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On adaptive modified projective synchronization of a supply chain management system

HAMED TIRANDAZ

Mechatronics Faculty, Hakim Sabzevari University, Sabzevar, Iran
E-mail: hamedtirandaz@gmail.com

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Abstract. In this paper, the synchronization problem of a chaotic supply chain management system is studied. A novel adaptive modified projective synchronization method is introduced to control the behaviour of the leader supply chain system by a follower chaotic system and to adjust the leader system parameters until the measurable errors of the system parameters converge to zero. The stability evaluation and convergence analysis are carried out by the Lyapanov stability theorem. The proposed synchronization and antisynchronization techniques are studied for identical supply chain chaotic systems. Finally, some numerical simulations are presented to verify the effectiveness of the theoretical discussions.

Keywords. Modified projective synchronization; supply chain system; chaotic behaviour.

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1. Introduction

1.1 *Motivation*

Chaos synchronization is an extension of the concept of chaos control. Nowadays, the control and synchronization of the chaotic systems has got a lot of attention by the researchers because of its unpredictable complex behaviour. The ultimate objective of the chaos synchronization is to design a feedback controller for the follower chaotic system such that the follower system tracks the trajectories of the leader chaotic system as time goes to infinity. However, the challenges occur when the chaotic systems are exposed to some uncertainty, unknown system parameters and have different initial values. Then, some actions have to be taken in order to stabilize and to improve synchronization.

Chaos phenomenon generally appears in nonlinear dynamical systems. A nonlinear dynamical system has chaotic behaviour if it is sensitive to the initial conditions. Since Lorenz [1] in 1963 has discovered his 3D chaotic system, many chaotic systems such as Chen system [2], Lü system [3], Liu system [4], Genesio system [5], Bhalekar Gejji system [6], supply chain system [7], and many other chaotic systems, were found and studied by the researchers.

Recently, supply chain system has got considerable attention in analysis, modelling and planning because of its many economical applications [8,9]. The purpose of this paper is to investigate the synchronization and the control of chaos in supply chain system.

1.2 *Literature review*

In the literature, the first method called OGY method on control of the chaotic system was developed by Ott *et al* [10], and the first identical synchronization method was developed by Pecora and Carroll in [11]. Since then, different approaches have been extended for the synchronization and antisynchronization of the chaotic systems, either identical or non-identical ones. Active method [12,13], impulsive method [14], projective method [15,16], lag method [17], sliding method [18,19] and backstepping control method [20] are some of the investigated synchronization methods. Nevertheless, more often the parameters of a chaotic system are fully or partially uncertain or unknown and also system states are exposed to some unknown disturbances, and so these methods are usually of no use. To overcome this problem, many researchers concentrate on adaptive methods [21–24], which are extensions of adaptive control theory, in order to estimate unknown parameters. Furthermore, some investigations are performed to cope with unknown disturbances [25–29].

Many adaptive methods have been developed for control and synchronization of different types of chaotic systems, due to their high performance in synchronization task. Adaptive synchronization of two identical Lü system in [30], Lorenz system in [31], Rössler system in [32], Chen system in [33], Chua system in [34], Genesio system in [35] and unified system in [36,37] are some of the adaptive methods studied. Adaptive synchronization of two different chaotic systems such as adaptive synchronization of the Lü and the Lorenz chaotic systems [38], Chen and the Lü systems as leader systems with the Lorenz system [39], Rössler and Chen [40], Chen and Chua [41] and so on are extensively investigated. Furthermore, some researchers investigated some sort of extended adaptive synchronization methods, such as adaptive backsteping method [42], adaptive sliding mode method [43], adaptive projective method [44] etc.

The synchronization between hyperchaotic Lü and Lorenz system [45], the Genesio–Tesi chaotic system [46], the unified chaotic system in [47], and also the synchronization between two typical unknown chaotic systems [48–52] are some of the researches based on the adaptive-MPS method.

Recently, supply chain system has attracted the attention of many researchers [9,53–57]. Supply chain system wants to afford of the customers demands accurately on time with minimum possible cost. Supply chain systems have usually some unknown/uncertain coefficient in their dynamical systems. The behaviour of the supply chain system may become chaotic in some situations depending on the customers or purchasing decisions. The deficiency of supply shortages, order batching, price fluctuations and lead times may result in a phenomenon called bullwhip effect [58]. A number of studies are devoted to find the bullwhip effect resources to reduce the uncertainty.

Chaotic behaviour of the supply chain system at the production or inventory levels is not pleasant. So the control of a supply chain system may eliminate its nonlinear factors of the system. And also the synchronization of the supply chain systems can equilibrate the demand and resource planning of the system.

Anne *et al* [59] have proposed an adaptive method for the synchronization of the supply chain system with unknown internal or external disturbances. In order to improve their competitiveness, every enterprise has to use supply chain management system. Goksu *et al* [60] have designed a linear feedback controller to control and to synchronize the supply chain system. Chaos synchronization of the supply chain system is carried out by using radial basis function in [7] to counteract the bullwhip effect. In [61], the bullwhip effect is challenged by the linear control theory. So far, there is no published article on adaptive-MPS synchronization of the supply chain chaotic systems, which is the novelty of this paper.

