}) \\ 3 - \mathrm{e}^{-3} + \mathrm{e}^{24 - t/5}\,\mathrm{N}, (t \geq 120\ \mathrm{s}) \end{cases} \tag{45}$$

While the angle fault for vT6 is

$$f_\alpha = \begin{cases} 0.25t - 20\,\mathrm{deg},\ (80\ \mathrm{s} \leq t < 100\ \mathrm{s}) \\ -0.25t + 30\,\mathrm{deg},\ (100\ \mathrm{s} \leq t < 140\ \mathrm{s}) \\ 0.25t - 40\,\mathrm{deg},\ (140\ \mathrm{s} \leq t < 160\ \mathrm{s}) \end{cases} \tag{46}$$

The designed faults are obviously unknown to the UUV system. Through equation (1), these faults produce an unknown vector $\boldsymbol{f}_\tau \triangleq [f_X, f_Y, f_Z, f_K, f_M, f_N]^{\mathrm{T}}$ of additive forces and moments in τ. The thust design for $\boldsymbol{u}_{\mathrm{d}}$ and fault observer for $\boldsymbol{f}_\tau$ are absent in this article. But a lemma is adopted to get the estimates of $\boldsymbol{f}_\tau$.[19]

Lemma 2. If the fault contained in τ from the formula (38) is expressed as

$$\tau = \tau_{\mathrm{d}} + \boldsymbol{f}_\tau \tag{47}$$

then $\boldsymbol{f}_\tau$ can be estimated by

$$\hat{\boldsymbol{f}} = \mathbf{x} + PM\nu \tag{48}$$

where the vector $\boldsymbol{x}$ satisfies

$$\dot{\boldsymbol{x}} = -P[\boldsymbol{x} + PM\nu - C(\nu)\nu - D(\nu)\nu - \boldsymbol{g}(\eta) + \tau_{\mathrm{d}}] \tag{49}$$

of which P is a positive definite diagonal matrix; and ν and η are measured states.

Using Lemma 2, $\boldsymbol{f}_\tau$ could be generated during the simulation as shown in Figure 3, where the variables with hats represent estimates of the elements of $\boldsymbol{f}_\tau$. Notice that Lemma 2 cannot be directly used to estimate $\boldsymbol{f}_{\mathbf{u}}$, because the rank of B is only five, which means B does not have an inverse matrix and cannot support the usage of Lemma 2 or other fault estimation/isolation methods.

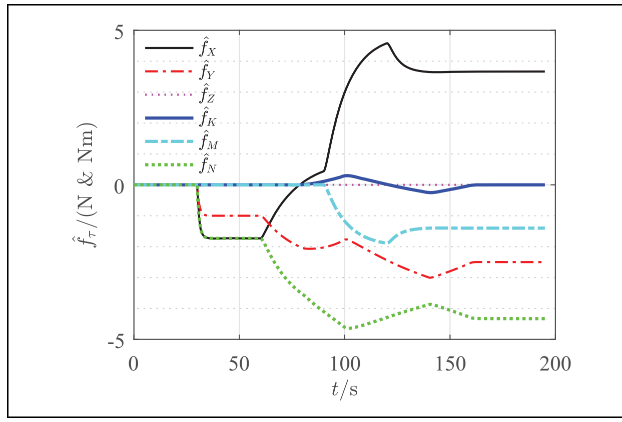

Figure 3. The estimated additive forces and moments in τ.

