

# Trajectory tracking control of unmanned surface vessels with input saturation and full-state constraints

International Journal of Advanced  
Robotic Systems  
September-October 2018: 1–9  
© The Author(s) 2018  
DOI: 10.1177/1729881418808113  
journals.sagepub.com/home/arx



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## Abstract

This article investigates the trajectory tracking control problem for unmanned surface vessels with input saturation and full-state constraints. The barrier Lyapunov function is used to solve the problem of state constraints, and the adaptive method is employed to handle the unknown random disturbances and saturation problems. The proposed control approach can guarantee that the control law and signals of closed-loop system are uniformly bounded and achieve the asymptotic tracking. Finally, simulation studies are provided to show the effectiveness of the proposed method.

## Keywords

Unmanned surface vessel, barrier Lyapunov function, adaptive control, input saturation, full-state constraints

Date received: 18 July 2018; accepted: 21 September 2018

Topic: Robot Manipulation and Control

Topic Editor: Yangquan Chen

Associate Editor: Ning Wang

## Introduction

Ocean accounts for about 70% of the earth surface, which contains a lot of resources. However, large-scale development of ocean has not been carried out due to various factors. In recent years, with the development of intelligent and automation technologies, the monitoring of the marine environment and the development of resources have received increasing attention. A variety of autonomous marine robots, as a kind of practical tools, have been applied widely, including unmanned surface vessels (USVs), remote operated vehicles (ROVs), and autonomous underwater vehicles (AUVs). Du et al.<sup>1</sup> proposed a motion planning approach which was based on trajectory unit, realizing the fine motion control. Cui et al.<sup>2</sup> presented an integral sliding mode controller for underwater robots with multiple-input and multiple-output (MIMO) extended-state-observer. It utilized an adaptive gain update algorithm for the uncertainties. Muñoz-Vázquez et al.<sup>3</sup> proposed a fractional-order robust control algorithm for the

ROV influenced by nonsmooth Hölder disturbances, achieving exponential tracking control. USVs are unmanned vehicles with high feasibility and wide application, which play an important role in the autonomous marine robots. Therefore, the research on the control design of USVs is of great significance.

In order to ensure that USVs have the ability to navigate as required, much research has been done on their motion control, such as trajectory tracking control, path following control, stabilization control, and formation control. Liu

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et al.<sup>4</sup> proposed an improved line-of-sight guidance algorithm with the adaptive method, and the path following control strategy was developed based on the backstepping approach and the Lyapunov stability theory. Liao et al.<sup>5</sup> transformed the tracking and stabilization problem of underactuated USVs into the stabilization problem of the trajectory tracking error equation and designed a nonlinear state feedback control protocol using the backstepping technique and Lyapunov direct method. Fu et al.<sup>6</sup> proposed a distributed control strategy based on virtual leader technique when considering the disturbances and solved the formation control problem of underactuated USV systems. Wang et al.<sup>7</sup> developed a formation control approach for USVs, which could sense the constraints moving in a leader–follower formation.

The main problem addressed in this study is the trajectory tracking control,<sup>8</sup> which means that the vessel navigates in the designed route, namely, the vessel needs to arrive at the destination during a specified time period. At present, the main approaches for trajectory tracking control include Lyapunov direct method, backstepping technique, sliding mode control, adaptive control, and state feedback method. Dong et al.<sup>9</sup> presented a backstepping control algorithm based on state feedback considering a nonlinear three degree-of-freedom (DOF) underactuated dynamic model for USVs, and it solved the trajectory tracking problem in the horizontal plane. Švec et al.<sup>10</sup> investigated the trajectory planning and tracking approach to follow the target which was differentially constrained enabling USVs to follow a moving target surrounded by obstacles. Mu et al.<sup>11</sup> designed an adaptive neural tracking control strategy based on backstepping technique, neural network approximation, and adaptive method for the tracking control of pod propulsion USVs, in which a novel neural shunting model was introduced to solve the effect of “explosion of complexity.” However, the results mentioned above only addressed the trajectory tracking control problem for USVs without considering some influencing factors, which may cause some effects on practical applications.

Due to the changes of navigation conditions in reality, the hydrodynamic derivatives of USVs will also change. And the dynamic parameters in the control design are the complex nonlinear functions of the hydrodynamic derivatives above. As a result, the uncertainty of the model dynamics takes place for the changes above. In addition, the randomness of the environment disturbances, including wind, wave, and ocean currents, also have a great influence on the motion of USVs, affecting its control performance.<sup>12–15</sup> Liao et al.<sup>16</sup> designed a backstepping based adaptive sliding mode control approach for underactuated surface vessels subject to uncertain influences and external disturbances. It utilized a virtual USV to generate the trajectory and achieved the trajectory tracking control. Park<sup>17</sup> employed multilayer neural networks to estimate the unknown model parameters and external disturbances and

designed a fault-tolerant control strategy based on the Nussbaum gain technique, realizing the trajectory tracking of underactuated surface vessel with thruster failure. Larrazabal and Peñas<sup>18</sup> presented a fuzzy logic controller for the dynamics uncertainties and designed an adaptive control law to address the trajectory tracking control for USVs.