1.3 *Approach and contribution*

In the following, the supply chain system and its chaotic behaviour are described. Then an adaptive-MPS scheme is developed for the synchronization of the leader–follower supply chain systems with or without unknown internal/external distortions; and also an appropriately designed feedback controller is proposed to track the trajectories of the leader supply chain system by the corresponding follower system. Then, chaos synchronization of the leader–follower systems are proved by the Lyapunov stability theorem. At the end, the validity of the proposed method is assessed by some numerical simulations.

The reminder of this article is as follows: In §2, a brief introduction of the supply chain chaotic system is provided. The proposed identical adaptive-MPS synchronization of the supply chain system is investigated in §3, for different unknown distortions such as internal distortions, external distortions and hybrid internal–external distortions. Section 4 includes the numerical simulation results of the represented approaches to study the effectiveness of their synchronization. Finally, in §5, some conclusions are provided.

2. Problem statement

A typical supply chain system can be constructed based on three main components: producers, distributors and final customers. In [59], the dynamic behaviour of the supply chain system is given by three-dimensional equations as follows:

$$\begin{aligned}\dot{x}_1 &= (m+\delta_m)x_2 - (n+1+\delta_n)x_1 + d_1,\\ \dot{x}_2 &= (r+\delta_r)x_1 - x_2 - x_1x_3 + d_2,\\ \dot{x}_3 &= x_1x_2 + (k-1-\delta_k)x_3 + d_3,\end{aligned} \tag{1}$$

where $\dot{X} = (\dot{x_1}, \dot{x_2}, \dot{x_3})$ is the time derivative of the state variable vector $X = (x_1, x_2, x_3)$. Linear disorders $\delta_m, \delta_n, \delta_r$ and δ_k are the amount of perturbance of the constant parameters m, n, r and k, respectively. And also d_1, d_2 and d_3 are the three nonlinear external distortions related to the states x_1, x_2 and x_3, which correspond to the three quantities as demand, inventory, and produced quantity, respectively. The component m indicates the distributor's delivery efficiency; the constant

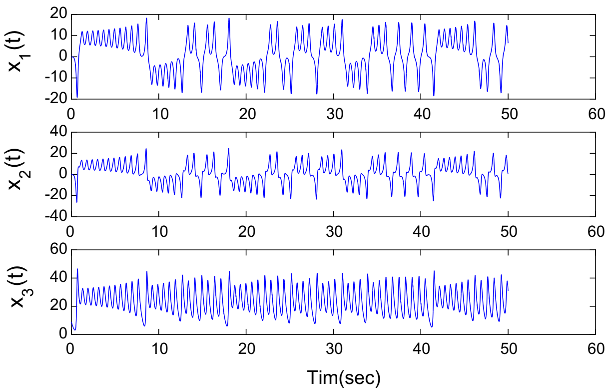

Figure 1. Time series phase portrait of the chaotic supply chain system for state values $x_1(t)$, $x_2(t)$ and $x_3(t)$, respectively.

parameter n denotes the customer demand rate; the constant parameter r implies the distortion coefficient and k is the safety stock coefficient.

Chaotic behaviour of the supply chain system is obtained with distributor values as: $m = 10$, $\delta_m = 0.1$, $n = 9$, $\delta_n = 0.1$, $r = 28$, $\delta_r = 0.2$, $k = -5/3$, $\delta_k = 0.3$ and external perturbation values as $d_1 = 0.2\sin(t)$, $d_2 = 0.1\cos(5t)$, $d_3 = 0.3\sin(t)$. The initial state values are considered as $x_1 = 0$, $x_2 = -0.11$ and $x_3 = 9$ throughout this paper.

Time series of the supply chain system is given in figure 1. The 2D and 3D phase plane behaviour of the system are shown in figures 2 and 3 respectively.

The 3D chaotic supply chain system presented in (1) can be rewritten as follows:

$$\dot{X} = (A_P + A_\Delta)X + x_1 \cdot BX + D, \tag{2}$$

where $D = (d_1, d_2, d_3)^T$ is the nonlinear distortion vector. $P = (m, n, r, k)$ denotes the constant parameter of the leader system (1) and $\Delta = (\delta_m, \delta_n, \delta_r, \delta_k)$ is the distribution vector of the leader system (1). The coefficient matrixes of $A_P, A_\Delta, B \in R^{3\times 3}$ are given as

$$A_P = \begin{bmatrix} -n-1 & m & 0 \\ r & -1 & 0 \\ 0 & 0 & k-1 \end{bmatrix}, \quad A_\Delta = \begin{bmatrix} -\delta_n & \delta_m & 0 \\ \delta_r & 0 & 0 \\ 0 & 0 & -\delta_k \end{bmatrix},$$
$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{3}$$

The matrices $(A_P + A_\Delta)$ as the coefficient of vector X in eq. (2) has three eigenvalues: $\lambda_1 = -23.0292$, $\lambda_2 = 11.9292$, $\lambda_3 = -2.9667$. Since λ_2 is positive, it can be concluded from the Lyapunov stability theory [62] that the supply chain attractor presented in (2) is not stable at its origin equilibrium point $E_0 = (0, 0, 0)$.

