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The propagation of waves in thin-film ferroelectric materials

ABBAGARI SOULEYMANOU[1,2,3,4], KHALID K ALI[5], HADI REZAZADEH[6,*], MOSTAFA ESLAMI[7], MOHAMMAD MIRZAZADEH[8] and ALPER KORKMAZ[9]

[1]National Advanced School of Engineering, University of Yaounde I, P.O. Box 8390, Yaoundé, Cameroon
[2]Laboratory of Mechanics, Materials and Structures, Department of Physics, University of Yaounde I, P.O. Box 812, Yaoundé, Cameroon
[3]Centre d'Excellence en Technologies de l'Information et de la Communication (CETIC), University of Yaounde I, P.O. Box 8390, Yaoundé, Cameroon
[4]Department of Basic Science, Faculty of Mines and Petroleum Industries, University of Maroua, P.O. Box 46, Maroua, Cameroon
[5]Mathematics Department, Faculty of Science, Al-Azhar University, Nasr-City, Cairo, Egypt
[6]Faculty of Modern Technologies Engineering, Amol University of Special Modern Technologies, Amol, Iran
[7]Department of Mathematics, Faculty of Mathematical Sciences, University of Mazandaran, Babolsar, Iran
[8]Department of Engineering Sciences, Faculty of Technology and Engineering, East of Guilan, University of Guilan, Rudsar-Vajargah, Iran
[9]Department of Mathematics, CAKU, Çankırı, Turkey
*Corresponding author. E-mail: H.rezazadeh@ausmt.ac.ir

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Abstract. The nonlinear evolution equation describing the propagation of waves in thin-film ferrroelectric materials is investigated in detail. The modified extended tanh method is used for the purpose and, as a result, novel soliton solutions are derived analytically which show the shape and the width of the waves. In the construction of the solutions obtained, it appears that bright and singular waves can be propagated in thin-film ferroelectric materials.

Keywords. Thin-film ferrroelectric materials; extended tanh method; bright soliton.

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1. Introduction

Wave phenomena exist in many areas of our environment, including wind, water and sound, just to name a few. Looking for materials that will improve the existing technological devices, researchers have investigated numerous materials and metamaterials in domains that encompass electric line, optical fibres and electronic devices [1,2]. While investigating these materials, mathematical equations are often derived, modelling the dynamics of wave moving in the considered material. In the case of optical fibres [3–5], scalar and vector short pulse equations have been derived, in the case of ferrites [2,6] and also in the case of thin-film ferroelectric materials [7], one-dimensional and two-dimensional equations have been derived. On having these equations at hand, the question of their integrability is posed, because their solutions are more expressive in describing the dynamics of waves in the material of interest. To answer these questions, a number of mathematical techniques are put forward, including the prolongation structure [8] and Painlevé analysis [9]. In the winding of these commonly handled methods, there are mathematical tools that are more direct in providing analytical expressions of the solution to nonlinear equations covering Hirota's bilinear method, Kudryashov's method, first integral approach, sub-equation technique, simple hyperbolic function ansatzes, hyperbolic tangent methods etc. [10–27].

With regard to thin-film ferroelctric materials, solution of nonlinear equation has been derived in [7] when considering the idealised model of a one-dimensional array of N identical ferroelectric domains [7,28,29]. While considering the Landau–Ginzburg–Devonshire mean-field theory, a nonlinear evolution equation has been derived, describing the propagation of polarisation in thin-film materials, which is shown to possess periodical soliton solutions. To provide the richness of the solution of such nonlinear equations, it is necessary to investigate its solution using alternative methods, including inverse scattering transform [30], Darboux [31] transform and tanh method just to name a few.

In the present study, we take into consideration the one-dimensional form of the well-known thin-film ferroelectric model:

$$\frac{m_d}{Q_d^2}\frac{\partial^2 p}{\partial t^2} - \left[(g_2 - 2\beta)p + g_4 p^3 + g_6 p^5\right] - k\Delta p = 0, \tag{1}$$

where m_d represents the mass density of the material, Q_d is the charge density and $g_i, i = 2, 4, 6$, are the parameters that are generally used to denote temperature and pressure [28,32].

Looking for a new soliton structure to the thin-film polarisation equation, we pay particular attention to the modified tanh method throughout this paper. Then, we present briefly the modified tanh method while applying it directly to the thin-film polarisation equation. We derive analytical expressions of the different solutions alongside the corresponding constraint and depict the solutions obtained. We end this paper with a brief conclusion.