Besides the dynamics uncertainties and the external disturbances, state constraints also need to be taken into account in the trajectory tracking control for USVs. If the constraints are violated, various accidents and dangers may occur, such as crash in the collision for straying off course and equipment failure for exceeding the speed limit. Liu et al.<sup>19</sup> proposed a nonlinear model predictive control (MPC) for the underactuated surface vessel, employing convex optimization based on MPC involving linear matrix inequalities to address the constrained input problem. Zheng et al.<sup>20</sup> adopted an asymmetric time-varying barrier Lyapunov function (BLF) for the output constraint, while the backstepping and adaptive methods were also used to realize the trajectory tracking control for a fully actuated surface vessel.

It can be found that the BLF is employed to address the problem of state constraints in most related results. Actually, the BLF is a control method for state constraints based on the thought of potential function. It can keep the state in the constraint boundary by ensuring the boundedness of the BLF in the closed-loop system. The BLF can be designed symmetrically or asymmetrically, and the expression of it is mainly in the logarithmic form with the square of constraint boundary as the numerator and the square difference between constraint boundary and bounded state vector as the denominator.

However, the research above still ignores an important factor, that is, the saturation problem. Due to some factors in practical applications, the actuators of USVs unlikely provide unlimited control forces and torques. For this engineering problem, it is necessary to introduce the saturation issue, which has a great effect on the control performance, to the design of control strategies. In fact, there is a difference  $\Delta\tau$  between the desired and actual control input. In this respect, one approach is to take the saturation function into the control law design, which is in the form of auxiliary variables. Liu et al.<sup>21</sup> designed a saturated coordinated control strategy on a closed curve for multiple underactuated USVs with a bounded neural network control law containing the saturation function. Park et al.<sup>22</sup> presented an adaptive output feedback control protocol based on the neural networks by considering the input saturation and underactuated problems realizing trajectory tracking for underactuated surface vessels. Park<sup>23</sup> designed a predefined performance function to simplify the structure of the control system and proposed an output-feedback control method for trajectory tracking. The underactuated and input saturation problems were addressed by auxiliary variables. Park and Yoo<sup>24</sup> proposed a fault accommodation control for underactuated surface vessels, in which the

auxiliary variables and their dynamics were derived to deal with the saturation problem of actuators with the nonholonomic property. The strategy achieved the trajectory tracking in the presence of nonlinear uncertainties. Zheng and Sun<sup>25</sup> used the auxiliary design system to analyze the effect of input saturation during the control law design. It addressed the path following control for USVs subject to model uncertainties and input saturation. The control approach was based on the backstepping technique with neural network. Wang et al.<sup>26</sup> employed a smooth function to adaptively approximate the input saturation nonlinearity and proposed the integral sliding mode control and homogeneous disturbance observer to realize the finite-time trajectory tracking control for USVs with input saturation and unknown disturbances. Yu et al.<sup>27</sup> adopted direct adaptive fuzzy control in the control loop to compensate for the influence of actuator saturation. It achieved the vertical-plane trajectory tracking for underactuated AUV subject to actuator saturation and external disturbances.

In this study, a trajectory tracking control algorithm based on the adaptive method and the BLF is proposed for USVs, so as to deal with the full-state constraints and input saturation. The characteristics of this study are listed in the following.

1. We take the external random disturbances into account during the trajectory tracking control and employ the adaptive method to address this problem.
2. The control approach is based on the BLF technique, which is used to handle the full-state constraint problem.
3. For the actuator saturation, the adaptive method is employed to estimate the upper bound of its norm and compensates for it in the control law.

The organization of this research is listed as follows. The second section gives the introduction of the basic knowledge including the dynamics of USVs, variable descriptions, and some relevant assumptions. The third section presents the derivation process of the trajectory tracking control which is mainly based on the BLF to address full-state constraint and adaptive method to handle the system uncertainties and saturation problem. Finally, the boundedness of the control law and the signals of closed-loop system are realized with the asymptotically tracking achieved. The full-state constraints are also satisfied. In the fourth section, we carry out the simulation studies to verify the effectiveness of the proposed control strategy. The fifth section draws the conclusion about the whole study.

## Problem formulation

In practical applications, the control force and torque that are provided by the actuators of USV are usually limited. Thus, it is necessary to consider the influence of input

saturation on the control performance in the control strategies design. The saturation function can generally be expressed as

$$\text{sat}(\tau) = [\text{sat}(\tau_1), \text{sat}(\tau_2), \text{sat}(\tau_3)]^T \quad (1)$$

where  $\text{sat}(\tau_i) = \text{sgn}(\tau_i) \min\{\tau_{i\max}, |\tau_i|\}$ ,  $i = 1, 2, 3$  and  $\tau_{i\max}$  is the amplitude of the saturation function.