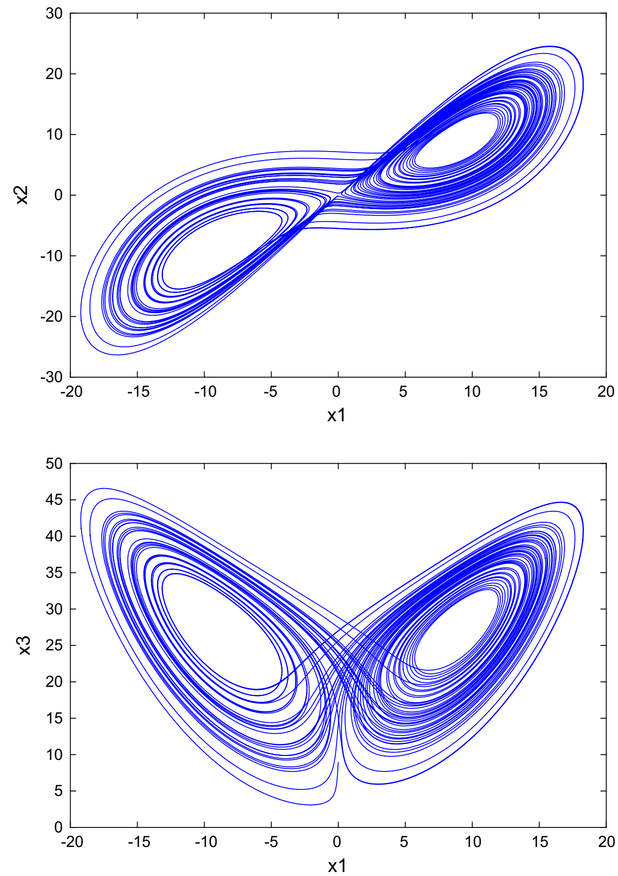

Figure 2. Two-dimensional phase portrait of the chaotic supply chain system.

3. Synchronization

Chaos synchronization of two identical and non-identical chaotic systems is discussed in this section. Some parameters of the leader system (2) are considered unknown. To this end, an adaptive-MPS method is extended to provide synchronization between the leader and follower supply chain systems.

3.1 *Identical synchronization with unknown internal linear distortions*

Chaos synchronization of the supply chain system with internal unknown linear distortion $\Delta = (\delta_m, \delta_n, \delta_k, \delta_r)$ is studied here.

Consider the 3D chaotic supply chain system presented in (2) as the leader system. Then the 3D chaotic follower system can be presented based on the supply chain system (2) as follows:

$$\dot{Y} = (A_P + A_{\hat{\Delta}})Y + y_1 \cdot BY + D + U, \tag{4}$$

where $Y = (y_1, y_2, y_3)$ is the state vector of the follower supply chain system and $U = (u_1, u_2, u_3)$ implicates

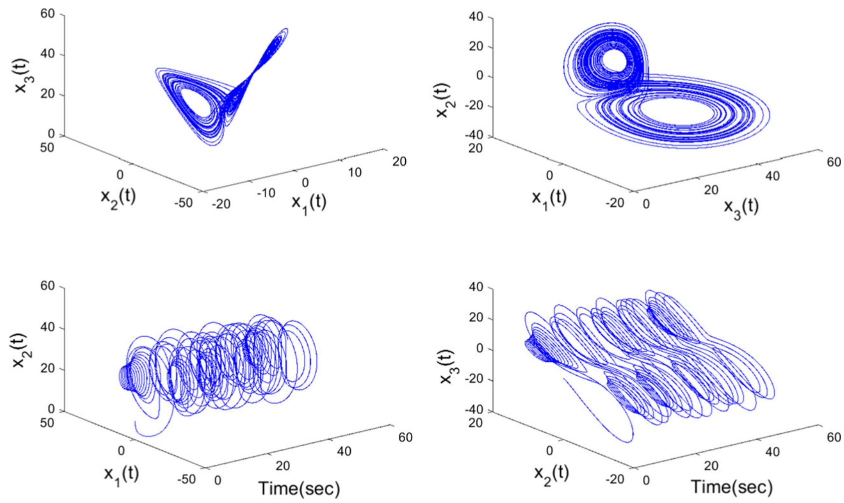

Figure 3. Three-dimensional phase portrait of the chaotic supply chain system.

the feedback control law of the closed-loop control system, which has to be designed in such a way that the behaviour of the follower system to track the trajectories of the leader system, would mean that two identical chaotic systems (2) and (4) would synchronize. The constant matrixes A_P and $B \in R^{3\times 3}$ can be defined based on eq. (3) and matrix $A_{\hat{\Delta}}$ can be set as

$$A_{\hat{\Delta}} = \begin{bmatrix} -\hat{\delta}_n & \hat{\delta}_m & 0 \\ \hat{\delta}_r & 0 & 0 \\ 0 & 0 & -\hat{\delta}_k \end{bmatrix}, \tag{5}$$

where $\hat{\Delta} = (\hat{\delta}_m, \hat{\delta}_n, \hat{\delta}_r, \hat{\delta}_k)$ is the estimation of the distribution parameter vector $\Delta = (\delta_m, \delta_n, \delta_r, \delta_k)$ of the leader system (2).