2. Method and solutions

To explain the principal idea of the method, consider the following partial differential equation with nonlinear terms of the form:

$$F(p, p_t, p_x, p_{tt}, p_{xt}, \ldots) = 0. \tag{2}$$

The simple wave transformation

$$p(x,t) = p(\xi), \quad \xi = x - ct,$$

where c is a non-zero constant and generally represents the velocity of the wave, reduces eq. (2) into an ordinary differential equation with integer orders as

$$H(p, p', p'', p''', \ldots) = 0, \tag{3}$$

where the derivatives are with respect to ξ.

Without any delay, the wave transform defined above reduces the governing equation (1) to give

$$\left(\frac{m_d c^2}{Q_d^2} - k\right)p'' - \left[(g_2 - 2\beta)p + g_4 p^3 + g_6 p^5\right] = 0. \tag{4}$$

Assume that the solution of eq. (4) is presented in a finite series:

$$p(\xi) = a_0 + \sum_{n=1}^{N}\left(a_n\phi^n(\xi) + b_n\phi^{-n}(\xi)\right), \tag{5}$$

where a_n and $b_n, n = 0, 1, 2, \ldots, N$, are constants with at least one of a_n or b_n being non-zero, which will be evaluated in the following steps, and $\phi(\xi)$ satisfies the following Riccati equation:

$$\phi' = w + \phi^2, \tag{6}$$

where w is a constant. The Riccati equation (6) has the following general solutions:

(i) If $w < 0$, then

$$\phi = -\sqrt{-w}\tanh\left(\sqrt{-w}\,\xi\right)$$

or

$$\phi = -\sqrt{-w}\coth\left(\sqrt{-w}\xi\right).$$

(ii) If $w > 0$, then

$$\phi = \sqrt{w}\tan\left(\sqrt{w}\xi\right) \quad \text{or} \quad \phi = -\sqrt{w}\cot\left(\sqrt{w}\xi\right).$$

(iii) If $w = 0$, then

$$\phi = \frac{-1}{\xi}.$$

The index limit positive integer N is found by the standard balance procedure between the linear highest order and nonlinear highest degree terms in (4). Substituting the assumed solution (5) and its derivatives

$$p' = \sum_{n=1}^{N}\left(a_n n\phi^{n-1}(w + \phi^2) - b_n n\phi^{-n-1}(w + \phi^2)\right),$$

$$p'' = \sum_{n=1}^{N}\Big(a_n n(n-1)\phi^{n-2}(w + \phi^2)^2 + 2na_n\phi^n(w+\phi^2) + b_n n(n+1)\phi^{-n-2}(w+\phi^2)^2 - 2b_n n\phi^{-n}(w + \phi^2)\Big),$$

into (4) yields

$$H(\phi(\xi)) = 0, \tag{7}$$

where $H(\phi(\xi))$ is a polynomial in $\phi(\xi)$. Equating the coefficients of each power of $\phi(\xi)$ in eq. (7) to zero leads to a system of algebraic equations.

The balance procedure p'' and p^5 in (4) results in $N + 2 = 5N$, and so $N = 1/2$. But we know that N must be a positive integer. Choosing the transformation function $p(\xi) = \sqrt{\psi(\xi)}$ and substituting into (4), we get

$$\left(\frac{m_d c^2}{Q_d^2} - k\right)\left(-\frac{1}{4}\psi'^2 + \frac{1}{2}\psi\psi''\right)$$

$$-(g_2-2\beta)\psi^2-g_4\psi^3-g_6\psi^4=0. \qquad (8)$$

Now, we balance $\psi\psi''$ and ψ^4 in (8) which results in $N+N+2=4N$, and so $N=1$. Thus, the solution takes the form

$$\psi(\xi)=a_0+a_1\phi(\xi)+b_1\phi^{-1}(\xi). \qquad (9)$$

Substituting eq. (9) into (8) and setting the coefficients of each power of $\phi(\xi)$ to zero, we obtain the following system of algebraic equations:

$$2\beta a_0^2+\frac{1}{4}kw^2a_1^2-3kwa_1b_1+4\beta a_1b_1+\frac{kb_1^2}{4}-a_0^2g_2$$
$$-2a_1b_1g_2-a_0^3g_4-6a_0a_1b_1g_4-a_0^4g_6$$
$$-12a_0^2a_1b_1g_6-6a_1^2b_1^2g_6-\frac{c^2w^2a_1^2m_d}{4Q_d^2}$$
$$+\frac{3c^2wa_1b_1m_d}{Q_d^2}-\frac{c^2b_1^2m_d}{4Q_d^2}=0,$$
$$-\frac{3kw^2b_1^2}{4}-b_1^4g_6+\frac{3c^2w^2b_1^2m_d}{4Q_d^2}=0,$$
$$-kw^2a_0b_1-b_1^3g_4-4a_0b_1^3g_6+\frac{c^2w^2a_0b_1m_d}{Q_d^2}=0,$$
$$-\frac{3kw^2a_1b_1}{2}-\frac{kwb_1^2}{2}+2\beta b_1^2-b_1^2g_2$$
$$-3a_0b_1^2g_4-6a_0^2b_1^2g_6$$
$$-4a_1b_1^3g_6+\frac{3c^2w^2a_1b_1m_d}{2Q_d^2}+\frac{c^2wb_1^2m_d}{2Q_d^2}=0,$$
$$-kwa_0b_1+4\beta a_0b_1-2a_0b_1g_2-3a_0^2b_1g_4-3a_1b_1^2g_4$$
$$-4a_0^3b_1g_6-12a_0a_1b_1^2g_6+\frac{c^2wa_0b_1m_d}{Q_d^2}=0,$$
$$-kwa_0a_1+4\beta a_0a_1-2a_0a_1g_2-3a_0^2a_1g_4$$
$$-3a_1^2b_1g_4-4a_0^3a_1g_6-12a_0a_1^2b_1g_6$$
$$+\frac{c^2wa_0a_1m_d}{Q_d^2}=0,$$
$$-\frac{1}{2}kwa_1^2+2\beta a_1^2-\frac{3}{2}ka_1b_1-a_1^2g_2-3a_0a_1^2g_4$$
$$-6a_0^2a_1^2g_6-4a_1^3b_1g_6+\frac{c^2wa_1^2m_d}{2Q_d^2}+\frac{3c^2a_1b_1m_d}{2Q_d^2}=0,$$
$$-ka_0a_1-a_1^3g_4-4a_0a_1^3g_6+\frac{c^2a_0a_1m_d}{Q_d^2}=0,$$
$$-\frac{3}{4}ka_1^2-a_1^4g_6+\frac{3c^2a_1^2m_d}{4Q_d^2}=0.$$

Solving the above system (see figure 1), gives:

Case 1

$$\beta=\frac{1}{128}\left(64g_2-\frac{15g_4^2}{g_6}\right),$$
$$k=\frac{1}{3}\left(-\frac{9g_4^2}{64wg_6}+\frac{3c^2m_d}{Q_d^2}\right),$$
$$b_1=\mp\frac{3\sqrt{w}g_4}{16g_6},\quad a_1=\mp\frac{3g_4}{16\sqrt{w}g_6},\quad a_0=-\frac{3g_4}{8g_6}.$$

Hence

$$p_1(x,t)=\sqrt{a_0+a_1\sqrt{w}\tan(\sqrt{w}\xi)+\frac{b_1}{\sqrt{\omega}}\cot\left(\sqrt{b}\xi\right)},\ w>0,$$
$$p_2(x,t)=\sqrt{a_0-a_1\sqrt{w}\cot(\sqrt{w}\xi)-\frac{b_1}{\sqrt{\omega}}\tan\left(\sqrt{b}\xi\right)},\quad w>0,$$
$$p_3(x,t)=\sqrt{a_0-a_1\sqrt{-w}\tanh(\sqrt{-w}\xi)-\frac{b_1}{\sqrt{-\omega}}\coth\left(\sqrt{-w}\xi\right)},$$
$$w<0,$$
$$p_4(x,t)=\sqrt{a_0-a_1\sqrt{-w}\coth(\sqrt{-w}\xi)-\frac{b_1}{\sqrt{-w}}\tanh\left(\sqrt{-w}\xi\right)},$$
$$w<0,$$

where $\xi=x-ct$.

Case 2

$$\beta=\frac{1}{32}\left(16g_2-\frac{3g_4^2}{g_6}\right),\quad k=\frac{1}{3}\left(\frac{9g_4^2}{64wg_6}+\frac{3c^2m_d}{Q_d^2}\right),$$
$$b_1=\mp\frac{3\mathrm{i}\sqrt{w}g_4}{16g_6},\quad a_1=\pm\frac{3\mathrm{i}g_4}{16\sqrt{w}g_6},\quad a_0=-\frac{3g_4}{8g_6}.$$

Hence

$$p_5(x,t)=\sqrt{a_0+a_1\sqrt{w}\tan(\sqrt{w}\xi)+\frac{b_1}{\sqrt{\omega}}\cot\left(\sqrt{w}\xi\right)},$$
$$w>0,$$
$$p_6(x,t)=\sqrt{a_0-a_1\sqrt{w}\cot(\sqrt{w}\xi)-\frac{b_1}{\sqrt{\omega}}\tan\left(\sqrt{w}\xi\right)},$$
$$w>0,$$