The desired control input  $\tau$  may be larger than the actual control input  $\text{sat}(\tau)$  provided by the actuator. Thus, there is a difference  $\Delta\tau$  between them. We propose  $\Delta\tau$  as

$$\Delta\tau = \text{sat}(\tau) - \tau \quad (2)$$

**Assumption 1.** For equations (1) and (2), there exists a non-negative real number  $\theta$  which satisfies the conditions described in the following

$$\|\Delta\tau\| \leq \theta \quad (3)$$

**Remark 1.** Assumption 1 is reasonable since when the input saturation occurs, if the difference between the desired control input and the actual control input is infinite, the system will be uncontrollable.

Taking saturation problem into consideration, the dynamics of an MIMO three-DOF USV is described as follows

$$\begin{cases} \dot{\eta} = J(\eta)\nu \\ M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \text{sat}(\tau) + H \end{cases} \quad (4)$$

where the state  $\eta = [\eta_x, \eta_y, \eta_\psi] \in \mathbb{R}^3$  represents the earth-fixed frame positions  $(\eta_x, \eta_y)$  and heading  $(\eta_\psi)$ , respectively.  $\nu = [\nu_x, \nu_y, \nu_\psi] \in \mathbb{R}^3$  denotes the USV surge, sway, and yaw velocities, respectively.  $M \in \mathbb{R}^{3 \times 3}$  is the symmetric positive definite inertia matrix,  $C(\nu) \in \mathbb{R}^{3 \times 3}$  is the Centripetal and Coriolis torques,  $D(\nu) \in \mathbb{R}^{3 \times 3}$  is the damping matrix,  $H$  represents the random disturbance of the surrounding environment, and  $g(\eta)$  represents the restoring forces caused by force of gravity, ocean currents, and floatage.  $J(\eta)$  is the transformation matrix which is assumed to be nonsingular and is defined as

$$J(\eta) = \begin{pmatrix} \cos\eta_\psi & -\sin\eta_\psi & 0 \\ \sin\eta_\psi & \cos\eta_\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Let  $x_1 = \eta, x_2 = \nu$ . Thus, the USV system (4) can be described as

$$\begin{cases} \dot{x}_1 = J(x_1)x_2 \\ \dot{x}_2 = M^{-1}[\text{sat}(\tau) - C(x_2)x_2 - D(x_2)x_2 - g(x_1) + H] = a \end{cases} \quad (5)$$

**Assumption 2.** For any positive vector  $b_a$ , there exist positive vectors  $A_1 = [A_{11}, A_{12}, A_{13}]^T$ ,  $A_2 = [A_{21}, A_{22}, A_{23}]^T$ , and

$S_0 = [S_{01}, S_{02}, S_{03}]^T$  satisfying  $A_1 \leq S_0 \leq b_a$ , such that, the desired trajectory  $x_d(t)$  and its time derivatives satisfy  $-A_1 \leq \dot{x}_d(t) \leq A_1$  and  $-A_2 \leq \ddot{x}_d(t) \leq A_2$  for  $\forall t \geq 0$ .

**Assumption 3.** The unknown random disturbance is bounded, that is, for  $\forall t \geq 0$ , there exist a constant  $h_m \in \mathbb{R}^+$  which makes  $\|H\| \leq h_m$ .

The control objective is to track the desired trajectory of the earth-frame positions  $x_d = [x_{d1}(t), x_{d2}(t), x_{d3}(t)]^T$  and the desired trajectory of the velocities  $x_{2d} = [x_{2d1}(t), x_{2d2}(t), x_{2d3}(t)]^T$  with the consideration of input saturation problem. Meanwhile, all signals should be bounded and the full-state constraints of tracking errors are not violated.

## Control law design

This section will present the trajectory tracking control algorithm design for the USV systems with input saturation and full-state constraints. We utilize the BLF to address the state constraints and use the adaptive method to compensate for the effect of the unknown random disturbances and saturation problem.

First, we denote the tracking errors as  $z_1 = [z_{11}, z_{12}, z_{13}]^T = x_1 - x_d$  and  $z_2 = [z_{21}, z_{22}, z_{23}]^T = x_2 - \alpha$ , where  $\alpha$  is the virtual control function to be designed. Substituting the above tracking errors into equation (5), we can obtain the closed-loop system

$$\begin{cases} \dot{z}_1 = J(z_2 + \alpha) - \dot{x}_d \\ \dot{z}_2 = M^{-1}[\text{sat}(\tau) - C(x_2)x_2 - D(x_2)x_2 - g(x_1) + H] - \dot{\alpha} \end{cases} \quad (6)$$

**Assumption 4.** When considering the actual system with input saturation described in equation (6), there should be a reasonable actual control input  $\text{sat}(\tau)$  to satisfy the tracking control requirement.

Choosing a BLF candidate for  $z_1$  as

$$V_1 = \sum_{i=1}^3 \frac{1}{2} \ln \frac{b_{1i}^2}{b_{1i}^2 - z_{1i}^2} \quad (7)$$

where  $b_1 = b_a - A_0 = [b_{11}, b_{12}, b_{13}]^T$  is the boundary of  $z_1$ . It is obvious that  $V_1$  is positive definite and continuous when  $|z_{1i}| \leq b_{1i}$ .