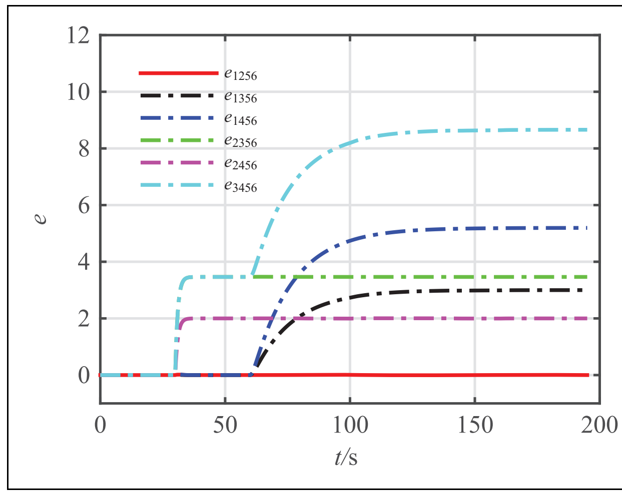

Figure 4. The consistency check errors.

Theorem 3 is now ready for FI. There is one redundant equation for each $B^\dagger$. Thus, $\binom{4}{2} = 6$ error variables are produced for all $B^\dagger$ to check the consistency, that is

$$e_{1256} = \sqrt{3}\hat{f}_Y - 3\sqrt{3}\hat{f}_K - \hat{f}_N \tag{50}$$

$$e_{1356} = \frac{\sqrt{3}}{3}\hat{f}_X - \hat{f}_Y + 2\hat{f}_K + \frac{2\sqrt{3}}{3}\hat{f}_M \tag{51}$$

$$e_{1456} = \hat{f}_X - \sqrt{3}\hat{f}_K + 2\hat{f}_M - \hat{f}_N \tag{52}$$

$$e_{2356} = -\hat{f}_X - \sqrt{3}\hat{f}_K - 2\hat{f}_M - \hat{f}_N \tag{53}$$

$$e_{2456} = -\frac{\sqrt{3}}{3}\hat{f}_X - \hat{f}_Y + 2\hat{f}_K - \frac{2\sqrt{3}}{3}\hat{f}_M \tag{54}$$

$$e_{3456} = -\sqrt{3}\hat{f}_Y + \sqrt{3}\hat{f}_K - \hat{f}_N \tag{55}$$

where e_{1256} is for $B^\dagger = [\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_5, \boldsymbol{b}_6]$ and the others are defined similarly. Among these six errors, only the one of

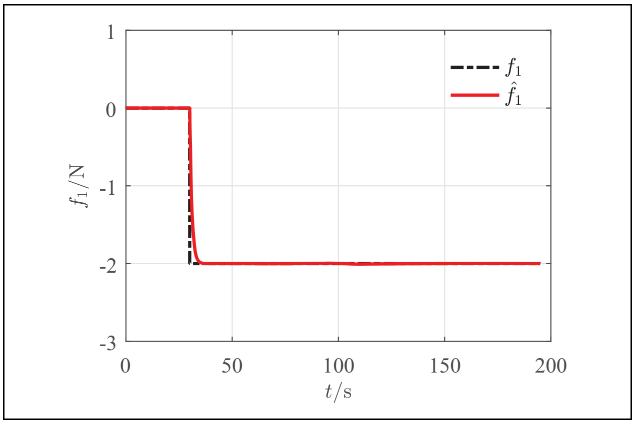

Figure 5. The additive thrust fault of sT1.

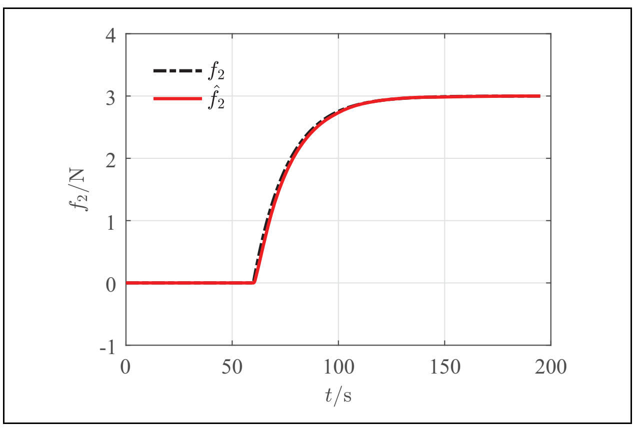

Figure 6. The additive thrust fault of sT2.

