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MARKOV RELIABILITY MODEL FOR HEAT METERS

MARKOWSKI MODEL NIEZAWODNOŚCI DLA LICZNIKÓW CIEPŁA

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Abstract: *The reliability of thermal energy meters is analysed using the Markov model which describes the operation of these meters in a large number of apartments and offices by a media accounting company. The data has been extracted from a relational database storing information on the operation, installation and exchange of these measures from the last 10 years. The built Markov model turned out to be ergodic, which allowed determining its limiting distribution. In addition, the probability distributions for the cumulated consumption were determined in the work - separately for all meters and meters' failures.*

Keywords: *reliability models, Markov model, exponential distribution, Weibull distribution*

Streszczenie*: Tematem publikacji jest analiza niezawodności mierników energii cieplnej z użyciem modelu Markowa. Omawiany model został zbudowany w oparciu o badanie procesu użycia tychże mierników w firmie zajmującej się rozliczaniem mediów. Dane zostały wyekstrahowane z relacyjnej bazy danych przechowującej informacje o eksploatacji, instalacji i wymianie tych mierników z ostatnich 10 lat. Zbudowany model Markowa okazał się być ergodyczny, co pozwoliło na wyznaczenie jego rozkładu granicznego. Oprócz tego w pracy wyznaczono rozkłady prawdopodobieństwa dla przebiegu mierników ciepła – osobno dla wszystkich mierników i osobno dla mierników uległych awarii.*

Słowa kluczowe*: modele niezawodności, model Markowa, rozkład wykładniczy, rozkład Weibulla*

1. Introduction

The analysed meters estimate heat consumption both in multi-family houses as well as in individual apartments or offices. To calculate the amount of received heat, the meter must measure the volume of the flowing medium[1] and - with two sensors - its temperature in the supply and return lines. The microcontroller, on grounds of these three values, determines the amount of heat in kilowatt-hours [kWh]. Due to the method of measuring the volume of the medium, we can distinguish two types of meters - volumetric and ultrasonic. The former calculates the volume based on the classical volumetric flow meter. The ultrasonic one (*Figure 1*) does not directly measure the volume, but measures the flow rate and knowing the cross-sectional area of the line is able to determine it [1].

Fig. 1 Example of a heat meter

The analysis of the reliability of heat meters is intended first to show the methodology of building Markov processes based on large data sets collected over many years. Finding a stationary or limiting distribution for such a model can also be used to plan inventory.

2. Source data

Information on the installation, operation and exchange of heat meters has been collected over the past 10 years in a relational database. This database also contains a lot of other information used for the settlement of heat and water, as well as data on other types of meters (water meters and heat cost allocators). The authors, however, decided to focus only on the heat meters discussed in the introduction.
Usually, preparation of data from such an enormous collection is a time-consuming task and cannot be easily automated (this was also the case here).

[1] *Most often it is water, although in the case when the system is also used for cooling, it can be water with appropriate additives to prevent freezing.*

Not all records' properties were standardized nor saved in the dictionary. In addition, the work was hindered by the usage of three different languages (German, French and Italian) to describe some of the parameters of the stored objects. Despite these obstacles, almost 367,000 historical records were identified, which corresponds to more than 50,000 meters including their possible failures.