Then differentiating  $V_1$  with respect to time, we have

$$\dot{V}_1 = \sum_{i=1}^3 \frac{z_{1i}\dot{z}_{1i}}{b_{1i}^2 - z_{1i}^2} \quad (8)$$

The virtual control function  $\alpha$  is designed as

$$\alpha = J^T(\dot{x}_d - Q_1) \quad (9)$$

where

$$Q_1 = \begin{bmatrix} (b_{11}^2 - z_{11}^2)k_{11}z_{11} \\ (b_{12}^2 - z_{12}^2)k_{12}z_{12} \\ (b_{13}^2 - z_{13}^2)k_{13}z_{13} \end{bmatrix} \quad (10)$$

and  $k_{1i}, i = 1, 2, 3$  are positive constants.

**Assumption 5.** The matrix  $J(x_1)$  is known, and there exists a boundary. From Assumption 2, it can be further assumed that there exist positive vectors  $B_0 = [B_{01}, B_{02}, B_{03}]^T$  and  $A_0 = [A_{01}, A_{02}, A_{03}]^T$ , which satisfy  $A_0 \leq B_0 \leq b_c$ , such that, for  $\forall t \geq 0$ ,  $\alpha(t)$  satisfies  $-A_0 \leq \alpha(t) \leq A_0$ .

Assumption 5 indicates that  $\alpha(t)$  is continuous and bounded.

Substituting equations (6), (9), and (10) into equation (8), we can obtain

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^3 \frac{z_{1i}\{J_i^T(z_2 + J_i^T(\dot{x}_{di} - B_{1i}) - \dot{x}_{di})\}}{b_{1i}^2 - z_{1i}^2} \\ &= -\sum_{i=1}^3 k_{1i}z_{1i}^2 + \sum_{i=1}^3 \frac{z_{1i}J_i z_{2i}}{b_{1i}^2 - z_{1i}^2} \end{aligned} \quad (11)$$

According to equation (11), it is obvious that if  $z_1 = 0$ ,  $\dot{V}_1 = -\sum_{i=1}^3 k_{1i}z_{1i}^2 \leq 0$ , which means that  $t \rightarrow \infty, z_1 \rightarrow 0$ .

In order to deal with the problem of random disturbance and input saturation, as well as to prevent the unnecessary chattering caused by the signum function, an adaptive algorithm is used to estimate the squares of the upper bounds of the unknown random disturbance and the difference of control input, such that,  $\kappa = h_m^2$  and  $\mu = \theta^2$ . In this research, adaptive estimation errors are defined as  $\tilde{\kappa} = \kappa - \hat{\kappa}$  and  $\tilde{\mu} = \mu - \hat{\mu}$ , and there exists  $\dot{\tilde{\kappa}} = -\dot{\hat{\kappa}}$  and  $\dot{\tilde{\mu}} = -\dot{\hat{\mu}}$ . Then choose the BLF as

$$V_2 = V_1 + \sum_{i=1}^3 \frac{1}{2} \ln \frac{b_{2i}^2}{b_{2i}^2 - z_{2i}^2} + \frac{1}{2} z_2^T M z_2 + \frac{1}{2\gamma_0} \tilde{\kappa}^2 + \frac{1}{2\beta_0} \tilde{\mu}^2 \quad (12)$$

where  $b_2 = b_c - A_1 = [b_{21}, b_{22}, b_{23}]^T$  is the boundary of  $z_2, \gamma_0 > 0, \beta_0 > 0$ . Then, differentiating equation (12) with respect to time, we have

$$\dot{V}_2 = \dot{V}_1 + \sum_{i=1}^3 \frac{z_{2i}\dot{z}_{2i}}{b_{2i}^2 - z_{2i}^2} + z_2^T M \dot{z}_2 + \frac{1}{\gamma_0} \tilde{\kappa}\dot{\tilde{\kappa}} + \frac{1}{\beta_0} \tilde{\mu}\dot{\tilde{\mu}} \quad (13)$$

According to the Moore–Penrose inverse, we obtain

$$z_2^T (z_2^T)^+ = \begin{cases} 0, & z_2 = [0, 0, 0]^T \\ 1, & \text{Otherwise} \end{cases} \quad (14)$$

Therefore, the control strategy and the adaptive law are designed as

$$\begin{aligned} \tau = & C(x_2)x_2 + D(x_2)x_2 + g(x_1) + M\dot{\alpha} - \sum_{i=1}^3 \frac{z_{1i}J_i^T}{b_{1i}^2 - z_{1i}^2} - (z_2^T)^+ \sum_{i=1}^3 \frac{z_{2i}(a_i - \dot{\alpha}_i)}{b_{2i}^2 - z_{2i}^2} \\ & - k_2 z_2 - (z_2^T)^+ \sum_{i=1}^3 \left( \frac{\hat{\kappa} z_{2i}^2}{2\varepsilon^2} + \frac{\hat{\mu} z_{2i}^2}{2\delta^2} - \frac{\varepsilon^2}{2} - \frac{\delta^2}{2} \right) \end{aligned} \quad (15)$$

$$\begin{cases} \dot{\hat{\kappa}} = \gamma_0 \sum_{i=1}^3 \frac{z_{2i}^2}{2\varepsilon^2} \\ \dot{\hat{\mu}} = \beta_0 \sum_{i=1}^3 \frac{z_{2i}^2}{2\delta^2} \end{cases} \quad (16)$$

where  $\varepsilon$  and  $\delta$  are positive constants and  $k_2$  is a positive control gain.