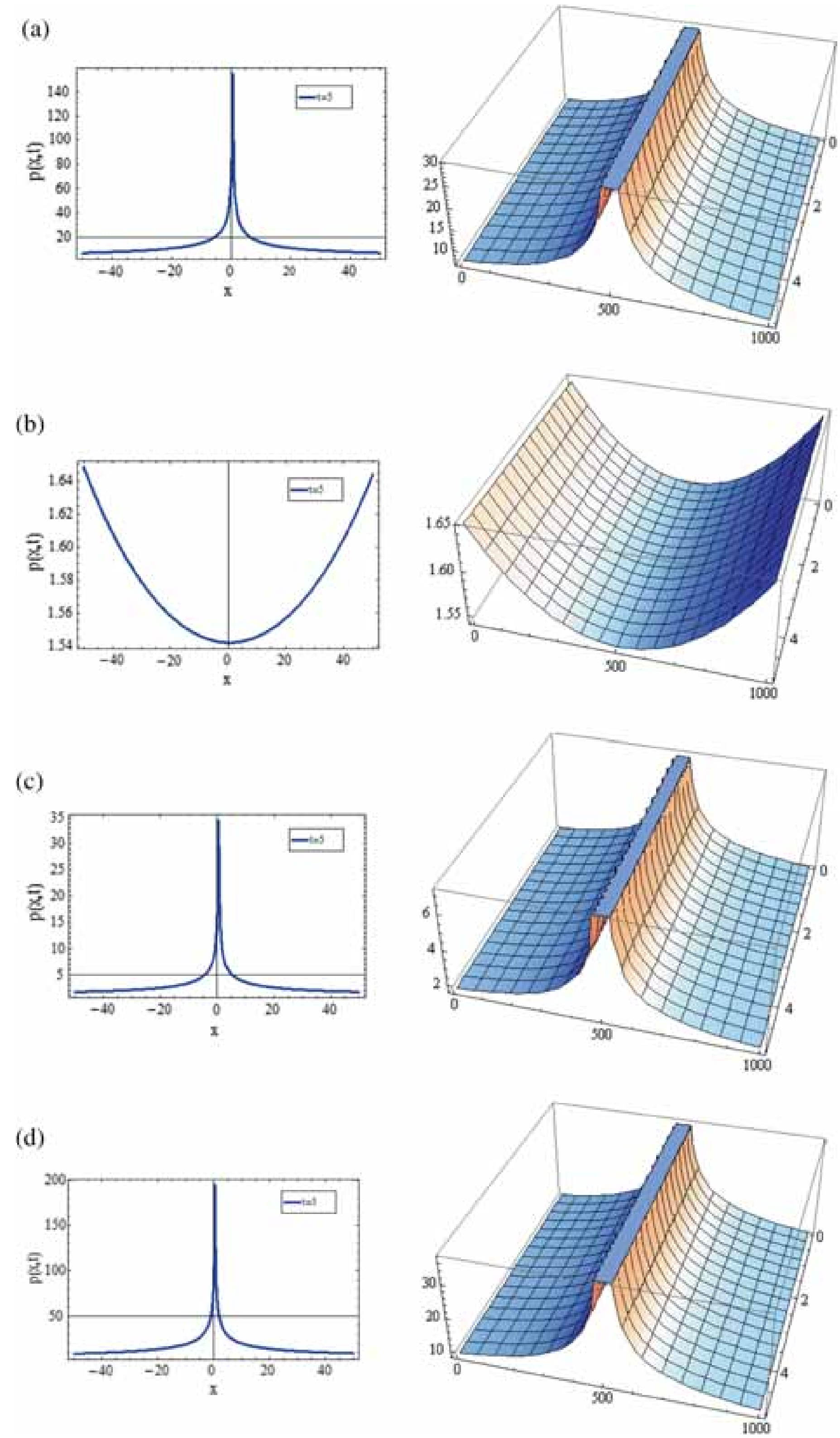

Figure 1. (**a–d**) Solutions for $p_3(x,t)$, $p_7(x,t)$, $p_{11}(x,t)$, $p_{14}(x,t)$ at $Q_d = 2 \times 10^5$, $T_c = 369$, $m_d = 6.02 \times 10^{-3}$, $T = T_c + 10^{-8}$, $\alpha_0 = 10.48 \times 10^4$, $g_2 = \alpha_0(T - T_c)$, $c = 0.1$, $p_0 = 1$, $p_1 = 1.5$.

$$p_7(x,t) = \sqrt{a_0 - a_1\sqrt{-w}\tanh(\sqrt{-w}\xi) - \frac{b_1}{\sqrt{-\omega}}\coth\left(\sqrt{-w}\xi\right)},$$
$w < 0,$

$$p_8(x,t) = \sqrt{a_0 - a_1\sqrt{-w}\coth(\sqrt{-w}\xi) - \frac{b_1}{\sqrt{-w}}\tanh\left(\sqrt{-w}\xi\right)},$$
$w < 0,$

where $\xi = x - ct$.