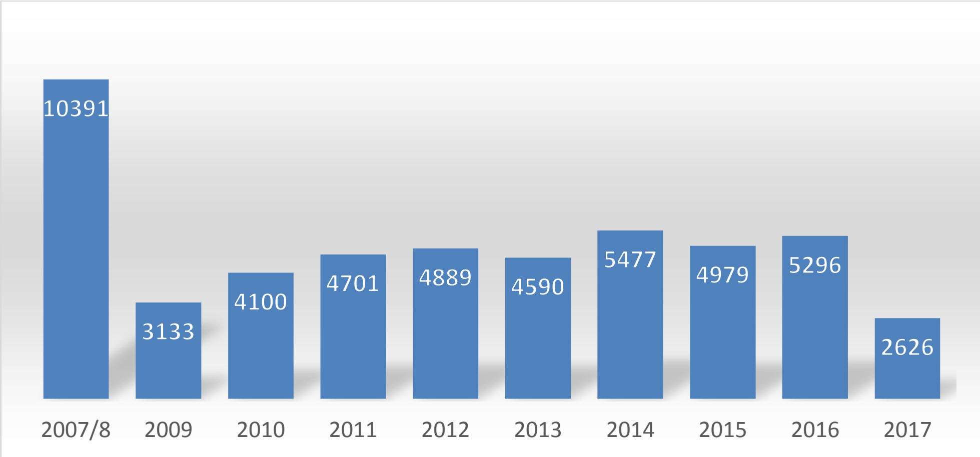

Fig. 2 Number of heat meters installed / registered per year

The database system was introduced in 2008, hence data from 2007 and 2008 are cumulative. For 2017 we own only partial data.

3. Analysis of meters

The Weibull distribution, and in particular the exponential distribution, is the standard distribution used in the survival and reliability analysis [2], [3]. We will show that the extracted data is also subject to this distribution.

Selected reliability characteristics

Let the random variable $X(t)$ takes value *1* when the object at time *t* is usable and *0* when the object at time *t* is unfit. Let us denote by $R(t)$ the reliability function of the object, which determines the probability that it will work correctly in time *[0, t]*. Then $F(t) = 1 - R(t)$ is a distributor of the distribution of a random variable $X(t)$ and $f(t) = F'(t)$ is its density. The failure rate function $\lambda(t)$ is defined in reliability analysis as follows:

$$\lambda(t) = \frac{f(t)}{R(t)} \tag{1}$$

The failure rate can be treated as a measure of the relative decrease of reliability of the object over time [4], [5], [6].

Data analysis

Figure 3 shows a histogram constructed using 35 bins for cumulated heat consumption of all 50,000 meters. This cumulated heat consumption is a feature that can be treated as their operating time.

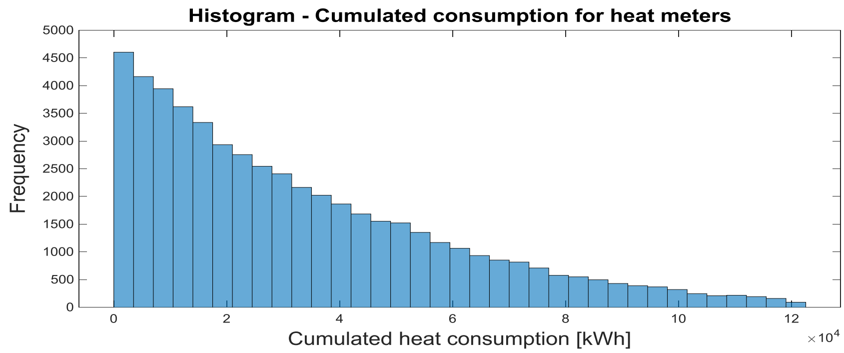

Fig. 3 Histogram – cumulated consumption for heat meters

The shape of the obtained graph is similar to the exponential distribution $f(t) = \lambda e^{-\lambda t}$. The parameter $\lambda = 3.08 * 10^{-5}$ of density function was calculated with the maximum likelihood method with confidence intervals of 95%. Figures 4-*6* show the fitting results.