Chaos synchronization errors extracted by adaptive-MPS synchronization between two identical leader–follower systems (2) and (4) can be obtained as

$$E_s = \sigma \cdot (Y - \lambda \cdot X), \quad E_\Delta = \hat{\Delta} - \lambda \cdot \Delta, \tag{6}$$

where E_s and E_Δ represent the state error vector and the parameter error vector, respectively. The coefficients $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ and the vector $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ are the multiple projective constant vectors. Then the error dynamic system between the leader and the follower systems (2) and (4) can be achieved by time derivatives of (6) as follows:

$$E_s' = \sigma \cdot (\dot{Y} - \lambda \cdot \dot{X}),$$
$$E_\Delta' = \dot{\hat{\Delta}} = (\dot{\hat{\delta}}_m, \dot{\hat{\delta}}_n, \dot{\hat{\delta}}_r, \dot{\hat{\delta}}_k). \tag{7}$$

Without designing an appropriate controller system, the state variable trajectories of the follower and the leader chaotic systems with different initial state values will quickly depart from each other. The objective of chaos synchronization is to design a feedback controller that can prevent such a bifurcation problem. Now, the control vector and the parameter estimation strategy can be defined based on the following illustrative theorem.

Theorem 1. *The trajectories of the leader chaotic system* (2) *with unknown internal distortion parameter vector* Δ *will be tracked asymptotically by an identical follower system* (4); *and unknown parameter vector* Δ *will be approximated by an estimated parameter vector* $\hat{\Delta}$; *for any initial state values and considering the feedback control and dynamical estimation parameters as follows*:

$$U = -(A_P + A_{\hat{\Delta}})Y - y_1 \cdot BY + x_1 \cdot \lambda \cdot BX + (\lambda - 1) \cdot D + A_{\hat{\Delta}} X + \lambda A_p X - \sigma \cdot E_s \tag{8}$$

and the elements of $\dot{\hat{\Delta}}$ *as*

$$\begin{aligned} \dot{\hat{\delta}}_m &= -x_2\sigma_1(y_1 - \lambda_1 x_1) - k_{\Delta 1}(\hat{\delta}_m - \lambda_1\delta_m), \\ \dot{\hat{\delta}}_n &= +x_1\sigma_1(y_1 - \lambda_1 x_1) - k_{\Delta 2}(\hat{\delta}_n - \lambda_1\delta_n), \\ \dot{\hat{\delta}}_r &= -x_1\sigma_2(y_2 - \lambda_2 x_2) - k_{\Delta 3}(\hat{\delta}_r - \lambda_2\delta_r), \\ \dot{\hat{\delta}}_k &= +x_3\sigma_3(y_3 - \lambda_3 x_3) - k_{\Delta 4}(\hat{\delta}_k - \lambda_3\delta_k), \end{aligned} \tag{9}$$

where $K_\Delta = (k_{\Delta 1}, k_{\Delta 2}, k_{\Delta 3}, k_{\Delta 4})$ *is a positive constant vector.*

Proof. The Lyapunov candidate function for stability analysis can be given as follows:

$$V = \frac{1}{2}\left(E_s^2 + E_\Delta^2\right) \tag{10}$$

which is a positive definite function organized based on the system state variable error vector E_s, and the parameter estimation error vector E_Δ, represented in (6).

Assuming that time derivative of eq. (10) exists, we have:

$$\begin{aligned}\dot{V} &= E_s E_s' + E_\Delta E_\Delta' \\ &= E_s\sigma[(A_P + A_{\hat{\Delta}})Y + y_1 \cdot BY + D + U \\ &\quad - \lambda(A_P + A_\Delta)X - x_1 \cdot \lambda BX - \lambda D] \\ &\quad + (\hat{\Delta} - \lambda\Delta)E_\Delta' \\ &= E_s\sigma[(A_{\hat{\Delta}} - \lambda A_\Delta)X - \sigma E_s] + (\hat{\Delta} - \lambda\Delta)E_\Delta' \\ &= -\sigma^2 E_s^2 - K_\Delta E_\Delta^2. \end{aligned} \tag{11}$$

Hence, the time derivative of V is negative definite. As a result, from the Lyapunov stability theorem, the dynamical error systems (7) will be stabilized at the origin in finite time. Thus, the trajectories of the state variables of the leader system will be tracked by the state variables of the follower system. So the theorem is proved.

3.2 *Identical synchronization with unknown external nonlinear distortions*

The purpose of this section is to perform identical synchronization of the supply chain chaotic system when $D = (d_1, d_2, d_3)^T$. The external nonlinear parameter vector of the leader system (2) is considered unknown.

Let the chaotic system presented in (2) is the leader system. Then the corresponding follower system with unknown external nonlinear distortion vector D can be represented as follows:

$$\dot{Y} = (A_P + A_\Delta)Y + y_1 \cdot BY + \hat{D} + U, \tag{12}$$

where $Y = (y_1, y_2, y_3)^T$ is the follower state vector. A_P, A_Δ and $B \in R^{3\times 3}$ are defined in eq. (3). $\hat{D} = (\hat{d}_1, \hat{d}_2, \hat{d}_3)$ is the estimated parameter vector of the unknown distribution vector D. And also the feedback controller vector is characterized by $U = (u_1, u_2, u_3)^T$. The objective is to design an appropriate feedback controller U and also an efficient external vector $\hat{D}$ such that the motion trajectories of the corresponding leader and follower state variables (2) and (12) asymptotically synchronize along the time domain.