Case 3

$$\beta = \frac{1}{32}\left(16g_2 - \frac{3g_4^2}{g_6}\right),$$
$$k = \frac{1}{3}\left(\frac{9g_4^2}{16wg_6} + \frac{3c^2m_d}{Q_d^2}\right),$$
$$b_1 = 0, \quad a_1 = \mp\frac{3\mathrm{i}g_4}{8\sqrt{w}g_6}, \quad a_0 = -\frac{3g_4}{8g_6}.$$

Hence

$$p_9(x,t) = \sqrt{a_0 + a_1\sqrt{w}\tan(\sqrt{w}\xi) + \frac{b_1}{\sqrt{\omega}}\cot\left(\sqrt{w}\xi\right)},$$
$w > 0,$

$$p_{10}(x,t) = \sqrt{a_0 - a_1\sqrt{w}\cot(\sqrt{w}\xi) - \frac{b_1}{\sqrt{\omega}}\tan\left(\sqrt{w}\xi\right)},$$
$w > 0,$

$$p_{11}(x,t) = \sqrt{a_0 - a_1\sqrt{-w}\tanh(\sqrt{-w}\xi) - \frac{b_1}{\sqrt{-\omega}}\coth\left(\sqrt{-w}\xi\right)},$$
$w < 0,$

$$p_{12}(x,t) = \sqrt{a_0 - a_1\sqrt{-w}\coth(\sqrt{-w}\xi) - \frac{b_1}{\sqrt{-w}}\tanh\left(\sqrt{-w}\xi\right)},$$
$w < 0,$

where $\xi = x - ct$.

Case 4

$$\beta = \frac{1}{32}\left(16g_2 - \frac{3g_4^2}{g_6}\right), \quad k = \frac{3g_4^2}{16wg_6} + \frac{c^2m_d}{Q_d^2},$$
$$a_1 = 0, \quad b_1 = \pm\frac{3\mathrm{i}\sqrt{w}g_4}{8g_6}, \quad a_0 = -\frac{3g_4}{8g_6}.$$

Hence

$$p_{13}(x,t) = \sqrt{a_0 + a_1\sqrt{w}\tan(\sqrt{w}\xi) + \frac{b_1}{\sqrt{\omega}}\cot\left(\sqrt{w}\xi\right)},$$
$w > 0,$

$$p_{14}(x,t) = \sqrt{a_0 - a_1\sqrt{w}\cot(\sqrt{w}\xi) - \frac{b_1}{\sqrt{\omega}}\tan\left(\sqrt{w}\xi\right)},$$
$w > 0,$

$$p_{15}(x,t) = \sqrt{a_0 - a_1\sqrt{-w}\tanh(\sqrt{-w}\xi) - \frac{b_1}{\sqrt{-\omega}}\coth\left(\sqrt{-w}\xi\right)},$$
$w < 0,$

$$p_{16}(x,t) = \sqrt{a_0 - a_1\sqrt{-w}\coth(\sqrt{-w}\xi) - \frac{b_1}{\sqrt{-w}}\tanh\left(\sqrt{-w}\xi\right)},$$
$w < 0,$

where $\xi = x - ct$.

3. Conclusions

Throughout this work, we have investigated the solutions of the thin-film polarisation equation, while using the modified tanh method and, as a result, a number of new analytical expressions of solutions have been derived along with associated constraints. While computing these solutions, it appeared that singular waves can propagate in the medium. This singularity originates from the hyperbolic tangent that occurs in analytical solutions. Further investigation may allow to distinguish which of the wave solutions can propagate in thin-film ferroelectric materials. Such an investigation will take into account numerical simulations. This issue deserves much attention and needs to be investigated further.

One should also note that the solutions reported in this study are derived with assistance from the predicted solutions. The finite series form solutions are constructed by the Riccati equation which are different from the solutions reported in [7] derived by some direct integrals together with some particular choices of the parameters used in the equation. On the other hand, the solutions reported in this study are of the form of tangent and hyperbolic tangent functions. This can also be another main difference between the solutions published in [7].

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