**Theorem 1.** Consider the USV system (5) under Assumptions 1 to 3. The signals of the closed-loop system are uniformly bounded with the adaptive control algorithm (15) and (16), if initial conditions satisfy  $z_{1i}(0) \in \mathcal{Q}_{01} \triangleq \{|z_{1i}| < b_{1i}, i = 1, 2, 3\}$ ,  $z_{2i}(0) \in \mathcal{Q}_{02} \triangleq \{|z_{2i}| < b_{2i}, i = 1, 2, 3\}$ .  $x_1(t) \rightarrow x_d(t)$  as  $t \rightarrow \infty$ , which means that the asymptotic tracking can be achieved.  $|x_1| < k_c, \forall t > 0$ , which means that the multiple output constraints are never violated. The tracking errors  $z_1$  and  $z_2$  will remain within the compact sets  $z_{1i} \in \mathcal{Q}_{z1} \triangleq \{|z_{1i}| \leq b_{1i}, i = 1, 2, 3\}$  and  $z_{2i} \in \mathcal{Q}_{z2} \triangleq \{|z_{2i}| \leq b_{2i}, i = 1, 2, 3\}$ . And  $z_1$  and  $z_2$  can converge to any small neighborhood of zero with the appropriate controller parameters.

**Proof.** According to equation (13) and (16),  $\dot{V}_1 = -\sum_{i=1}^3 k_{1i} z_{1i}^2 \leq 0$  when  $z_2 = [0, 0, 0]^T$ . Then, the asymptotic stability of the system (5) can be proved by the Barbalat's lemma.<sup>28</sup>

Otherwise, in the case of  $z_2 \neq [0, 0, 0]^T$ , substituting equations (14) to (16) and (6) into equation (13), we have

$$\begin{aligned} \dot{V}_2 = & -\sum_{i=1}^3 k_{1i} z_{1i}^2 + \sum_{i=1}^3 \frac{z_{1i} J_i z_2}{b_{1i}^2 - z_{1i}^2} + \sum_{i=1}^3 \frac{z_{2i} \dot{z}_{2i}}{b_{2i}^2 - z_{2i}^2} - \frac{1}{\gamma_0} \tilde{\kappa} \dot{\hat{\kappa}} - \frac{1}{\beta_0} \tilde{\mu} \dot{\hat{\mu}} \\ & + z_2^T \left[ -\sum_{i=1}^3 \frac{z_{1i} J_i^T}{b_{1i}^2 - z_{1i}^2} - (z_2^T)^+ \sum_{i=1}^3 \frac{z_{2i} \dot{z}_{2i}}{b_{2i}^2 - z_{2i}^2} + H + \Delta\tau - k_2 z_2 \right. \\ & \left. - (z_2^T)^+ \sum_{i=1}^3 \left( \frac{\hat{\kappa} z_{2i}^2}{2\varepsilon^2} + \frac{\hat{\mu} z_{2i}^2}{2\delta^2} - \frac{\varepsilon^2}{2} - \frac{\delta^2}{2} \right) \right] \\ = & -\sum_{i=1}^3 k_{1i} z_{1i}^2 - z_2^T k_2 z_2 + z_2^T H + z_2^T \Delta\tau - \sum_{i=1}^3 \left( \frac{\hat{\kappa} z_{2i}^2}{2\varepsilon^2} + \frac{\hat{\mu} z_{2i}^2}{2\delta^2} - \frac{\varepsilon^2}{2} - \frac{\delta^2}{2} \right) - \frac{1}{\gamma_0} \tilde{\kappa} \dot{\hat{\kappa}} - \frac{1}{\beta_0} \tilde{\mu} \dot{\hat{\mu}} \\ \leq & -\sum_{i=1}^3 k_{1i} z_{1i}^2 - z_2^T k_2 z_2 + \|z_2\| h_m + \|z_2\| \theta - \sum_{i=1}^3 \left( \frac{\hat{\kappa} z_{2i}^2}{2\varepsilon^2} + \frac{\hat{\mu} z_{2i}^2}{2\delta^2} - \frac{\varepsilon^2}{2} - \frac{\delta^2}{2} \right) - \frac{1}{\gamma_0} \tilde{\kappa} \dot{\hat{\kappa}} - \frac{1}{\beta_0} \tilde{\mu} \dot{\hat{\mu}} \end{aligned} \quad (17)$$

According to Young's inequality, we can obtain

$$\begin{cases} \|z_2\| h_m \leq \sum_{i=1}^3 \left( \frac{\kappa z_{2i}^2}{2\varepsilon^2} + \frac{\varepsilon^2}{2} \right) \\ \|z_2\| \theta \leq \sum_{i=1}^3 \left( \frac{\mu z_{2i}^2}{2\delta^2} + \frac{\delta^2}{2} \right) \end{cases} \quad (18)$$