The synchronization error vectors can be defined similar to eq. (6), as follows:

$$E_s = \sigma(Y - \lambda X), \quad E_D = \hat{D} - \lambda D, \tag{13}$$

where E_s, the same as in the previous section, represents the state error vector and E_D describes the estimation error of the unknown external disturbance vector D. The coefficient vector $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ and vector $\lambda = (\lambda_1, \lambda_2, \lambda_3)$ denotes the modified projective synchronization factors. Then the error dynamical system between the leader chaotic system (2) and the follower chaotic system (12) can be described as

$$E_s' = \sigma(\dot{Y} - \lambda\dot{X}), \quad E_D' = \dot{\hat{D}} = (\dot{\hat{d}}_1, \dot{\hat{d}}_2, \dot{\hat{d}}_3). \tag{14}$$

Now, the following theorem presents an appropriate feedback controller and also an efficient external parameter estimation scheme to address the synchronization problem.

Theorem 2. *The motion trajectories of the leader and the follower chaotic systems* (2) *and* (12) *will asymptotically synchronize with any initial state values and considering the feedback controller vector and the dynamic representation of the unknown external parameter estimation vector as follows*:

$$\begin{aligned} U = &- (A_P + A_\Delta)Y - y_1 \cdot BY + \lambda(A_P + A_\Delta)X \\ &+ x_1 \cdot \lambda BX - \sigma E_s \end{aligned} \tag{15}$$

and

$$\dot{\hat{D}} = -\sigma E_s - K_d E_D, \tag{16}$$

where $K_d = (k_{d1}, k_{d2}, k_{d3})$ *is a positive constant vector.*

Proof. The Lyapunov candidate function can be described as follows:

$$V = \frac{1}{2}\left(E_s^2 + E_D^2\right) \tag{17}$$

which is clearly positive definite. Furthermore, the time derivative of eq. (17) can be extended as

$$\begin{aligned}\dot{V} &= E_s E_s' + E_D E_D' \\ &= \sigma E_s\left[\dot{Y} - \lambda\dot{X}\right] + E_D\dot{\hat{D}} \\ &= \sigma E_s\left[(A_P + A_\Delta)Y + y_1 \cdot BY + \hat{D}\right. \\ &\quad + U - \lambda(A_P + A_\Delta)X \\ &\quad \left. - x_1 \cdot \lambda BX - \lambda D\right] + (\hat{D} - \lambda D)\dot{\hat{D}} \\ &= \sigma E_s\left[\hat{D} - \lambda D - \sigma E_s\right] + (\hat{D} - \lambda D)\dot{\hat{D}} \\ &= -\sigma^2 E_s^2 - K_d E_D^2 < 0. \end{aligned} \tag{18}$$

Therefore, the derivative of V is negative definite. Then according to the Lyapunov stability theorem, the synchronization errors E_s between the leader and the follower state variables and E_d between the estimated external disturbance and its true values converge to zero. This completes the proof.

3.3 *Identical synchronization with unknown internal and external distortions*

In the following, the identical synchronization of the supply chain system is discussed by designing an adaptive-MPS synchronization method. For synchronization purposes, both the internal linear distortion and the external nonlinear distortions are considered unknown.

Let the supply chain system represented in (2) is the leader chaotic system, then the follower chaotic system with unknown internal and external distortions can be described as

$$\dot{Y} = (A_P + A_{\hat{\Delta}})Y + y_1 \cdot BY + \hat{D} + U, \tag{19}$$

where A_P and B can be determined by eqs (3) and the estimated internal and external parameter vectors $\hat{\Delta}$ and $\hat{D}$ can be determined by eqs (9) and (16) respectively. The synchronization error vectors can be obtained based on the error vectors given in (6) and (13) as follows:

$$E_s = \sigma(Y - \lambda X),$$
$$E_\Delta = \hat{\Delta} - \lambda\Delta, \quad E_D = \hat{D} - \lambda D. \tag{20}$$

Then the dynamical error vectors can be described as

$$E'_s = \sigma(\dot{Y} - \lambda\dot{X}), \quad E'_\Delta = \dot{\hat{\Delta}} = (\dot{\hat{\delta}}_m, \dot{\hat{\delta}}_n, \dot{\hat{\delta}}_r, \dot{\hat{\delta}}_k),$$
$$E'_D = \dot{\hat{D}} = (\dot{\hat{d}}_1, \dot{\hat{d}}_2, \dot{\hat{d}}_3). \tag{21}$$

Theorem 3. *The dynamical estimated internal error vector* (9)*, the dynamical external error vector* (16)*, and the adaptive-MPS feedback controller law*:

$$U = (A_P + A_{\hat{\Delta}})Y + y_1 \cdot BY$$
$$- \lambda A_P X - x_1 \cdot \lambda BX - \sigma E_s \tag{22}$$

when applied to the follower chaotic system (19) *guarantee the asymptotical synchronization of the leader and the follower chaotic systems and also assure the convergence of the synchronization errors E_Δ and E_D to zero, as time tends to infinity.*