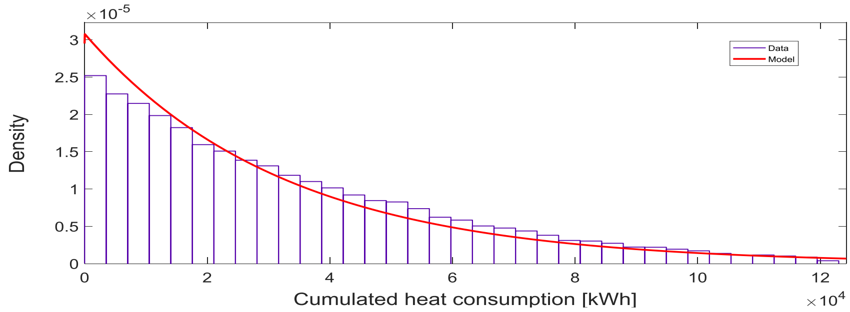

Fig. 4 Fitting – exponential distribution PDF

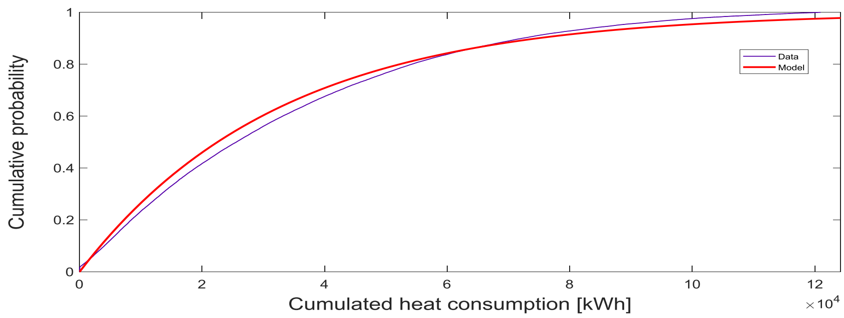

Fig. 5 Fitting – cumulative density function CDF

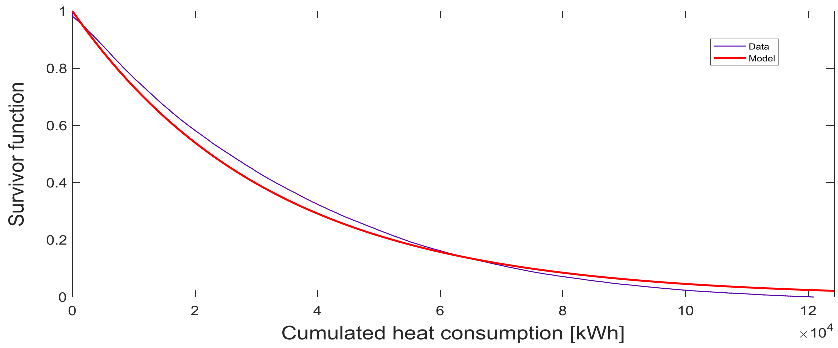

Fig. 6 Fitting – survivor function

As you can see, the empirical data fit the exponential model quite well. Several interesting conclusions can be drawn on this basis. The exponential model is mainly characterised by a constant intensity of damage, i.e. $\lambda(t) = \lambda$. This means that failures are external random events and do not depend on the time of usage – they appear accidentally with a constant intensity. In addition, the reliability function $R(t) = e^{-\lambda t}$ has the property that $R(t + x) = R(t)R(x)$. It is called "memorylessness" of exponential distribution and means, that "waiting time" until the failure occurs, does not depend on how much time has elapsed already. In other words, if we know that at the time x the element was fit, then from now the expiration time of this element has the same distribution as the new element [4], [6], [7], [8]. The calculated parameter $\lambda = 3.08 * 10^{-5}$ of that distribution can also be interpreted as the average time between failures.

The next step was to investigate the distribution of cumulated heat consumption for damaged meters. The collection contained over 11700 records. The obtained histogram is shown in *Figure 7*.

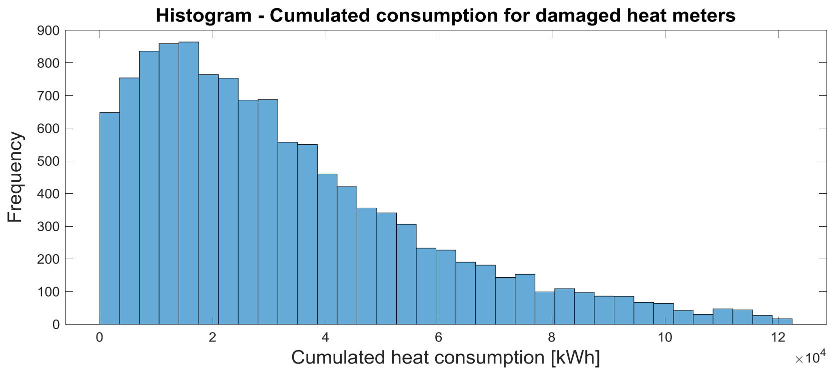

Fig. 7 Histogram – cumulated consumption for damaged heat meters

This time the shape of the histogram does not resemble the exponential distribution, but rather the Weibull distribution $f(t) = \lambda^k t^{k-1} e^{-(\lambda t)^k}$. Since the maximum likelihood method is best suited for the estimation of the parameters of this distribution [9], it was used again and parameters were obtained $k = 1.1269$, $\lambda = 2.9691 * 10^{-5}$. A comparison of the data and the model is shown in figures *8-10*.