Substituting equation (18) into equation (17), we have

$$\begin{aligned}
\dot{V}_2 &\leq -\sum_{i=1}^3 k_{1i} z_{1i}^2 - z_2^T k_2 z_2 + \sum_{i=1}^3 \left( \frac{\kappa z_{2i}^2}{2\varepsilon^2} + \frac{\varepsilon^2}{2} \right) \\
&\quad + \sum_{i=1}^3 \left( \frac{\mu z_{2i}^2}{2\delta^2} + \frac{\delta^2}{2} \right) - \sum_{i=1}^3 \left( \frac{\hat{\kappa} z_{2i}^2}{2\varepsilon^2} + \frac{\hat{\mu} z_{2i}^2}{2\delta^2} - \frac{\varepsilon^2}{2} - \frac{\delta^2}{2} \right) \\
&\quad - \frac{1}{\gamma_0} \tilde{\kappa} \gamma_0 \sum_{i=1}^3 \frac{z_{2i}^2}{2\varepsilon^2} - \frac{1}{\beta_0} \tilde{\mu} \beta_0 \sum_{i=1}^3 \frac{z_{2i}^2}{2\delta^2} \\
&\leq -\sum_{i=1}^3 k_{1i} z_{1i}^2 - z_2^T k_2 z_2
\end{aligned} \tag{19}$$

According to Assumption 5, we can find that  $z_1$  still remains within the interval  $-b_1 \leq z_1 \leq b_1, \forall t > 0$  while  $z_2$  remains within the interval  $-b_2 \leq z_2 \leq b_2, \forall t > 0$ . As a result, high-accuracy tracking control for USV can be achieved, in the presence of the input saturation and state constraints.

## Numerical simulations

In order to verify the effectiveness of the control law (15) and (16) in the trajectory tracking control for the USV system (5) with input saturation and full-state constraints, the simulation study is conducted in this section.

The model vessel for the simulations is Cybership II, which is a 1:70 scale replica of a supply vessel built in the Norwegian University of Science and Technology.<sup>29</sup>

The desired trajectories are chosen as follows

$$x_{1d}(t) = [x_{1xd}(t), x_{1yd}(t), x_{1\psi d}(t)]^T \tag{20}$$

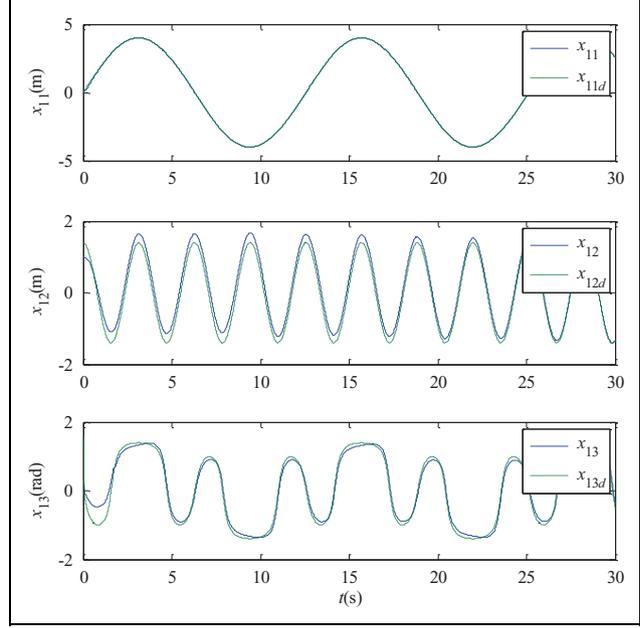


Figure 1. Comparison between  $x_1$  and  $x_d$ .

$$\begin{cases} x_{1xd}(t) = 4 \sin 0.5t \\ x_{1yd}(t) = 1.4 \cos 2t \\ x_{1\psi d}(t) = \tan^{-1}(\dot{x}_{1yd}/\dot{x}_{1xd}) \end{cases} \tag{21}$$

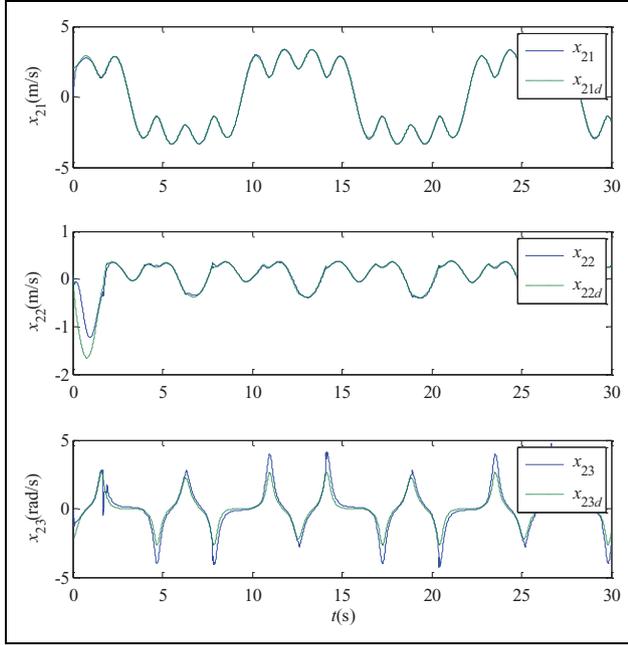
$$x_{2d} = J^{-1} \dot{x}_{1d} \tag{22}$$