Proof. Define the Lyapunov stability function based on the system errors (20) as

$$V = \frac{1}{2}\left(E_s^2 + E_\Delta^2 + E_D^2\right). \tag{23}$$

which is a positive definite function on R. Furthermore, the time derivative of V can be simplified as

$$\dot{V} = E_s E'_s + E_\Delta E'_\Delta + E_D E'_D. \tag{24}$$

By substituting (20) and (21) in (24), we obtained:

$$\dot{V} = \sigma E_s\left[\dot{Y} - \lambda\dot{X}\right] + (\hat{\Delta} - \lambda\Delta)\dot{\hat{\Delta}} + (\hat{D} - \lambda D)\dot{\hat{D}}$$
$$= \sigma E_s\left[(A_P + A_{\hat{\Delta}})Y + y_1 \cdot BY + \hat{D}\right.$$
$$\left. + U - \lambda(A_P + A_\Delta)X - x_1 \cdot \lambda BX - \lambda D\right]$$
$$+ (\hat{\Delta} - \lambda\Delta)\dot{\hat{\Delta}} + (\hat{D} - \lambda D)\dot{\hat{D}}$$
$$= \sigma E_s\left[\hat{D} + (A_{\hat{\Delta}} - \lambda A_\Delta)X - \lambda D\right]$$
$$+ (\hat{\Delta} - \lambda\Delta)\dot{\hat{\Delta}} + (\hat{D} - \lambda D)\dot{\hat{D}}$$
$$= -\sigma^2 E_s^2 - K_\Delta E_\Delta^2 - K_d E_D^2 \tag{25}$$

which is negative definite. So the theorem is proved.

4. Numerical simulations

The objective of the numerical simulations is to validate the effectiveness and feasibility of the proposed approach for synchronization of two chaotic systems and also identification of unknown distributions. In this section, some numerical results related to the synchronization of the identical supply chain system are given.

Numerical simulations have been carried out using Matlab Simulink. The implementation program is written based on fourth-order Runge–Kutta iterative method with a fixed time-step size and a tolerance of 10^{-6}.

For simulation purposes, the supply chain system presented in (1) is considered as the leader system. Then the synchronization between the leader and the corresponding follower systems are done based on the designed feedback controllers and parameter estimation strategies.

For chaotic behaviour of the supply chain system (1), parameters are selected as: $m = 10, \delta_m = 0.1, n = 9, \delta_n = 0.1, r = 28, \delta_r = 0.2, k = -5/3, \delta_k = 0.3$ and external perturbation values as $d_1 = 0.2\sin(t), d_2 = 0.1\cos(5t), d_3 = 0.3\sin(t)$.

4.1 *Synchronization results with unknown internal distortions*

The initial state values are assumed typically as: $X(0) = (0, -0.11, 9)^T$ and $Y(0) = (7, 8, 2)^T$. The initial values of internal distortions are considered as: $\hat{\Delta} = (\hat{\delta}_m, \hat{\delta}_n, \hat{\delta}_r, \hat{\delta}_k) = (0.3, 0.1, 0.5, 0.6)$.

The behaviour of the leader and the follower systems are given in figure 4 and antisynchronization are presented in figure 5. Parameter estimation errors are depicted in figure 6. It is clearly evident from figures 4–6 that the expected synchronization and antisynchronization between the leader and the follower systems are obtained; and the synchronization errors converge to zero as time tends to infinity.

4.2 *Synchronization results with unknown external nonlinear distortions*

The initial condition of the leader and the follower systems are considered as: $X(0) = (3, 8, 2)^T$ and $Y(0) = (9, 0, 10)^T$. The initial values for the external distortions are initially considered as: $D = (\hat{d}_1, \hat{d}_2, \hat{d}_3) = (0.3, 0.1, 0.5)$.

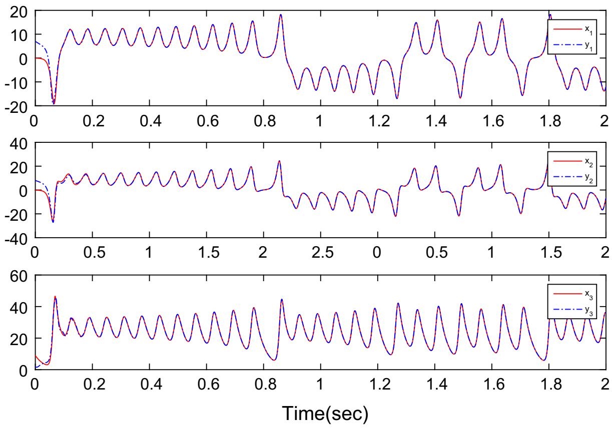

Figure 4. Motion trajectories of the state variables of the leader and the follower chaotic supply chain systems for multiple projective coefficient $\lambda = (1, 1, 1)$ or the complete synchronization with unknown internal distortions.

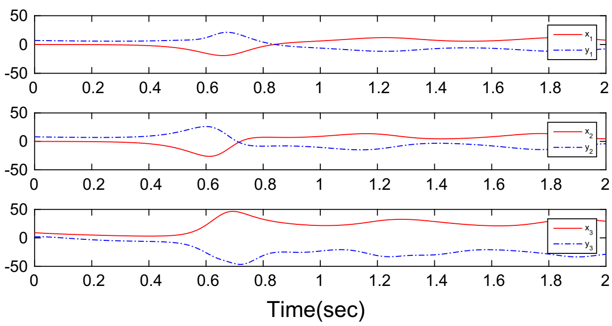

Figure 5. Motion trajectories of the state variables of the leader and the follower chaotic supply chain systems for multiple projective coefficient $\lambda = (-1, -1, -1)$ or the anti-synchronization with unknown internal distortions.