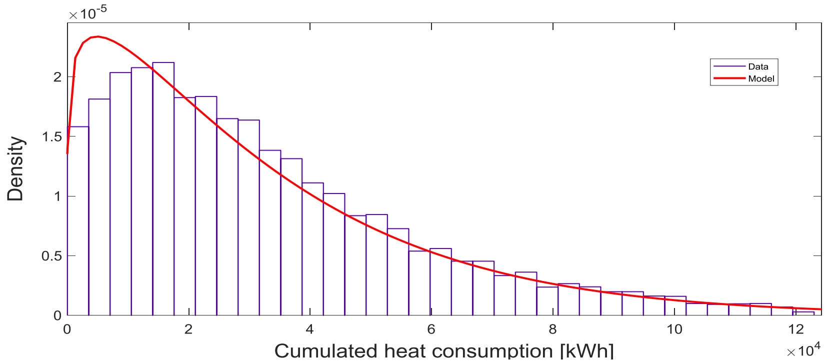

Fig. 8 Fitting –Weibull distribution

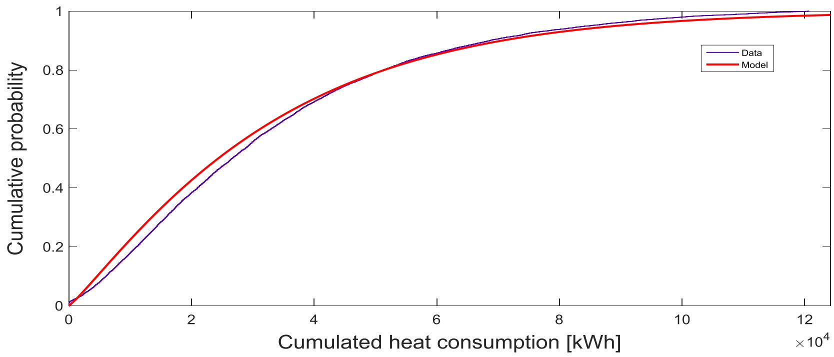

Fig. 9 Fitting – cumulative density function CDF

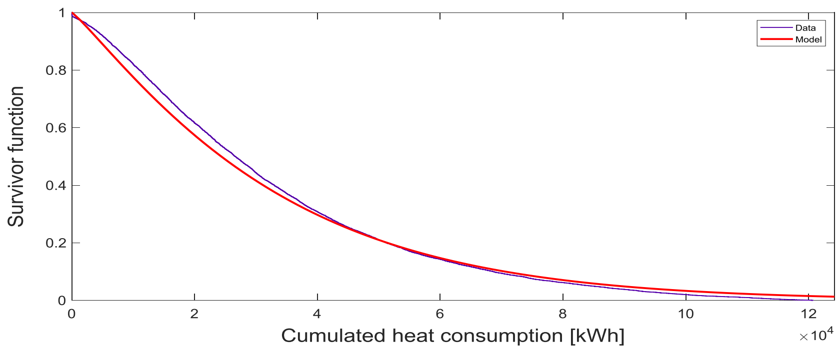

Fig. 10 Fitting – survivor function

The Weibull distribution for data related only to meter failures allows us to analyse in more detail the nature of these events and better determine the reliability of heat meters. The reliability function in this case is in the form of $R(t) = e^{-(\lambda t)^k}$ and the intensity of damage is no longer a constant $\lambda(t) = k\lambda t^{k-1}$. [10], [11]. Due to the fact that in our model, calculated parameter $k > 1$, the probability of failure increases over time. This is in line with the observations that the equipment is wearing down. On the one hand, this is partly contradicted by the analysis carried out for all meters, where we found that the probability of failure is time-independent. However, it should be noted that firstly, the data used in the first analysis are five times larger and include cases where the meter was replaced despite the failure. Secondly, in the Weibull model, the calculated *k* is very close to 1, which corresponds to a special case when the Weibull distribution coincides with the exponential distribution. In conclusion – it can be said that over 10 years, the intensity of damage of heat meters is poorly dependent on the time of use. The reliability function shows the "memorylessness" property, which encourages further analysis using the Markov processes.