The symmetric positive definite inertia matrix  $M$ , the Centripetal and Coriolis torques  $C(\nu)$ , and the damping matrix  $D(\nu)$  are given as follows

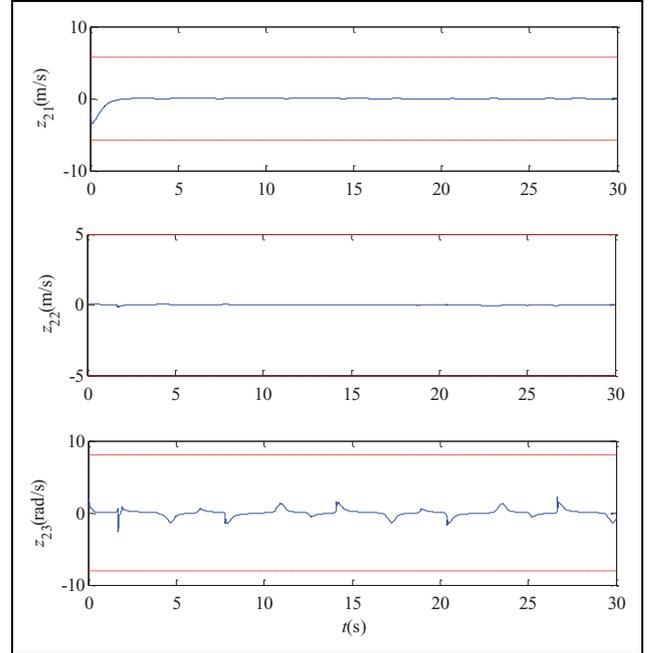
$$M = \begin{bmatrix} m - X_{du} & 0 & 0 \\ 0 & m - Y_{dv} & mx_g - Y_{dr} \\ 0 & mx_g - Y_{dr} & I_z - N_{dr} \end{bmatrix}$$

$$C(\nu) = \begin{bmatrix} 0 & 0 & (-m - Y_{dv})\nu_y - (mx_g - Y_{dr})\nu_\psi \\ 0 & 0 & (m - X_{du})\nu_x \\ (m - Y_{dv})\nu_y + (mx_g - Y_{dr})\nu_\psi & (m - X_{du})\nu_x & 0 \end{bmatrix}$$

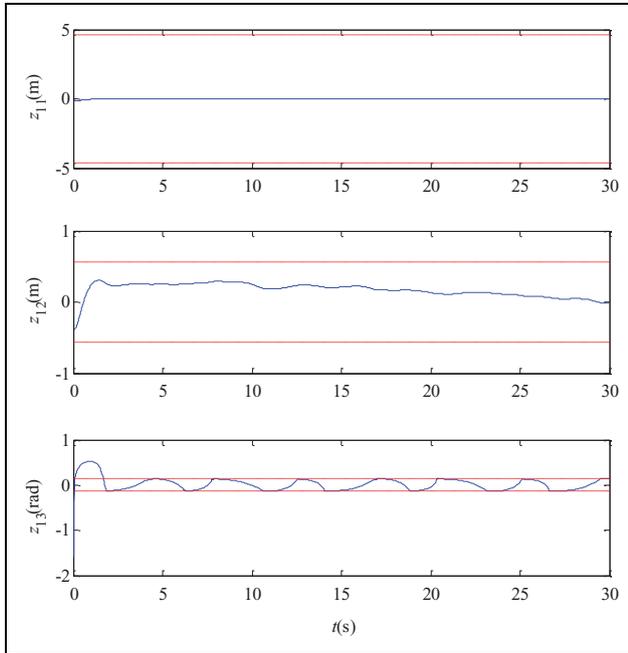
$$D(\nu) = \begin{bmatrix} -X_u - X_{uu}|\nu_x| - X_{uuu}\nu_x^2 & 0 & 0 \\ 0 & -Y_v - Y_{vv}|\nu_y| - Y_{rv}|\nu_\psi| & -Y_r - Y_{vr}|\nu_y| - Y_{rr}|\nu_\psi| \\ 0 & -N_v - N_{vv}|\nu_y| - N_{rv}|\nu_\psi| & -N_r - N_{vr}|\nu_y| - N_{rr}|\nu_\psi| \end{bmatrix}$$