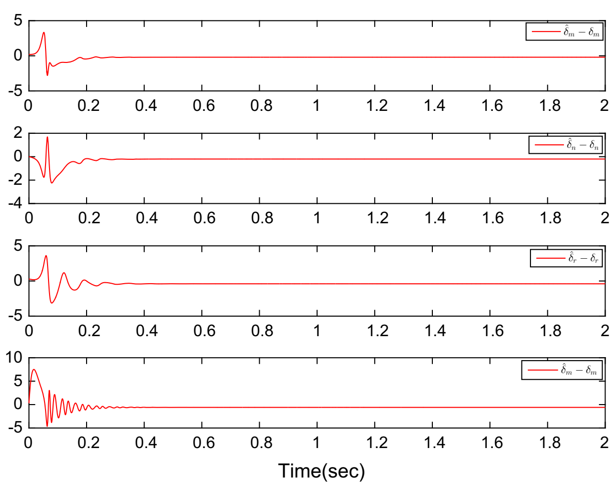

Figure 6. Estimation errors of unknown internal distortion parameters $\delta_m, \delta_n, \delta_r$ and δ_k.

The time responses of the leader and the follower systems are shown in figure 7 and also antisynchronization are given in figure 8. Unknown external parameter

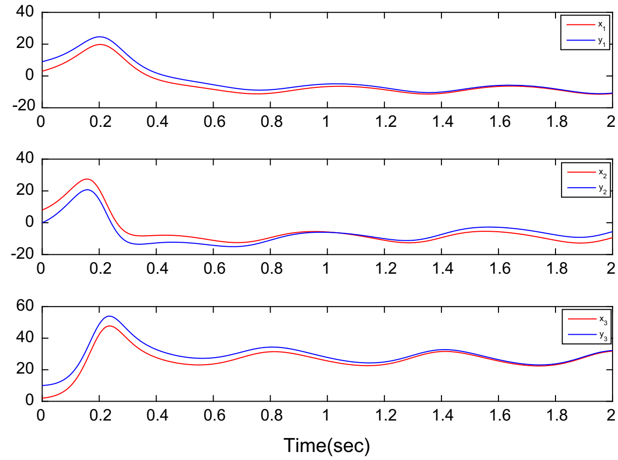

Figure 7. Motion trajectories of the state variables of the leader and the follower chaotic supply chain systems for multiple projective coefficient $\lambda = (1, 1, 1)$ or the complete synchronization with unknown external distortions.

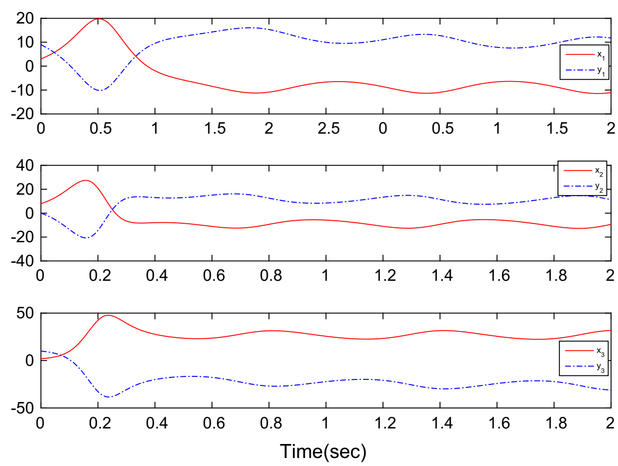

Figure 8. Motion trajectories of the state variables of the leader and the follower chaotic supply chain systems for multiple projective coefficient $\lambda = (-1, -1, -1)$ or the anti-synchronization with unknown external distortions.

estimation errors are given in figure 9. It is clearly apparent from figures 7–9 that the expected synchronization between the leader and the follower systems is achieved.

4.3 *Synchronization results with unknown internal and external distortions*

Let the initial state values for the leader and the follower chaotic systems are $X(0) = (7, 12, 8)^T$ and $Y(0) = (-3, 2, 1)^T$, respectively. The initial values for the internal and the external distortions are considered as $\hat{\Delta} = (\hat{\delta}_m, \hat{\delta}_n, \hat{\delta}_r, \hat{\delta}_k) = (0.2, 0.7, 0.3, 0.7)$ and $D = (\hat{d}_1, \hat{d}_2, \hat{d}_3) = (0.6, 0.4, 0.6)$, respectively.

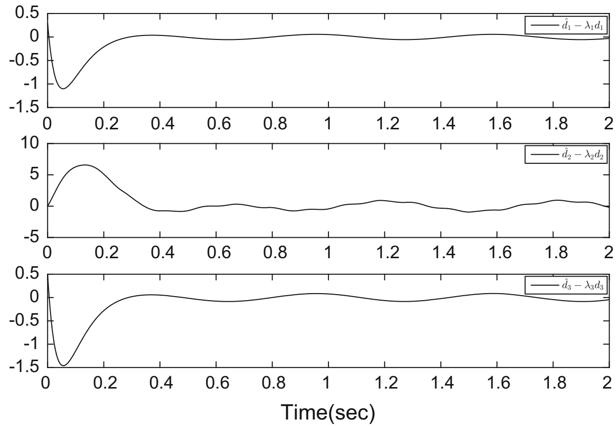

Figure 9. Estimation errors of the unknown external distortion parameters d_1, d_2 and d_3.