4. Markov processes

Let $E \subset \mathbb{R}^d$ be a finite or countable set. Stochastic process $\{X_n\}$ will be called a Markov process, if for every $n \geq 0$ the following equality is true:

$$P\big(X_{n+1} = i_{n+1} \big| (X_0 = i_0, \dots, X_n = i_n)\big) = P(X_{n+1} = i_{n+1} | X_n = i_n) \qquad (2)$$

where $\forall_{k\geq 0}\ i_k \in E$.

The set *E* can be understood as a set of all possible states of a certain system. Then, a random vector X_n means the state in which our system is located at the moment of time *n*. On the ground that X_n is a random vector, in fact we do not know exactly its "position" but only the probability distribution of the event, that the system is in one of the states.

Equation *2* can be interpreted as a condition that the probability of transition from one state to another state in a time unit depends only on these states and does not depend on the history of the system or the specific moment in which this transition occurs. This property is often called the "memorylessness" or Markov property [12].

Let us denote by $p_{i,j}$ conditional probability $P(X_{n+1} = i_{n+1} | X_n = i_n)$ from equation *2*. In the case when the set of states is finite, the square matrix

$$\boldsymbol{P} = [p_{i,j} : i, j \in E] \qquad (3)$$

is a stochastic matrix whose rows add up to unity.

Classification of states

Let us denote by $f_{j,i}$ the probability of getting from state *j* to state *i*. Then, from the formula for the total probability we have: $f_{j,i} = \sum_{k\epsilon E\setminus i} p_{j,k} f_{k,i} + p_{j,i}$. The state *i* will be called recurrent if $f_{i,i} = 1$, i.e. if we return to it with probability 1. The state is transient if it is not recurrent.
The state *j* is accessible from state *i*, if $\exists\, k > 0 \colon p_{i,j}(k) > 0$, where $p_{i,j}(k)$ means the probability of reaching from state *i* to *j* after *k* steps – in particular, $p_{i,j}(k)$ is an appropriate element of matrix $\boldsymbol{P}^k$. A set of all states accessible from the state *i* we mark $A(i)$. The states *i* and *j* are said to communicate if $i \in A(j)$ and $j \in A(i)$. The state *i* is periodic, if there is a natural number *d* greater than one such that, if $p_{i,i}(k) > 0$ for some *k*, then *d* is the divisor of *k*.

Ergodicity

Distribution of probabilities π on the space of states *E* is called stationary if and only if the condition is met: $\pi_j = \sum_{i\in E} \pi_i p_{i,j}$ (or in matrix notation $\pi\boldsymbol{P} = \pi$), where π is such a line vector that $\forall_{i\in E}\, \pi_i \geq 0$ and $\sum_{i\in E} \pi_i = 1$.
If the finite Markov chain is irreducible (i.e. it has only one class of communicating states), and does not contain periodic states, then there exists a stationary distribution π such as $\lim_{n\to\infty} p_{i,j}(n) = \pi_j$ ([13]).
This distribution is called equilibrium distribution, and Markov chain – ergodic. It is worth noting that the equilibrium distribution does not depend on the initial distribution (which is not necessarily true for stationary distributions). In addition, the equilibrium distribution π is the only non-zero system solution $(P^T - I)\pi^T = 0$ fulfilling the condition $\sum_i \pi_i = 1$ ([14], [3]).

5. Markov model

The operation process of the meters has been presented in *Figure 11*. After installation, we enter the read – validation cycle, which corresponds to one accounting period, i.e. the most often 12 months. In the event of a failure being detected, we proceed to the repair or renewal status, that is, the meter replacement.