**Figure 2.** Comparison between  $x_2$  and  $x_{2d}$ .



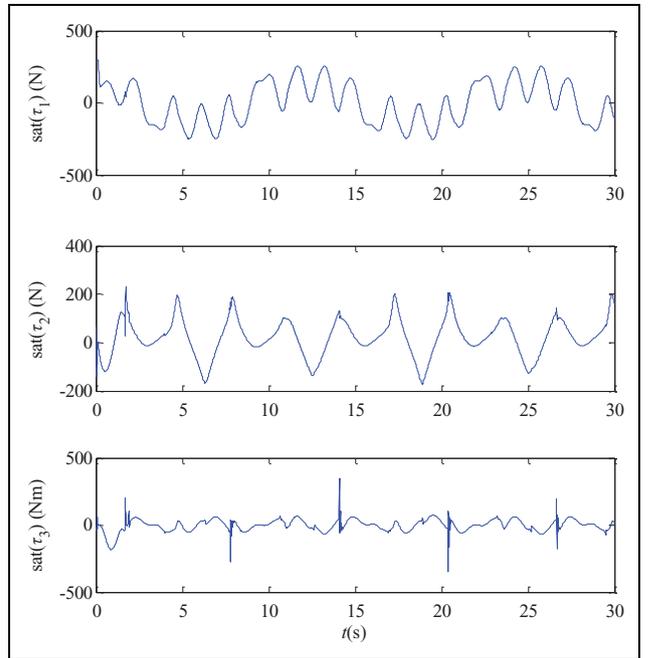
**Figure 4.** Tracking error  $z_2$ .



**Figure 3.** Tracking error  $z_1$ .

The corresponding hydrodynamic parameters are given as  $m = 23.8$ ,  $I_z = 1.76$ ,  $x_g = 0.046$ ,  $X_u = -0.7225$ ,  $X_{uu} = -1.3274$ ,  $X_{uuu} = -5.8664$ ,  $Y_v = -0.8612$ ,  $Y_{vv} = -36.2823$ ,  $Y_r = 0.1079$ ,  $N_v = 0.1052$ ,  $N_{vv} = 5.0437$ ,  $X_{du} = -2.0$ ,  $Y_{dv} = -10.0$ ,  $Y_{dr} = -0$ ,  $N_{dv} = 0$ ,  $N_{dr} = -1.0$ ,  $Y_{rv} = 2$ ,  $Y_{vr} = 1$ ,  $Y_{rr} = 3$ ,  $N_{rv} = 5$ ,  $N_r = 4$ ,  $N_{vr} = 0.5$ , and  $N_{rr} = 0.8$ .

The initial conditions are chosen as  $x_1(0) = [0.01, 1, -0.012]^T$  and  $x_2(0) = [0.08, 0.08, -0.1]^T$ . The control parameters are selected as  $K_1 = \text{diag}[1, 10, 0.1]$



**Figure 5.** Control input  $\text{sat}(\tau)$ .

and  $K_2 = \text{diag}[0.4, 40, 0.001]$ . The constraint boundaries are chosen as  $b_1 = [4.6, 0.57, 0.14]^T$  and  $b_2 = [5.8, 5, 8]^T$ . The adaptive parameters are set as  $\gamma_0 = 0.1$ ,  $\beta_0 = 2.5$ ,  $\varepsilon = 0.5$ , and  $\delta = 0.5$ . And the amplitudes of the saturation function are chosen as  $\tau_{1\max} = 300$  N,  $\tau_{2\max} = 350$  N, and  $\tau_{3\max} = 350$  Nm.

The results of the simulation studies are shown from Figures 1 to 5. From Figures 1 and 2, we can find that  $x_{12}$  and  $x_{23}$  track the desired trajectory with slight

deviations, while the rest trajectories including  $x_{11}$ ,  $x_{13}$ ,  $x_{21}$ , and  $x_{22}$  can successfully track their desired trajectories with high accuracy. The tracking errors are shown in Figures 3 and 4, from which we can see that all of them can converge to zero with slight fluctuations, although  $z_{12}$ ,  $z_{13}$ , and  $z_{21}$  are a little far away from the origin at the initial state and  $z_{13}$  is the only signal which violates the constraint in the first 2 s. The corresponding control input  $\text{sat}(\tau)$  are shown in Figure 5. It can be seen that  $\text{sat}(\tau)$  could remain within the sets  $-500 < \text{sat}(\tau_i) < 500$ ,  $i = 1, 2, 3$ , while  $\text{sat}(\tau_3)$  increases sharply every few seconds.

## Conclusion

This study investigates the control approach for trajectory tracking of USV with input saturation and full-state constraints. For the saturation problem, the upper bound value of saturation input is estimated by the adaptive method, and its influence can be compensated in the control law. In addition, the BLF is used to deal with the state constraints making the tracking errors in the boundary. The actual trajectory is proved to track the desired trajectory successfully. The control approach proposed in this research can guarantee that the signals of the closed-loop system are uniformly bounded and the asymptotic tracking is almost achieved without violating the state constraints. Simulation results verify the facticity and effectiveness of the proposed method.

## Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

## Funding

The author(s) declared the following potential conflicts of interest with respect to the research, authorship, and/or publication of this article: This work was supported by the National Natural Science Foundation of China under grant (nos U1713205 and 61803119).

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