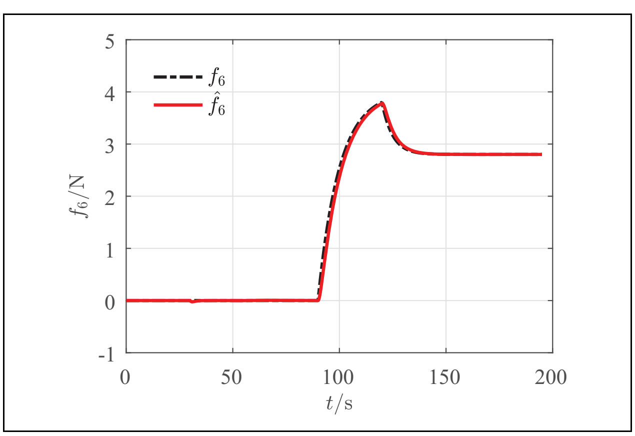

Figure 7. The additive thrust fault of vT6.

which the corresponding $B^{\dagger}$ contains the column vectors of all the faulty thrusters will converge to 0, that is, the consistency check of equation (29) is not conflicting.

Figure 4 gives the consistency check errors throughout this simulation, where the red solid line corresponding to e_{1256} always tends to 0. However, the green, magenta, and

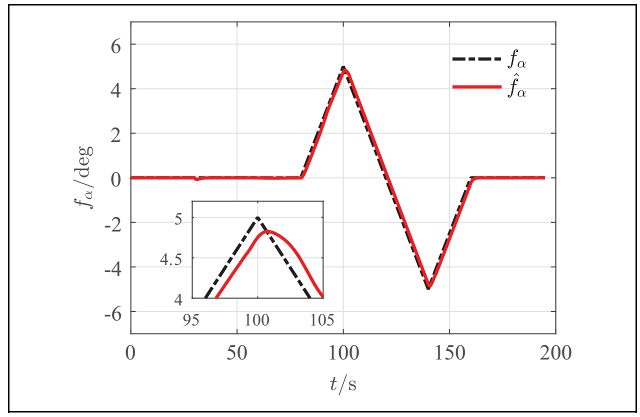

Figure 8. The additive angle fault of vT6.

cyan dash-dot lines begin to leave apart from 0 at time instant 30 s because thruster sT1 gets into faulty; the black and blue ones begin to change at time instant 60 s because thruster sT2 gets faulty. These calculating results coincide with the given faults, that is, only the $B^{\dagger}$ which contains all the vectors that correspond to the faulty thrusters can pass the consistency check. While thruster vT6 gets into faulty, there is no change in Figure 4, because it is an individual thruster that has been contained by all $B^{\dagger}$ and the faults are derived directly.

The calculated additive thrust and angle faults are shown in Figures 5 to 8. In these figures, the black dash-dot lines and the red solid lines represent the given and the calculated faults, respectively.

The overlaps of these lines illustrate the effectiveness of the aforementioned methodologies. While proper thresholds are defined for all thrusters, the red line that departs from 0 and reaches the threshold line indicates fault of the corresponding thruster.

Conclusion

The FI issue for the frame-structure UUVs mounted with redundant thrusters has been revealed in this article, which was carried out against the control input equation where the faults were expressed as additive terms. S_{maxLIV} of the control matrix were generated by an existing algorithm which led to the first theorem for analyzing the redundant relations among thrusters. The second algorithm brought forward the local S_{maxLIV} and the individual vectors. With some assumptions, the second theorem was built for restricting the faulty thrusters to local S_{maxLIV} and individual thrusters. Then, the final theorem was generated to realize the FI, of which the core was to solve a reformed control input equation by taking into the local S_{maxLIV} and the individual vectors. The faulty thrusters were isolated while the consistency check was passed with right $B^{\dagger}$.

Future research efforts will be devoted to isolate the faults with faulty thrust directions, and analyze the disturbance robustness for the FI methodology.

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