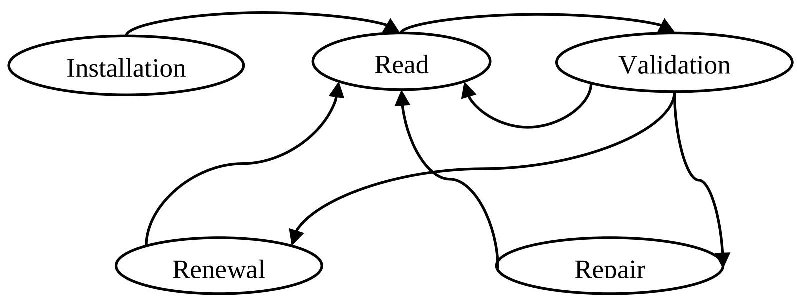

Fig. 11 Operation process

Based on this, the following Markov model was built (*Figure 12*).

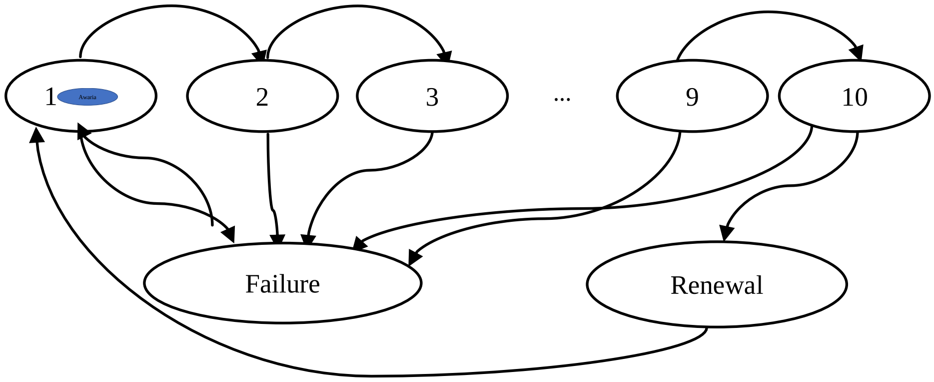

Fig. 12 Markov model

The states with numbers from 1 to 10 indicate the number of the billing cycle in which the counter is currently located. After 10 cycles, the meter automatically changes to the "Renewal" status and is replaced. In each of these states, the meter may fail, as illustrated by the "Failure" status. The above directed graph allows to build a system of differential equations, known as the Kolmogorov equation.

$$
\begin{gathered}
\frac{dP_1(t)}{dt} = -(\lambda_{1,2} + \lambda_{1,A})P_1(t) + \lambda_{A,1}A(t) + \lambda_{O,1}O(t) \\
\frac{dP_2(t)}{dt} = -(\lambda_{2,3} + \lambda_{2,A})P_2(t) + \lambda_{1,2}P_1(t) \\
\frac{dP_3(t)}{dt} = -(\lambda_{3,4} + \lambda_{3,A})P_3(t) + \lambda_{2,3}P_2(t) \\
\frac{dP_4(t)}{dt} = -(\lambda_{4,5} + \lambda_{4,A})P_4(t) + \lambda_{3,4}P_3(t) \\
\frac{dP_5(t)}{dt} = -(\lambda_{5,6} + \lambda_{5,A})P_5(t) + \lambda_{4,5}P_4(t) \\
\frac{dP_6(t)}{dt} = -(\lambda_{6,7} + \lambda_{6,A})P_6(t) + \lambda_{5,6}P_5(t) \\
\frac{dP_7(t)}{dt} = -(\lambda_{7,8} + \lambda_{7,A})P_7(t) + \lambda_{6,7}P_6(t) \\
\frac{dP_8(t)}{dt} = -(\lambda_{8,9} + \lambda_{8,A})P_8(t) + \lambda_{7,8}P_7(t) \\
\frac{dP_9(t)}{dt} = -(\lambda_{9,10} + \lambda_{9,A})P_9(t) + \lambda_{8,9}P_8(t) \\
\frac{dP_{10}(t)}{dt} = -(\lambda_{10,\mathrm{O}} + \lambda_{10,A})P_{10}(t) + \lambda_{9,10}P_9(t) \\
\frac{dO(t)}{dt} = -\lambda_{O,1}O(t) + \lambda_{10,\mathrm{O}}P_{10}(t) \\
\frac{dA(t)}{dt} = -\lambda_{A,1}A(t) + \sum_{k=1}^{10} \lambda_{k,A} P_k(t)
\end{gathered}
\tag{4}
$$