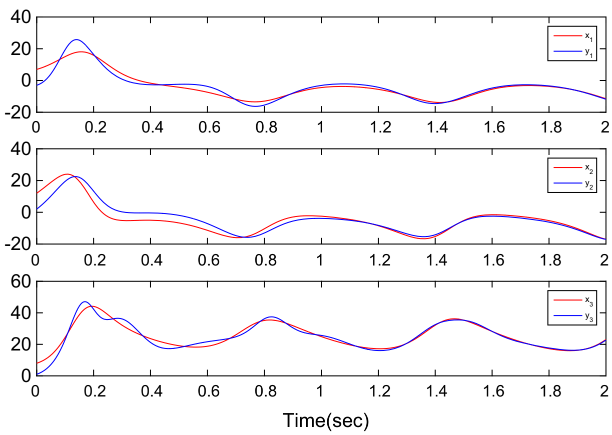

Figure 10. Motion trajectories of the state variables of the leader and the follower chaotic supply chain systems for multiple projective coefficient $\lambda = (1, 1, 1)$ or the complete synchronization with unknown internal and external distortions.

The time response of the leader and the follower systems are shown for synchronization in figure 10 and also for antisynchronization in figure 11. The internal and external parameter estimation errors are given in figures 12 and 13, respectively. It is clearly apparent from figures 10 to 13 that the expected synchronization between the leader and the follower systems is obtained.

5. Conclusion

Nonlinear behaviour and internal/external distortions are not desirable factors in a supply chain system. The undesirable behaviour of the system may be caused by the phenomenon called bullwhip effect which causes chaos in the supply chain system. Bullwhip effect can be detected sometimes under certain circumstances. One of the most effective ways to reduce the bullwhip effect is the demand driven supply chain management. Hence, an appropriate synchronization method can reduce the nonlinear behaviours and bullwhip effect of the supply chain system.

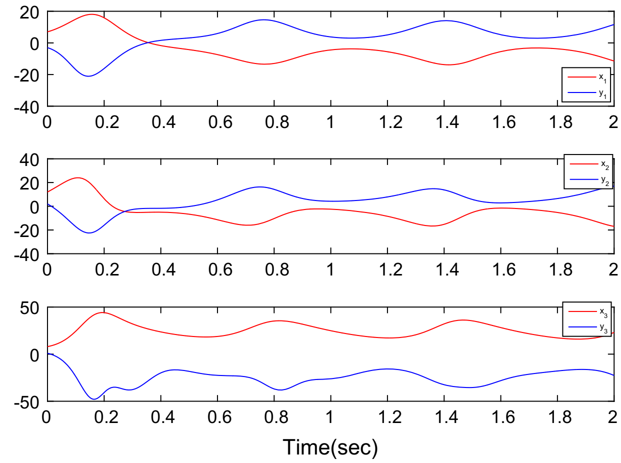

Figure 11. Motion trajectories of the state variables of the leader and the follower chaotic supply chain systems for multiple projective coefficient $\lambda = (-1, -1, -1)$ or the antisynchronization with unknown internal and external distortions.

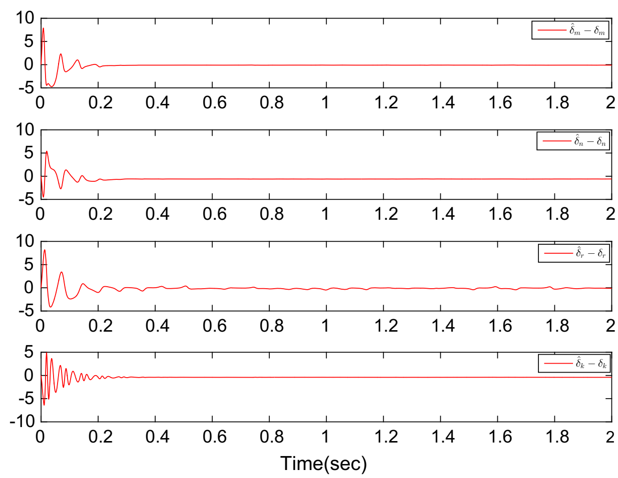

Figure 12. Estimation errors of the unknown internal and external distortion parameters δ_m, δ_n, δ_r and δ_k.

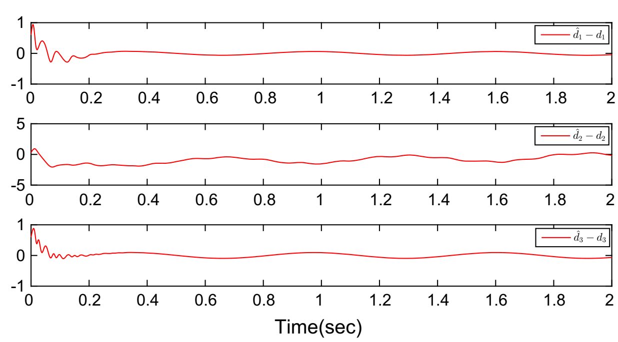

Figure 13. Estimation errors of the unknown internal and external distortion parameters d_1, d_2 and d_3.

Chaos synchronization of the supply chain system is addressed in this paper. An adaptive-MPS synchronization approach is used for identifying an appropriate feedback controller and also an estimated unknown parameter vector. Investigation of synchronization method is carried out for different types of unknown distortions: internal, external or both of them. The performance of the proposed feedback controller and the developed synchronization is proved by Lyapunov stability theorem. Furthermore, as we can see from the simulation results, the synchronization errors of the two identical supply chain systems for either internal or external unknown parameters converges asymptotically to zero.

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