where $P_k(t)$ is the probability of remaining in the state *k*, $A(t)$ and $O(t)$ indicate the probabilities of being in a state of failure, and state of renewal respectively. The parameters $\lambda_{x,y}$ means the intensity of the transition from state *x* to state *y* at what $x, y \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, A, O\}$.
In matrix notation, this system shall take the form:

$$\frac{d}{dt}P(t) = \Lambda P(t) \tag{5}$$

where $P(t) = [P_1(t), P_2(t), P_3(t) \ldots P_{10}(t), A(t), O(t)]^T$ is a column vector and matrix Λ looks as follows

$$\Lambda= \begin{array}{cccccccc}
-(\lambda_{1,2}+\lambda_{1,A}) & 0 & 0 & 0 & 0 & 0 & \lambda_{A,1} & \lambda_{0,1} \\
\lambda_{1,2} & -(\lambda_{2,3}+\lambda_{2,A}) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_{2,3} & -(\lambda_{3,4}+\lambda_{3,A}) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_{3,4} & \ldots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -(\lambda_{9,10}+\lambda_{9,A}) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_{9,10} & -(\lambda_{10,0}+\lambda_{10,A}) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \lambda_{10,0} & \lambda_{0,1} & 0 \\
\lambda_{1,A} & \lambda_{2,A} & \lambda_{3,A} & \lambda_{i,A} & \lambda_{9,A} & \lambda_{10,A} & 0 & \lambda_{A,1}
\end{array} \tag{6}$$

The values of transition intensities $\lambda_{x,y}$ can be calculated based on available empirical data (see table in Tab 1). Then, using the Laplace transform $\mathcal{L}$ and using its property

$$\mathcal{L}[f'(t)] = s\hat{f}(s) - f(0) \tag{7}$$

a system of linear equations was obtained

$$s\hat{P}(s) - P(0) = \Lambda\hat{P}(s) \tag{8}$$

Solving it with respect to $\hat{P}(s)$ and using the inverse Laplace transform $\mathcal{L}^{-1}$, one can get the searched vector of probabilities $P(t)$.

Tab. 1 Intensity of transitions between individual model states [1/d]

$\lambda_{i,j}$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**A**	**O**
1	0	7,19	0	0	0	0		0	0	0	1,39	0
2	0	0	14,51	0	0	0	0	0	0	00	1,89	0
3	0	0	0	13,64		0	0	0	0	0	3,80	0
4	0	0	0	0	15,00	0	0	0	0	0	3,46	0
5	0	0	0	0	0	12,57	0	0	0	0	3,58	0
6	0	0	0	0	0	0	13,39		0	0	4,44	0
7	0	0	0	0	0	0	0	12,87	0	0	5,11	0
8	0	0	0	0	0	0	0	0	11,23	0	3,68	0
9	0	0	0	0	0	0	0	0	0	8,58	3,20	0
10	0	0	0	0	0	0	0	0	0	0	3,77	15,43
A	34,32	0	0	0	0	0	0	0	0	0	0	0
O	15,43	0	0	0	0	0	0	0	0	0	0	0

In practice, we are interested in the asymptotic values of probabilities $P(t)$ at $t \to \infty$, which in case of discrete time finite Markov process, comes down to the solution of the system of equations:

$$(\mathrm{P}^T - I)\pi^T = 0 \tag{9}$$

where *P* is the matrix of probabilities of transitions between states (Table 2), while *I* is the unit matrix.

Tab. 2 Transition probabilities matrix

Stan	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**A**	**O**
1	0	0,947	0	0	0	0		0	0	0	0,053	0
2	0	0	0,888	0	0	0	0	0	0	00	0,112	0
3	0	0	0	0,882		0	0	0	0	0	0,118	0
4	0	0	0	0	0,853	0	0	0	0	0	0,147	0
5	0	0	0	0	0	0,856	0	0	0	0	0,144	0
6	0	0	0	0	0	0	0,820		0	0	0,180	0
7	0	0	0	0	0	0	0	0,789	0	0	0,211	0
8	0	0	0	0	0	0	0	0	0,767	0	0,233	0
9	0	0	0	0	0	0	0	0	0	0,768	0,232	0
10	0	0	0	0	0	0	0	0	0	0	0,238	0,762
A	1	0	0	0	0	0	0	0	0	0	0	0
O	1	0	0	0	0	0	0	0	0	0	0	0

Using the Matlab package, the following solution was obtained (asymptotic vector)

$$\pi = [0{,}1432; 0{,}1357; 0{,}1206; 0{,}1064; 0{,}0908; 0{,}0777; 0{,}0637; 0{,}0503; 0{,}0386; 0{,}0297; 0{,}1206; 0{,}0226] \quad (8)$$

which is shown in *Fig 13*.

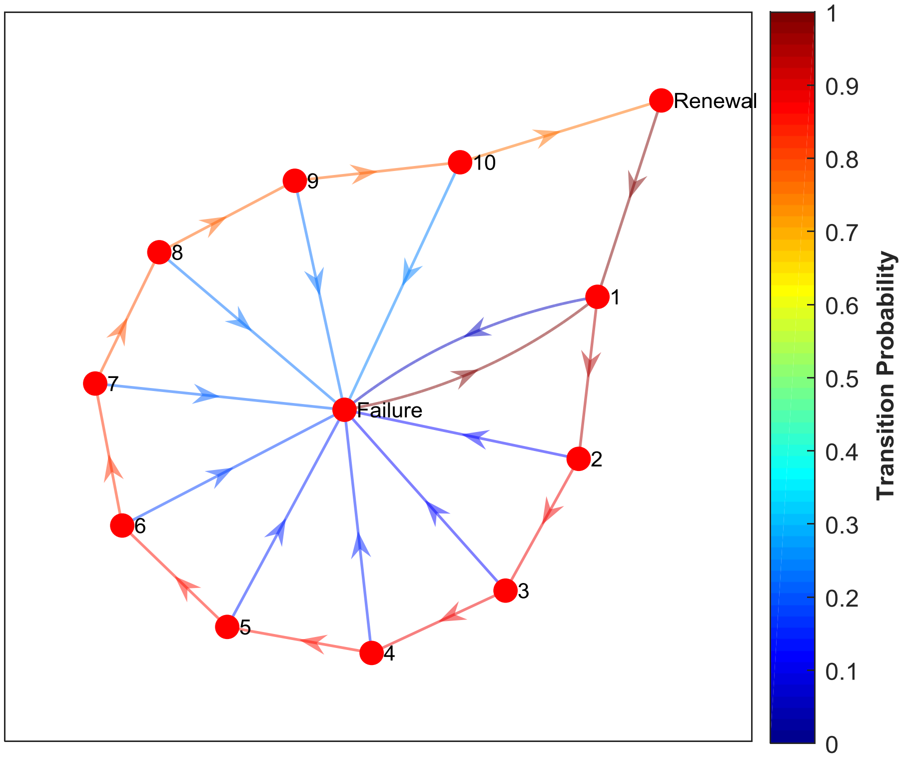

Fig. 13 Transition probability

6. Summary

Analysis of reliability characteristics for heat meters provide interesting information about their maintenance and failure rate. It turns out that the intensity of damage is poorly dependent on their operation time - at least during the first 10 years after installation. This leads to rethinking of the decision on the standard replacement of meters after this period and extending it until noticing a significant increase in failure rate. Analysis of the presented Markov model allows to confirm the correctness of the conclusions drawn from the analysis of the characteristics. It can be seen that the equilibrium probabilities in subsequent cycles decrease almost linearly. The values of these probabilities can be used to plan inventory and company resources for handling crisis situations.

Well, regardless of their age, we have about 12% of the measures requiring replacement in each annual accounting period. Also, the value of the asymptotic vector in the state of renewal: *0.0226* in relation to its value in state no. *10 – 0.0297* suggests the sense of extending their lifetime.

The availability of such a large amount of data leads to the extension of the above analysis and possible breakdown of it to individual producers or geographic regions.

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